Design of an adaptive sliding mode controller with a sliding mode Luenberger observer for the MEMS capacitive plates

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Abstract

Uncertain parameters in a MEMS tunable capacitor, as one of the fundamental elements in a MEMS AC voltage reference source (VRS) structure, is an important issue for acquisition of a rigorous voltage in the VRS output. The output is mostly supposed to be destroyed by harmful disturbance and noise. Even though the manufacturers make attempts to build MEMS tunable capacitor with desired characteristics in thorough compliance with the initial major design, uncertainties in the parameters inescapably do exist in manufacturing process on the ground of deficiencies in various procedures. This paper mainly remarks an adaptive sliding mode controlling design with a sliding mode Luenberger observer and its application for a MEMS tunable capacitor. The control design is done considering matched and unmatched terms of uncertainties and disturbance. The advantageous outcome is that the VRS could be generating a stable precise regulated output voltage owing to correct distance adjustment of the capacitor charged plates. The design exploits the sliding mode observer to get rid of extra usage of plate position measurement in the control structure. The controller alleviates the system output of the matched and unmatched disturbance which could exist dynamically because of cross stiffness and damping in addition to other inertial and electrostatic disturbing forces. The matched uncertainties, such as stiffness and damping change, might appear due to variation in fabrication procedures or working condition. The simulation yields are satisfactory and depict a persuasive tracking performance as well as regulation about the pull-in point.

Keywords MEMS · Adaptive sliding mode control · Sliding mode observer · State estimation · Matched and unmatched disturbance and uncertainty · Stiffness

1 Introduction

Suhonen firstly presented essential principles of AC and DC MEMS-based VRS [27]. A micro parallel-moving plate capacitor was put forward which is practiced in an uncomplicated circuitry to bring about a fixed voltage. The capacitor specifications are normally established on noticeable mechanical stability of single crystalline silicon. It sources rigorous output voltage between its plates. Additionally, inconsiderable production cost and tiny size make VRS outstanding miniaturized apparatus for electronic

purposes. Many researchers initially worked on the VRS to develop their veracity. Associatively, some other papers more concentratively investigate micro-machining procedures of MEMS tunable capacitors [2–5, 13, 14] considering important role of the MEMS tunable capacitor equity as the essential part of MEMS VRS. The researchers' essential objectives in these works is proposal and implementation of novel schemes for small tunable capacitors as well as quality advancement in the final MEMS product.

Pull-in point is the most important phenomenon while driving tunable capacitive plates in the MEMS VRS.

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It comes from electromechanical coupling between the applied electrostatic and the spring reaction forces. Figure 1a displays the simplified structure and the lumped model. Figure 1b also demonstrates that the electrostatic force rises as the electrical charge accumulation on the capacitor plates gets increased. Therefore, application of all the mentioned forces moves the movable plate of the capacitor towards the immobile plate progressively, until the spring force cannot endure the electrostatic force. It brings about an operation point which is entitled pull-in point. Here, the voltage changes between capacitor plates is corresponding to the square of the displacement in the moving plate. As a consequence, a well regulated output is given out by the VRS owing to operation around the pull-in point. The desired working condition is finally functioning adjacent to the pull-in point of the MEMS tunable capacitor. The pull-in point instability is engendered by mechanical stress and electrical charge over dielectric layer [12]. It is asserted by [12] that AC voltage exploitation, instead of DC voltage, reduces the charge pile-up over the dielectric layer. In fact, fabrication of the MEMS AC VRS is a serious consequence of AC voltage input utilization. Here, we are concerned with the MEMS AC VRS function.

By and large, MEMS industry and fabrication of the tunable capacitors greatly owes to the modern techniques of micro-fabrication. The modern methods are capable of presenting noticeable rigorous size, structure, and less mechanical stress. Nevertheless, control application to the MEMS VRS is required since it could play an efficacious role for establishing long-lasting stability and veracity of reference voltage. Application of efficient control approaches for MEMS, robots and other mechatronic systems to engender error downturn and boosted performance is a noticeable challenge for researchers studying in the field of control engineering application [1, 9, 19, 21, 28]. Adaptive and robust control approaches have been drawing the researchers' curiosity and attention since deficiencies and irregularities in MEMS modeling as well as ambivalence

in different parameters arise in the manufacturing procedures [8, 10, 11, 29].

Unlike our previous works on MEMS capacitive plates modeling and control issues such as [22], in which we were concerned with a sinusoidal trajectory tracking, our focus in the MEMS tunable capacitor is reaching a regulation static point. We are following a harmless trajectory in operation of the model, similar to [23]. Mehrnezhad presented a state space model in [18] in order to design control techniques for MEMS tunable capacitors. A controller might be used to make the capacitor plate move and get maintained at the pull-in point whereas the capacitor parameters, such as spring stiffness, shift during the operation. If the system parameters were recognized just as constants, an uncomplicated pole-placement state feedback control design could be exploited to access this objective easily. Nevertheless, imminent uncertainties considerably appear among the capacitor parameters, requiring application of more modern complicated controllers. Exemplifying this issue, one may refer to the spring stiffness factor which might not be decisive. Other examples are mechanical noise, electrical measurement noise and time-variant parameters. They are observed as factors having great impact over the device performance in implementation.

In this paper, we are mostly concerned with design of an Adaptive Sliding Mode Controller (ASMC) equipped with a Sliding Mode Luenberger Observer (SMLO) to cope with matched and unmatched disturbance as well as uncertainty in the capacitive plate dynamical system. Our target is obtainment of operation near the pull-in point which is brought about by perfect tracking of the movable plate displacement towards the pull-in region. Utilizing a Proportional Integral (PI) sliding surface for the ASMC design, displacement state is estimated by the Luenberger observer all in order to grapple the matched and unmatched uncertainties and disturbance ultimately. Consequently, the MEMS tunable capacitor dynamics is manipulated in such a way that the movable electrode is

Fig. 1 **a** A lumped model of the MEMS tunable capacitor is displayed. **b** Moving plate attains one 3rd of its original gap to the stationary plate at the pull-in stop

placed at the right point while it is covering an innocuous displacement trajectory. The leading contribution of our work is dealing with the matched and unmatched uncertainty and disturbance without any need for read-out of the movable-plate displacement, owing to the SLMO.

Actually, three estimators are embedded in our design: two for parameter estimation and one for state estimation. Contrary to [7, 24], just velocity of the movable plate is assumed to be at the access. The simulation yields rise so persuasive. It should be reminded that the stiffness and damping factors are prone to shift due to any variation in environmental and operational conditions such as temperature and humidity trend. Mechanical harmonics and noise in addition to cross stiffness and damping are of great impact over the damping and stiffness factors, and finally on the device performance. In the designed control system, the two parameter estimation errors as well as state estimation error remain bounded. It is a rigorous hint of the system stability. The unmatched disturbance includes stiffness and damping cross-talk along with gravity weight force as well as external inertial force. With regard to the model of the system, matched disturbance and matched uncertainty are respectively comprised of the electrostatic disturbing force, stiffness and damping mismatch.

Organization of the article goes as following. A short review and fundamentals of the MEMS tunable capacitor structure and its lumped model are all presented in Sect. 2. The mathematical prerequisites for controller design are put forth subsequently in Sect. 3. Afterwards, simulation outputs are conclusively given in Sect. 4. Eventually, Sect. 5 terminates the paper bringing forth a conclusive paragraph.

2 A review on operating fundamentals and modeling

Two plates, a movable and an unmovable plate, fundamentally constitute a tunable capacitor. Here, one of the typical tunable capacitors, which is displayed on Fig. 1a, b, is taken into account and studied. The MEMS parallel plate tunable capacitor includes a movable plate which shifts the inter-plate gap. The most addressed point in our article will be augmented tracking performance characteristics associated with the electrically charge-accumulated movable plate in the tunable capacitor in existence of both matched and unmatched parameter uncertainty and disturbance. In our innovative design, the plate displacement state is not measured and is evaluated with a Luenberger state observer instead. The movable plate could be displaced and kept fixed at the pull-in point region [24]. The facilitating presumptions in the model possibly cause

parameter uncertainty in stiffness factor in the suspension system. Other error sources come from exhaustion of desired mechanical characteristics due to aging and variation of the working condition. These defects lead to parameter uncertainty. Unfortunately and on the other hand, deficiencies in fabrication process appear as another cause of uncertainty in dominant parameters such as coefficients of stiffness and damping as well as plate mass. On these occasions, design of a controller and its application in order to handle parameter uncertainty is propitious enough to intensify one's focus.

