Characterization of hyperelastic deformation behavior of rubber-like materials

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Abstract

This paper proposes a new hyperelastic model for rubber-like materials applicable over the entire modes of the large deformations. The whole material parameters have been identified using the mathematical approach combined with the genetic algorithm for the best method for the structure of hyperelastic models applicable to incompressible rubber-like materials. The step-by-step method is used on the restricted form by gradually adding a term of the highest degree of the hyperelastic model when the deformation increases. This combination greatly minimizes the difference between experimental data and analytical solutions. It is about a method that is applied to nonlinear behaviors leading to an optimal solution. The power of the model is demonstrated via comparisons with different phenomenological hyperelastic models using the experimental data from the Treloar and Nunes works. According to the literature, many authors have proposed a model based on both strain invariants but unable to reproduce the equibiaxial deformation mode. This work offers the constitutive hyperelastic model based on both strain invariants I_1 and I_2 with a minimal number of rheological parameters.

Keywords Hyperelastic parameters · Approach-in-stage · Genetic algorithm · Rubber-like materials · Model

1 Introduction

Elastomers, like-rubber materials are much more used in diverse fundamental rubber manufacturing industries ranging from the tire, seal or vibration mounts to aerospace. The ability to maintain the original configuration after a large deformation and their power to withstand shocks are their most fascinating properties appreciated in several fields of use. Indeed, their mechanical and chemical properties make them the good component against pressure, humidity and temperature and also, they have very good energy absorption properties. The implementation of this kind of materials requires an experimental [1–3] and theoretical characterization behavior. On this subject, several isotropic and incompressible hyperelastic

models have been proposed in the literature during the last eighty years [4–15].

There are several hyperelastic constitutive models based on classical Gaussian law [16–21], one on the strain energy density function adopting the Langevin statistical approach [22–27] and the others using the phenomenological technique. On this issue, Steinman [28] and Nguesong [29] have widely explained. Valanis [30], Ogden [6] are used the principal stretches for expressing the strain energy functions. Beda and Chevalier [31] have proposed a model combining both the classical strain invariants like [14, 32–35] and the principal stretches.

In the literature, many hyperelastic models have been proposed which give better agreement with experimental data only in simple extension and pure shear [5, 9,

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11, 22]. This work proposes a robust model that will take into account the simple and equibiaxial extension, simple and pure shear.

In this regard, the main aim of this paper is to build a new hyperelastic model that is simple and contains rheological parameters which may be given a numerical and physical interpretations. In particular, this strain energy density function is able to correctly characterize and predict the common deformation modes such as simple tension, equibiaxial extension, simple and pure shear. Among the various important contributions in the literature [8, 11, 36–39], this empirical model has not only the advantages of mathematical simplicity and but also shows its effectiveness in characterizing the hyperelastic behavior and many elastomeric and biological materials.

2 Experimental setup and measurement techniques

In continuum mechanics, the property of rubber-like materials is modeled by the strain energy potential W . According to the isotropic material, that energy depends on the three strain invariants $W(I_1, I_2, I_3)$ or the principal stretches $W(\lambda_1, \lambda_2, \lambda_3)$ [4, 40]. The strain invariants of the left Cauchy-Green B are related to the principal stretches λ_i by the following relations:

$$\begin{cases} I_1 = \sum_{i=1}^3 \lambda_i^2 \\ I_2 = \sum_{i,j=1}^3 (\lambda_i \lambda_j)^2 \quad (i \neq j) \\ I_3 = \prod_{i=1}^3 \lambda_i^2 \end{cases} \quad (1)$$

$$\sigma = \left[\frac{\partial W}{\partial I_1} + I_1 \frac{\partial W}{\partial I_2} \right] B - 2 \frac{\partial W}{\partial I_2} B^2 - pI \quad (2)$$

Where p is the hydrostatic pressure term that is related with the incompressible constraint and B is the left Cauchy-Green tensor. These equations can also be written in terms of the principal stretches and stresses for detailed treatment, see [41–43]. Assuming that the material is isotropic and incompressible $I_3 = 1$:

$$\sigma = \lambda_i \frac{\partial W}{\partial \lambda_j} - pI, \quad i = 1, 2, 3 \quad (3)$$

The Eq. (3) can be explained as:

$$\frac{\partial W}{\partial \lambda_i} = \frac{\partial W}{\partial I_1} \frac{\partial I_1}{\partial \lambda_i} + \frac{\partial W}{\partial I_2} \frac{\partial I_2}{\partial \lambda_i} \quad (4)$$