Equation (1) defines the capacitance in a MEMS tunable capacitor. It shifts with the inter-plate gap;

$$C = \epsilon A / (d - x). \quad (1)$$

The parameters in Eq. (1), consisting of ϵ , A , d and x , are respectively the medium permittivity, the electrode area, the original gap between the plates and the movable plate deviation. The capacitor actuation takes place by an AC current presented by Eq. (2);

$$I = I_m \sin(\omega t). \quad (2)$$

The captivating electrostatic force, entitled F_E , brings about attraction of electrodes towards each other. Counteractively, the spring force of kx prevents the plates from extra approaching and collapsing over. A balance will be established between the two forces. It is described by Eq. (3) with regard to Fig. 1a [15];

$$F_E = \left| -\frac{\partial E_l}{\partial x} \right| = \frac{I_m^2 \cos^2(\omega t)}{2\epsilon A \omega^2} = kx + b \frac{dx}{dt} + m \frac{d^2x}{dt^2}. \quad (3)$$

In Eq. (3), numerous variables are introduced. They are designated as F_E , E_l , x , k , m and b which are respectively the electrostatic force engendered by the AC current, the electrostatic energy, plate distance, spring constant, movable electrode mass, and damping factor because of existing gas between plates. The actuation current frequency is $\omega/2\pi$. It is roughly speculated to be greater than the mechanical resonance frequency delineated by $\omega_0/2\pi = \sqrt{k/m}/2\pi$. Additionally, the movable plate gets a damped state when stability is reached. The damping factor is resultant of the gas damping. The quality factor, which is symbolized by Q , is controllable via gas pressure adjustment within the device packaging;

$$Q = 1/2\zeta. \quad (4)$$

Looking down on the slide film damping as well as esteeming limitations on plate movement, squeeze film damping can be typified by b . Nevertheless, this effect is substantially conducted by taking measures in a preventative manner via tricky fabrication techniques. The gas

compressibility requires to be esteemed quantitatively. These watchful actions are pointed out in [24] and [15]. There is a struggle between the electrostatic force and all the resistive forces such as stiffness and damping ones. This balanced struggle keeps the movable electrode position fixed while there are sinusoidal alterations in the AC current. If the electrostatic force in Eq. (3) is replaced by the electrostatic force mean value, Eq. (5) will be yielded where $\omega = 2\pi/T$, $\ddot{x} = \dot{x} \approx 0$. The reason for the latter second and third equations is that the device is operated around the pull-in point and a steady state response is put into consideration.

$$\bar{F}_E = \frac{2}{T} \int_0^{\frac{T}{2}} \frac{I_m^2 \cos^2(\omega t)}{2\epsilon A \omega^2} dt = \frac{1}{\pi} \int_0^{\pi} \frac{I_m^2 (1 + \cos(2\alpha))}{4\epsilon A \omega^2} d\alpha \quad (5)$$

$$d\alpha = \frac{I_{RMS}^2}{2\epsilon A \omega^2 \pi} = kx$$

There is an estimation term for the electrostatic force which enforces the movable plate. It is given by Eq. (6);

$$\bar{F}_E = \left| -\frac{\partial E_V}{\partial x} \right| = \frac{\epsilon A V_{RMS}^2}{2(d-x)^2} = kx. \quad (6)$$

The AC voltage Root Mean Square (RMS) is represented by V_{RMS} which is applied between the plates. The symbol of E_V above typifies the electrostatic energy. Obtaining x in terms of I_{RMS} from Eq. (5), and superseding x in Eq. (6) with the latter term, generates Eq. (7). It states V_{RMS} in terms of I_{RMS} [18];

$$V_{RMS} = \frac{d}{\epsilon A \omega} I_{RMS} - \frac{1}{2k\epsilon^2 A^2 \omega^3} I_{RMS}^3. \quad (7)$$

Derivative of V_{RMS} with respect to I_{RMS} is obtained;

$$\frac{\partial V_{RMS}}{\partial I_{RMS}} = \frac{d}{\epsilon A \omega} - \frac{3}{2k\epsilon^2 A^2 \omega^3} I_{RMS}^2. \quad (8)$$

The value of I_{RMS}^{max} is accomplished setting Eq. (8) to zero. The result is $\sqrt{2kd\epsilon A \omega^2/3}$. The reference or pull-in voltage, entitled V_{RMS}^{max} , is specified via replacement of I_{RMS} in Eq. (7) with its corresponding term of I_{RMS}^{max} .

$$V_{RMS}^{max} = \sqrt{8kd^2/27C_0} = \sqrt{8kd^3/27\epsilon A}. \quad (9)$$

x is accomplished by Eq. (10), esteeming Eqs. (5), and (6) as well as consideration of the attained maximum V_{RMS}^{max} and its respective value of I_{RMS}^{max} [18];

$$x = d/3. \quad (10)$$

The dominant operational region in MEMS tunable capacitor function, appears as the pull-in point. Consequently and importantly, it is necessary to examine tiny shift in the

input current around the pull-in point and its impact over the output voltage closely. Embedding $I_{RMS} + \Delta I$ into Eq. (7) instead of I_{RMS} , outcomes Eq. (11);

$$\Delta V = \frac{d\Delta I}{\epsilon A \omega} - \frac{(\Delta I^3 + 3I_{RMS}^{max} \Delta I + 3I_{RMS}^{max} \Delta I^2)}{2k\epsilon^2 A^2 \omega^3}. \quad (11)$$

Equation (12) is accomplished as an output of division of Eq. (11) by V_{RMS}^{max} ,

$$\frac{\Delta V_{RMS}}{V_{RMS}^{max}} = -\frac{3}{2} \left(\frac{\Delta I}{I_{RMS}^{max}} \right)^2 - \frac{1}{2} \left(\frac{\Delta I}{I_{RMS}^{max}} \right)^3 \approx -\frac{3}{2} \left(\frac{\Delta I}{I_{RMS}^{max}} \right)^2. \quad (12)$$

Esteeming Eq. (12), it is affirmed that the voltage variations in the pull-in point is roughly equitable to the square of the current changes. For that reason, small variations in the input current with respect to the pull-in current bring about a tiny alteration in the pull-in voltage. It backs the concept of taking advantage of the tunable capacitor as the leading part in a VRS. Since the actuation frequency exceeds the mechanical resonance frequency, the electrostatic force in Eq. (3) and the electrostatic force mean value are exchangeable. It contributes to Eq. (13);

$$\bar{F}_E = \frac{I_{RMS}^2}{2\epsilon A \omega^2 \pi} = u = m\ddot{x}(t) + b\dot{x}(t) + kx(t). \quad (13)$$

A tunable capacitor is demonstrated on Fig. 1 with its three cantilevers named L_1 , L_2 and L_3 as the suspension system which hang over the movable plate of the tunable capacitor. The minor discrepancies among their lengths lead to aberrational movement of the plate mass center, straying from x direction into y and z directions slightly. They are modeled as stiffness and damping crosstalk enforcing the plate along the x direction. This minor divergence into y and z directions might be esteemed in the plate dynamics so that tracking performance veracity could be upturned. One may examine Fig. 2 to have a better understanding of the various forces and disturbance which might be applied over the system. Actually, the conversed dynamics is stated by (14):

Fig. 2 Resolution of the plate weight force vector to its various components

$$u + d_{Ed} + mg \cos \psi \cos \theta + F_{Ein} \\ = \ddot{x} + (b_n + \Delta b)\dot{x} + (k_n + \Delta k)x \\ + b_{xy}\dot{y} + b_{xz}\dot{z} + k_{xy}y + k_{xz}z, \quad (14)$$

where Δb as well as Δk are respectively bounded uncertain values of damping and stiffness factors. Furthermore, d_{Ed} is the bounded value for the electrostatic disturbance. One element of the weight force of the tunable cap along the x direction includes $mg \cos \psi \cos \theta$. Angle ψ is the sweeping angle between the weight force vector and the plane of XY. The other angle, also seen in Fig. 2 and designated as θ , is a sweep between the previous mentioned element of the weight force vector ($mg \cos \psi$) and x direction. If the VRS is employed in a movable accelerated system, an inertial force may come up. The terms of k_{xy} , k_{xz} , d_{xy} and d_{xz} are correspondingly stiffness and damping crosstalk coefficients between "x-and-y" direction as well as "x-and-z". We are going to address a controller to force the movable capacitive plate to follow a desired reference model so that it could be transiently guided towards the pull-in region with little corruption to its cantilever micro structure. It should be reminded that the plate is occasionally prone to possible depicted uncertain and disturbing situation. The reference model for the plate movement goes as follows:

$$\ddot{x}_r + a_r \dot{x}_r + b_r x_r = c_r u_r(t). \quad (15)$$

The reference model stability is established by constants of a_r and b_r . The constant of c_r with the previous ones also adjust transient and steady state response of the reference model.

One may esteem system dynamics in state space model format. On this approach, he would define the state space vector of the system as $X = [x_1 \ x_2]^T = [x \ \dot{x}]^T$;

$$\dot{X} = AX + BU, \quad (16)$$

$$Y = CX, \quad (17)$$

where

$$A_{2 \times 2} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}, \quad B_{2 \times 1} = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}, \quad C_{1 \times 2} = [0 \ 1] \quad (18)$$

and $U = u$. In our study, the dynamics is a single input one. The state space vector of the reference model is also esteemed as $X = [x_{r1} \ x_{r2}]^T = [x_r \ \dot{x}_r]^T$. It brings forth the following state space model;

$$\dot{X}_r = A_r X_r + B_r U_r, \quad (19)$$

$$Y_r = C_r X_r, \quad (20)$$

where

$$A_r = \begin{bmatrix} 0 & 1 \\ -b_r & -a_r \end{bmatrix}, \quad B_r = \begin{bmatrix} 0 \\ c_r \end{bmatrix}, \quad C_r = [0 \ 1], \quad U_r = u_r. \quad (21)$$

The parameter uncertainties and disturbance are speculated to come up and be joined to the model with this format;

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -\frac{k_n}{m} & -\frac{b_n}{m} \end{bmatrix} X + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} d_{Ed} + \begin{bmatrix} 0 & 1 \\ -\frac{\Delta k}{m} & -\frac{\Delta b}{m} \end{bmatrix} X \\ + \begin{bmatrix} 0 \\ -\frac{k_{xy}y + k_{xz}z}{m} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{b_{xy}\dot{y} + b_{xz}\dot{z}}{m} \end{bmatrix} \\ + \begin{bmatrix} 0 \\ g \cos \psi \cos \theta + \frac{F_{Ein}}{m} \end{bmatrix}. \quad (22)$$

3 Design of the controller and its mathematics

Total dynamics of the movable plate in the tunable capacitor could be esteemed in another form:

$$\dot{X} = (A + \Delta A)X(t) + Bu(t) + d(t), \quad (23)$$

in which $A_{n \times n}$, $X_{n \times 1} = [x \ \dot{x}]^T$, $B_{n \times m}$, $[u(t)]_{m \times 1}$ are accordingly system matrix, state space vector, input manipulation matrix, and system input. ΔA and $d(t)$ are mutually unexplored parameter uncertainties as well as disturbance. They are enforced over the system in both unmatched and matched way, being concerned with input manipulation matrix. One may adopt $n = 2$ and $m = 1$ in our case. The ensuing equation of (24) indicates the reference model;

$$\dot{X}_r = A_r X_r(t) + B_r r(t), \quad (24)$$

where $[A_r]_{n \times n}$, $[X_r]_{n \times 1} = [x_r \ \dot{x}_r]^T$, $[B_r]_{n \times m}$, $[r(t)]_{m \times 1}$ are jointly reference model system matrix, desired trajectory vector, input matrix, and reference input. Equations (25) and (26) represent the mathematical characterization of the matched and unmatched disturbance and uncertainty;

$$\Delta A(t) = B \Delta A_m(t) + \Delta A_{um}(t), \quad (25)$$

$$d(t) = B d_m(t) + d_{um}(t). \quad (26)$$

Esteeming Eqs. (23), (25) and (26), the plate dynamics will be streamlined in the subsequent manner

$$\begin{aligned}
\dot{X} &= AX + Bu + \Delta AX + d = AX + Bu + B\Delta A_m X \\
&\quad + \Delta A_{um} X + B d_m + d_{um} \\
&= AX + Bu + B[\Delta A_m X + d_m] + \Delta A_{um} X + d_{um} \Rightarrow \\
\dot{X} &= AX + Bu + B l_m + l_{um},
\end{aligned} \quad (27)$$

where l_u and l_m are accordingly the lumped unmatched and matched uncertainty and disturbance;

$$l_m = \Delta A_m(t)X(t) + d_m(t), \quad (28)$$

$$l_{um} = \Delta A_{um}(t)X(t) + d_{um}(t). \quad (29)$$

Commonly, it is speculated that the unmatched and matched terms of disturbances and uncertainties are limited:

$$\|\Delta A_m\| \leq \beta_{m1} \quad \|d_m\| \leq \beta_{m0} \quad (30)$$

$$\|\Delta A_{um}\| \leq \beta_{um1} \quad \|d_{um}\| \leq \beta_{um0} \quad (31)$$

Hence,

$$\|l_m\| = \|\Delta A_m X + d_m\| \leq \beta_{m1} \|X\| + \beta_{m0} \quad (32)$$

$$\|l_{um}\| = \|\Delta A_{um} X + d_{um}\| \leq \beta_{um1} \|X\| + \beta_{um0} \quad (33)$$

Before proceeding with the controller design, it is necessary to redefine $\text{sgn}(\cdot)$ function which its argument might be a matrix or vector;

$$\text{sgn}(\mu) = \begin{cases} -\frac{\mu}{\|\mu\|} & \text{if } \mu \neq 0, \\ 0 & \text{otherwise} \end{cases} \quad (34)$$

Considering $C_{q \times n}$ as the output matrix, SMLO structure goes as follows

$$\begin{aligned}
\dot{\hat{X}} &= A\hat{X} + Bu - L(C\hat{X} - Y) + B\eta_{ml} \text{sgn}(WC\tilde{X}) \\
&\quad + B\eta_{ul} \text{sgn}(GC\tilde{X}),
\end{aligned} \quad (35)$$

or

$$\begin{aligned}
\dot{\hat{X}} &= A\hat{X} + Bu - L(C\hat{X} - Y) + B\eta_{ml} \text{sgn}(WC\hat{X} - WY) \\
&\quad + B\eta_{ul} \text{sgn}(GC\hat{X} - GY)
\end{aligned} \quad (36)$$

or even

$$\dot{\hat{X}} = A\hat{X} + Bu - LC\tilde{X} + B\eta_{ml} \text{sgn}(WC\tilde{X}) + B\eta_{ul} \text{sgn}(GC\tilde{X}), \quad (37)$$

in which $\hat{X}$ is the estimation of X ($\tilde{X} = \hat{X} - X$), and $L_{n \times q}$ is the Luenberger observer matrix satisfying the following equation:

$$(A - LC)^T P + P(A - LC) = -Q. \quad (38)$$

The number of system outputs is q . In the above equation, $Q_{n \times n}$ and $P_{n \times n}$ are positive definite symmetric matrices; $Q^T = Q > 0$ and $P^T = P > 0$. Constants of η_{ml} and η_{ul} are positive. Note that $W_{m \times q}$ and $G_{m \times q}$ are matrices holding true the following equations:

$$WC = B^T P, \quad (39)$$

$$GC = B^T P. \quad (40)$$

Note that there may be no need for adopting two different variables including W and G . Adding and subtracting the term of AX from Eq. (35), results in

$$\begin{aligned}
\dot{\tilde{X}} &= A\tilde{X} + Bu - LC\tilde{X} + B\eta_{ml} \text{sgn}(WC\tilde{X}) + B\eta_{ul} \text{sgn}(GC\tilde{X}) \\
&\quad - AX + AX \xrightarrow{\tilde{X}=\hat{X}-X} \\
\dot{\tilde{X}} &= (A - LC)\tilde{X} + B\eta_{ml} \text{sgn}(WC\tilde{X}) \\
&\quad + B\eta_{ul} \text{sgn}(GC\tilde{X}) + AX + Bu
\end{aligned} \quad (41)$$