The derivatives of the different strain invariants to three stretches can be expressed by the following functions:

$$\frac{\partial I_1}{\partial \lambda_i} = 2\lambda_i, \quad i = 1, 2, 3 \quad (5)$$

and

$$\frac{\partial I_2}{\partial \lambda_i} = 2\lambda_i (\lambda_j^2 + \lambda_k^2) \quad (6)$$

with $(i, j, k) = (1, 2, 3), (2, 3, 1), (3, 1, 2)$. The Eq. (3) can be expressed as:

$$p = \lambda_3 \frac{\partial W}{\partial \lambda_3} \quad (7)$$

then

$$\sigma_1 = \lambda_1 \frac{\partial W}{\partial \lambda_1} - \lambda_3 \frac{\partial W}{\partial \lambda_3} \quad (8)$$

by introducing the relations (4), (5) and (6) into the Eq. (8), we can get:

$$\begin{cases} \sigma_1 = 2 \left[\lambda_1^2 - \frac{1}{(\lambda_1 \lambda_2)^2} \right] \left[\frac{\partial W}{\partial I_1} + \lambda_2^2 \frac{\partial W}{\partial I_2} \right] \\ \sigma_2 = 2 \left[\lambda_2^2 - \frac{1}{(\lambda_1 \lambda_2)^2} \right] \left[\frac{\partial W}{\partial I_1} + \lambda_1^2 \frac{\partial W}{\partial I_2} \right] \\ \sigma_3 = 0 \end{cases} \quad (9)$$

The Eq. (9) express the stress-energy relationship that describe the different modes of hyperelastic deformation behavior.

3 Common modes of hyperelastic deformation

In the large strain domain, the common modes of deformation that characterize the rubber-like materials are: uniaxial and equibiaxial extension and also the pure shear test [1].

3.1 Uniaxial extension test

Assuming that the material is incompressible i.e., $I_3 = (\lambda_1 \lambda_2 \lambda_3)^2 = 1$, therefore, the both strain invariants can be expressed as:

$$\begin{cases} I_1 = \lambda^2 + \frac{2}{\lambda} \\ I_2 = 2\lambda + \frac{1}{\lambda^2} \end{cases} \quad (10)$$

With $\lambda_1 = \lambda$ and $\lambda_2 = \lambda_3 = \frac{1}{\sqrt{\lambda}}$. Consider a sample of isotropic hyperelastic rubber-like materials, the principal stresses lead to:

$$\begin{cases} \sigma_1 = 2 \left[\lambda - \frac{1}{\lambda^2} \right] \left[\frac{\partial W}{\partial I_1} + \frac{1}{\lambda} \frac{\partial W}{\partial I_2} \right] \\ \sigma_2 = \sigma_3 = 0 \end{cases} \quad (11)$$

3.2 Equibiaxial extension test

According to the equibiaxial extension test, the equation that characterizes the two first strain invariants are:

$$\begin{cases} I_1 = 2\lambda^2 + \frac{1}{\lambda^4} \\ I_2 = \lambda^4 + \frac{2}{\lambda^2} \end{cases} \quad (12)$$

The relationship between the three stretches is given by the following expression: $\lambda_1 = \lambda_2 = \lambda$ and $\lambda_3 = \lambda^{-2}$. The different principal stresses are:

$$\begin{cases} \sigma_1 = \sigma_2 = 2 \left[\lambda - \frac{1}{\lambda^5} \right] \left[\frac{\partial W}{\partial I_1} + \lambda^2 \frac{\partial W}{\partial I_2} \right] \\ \sigma_3 = 0 \end{cases} \quad (13)$$

3.3 Pure shear test

The pure shear conditions are: $\lambda_1 = \lambda$, $\lambda_2 = 1$ and $\lambda_3 = \lambda^{-1}$ since $(\lambda_1 \lambda_2 \lambda_3)^2 = 1$. The two strain invariants in this kind of deformation mode have the same expression.

$$I_1 = I_2 = \lambda^2 + \frac{1}{\lambda^2} + 1 \quad (14)$$

The principal stress is leading by the following equation:

$$\begin{cases} \sigma_1 = 2 \left[\lambda - \frac{1}{\lambda^3} \right] \left[\frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2} \right] \\ \sigma_2 = 2 \left[1 - \frac{1}{\lambda^2} \right] \left[\frac{\partial W}{\partial I_1} + \lambda^2 \frac{\partial W}{\partial I_2} \right] \\ \sigma_3 = 0 \end{cases} \quad (15)$$

4 Hyperelastic models

4.1 Edwards and Vilgis model

The similar model that had been proposed before Gent was the Edwards and Vilgis model [44] given by the following equation:

$$W = \frac{\mu}{2} \left[\frac{I_m(I_m - 5)(I_1 - 3)}{(I_m - 3)(I_m - I_1)} \right] + \ln \left[1 - \frac{I_1 - 3}{I_m - 3} \right] \quad (16)$$

Where $\mu/2$ is the hyperelastic parameter and I_m is the limiting value of I_1 that corresponds to the deformation when the network is fully stretched.