Regarding Eqs. (27), (41) changes into

$$\begin{aligned}
\dot{\tilde{X}} &= (A - LC)\tilde{X} + B\eta_{ml} \text{sgn}(WC\tilde{X}) + B\eta_{ul} \text{sgn}(GC\tilde{X}) \\
&\quad + \dot{\tilde{X}} - B l_m - l_{um},
\end{aligned} \quad (42)$$

which could be expressed as

$$\begin{aligned}
\dot{\tilde{X}} &= (A - LC)\tilde{X} + B\eta_{ml} \text{sgn}(WC\tilde{X}) + B\eta_{ul} \text{sgn}(GC\tilde{X}) \\
&\quad - B l_m - l_{um},
\end{aligned} \quad (43)$$

regarding

$$\tilde{X}(t) = \hat{X}(t) - X(t). \quad (44)$$

Equation (43) introduces state estimation error of the SMLO which is normally called observer error dynamics. Hence, $[A - LC]$ has to be a Hurwitz matrix. Regarding Eq. (27) as well as (24), and considering estimated tracking error as

$$\hat{e}(t) = \hat{X}(t) - X_r(t), \quad (45)$$

the time-derivative of the estimated tracking error will be worked out in the ensuing manner, esteeming Eqs. (35) and (24):

$$\begin{aligned}
\dot{\hat{e}} &= \dot{\hat{X}} - \dot{X}_r = A\hat{X} + Bu - LC\tilde{X} + B\eta_{ml} \text{sgn}(WC\tilde{X}) \\
&\quad + B\eta_{ul} \text{sgn}(GC\tilde{X}) - A_r X_r \\
&\quad - B_r r \pm A_r \hat{X} = A_r \hat{X} - A_r X_r + (A - A_r)\hat{X} + Bu - LC\tilde{X} \\
&\quad + B\eta_{ml} \text{sgn}(WC\tilde{X}) + B\eta_{ul} \text{sgn}(GC\tilde{X}) - B_r r \Rightarrow \\
\dot{\hat{e}} &= A_r \hat{e} + (A - A_r)\hat{X} + Bu - LC\tilde{X} + B\eta_{ml} \text{sgn}(WC\tilde{X}) \\
&\quad + B\eta_{ul} \text{sgn}(GC\tilde{X}) - B_r r
\end{aligned} \quad (46)$$

Contrariwise [24], estimated sliding surface vector, designated as $[s(t)]_{m \times 1}$, is constructed with a proportional and integral combination of estimated tracking error such as

$$\hat{s}(t) = \Lambda \hat{e}(t) - \int_0^t \Lambda A_r \hat{e}(\tau) d\tau, \quad (47)$$

where $[\Lambda]_{m \times n}$ is a constant matrix (vector). Here, it could be esteemed as

$$\Lambda = [\lambda_1 \quad \lambda_2] = [\lambda \quad \lambda], \quad (48)$$

in which λ_1, λ_2 and $\lambda > 0$. It should be clarified that they are the tuning transient constants for the sliding error dynamics. Similar to [25], they are adopted equal just for simplification of the simulation ($\lambda_1 = \lambda_2 = \lambda = 0.000001$) while they can be attributed different values. Furthermore, Λ is selected so that $[\Lambda B]_{m \times m}^{-1}$ exists. Time derivative of $\hat{s}(t)$ is

$$\dot{\hat{s}}(t) = \Lambda \dot{\hat{e}}(t) - \Lambda A_r \hat{e}(t). \quad (49)$$

Regarding Eqs. (46) and (49), $\dot{\hat{s}}(t)$ will be

$$\begin{aligned} \dot{\hat{s}}(t) &= \Lambda A_r \hat{e} + \Lambda (A - A_r) \hat{X} + \Lambda B u - \Lambda L C \tilde{X} \\ &\quad + \Lambda B \eta_{ml} \text{sgn}(W C \tilde{X}) + \Lambda B \eta_{ul} \text{sgn}(G C \tilde{X}) \\ &\quad - \Lambda B_r r - \Lambda A_r \hat{e} \Rightarrow \\ \dot{\hat{s}}(t) &= \Lambda (A - A_r) \hat{X} + \Lambda B u - \Lambda L C \tilde{X} + \Lambda B \eta_{ml} \text{sgn}(W C \tilde{X}) \\ &\quad + \Lambda B \eta_{ul} \text{sgn}(G C \tilde{X}) \\ &\quad - \Lambda B_r r \end{aligned} \quad (50)$$

The minus sign, in the estimated sliding surface error definition in Eq. (47), is to eliminate term of $\Lambda A_r \hat{e}$ in $\dot{\hat{s}}$. Bounded-ness of $\hat{e}(t)$ and $\dot{\hat{s}}(t)$ will be accordingly proved later. Putting $\dot{\hat{s}}(t)$ equal to zero, $u_{eq}(t)$ will be obtained as

$$u_{eq}(t) = \Phi_1 \hat{X} + \Phi_2 r - [\Lambda B]^{-1} \Lambda [B \eta_{ml} \text{sgn}(W C \tilde{X}) + B \eta_{ul} \text{sgn}(G C \tilde{X})] \quad (51)$$

where

$$[\Phi_1]_{m \times n} = -[\Lambda B]^{-1} \Lambda (A - A_r), \quad (52)$$

$$[\Phi_2]_{m \times m} = [\Lambda B]^{-1} \Lambda B_r. \quad (53)$$

As stated before, the matrix Λ is specified in such a way that $[\Lambda B]$ would be non-singular. It should also be notified that the value of the matched and unmatched uncertainties are not known so they are managed and suppressed with the term of $-\eta \frac{s(t)}{\|s(t)\|}$ or $\eta \text{sgn}(s)$. Thenceforth,

$$u(t) = \Phi_1 \hat{X} + \Phi_2 r - \eta \frac{\hat{s}(t)}{\|\hat{s}(t)\|}, \quad (54)$$

where $\hat{\Phi}_1$ and $\hat{\Phi}_2$ are estimates of Φ_1 and Φ_2 accordingly. It is worthwhile being mentioned that

$$[\eta]_{m \times m} = \text{diag}\{\eta_1, \dots, \eta_m\}, \quad (\eta_i > 0 \quad i = 1, \dots, m). \quad (55)$$

Estimation errors of the so-called feedback and feed-forward gains in the control law statement are esteemed in ensuing equations:

$$\tilde{\Phi}_1 = \hat{\Phi}_1 - \Phi_1, \quad (56)$$

$$\tilde{\Phi}_2 = \hat{\Phi}_2 - \Phi_2. \quad (57)$$

Evidently, the control law, expressed by Eq. (54), does not let $\dot{\hat{s}}$ be zero. Nevertheless, it would be of great assistance to figure out the term for $\dot{\hat{s}}$, esteeming u in Eq. (54). Our approach will be provision of a guarantee for boundedness of $\dot{\hat{s}}$ in the first stage. Secondly, we will try to substantiate zero-convergence of $\hat{s}(t)$, adopting appropriate Lyapunov candidate function of the estimated sliding surface. Before covering these stages, it is more beneficial to acquire a more developed statement for $\dot{\hat{s}}$. The time-derivative of $\hat{e}$ is appraised in the ensuing format, esteeming Eqs. (35) and (24):

$$\begin{aligned} \dot{\hat{e}} &= \dot{\hat{X}} - \dot{X}_r = A \hat{X} + B u - L C \tilde{X} \\ &\quad + B \eta_{ml} \text{sgn}(W C \tilde{X}) + B \eta_{ul} \text{sgn}(G C \tilde{X}) - A_r X_r - B_r r. \end{aligned} \quad (58)$$

Envisaging Eq. (54),

$$\begin{aligned} \dot{\hat{e}} &= A \hat{X} + B [\hat{\Phi}_1 \hat{X} + \hat{\Phi}_2 r] - B \eta \frac{s(t)}{\|s(t)\|} - L C \tilde{X} \\ &\quad + B \eta_{ml} \text{sgn}(W C \tilde{X}) + B \eta_{ul} \text{sgn}(G C \tilde{X}) - A_r X_r - B_r r. \end{aligned} \quad (59)$$

Combining Eqs. (56), (57), and (59),

$$\begin{aligned} \dot{\hat{e}} &= A \hat{X} + B [(\tilde{\Phi}_1 + \Phi_1) \hat{X}] + B [(\tilde{\Phi}_2 + \Phi_2) r] \\ &\quad - B \eta \frac{\hat{s}(t)}{\|\hat{s}(t)\|} - L C \tilde{X} \\ &\quad + B \eta_{ml} \text{sgn}(W C \tilde{X}) + B \eta_{ul} \text{sgn}(G C \tilde{X}) - A_r X_r - B_r r, \end{aligned} \quad (60)$$

or

$$\begin{aligned} \dot{\hat{e}} &= A \hat{X} + B \tilde{\Phi}_1 \hat{X} + B \tilde{\Phi}_2 r + B \Phi_1 \hat{X} + B \Phi_2 r \\ &\quad - B \eta \frac{\hat{s}(t)}{\|\hat{s}(t)\|} - L C \tilde{X} \\ &\quad + B \eta_{ml} \text{sgn}(W C \tilde{X}) + B \eta_{ul} \text{sgn}(G C \tilde{X}) - A_r X_r - B_r r. \end{aligned} \quad (61)$$

The time-derivative of the estimated sliding surface will turn equal to Eq. (62) with regard to Eqs. (49) and (45);

$$\begin{aligned}\dot{\hat{s}} &= \Lambda A \hat{X} + \Lambda B \tilde{\Phi}_1 \hat{X} + \Lambda B \tilde{\Phi}_2 r + \Lambda B \Phi_1 \hat{X} + \Lambda B \Phi_2 r \\ &\quad - \Lambda B \eta \frac{\hat{s}(t)}{\|\hat{s}(t)\|} - \Lambda L C \tilde{X} \\ &\quad + \Lambda B \eta_{ml} \operatorname{sgn}(WC \tilde{X}) + \Lambda B \eta_{ul} \operatorname{sgn}(GC \tilde{X}) - \Lambda A_r X_r \\ &\quad - \Lambda B_r r - \Lambda A_r \hat{X} + \Lambda A_r X_r \Rightarrow \\ \dot{\hat{s}} &= \Lambda A \hat{X} + \Lambda B \tilde{\Phi}_1 \hat{X} + \Lambda B \tilde{\Phi}_2 r + \Lambda B \Phi_1 \hat{X} + \Lambda B \Phi_2 r \\ &\quad - \Lambda B \eta \frac{\hat{s}(t)}{\|\hat{s}(t)\|} - \Lambda L C \tilde{X} \\ &\quad + \Lambda B \eta_{ml} \operatorname{sgn}(WC \tilde{X}) + \Lambda B \eta_{ul} \operatorname{sgn}(GC \tilde{X}) \\ &\quad - \Lambda B_r r - \Lambda A_r \hat{X}.\end{aligned}\quad (62)$$

Esteeming a Lyapunov function of $\tilde{\Phi}_1$ and $\tilde{\Phi}_2$ besides $\hat{s}$, it implies that it would be advantageous to dump feedback and feed-forward terms of Φ_1 and Φ_2 , while exterminating terms of AX and $A_r X$. Namely, presuming

$$B \Phi_2 = B_r, \quad (63)$$

$$B \Phi_1 = A_r - A, \quad (64)$$

engenders $\hat{s}$ in the subsequent statement:

$$\begin{aligned}\dot{\hat{s}} &= \Lambda B \tilde{\Phi}_1 \hat{X} + \Lambda B \tilde{\Phi}_2 r - \Lambda B \eta \frac{\hat{s}}{\|\hat{s}\|} - \Lambda L C \tilde{X} \\ &\quad + \Lambda B \eta_{ml} \operatorname{sgn}(WC \tilde{X}) + \Lambda B \eta_{ul} \operatorname{sgn}(GC \tilde{X}).\end{aligned}\quad (65)$$

The Lyapunov candidate function is thought to be

$$\begin{aligned}V(\hat{s}, \tilde{\Phi}_1, \tilde{\Phi}_2, \tilde{X}) &= \frac{1}{2} \hat{s}^T \hat{s} + \frac{1}{2} \operatorname{tr}\{\tilde{\Phi}_1 \Gamma_1 \tilde{\Phi}_1^T\} \\ &\quad + \frac{1}{2} \operatorname{tr}\{\tilde{\Phi}_2 \Gamma_2 \tilde{\Phi}_2^T\} + \frac{1}{2} \tilde{X}^T P \tilde{X},\end{aligned}\quad (66)$$

in which $[\Gamma_1]_{n \times n}$ and $[\Gamma_2]_{m \times m}$ are fixed symmetric positive definite matrixes. The term of tr implies trace of a square matrix. Its time-derivative will be appraised in the subsequent terms;

$$\begin{aligned}\dot{V} &= \frac{1}{2} \dot{\hat{s}}^T \hat{s} + \frac{1}{2} \hat{s}^T \dot{\hat{s}} + \frac{1}{2} \operatorname{tr}\{\dot{\tilde{\Phi}}_1 \Gamma_1 \tilde{\Phi}_1^T\} + \frac{1}{2} \operatorname{tr}\{\tilde{\Phi}_1 \Gamma_1 \dot{\tilde{\Phi}}_1^T\} \\ &\quad + \frac{1}{2} \operatorname{tr}\{\dot{\tilde{\Phi}}_2 \Gamma_2 \tilde{\Phi}_2^T\} + \frac{1}{2} \operatorname{tr}\{\tilde{\Phi}_2 \Gamma_2 \dot{\tilde{\Phi}}_2^T\} + \frac{1}{2} \dot{\tilde{X}}^T P \tilde{X} \\ &\quad + \frac{1}{2} \tilde{X}^T P \dot{\tilde{X}}.\end{aligned}\quad (67)$$

Scalar terms constitute $\dot{V}$ in Eq. (67). Moreover, it should be notified that $\operatorname{tr}\{Q^T\} = \operatorname{tr}\{Q\}$ [17]. Thence, $\dot{V}$ will be equivalent to

$$\dot{V} = \hat{s}^T \dot{\hat{s}} + \operatorname{tr}\{\dot{\tilde{\Phi}}_1 \Gamma_1 \tilde{\Phi}_1^T\} + \operatorname{tr}\{\tilde{\Phi}_2 \Gamma_2 \dot{\tilde{\Phi}}_2^T\} + \frac{1}{2} \dot{\tilde{X}}^T P \tilde{X} + \frac{1}{2} \tilde{X}^T P \dot{\tilde{X}}. \quad (68)$$

Replacement of $\dot{\hat{s}}$ with its exchangeable term in Eq. (65) is conclusive of

$$\begin{aligned}\dot{V} &= \hat{s}^T \Lambda B \tilde{\Phi}_1 \hat{X} + \hat{s}^T \Lambda B \tilde{\Phi}_2 r - \hat{s}^T \Lambda B \eta \frac{\hat{s}}{\|\hat{s}\|} - \hat{s}^T \Lambda L C \tilde{X} \\ &\quad + \hat{s}^T \Lambda B \eta_{ml} \operatorname{sgn}(WC \tilde{X}) + \hat{s}^T \Lambda B \eta_{ul} \operatorname{sgn}(GC \tilde{X}) \\ &\quad + \operatorname{tr}\{\dot{\tilde{\Phi}}_1 \Gamma_1 \tilde{\Phi}_1^T\} + \operatorname{tr}\{\tilde{\Phi}_2 \Gamma_2 \dot{\tilde{\Phi}}_2^T\} + \frac{1}{2} \dot{\tilde{X}}^T P \tilde{X} + \frac{1}{2} \tilde{X}^T P \dot{\tilde{X}}.\end{aligned}\quad (69)$$

One may intentionally restate and rearrange Eq. (69) as the new equation (70);

$$\begin{aligned}\dot{V} &= \hat{s}^T \Lambda B \tilde{\Phi}_1 \hat{X} + \hat{s}^T \Lambda B \tilde{\Phi}_2 r + \operatorname{tr}\{\dot{\tilde{\Phi}}_1 \Gamma_1 \tilde{\Phi}_1^T\} + \operatorname{tr}\{\tilde{\Phi}_2 \Gamma_2 \dot{\tilde{\Phi}}_2^T\} \\ &\quad - \hat{s}^T \Lambda B \eta \frac{\hat{s}}{\|\hat{s}\|} - \hat{s}^T \Lambda L C \tilde{X} + \hat{s}^T \Lambda B \eta_{ml} \operatorname{sgn}(WC \tilde{X}) \\ &\quad + \hat{s}^T \Lambda B \eta_{ul} \operatorname{sgn}(GC \tilde{X}) \\ &\quad + \frac{1}{2} \dot{\tilde{X}}^T P \tilde{X} + \frac{1}{2} \tilde{X}^T P \dot{\tilde{X}}.\end{aligned}\quad (70)$$