4.2 Gent model

Ten years later, Gent proposed a phenomenological model in terms of the first strain invariant. like Yamashita [22, 45] and Yeoh [2]. Gent neglected the first term of the Edwards and Vilgis model. Therefore, the simple strain energy density function that Gent noted out [8] has only two parameters:

$$W = -\frac{\mu}{2} (I_m - 3) \ln \left[1 - \frac{I_1 - 3}{I_m - 3} \right] \quad (17)$$

The Gent's model has been modified by several authors in their hyperelastic models as follows like Yeoh that proposed a similar model which involves three parameters [46].

4.3 Yeoh–Fleming model

The Yeoh–Fleming model is the net result from the combination of the exponential form and the Gent model [9]. Their strain energy is written as:

$$W = \frac{A}{\beta} \left[1 - e^{-\beta(I_1 - 3)} \right] - \frac{\mu}{2} (I_m - 3) \ln \left[1 - \frac{I_1 - 3}{I_m - 3} \right] \quad (18)$$

4.4 Pucci–Saccomandi model

Pucci and Saccomandi proposed a phenomenological hyperelastic model that is also called Gent–Gent model [11]. Adding just the logarithmic part in terms of second strain invariant. Pucci and Saccomandi noted that the Gent model has a good agreement in uniaxial tension for the large values, but very poor for the small values of the strain energy. When the material is incompressible, the model takes into account the both strain invariants

ones. In this case, the strain energy density function formulates like:

$$W = K \ln \frac{l_2}{3} - \frac{\mu}{2} (l_m - 3) \ln \left[1 - \frac{l_1 - 3}{l_m - 3} \right] \quad (19)$$

K , $\mu/2$ and l_m are the hyperelastic parameters.

Pucci and Saccomandi generalized the Gent model by taking a linear term in the second invariant [11] that presents as:

$$W = K(l_2 - 3) - \frac{\mu}{2} (l_m - 3) \ln \left[1 - \frac{l_1 - 3}{l_m - 3} \right] \quad (20)$$

4.5 Beda model

The strain energy density function proposed by Yeoh [2] with three hyperelastic parameters was generalized by Beda [14]. Assuming that the first part of the strain energy function like an infinite convergent power series based on the first strain invariant and the other one which expresses in the second strain invariant is the logarithmic form:

$$W = \sum_{p=1}^N \frac{C_p}{p} (l_1 - 3)^p + K \ln \frac{l_2}{3} \quad (21)$$

5 Experimental results and model fitting

In the previous works, many authors have been used the classical least squares [14, 34–36, 47, 48]. Therefore, the approach-in-stage combining by the genetic algorithm [49–52] and direct search toolbox for Matlab software user's guide [53] methods will be used for identifying all model parameters in the present paper. The approach-in-stages method was widely used and explained by Beda [34, 35, 37, 47, 48]. The strategy of the approach-in-stages consists in identifying step by step the generating function. The method consists in linearizing the function $\psi(x)$ that may be written as $\gamma\theta(x)$, finally, the linearized function corresponds to $\psi(x) = \gamma\theta(x)$. This strategy consists in plotting $\psi(x)$ versus $\theta(x)$ and ought to be linear, with slope equal to γ . In this case, $\psi(x)$ and $\theta(x)$ are just the simple arbitrary functions. Let us use the approach method that explains in the previous section in the uniaxial test. According to Pucci–Saccomandi and Beda models, the different stress Laws can be respectively expressed by considering the relations (19) and (21):

$$\sigma = \left(\lambda - \frac{1}{\lambda^2} \right) \left[\frac{\mu}{2} \frac{l_m - 3}{l_m - l_1} + \frac{K}{\lambda l_2} \right] \quad (22)$$

and

$$\sigma = \left(\lambda - \frac{1}{\lambda^2} \right) \left[\sum_{p=1}^N C_p (l_1 - 3)^{p-1} + \frac{K}{\lambda l_2} \right] \quad (23)$$

The reduced stresses can be taken respectively the form:

$$\frac{\phi}{2} = \frac{\mu}{2} \frac{l_m - 3}{l_m - l_1} + \frac{K}{\lambda l_2} \quad (24)$$

and the other one is:

$$\frac{\phi}{2} = \sum_{p=1}^N C_p (l_1 - 3)^{p-1} + \frac{K}{\lambda l_2} \quad (25)$$