Equation (70) is constituted of separate scalar terms so it could be restated with the following form;

$$\begin{aligned}\dot{V} &= \operatorname{tr}\{\dot{\tilde{\Phi}}_1 \Gamma_1 \tilde{\Phi}_1^T\} + \operatorname{tr}\{\tilde{\Phi}_2 \Gamma_2 \dot{\tilde{\Phi}}_2^T\} \\ &\quad + \operatorname{tr}\left\{\left[\hat{s}^T \Lambda B \tilde{\Phi}_1\right]_{1 \times n} \hat{X}\right\} + \operatorname{tr}\left\{\left[\hat{s}^T \Lambda B \tilde{\Phi}_2\right]_{1 \times m} r\right\} \\ &\quad - \hat{s}^T \Lambda B \eta \frac{\hat{s}}{\|\hat{s}\|} - \hat{s}^T \Lambda L C \tilde{X} + \hat{s}^T \Lambda B \eta_{ml} \operatorname{sgn}(WC \tilde{X}) \\ &\quad + \hat{s}^T \Lambda B \eta_{ul} \operatorname{sgn}(GC \tilde{X}) \\ &\quad + \frac{1}{2} \dot{\tilde{X}}^T P \tilde{X} + \frac{1}{2} \tilde{X}^T P \dot{\tilde{X}}.\end{aligned}\quad (71)$$

The trace properties are given by Eqs. (72) and (73) [17]:

$$\operatorname{tr}\{Q\} = \operatorname{tr}\{Q^T\}, \quad (72)$$

$$\operatorname{tr}\{Q_{m \times n} P_{n \times m}\} = \operatorname{tr}\{P_{n \times m} Q_{m \times n}\}. \quad (73)$$

On this occasion, Eq. (71) easily changes into Eq. (74);

$$\begin{aligned}\dot{V} &= \operatorname{tr}\{\tilde{\Phi}_1 \left[\Gamma_1 \dot{\tilde{\Phi}}_1^T\right]_{n \times m}\} + \operatorname{tr}\{\tilde{\Phi}_2 \left[\Gamma_2 \dot{\tilde{\Phi}}_2^T\right]_{m \times m}\} \\ &\quad + \operatorname{tr}\{\hat{X} \hat{s}^T \Lambda B \tilde{\Phi}_1\} + \operatorname{tr}\{r \hat{s}^T \Lambda B \tilde{\Phi}_2\} \\ &\quad - \hat{s}^T \Lambda B \eta \frac{\hat{s}}{\|\hat{s}\|} - \hat{s}^T \Lambda L C \tilde{X} + \hat{s}^T \Lambda B \eta_{ml} \operatorname{sgn}(WC \tilde{X}) \\ &\quad + \hat{s}^T \Lambda B \eta_{ul} \operatorname{sgn}(GC \tilde{X}) \\ &\quad + \frac{1}{2} \dot{\tilde{X}}^T P \tilde{X} + \frac{1}{2} \tilde{X}^T P \dot{\tilde{X}}.\end{aligned}\quad (74)$$

Again, using trace properties of Eq. (73) twice engenders

$$\begin{aligned}\dot{V} = & \operatorname{tr}\{\Gamma_1 \dot{\tilde{\Phi}}_1^T \tilde{\Phi}_1\} + \operatorname{tr}\{\Gamma_2 \dot{\tilde{\Phi}}_2^T \tilde{\Phi}_2\} + \operatorname{tr}\{\hat{X} \dot{\hat{s}}^T \Lambda B \tilde{\Phi}_1\} \\ & + \operatorname{tr}\{r \dot{\hat{s}}^T \Lambda B \tilde{\Phi}_2\} \\ & - \hat{s}^T \Lambda B \eta \frac{\hat{s}}{\|\hat{s}\|} - \hat{s}^T \Lambda L C \tilde{X} + \hat{s}^T \Lambda B \eta_{ml} \operatorname{sgn}(W C \tilde{X}) \\ & + \hat{s}^T \Lambda B \eta_{ul} \operatorname{sgn}(G C \tilde{X}) \\ & + \frac{1}{2} \dot{\tilde{X}}^T P \tilde{X} + \frac{1}{2} \tilde{X}^T P \dot{\tilde{X}}.\end{aligned}\quad (75)$$

Estimating the subsequent trace property [17],

$$\operatorname{tr}\{A + B\} = \operatorname{tr}\{A\} + \operatorname{tr}\{B\}, \quad (76)$$

the time-derivative of the Lyapunov function will get equal to

$$\begin{aligned}\dot{V} = & \operatorname{tr}\left\{\left(\Gamma_1 \dot{\tilde{\Phi}}_1^T + \hat{X} \dot{\hat{s}}^T \Lambda B\right) \tilde{\Phi}_1\right\} \\ & + \operatorname{tr}\left\{\left(\Gamma_2 \dot{\tilde{\Phi}}_2^T + r \dot{\hat{s}}^T \Lambda B\right) \tilde{\Phi}_2\right\} \\ & - \hat{s}^T \Lambda B \eta \frac{\hat{s}}{\|\hat{s}\|} - \hat{s}^T \Lambda L C \tilde{X} + \hat{s}^T \Lambda B \eta_{ml} \operatorname{sgn}(W C \tilde{X}) \\ & + \hat{s}^T \Lambda B \eta_{ul} \operatorname{sgn}(G C \tilde{X}) \\ & + \frac{1}{2} \dot{\tilde{X}}^T P \tilde{X} + \frac{1}{2} \tilde{X}^T P \dot{\tilde{X}}.\end{aligned}\quad (77)$$

Two update laws have to be considered with consideration of slow variation in the plate parameters;

$$\dot{\tilde{\Phi}}_1 = \dot{\tilde{\Phi}}_1 = -B^T \Lambda^T \hat{s} \hat{X}^T \Gamma_1^{-1}, \quad (78)$$

$$\dot{\tilde{\Phi}}_2 = \dot{\tilde{\Phi}}_2 = -B^T \Lambda^T \hat{s} r^T \Gamma_2^{-1}. \quad (79)$$

On this occasion, $\dot{V}$ gets equal to

$$\begin{aligned}\dot{V} = & -\hat{s}^T \Lambda B \eta \frac{\hat{s}}{\|\hat{s}\|} - \hat{s}^T \Lambda L C \tilde{X} + \hat{s}^T \Lambda B \eta_{ml} \operatorname{sgn}(W C \tilde{X}) \\ & + \hat{s}^T \Lambda B \eta_{ul} \operatorname{sgn}(G C \tilde{X}) \\ & + \frac{1}{2} \dot{\tilde{X}}^T P \tilde{X} + \frac{1}{2} \tilde{X}^T P \dot{\tilde{X}}.\end{aligned}\quad (80)$$

Note that $[\Lambda B \eta]_{m \times m}$ is a non-singular matrix. It is accepted from linear algebra that [6, 16, 26]

$$\sigma_{\min}\{\Lambda B \eta\} \|\hat{s}\|^2 \leq \hat{s}^T \Lambda B \eta \hat{s} \leq \sigma_{\max}\{\Lambda B \eta\} \|\hat{s}\|^2, \quad (81)$$

where σ is the eigenvalue of $[\Lambda B \eta]$. The inequality (81) could be worked out more;

$$-\sigma_{\max}\{\Lambda B \eta\} \|\hat{s}\| \leq -\hat{s}^T \Lambda B \eta \frac{\hat{s}}{\|\hat{s}\|} \leq -\sigma_{\min}\{\Lambda B \eta\} \|\hat{s}\|. \quad (82)$$

The exploitable part of the inequality (82) is

$$-\hat{s}^T \Lambda B \eta \frac{\hat{s}}{\|\hat{s}\|} \leq -\sigma_{\min}\{\Lambda B \eta\} \|\hat{s}\|. \quad (83)$$

Estimating Eqs. (80) and (83) all together,

$$\begin{aligned}\dot{V} \leq & -\sigma_{\min}\{\Lambda B \eta\} \|\hat{s}\| - \hat{s}^T \Lambda L C \tilde{X} + \hat{s}^T \Lambda B \eta_{ml} \operatorname{sgn}(W C \tilde{X}) \\ & + \hat{s}^T \Lambda B \eta_{ul} \operatorname{sgn}(G C \tilde{X}) \\ & + \frac{1}{2} \dot{\tilde{X}}^T P \tilde{X} + \frac{1}{2} \tilde{X}^T P \dot{\tilde{X}}.\end{aligned}\quad (84)$$