The hyperelastic parameter K is evaluated at the first stage from relations (24) and (25). K is the slope giving by the straight line that describes in Fig. 1. Let us estimate the remains of the hyperelastic parameters of the relation (24), the Pucci–Saccomandi model parameters. The reduced stress can be rewritten as:

$$\frac{\phi}{2} - \frac{K}{\lambda l_2} = \frac{\mu}{2} \frac{l_m - 3}{l_m - l_1} \quad (26)$$

By identifying each term of the Eq. (26) with the relation (11):

$$\frac{\partial W}{\partial l_1} = \frac{\mu}{2} \frac{l_m - 3}{l_m - l_1} \quad (27)$$

Now let us estimate the different hyperelastic parameters by the step-by-step process in the relation (27). Using the inverse method, the straight line will be gotten.

Fig. 1 Estimation of parameter K

$$\left(\frac{\partial W}{\partial I_1}\right)^{-1} = \pi = \alpha I_1 + \beta \quad (28)$$

α and β which are respectively the slope and intercept of the line are evaluated on the linear part by plotting π versus I_1 . This strategy is summarized in Fig. 2. These different parameters can now be calculated by using α and β those are estimated in the second stage. Now the hyperelastic parameters μ and I_m are easily computed by the following relationships:

$$\begin{cases} \mu = \frac{2}{3\beta + \alpha} \\ I_m = -\frac{\alpha}{\beta} \end{cases} \quad (29)$$

The inverse method that is talked about here was widely explained and performed by Beda [31, 34] on the bilogarithmic plot curve.

The common nonlinear deformation modes are the simple tension, equibiaxial extension and the pure shear tests [1–3]. For each mode of deformation, let us analyze and compare with the theoretical modes.

Fig. 2 Identification of α and β

5.1 Simple extension and a note on the Gent model

As demonstrated by Rivlin and Saunders [54], the hyperelastic model mostly takes into account both strain invariants I_1 and I_2 . The model can be expressed by the compound functions:

$$W(I_1, I_2) = f_1(I_1) + f_2(I_2) \quad (30)$$

Where $f_1(I_1)$ and $f_2(I_2)$ must satisfy and verify the following circumstances:

$$\frac{\partial W(I_1, I_2)}{\partial I_1} = \frac{\partial f_1(I_1)}{\partial I_1} \quad (31)$$

and

$$\frac{\partial W(I_1, I_2)}{\partial I_2} = \frac{\partial f_2(I_2)}{\partial I_2} \quad (32)$$

The Gent hyperelastic constitutive model satisfied the first condition and given by the relation (27). According to $f_2(I_2)$, it is experimentally verified that $\partial f_2 / \partial I_2$ is a positive and decreasing function of I_2 . In agreement with conditions mentioned above, the second part of the strain energy density function can be pointed by:

$$f_2(I_2) = K \left(I_2^{\frac{1}{2}} - 3^{\frac{1}{2}} \right) \quad (33)$$

and

$$\frac{\partial f_2(I_2)}{\partial I_2} = \frac{K}{\sqrt{I_2}} \quad (34)$$

Then, the whole strain energy can be summarised by the simplest expression:

$$W = K \left(I_2^{\frac{1}{2}} - 3^{\frac{1}{2}} \right) - \frac{\mu}{2} (I_m - 3) \ln \left[1 - \frac{I_1 - 3}{I_m - 3} \right] \quad (35)$$

In accordance with the Eq. (11), the uniaxial behavior law is given by the next relationship:

$$\sigma = \left(\lambda - \frac{1}{\lambda^2} \right) \left[\frac{\mu}{2} \frac{I_m - 3}{I_m - I_1} + \frac{K}{\lambda \sqrt{I_2}} \right] \quad (36)$$

The proposed model has been performed by comparing to the previous models in particular: the phenomenological Pucci–Saccomandi model [11] and the Beda model [14]. All of these models are in comparison with the Treloar experimental data [1]. It is observed in Fig. 3 that, each model shows a good agreement with the Treloar experimental test.

The main aim of the method in stage is to determine if, for each behavior law, a unique set of hyperelastic

Fig. 3 Comparison of models with Treloar uniaxial data

Fig. 4 Plots of relative errors against λ according to uniaxial tension

parameters capable to reproduce simultaneously the different modes of deformations. These different identified hyperelastic parameters are used to characterize the hyperelastic behaviors in simple extension Fig. 3, equibiaxial tension see Fig. 7 and pure shear test see also Fig. 5. The Corresponding optimal rheological parameters are given in Tables 1 and 2 for the various hyperelastic models using in this work. All hyperelastic parameters obtained by using the Treloar [1] and Nunes [3] experimental data. The method in stage is the method that optimizes directly the number of rheological parameters, i.e., the number of terms, characteristic of rubber-like materials [34, 35, 37]. The coefficients from Tables 1 and 2 are the results from the simulation of the Pucci–Saccomandi, Beda and proposed models. These hyperelastic parameters allow to validate the model and to compare the models to each other.