Equations (84) and (43) cooperatively engender

$$\begin{aligned}\dot{V} \leq & -\sigma_{\min}\{\Lambda B \eta\} \|\hat{s}\| - \hat{s}^T \Lambda L C \tilde{X} + \hat{s}^T \Lambda B \eta_{ml} \operatorname{sgn}(W C \tilde{X}) \\ & + \hat{s}^T \Lambda B \eta_{ul} \operatorname{sgn}(G C \tilde{X}) \\ & + \frac{1}{2} [(A - LC) \tilde{X} + B \eta_{ml} \operatorname{sgn}(W C \tilde{X}) \\ & + B \eta_{ul} \operatorname{sgn}(G C \tilde{X}) - B l_m - l_{um}]^T P \tilde{X} \\ & + \frac{1}{2} \tilde{X}^T P [(A - LC) \tilde{X} + B \eta_{ml} \operatorname{sgn}(W C \tilde{X}) \\ & + B \eta_{ul} \operatorname{sgn}(G C \tilde{X}) - B l_m - l_{um}]\end{aligned}\quad (85)$$

or

$$\begin{aligned}\dot{V} \leq & -\sigma_{\min}\{\Lambda B \eta\} \|\hat{s}\| - \hat{s}^T \Lambda L C \tilde{X} + \hat{s}^T \Lambda B \eta_{ml} \operatorname{sgn}(W C \tilde{X}) \\ & + \hat{s}^T \Lambda B \eta_{ul} \operatorname{sgn}(G C \tilde{X}) \\ & + \frac{1}{2} \tilde{X}^T (A - LC)^T P \tilde{X} + \frac{1}{2} \tilde{X}^T P B \eta_{ml} \operatorname{sgn}(W C \tilde{X}) \\ & + \frac{1}{2} \tilde{X}^T P B \eta_{ul} \operatorname{sgn}(G C \tilde{X}) \\ & - \frac{1}{2} \tilde{X}^T P B l_m - \frac{1}{2} \tilde{X}^T P l_{um} \\ & + \frac{1}{2} \tilde{X}^T P (A - LC) \tilde{X} + \frac{1}{2} \tilde{X}^T P B \eta_{ml} \operatorname{sgn}(W C \tilde{X}) \\ & + \frac{1}{2} \tilde{X}^T P B \eta_{ul} \operatorname{sgn}(G C \tilde{X}) \\ & - \frac{1}{2} \tilde{X}^T P B l_m - \frac{1}{2} \tilde{X}^T P l_{um}.\end{aligned}\quad (86)$$

Equation (86) is simplified with regard to Eq. (38);

$$\begin{aligned}\dot{V} \leq & -\sigma_{\min}\{\Lambda B \eta\} \|\hat{s}\| - \hat{s}^T \Lambda L C \tilde{X} + \hat{s}^T \Lambda B \eta_{ml} \operatorname{sgn}(W C \tilde{X}) \\ & + \hat{s}^T \Lambda B \eta_{ul} \operatorname{sgn}(G C \tilde{X}) \\ & - \frac{1}{2} \tilde{X}^T Q \tilde{X} + \tilde{X}^T P B \eta_{ml} \operatorname{sgn}(W C \tilde{X}) \\ & + \tilde{X}^T P B \eta_{ul} \operatorname{sgn}(G C \tilde{X}) \\ & - \tilde{X}^T P B l_m - \tilde{X}^T P l_{um}.\end{aligned}\quad (87)$$

It is also more developed with consideration of Eqs. (39) and (40);

$$\begin{aligned} \dot{V} \leq & -\sigma_{\min}\{AB\eta\}\|\hat{s}\| - \hat{s}^T \Lambda LC\tilde{X} + \hat{s}^T \Lambda B\eta_{ml} \operatorname{sgn}(WC\tilde{X}) \\ & + \hat{s}^T \Lambda B\eta_{ul} \operatorname{sgn}(GC\tilde{X}) \\ & - \frac{1}{2} \tilde{X}^T Q \tilde{X} + \tilde{X}^T C^T W^T \eta_{ml} \operatorname{sgn}(WC\tilde{X}) \\ & + \tilde{X}^T C^T G^T \eta_{ul} \operatorname{sgn}(GC\tilde{X}) \\ & - I_m^T B^T P \tilde{X} - I_{um}^T P \tilde{X}, \end{aligned} \quad (88)$$

or

$$\begin{aligned} \dot{V} \leq & -\sigma_{\min}\{AB\eta\}\|\hat{s}\| - \hat{s}^T \Lambda LC\tilde{X} + \hat{s}^T \Lambda B\eta_{ml} \operatorname{sgn}(WC\tilde{X}) \\ & + \hat{s}^T \Lambda B\eta_{ul} \operatorname{sgn}(GC\tilde{X}) \\ & - \frac{1}{2} \tilde{X}^T Q \tilde{X} + (WC\tilde{X})^T \eta_{ml} \operatorname{sgn}(WC\tilde{X}) \\ & + (GC\tilde{X})^T \eta_{ul} \operatorname{sgn}(GC\tilde{X}) \\ & - I_m^T WC\tilde{X} - I_{um}^T P \tilde{X}. \end{aligned} \quad (89)$$

Equation (89) could be restated like this, regarding Eq. (34):

$$\begin{aligned} \dot{V} \leq & -\sigma_{\min}\{AB\eta\}\|\hat{s}\| - \hat{s}^T \Lambda LC\tilde{X} + \hat{s}^T \Lambda B\eta_{ml} \operatorname{sgn}(WC\tilde{X}) \\ & + \hat{s}^T \Lambda B\eta_{ul} \operatorname{sgn}(GC\tilde{X}) \\ & - \frac{1}{2} \tilde{X}^T Q \tilde{X} - \eta_{ml} \|WC\tilde{X}\| - \eta_{ul} \|GC\tilde{X}\| \\ & - I_m^T WC\tilde{X} - I_{um}^T P \tilde{X}. \end{aligned} \quad (90)$$

Since the term of $\tilde{X}^T Q \tilde{X}$ is positive, it could be omitted and

$$\begin{aligned} \dot{V} \leq & -\sigma_{\min}\{AB\eta\}\|\hat{s}\| + \|\hat{s}\| \|\Lambda LC\tilde{X}\| + \|\hat{s}\| \|\Lambda B\eta_{ml}\| \\ & + \|\hat{s}\| \|\Lambda B\eta_{ul}\| \\ & - \eta_{ml} \|WC\tilde{X}\| - \eta_{ul} \|GC\tilde{X}\| + \|I_m\| \|WC\tilde{X}\| \\ & + \|I_{um}\| \|P\| \|\tilde{X}\|, \end{aligned} \quad (91)$$

or

$$\begin{aligned} \dot{V} \leq & -\|\hat{s}\| [\sigma_{\min}\{AB\eta\} - \|\Lambda LC\tilde{X}\| - \|\Lambda B\eta_{ml}\| - \|\Lambda B\eta_{ul}\|] \\ & - \|WC\tilde{X}\| (\eta_{ml} - \|I_m\|) - \|\tilde{X}\| (\|GC\eta_{ul}\| - \|I_{um}\| \|P\|). \end{aligned} \quad (92)$$

It is assumed that Λ and η are set in such a way that

$$0 < \sigma_{\min}\{AB\eta\} < \varepsilon \quad \text{or} \quad \frac{1}{\sigma_{\min}\{AB\eta\}} > \frac{1}{\varepsilon}. \quad (93)$$

It could also be supposed that

$$\eta_{ml} > \|I_m\|_{\max} = \beta_{m1} \|X\| + \beta_{m0}, \quad (94)$$

$$\eta_{ul} > \frac{\|I_{um}\|_{\max} \|P\|}{\|GC\|} = \frac{(\beta_{um1} \|X\| + \beta_{um0}) \|P\|}{\|GC\|}, \quad (95)$$

$$\|\Lambda LC\tilde{X}\| + \|\Lambda B\eta_{ml}\| + \|\Lambda B\eta_{ul}\| < \sigma_{\min}\{AB\eta\} < \varepsilon. \quad (96)$$

Regarding Eqs. (32) and (33) as well as the above equations,

$$\dot{V} \leq -\|\hat{s}\| [\sigma_{\min}\{AB\eta\} - \|\Lambda LC\tilde{X}\| - \|\Lambda B\eta_{ml}\| - \|\Lambda B\eta_{ul}\|]. \quad (97)$$

It is also known that

$$\max\{\sigma_{\min}\{AB\eta\} - \|\Lambda LC\tilde{X}\| - \|\Lambda B\eta_{ml}\| - \|\Lambda B\eta_{ul}\|\} = \varepsilon. \quad (98)$$