In this simulation, using the simple tension deformation, the relative errors are about:

- 0 to 4% on the Pucci–Saccomandi model;
- 0 to 3% on the Beda model;
- 0 to 2% on the proposed model.

The values thus presented in Table 1 are optimal because the errors determined on each model are insignificant, the average of these values is around 3%. Considering the

Fig. 5 Comparison of the models using the Treloar data according to the pure shear

Fig. 6 Plots of relative errors against λ according to pure shear

Fig. 8 Plots of relative errors against λ for the equibiaxial extension

Fig. 7 Comparison of the models using the Treloar data according to the Equibiaxial tension

Table 1 The optimal hyperelastic parameters of the models using the Treloar experimental data [1]

Parameters	Model	Bede 2007	Pucci-Sacc
μ (MPa)	2.310^{-2}	–	2.3610^{-3}
J_m	80.440	–	83.500
K (MPa)	6.7010^{-3}	2.12310^{-3}	2.110^{-2}
C_1 (MPa)	–	1.33610^{-1}	–
C_2 (MPa)	–	6.6010^{-6}	–
C_3 (MPa)	–	7.6010^{-5}	–

different hyperelastic models evaluated, the estimated error on each model is relatively insignificant.

The explanation is resumed in Fig. 4 where the relative errors are plotted against stretch λ .

5.2 Pure shear and equibiaxial extension compared

The corresponding simulations are observed in Figs. 5 and 7 according respectively to the pure shear and equibiaxial tension deformation modes. The previous analysis that are plotted against stretch λ are given the relative errors around 10%. The relative errors of both modes of deformation are shown in Figs. 6 and 8. The hyperelastic parameters of these models were identified with the

Fig. 9 Comparison of the model proposed with Nunes data

Table 2 Identification of hyperelastic parameters of the models using Nunes simple shear data [3]

Parameters	Model	Beda 2007	Pucci-Sacc
μ (MPa)	2.4810^{-2}	–	1.30210^{-1}
J_m	140.500	–	80.380
K (MPa)	4.8010^{-2}	5.58610^{-1}	3.43510^{-1}
C_1 (MPa)	–	8.3910^{-2}	–
C_2 (MPa)	–	1.10310^{-2}	–
C_3 (MPa)	–	-6.8510^{-4}	–

approach-in-stage performed with the genetic algorithm method [52] are displayed in Table 1.

C_1 , C_2 , α , et β are the rheological parameters

5.3 Simple shear models compared with Nunes experimental data

The simple shear is the deformation mode that is expressed by the following relationship:

$$\sigma_{12} = 2\gamma \left[\frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2} \right] \quad (37)$$

By applying the relation (37) to the models (19), (21) and to (35), we have obtained the relationships that are plotted in

Fig. 9. The optimal hyperelastic parameters values of the models using Nunes data [3] are given in Table 2.

It comes out from the above analysis that, many models [2, 9, 11, 14, 18] are valid only in simple tension and pure shear domains of hyperelastic rubber-like materials. The model that has been proposed successfully verifies experimental and theoretical requirements. It can be used in both small and large deformations behavior. However, this stored-energy function has a disadvantage especially in equibiaxial tension. It presents a significant error in small deformation for $1 \leq \lambda \leq 2$, see Fig. 8.

6 Conclusion

The main aim of the present work was to study the recent models that are related to the Gent strain energy density function, in order to propose the stored-energy function that can characterize all deformation modes. For this purpose, some models have been compared such as Gent, Pucci-Saccomandi, Beda models. Nevertheless, the hyperelastic model has been proposed to improve the Gent and Pucci-Saccomandi models which are unable to fit the equibiaxial deformation mode. The proposal model covers the whole domain of large strain like the uniaxial, equibiaxial tension, the pure and simple shear. The approach-in-stage and the genetic algorithm methods were combined to identify the different rheological parameters of the models. This new combination provided the optimal parameters. The result presents a good agreement with the experimental data from the Treloar and the Nunes works. This simple stored-energy function can be apply to the homogeneous and the inhomogeneous deformation modes. It can also be used to the modeling of soft biological tissues and the characterization of the response of arterial walls.

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