Thence,

$$\dot{V}(\hat{s}, \tilde{\Phi}_1, \tilde{\Phi}_2, \tilde{X}) \leq -\varepsilon \|\hat{s}\| \quad (99)$$

Equation (99) is the sliding condition and results in the following consecutive equations;

$$\hat{s}(t) \in L_{\infty}, \quad (100)$$

$$\tilde{\Phi}_1(t) \in L_{\infty}, \quad (101)$$

$$\tilde{\Phi}_2(t) \in L_{\infty}, \quad (102)$$

$$\tilde{X}(t) \in L_{\infty}, \quad (103)$$

since $\dot{V}(s, \tilde{\Phi}_1, \tilde{\Phi}_2, \tilde{X})$ is negative semi-definite. It should be clarified that when a time-variant variable or parameter such as g is a member of L_{∞} ($g \in L_{\infty}$), it means that the variable or parameter is bounded. In other terms, L_{∞}^m is the space of the bounded functions, which is defined by the following sets; $L_{\infty}^m := \{f(t) : [0, \infty) \rightarrow \mathbb{R}^m \mid \|f(t)\|_{L_{\infty}} < \infty\}$ and $L_{\infty} := \{f(t) : [0, \infty) \rightarrow \mathbb{R} \mid \|f(t)\|_{L_{\infty}} < \infty\}$. In the mentioned sets, $\|f(t)\|_{L_{\infty}} := \sup_{t \geq 0} \|f(t)\|$. Consequently, by Eqs. (100), (101), (102) and (103), we intend to imply $\hat{s}$, $\tilde{\Phi}_1$, $\tilde{\Phi}_2$, $\tilde{X}$, and their supremum are all bounded.

Integrating over the sliding condition, expressed by Eq. (99), brings about the subsequent inequality;

$$V(t) - V(0) < - \int_0^t \varepsilon \|\hat{s}(\tau)\| d\tau, \quad (104)$$

or

$$\int_0^t \varepsilon \|\hat{s}(\tau)\| d\tau < V(0) - V(t). \quad (105)$$

Since V is positive definite and it is non-increasing due to semi-indefiniteness, one may state that

$$\int_0^{\infty} \|\hat{s}(\tau)\| d\tau < \infty, \quad (106)$$

which indicates

$$\hat{s}(t) \in L_1. \quad (107)$$

At this stage as the final step in our presented mathematics, it is a must to check whether $\hat{s}(t) \in L_{\infty}$ or not. All terms,

in Eq. (65), have been demonstrated or speculated to be limited excluding the terms which is comprised of estimated states of $\hat{X}$ such as $\Lambda B \tilde{\Phi}_1 \hat{X}$. Hence, it is required to establish the truth of limitation of $\hat{X}$. Equations (78) and (79) affirm that

$$\hat{\Phi}_1(t) = - \int_0^t B^T \Lambda^T \hat{s}(\tau) \hat{X}^T(\tau) \Gamma_1^{-1} d\tau + \hat{\Phi}_1(0), \quad (108)$$

or

$$\tilde{\Phi}_1(t) = - \int_0^t B^T \Lambda^T \hat{s}(\tau) \hat{X}^T(\tau) \Gamma_1^{-1} d\tau + \tilde{\Phi}_1(0), \quad (109)$$

and

$$\hat{\Phi}_2(t) = - \int_0^t B^T \Lambda^T \hat{s}(\tau) r^T(\tau) \Gamma_2^{-1} d\tau + \hat{\Phi}_2(0), \quad (110)$$

or

$$\tilde{\Phi}_2(t) = - \int_0^t B^T \Lambda^T \hat{s}(\tau) r^T(\tau) \Gamma_2^{-1} d\tau + \tilde{\Phi}_2(0). \quad (111)$$

Since $\tilde{\Phi}_1(t)$ and $\tilde{\Phi}_2(t)$ are bounded terms and the initial values $\tilde{\Phi}_1(0)$ and $\tilde{\Phi}_2(0)$ can be set limited, the integrals over the right side of Eqs. (109) and (111) will be limited. Thence, the term of $\hat{X}(t)$ will be bounded due to boundedness of $\hat{s}(\tau)$ and all other terms inside the integrals excluding $\hat{X}(\tau)$. The logic gets back to [20] and Riemann Integrability Theorem. Consequently, $\hat{X}(t)$ would be bounded in any definite finite time period. Therefore,

$$\dot{\hat{s}}(t) \in L_\infty. \quad (112)$$

Taking advantage of Eqs. (107), (100) and (112) as prerequisites for Barbalat's Lemma, zero-convergence of $s(t)$ is held true regarding [26];

$$\left. \begin{array}{l} \hat{s}(t) \in L_1 \\ \hat{s}(t) \in L_\infty, \quad \dot{\hat{s}}(t) \in L_\infty \end{array} \right\} \longrightarrow \lim_{t \rightarrow \infty} \hat{s}(t) = 0. \quad (113)$$

4 Simulation results

Parameter values of Table 1 are used for performing a simulation study. The comprehensive control scheme of an ASMC with SMLO is illustrated on Fig. 3. Five main parts, including model reference, sliding surface generator, parameter update law, control law generator, and Luenberger observer, thoroughly constitute the simulation platform. The simulation block diagram of the suggested ASMC with SMLO (ASMC-SMLO) is delineated on Fig. 4. There is a table in association with the ASMC-SMLO simulation scheme below. It is intended to describe the equations inside the simulation blocks.

The fixed exogenous disturbance, designated with d_{Ed} , is about 850 μN and has been applied over the system in simulation. The uncertainty is considered to be 5%. In other terms, it is as much as 1.05 of the nominal value in the stiffness and damping factors;

$$k = 1.05k_n, \quad b = 1.05b_n, \quad (114)$$

where k_n and b_n are the nominal values of the stiffness and damping coefficients in the movable capacitive plate dynamics. In addition, inside block No. 1, some other external forces and disturbances are speculated to get applied over the system simulatively—including weight force as well as cross stiffness and damping forces. In fact, they are added impacts over the plate mass inertia. Angle θ is roughly supposed to be 15° . The cross talk coefficients are set with the following values:

Table 1 Simulation parameters considered in the plate dynamics

Parameter	Real value	Parameter	Real value
m	1.11×10^{-7} Kg	k	2338 N/m
b	0.01 Ns/m	d	2 μm

Fig. 3 The general block diagram for an ASMC-SMLO design

Fig. 4 Simulation scheme of the ASMC-SMLO design

$$d_{xy} = d_{xz} = 0.001 \text{ N s/m}, \quad d_{xy} = d_{xz} = 0.001 \text{ N s/m}. \quad (115)$$

The corresponding cross-talk position deviation and respective divergence velocities are presumed to be:

$$\dot{y} = \dot{z} = 0.1 \text{ } \mu\text{m/s}, \quad y = z = 0.1 \text{ } \mu\text{m}. \quad (116)$$

Simulation parameters for the controller are set as following: $A = [0.000001 \quad 0.000001]$, $\eta = 773 \times 10^{-6}$, $\eta_{ml} = 0.001$, $\eta_{ul} = 0.001$, $I_1 = 1000I_2$ and $I_2 = 1000$. Reference model parameters are considered as $a_r = 10000$, $b_r = 25000000$ and $c_r = 25000000$ with zero initial conditions. The poles of the Luenberger observer are considered as $p_{1,2} = -800 \pm i$. If $L = [l_{11} \quad l_{21}]^T$ and $C = [0 \quad 1]$, the matrix of $A - LC$ will be:

$$A - LC = \begin{bmatrix} 0 & 1 - l_{11} \\ -\frac{k_n}{m} & -\frac{b_n}{m} - l_{21} \end{bmatrix}. \quad (117)$$

Setting determinant of the matrix of $[pl_2 - A + LC]$ to zero, leads to the following equation:

$$-\frac{k_n}{m}l_{11} + pl_{21} = -p^2 - \frac{b_n}{m}p - \frac{k_n}{m}. \quad (118)$$

Note that $\det(pl_2 - A + LC)$ should be Hurwitz in terms of Laplace variable of p . Adopting $p = -800 \pm i$ results in a linear equation set. Solving this set yields $l_{11} = 1.0000$ and $l_{21} = -8.8490 \times 10^4$. Supposing $Q = 10I_2$ and solving Eq. (38) brings about the following values for the components of matrix P ;

$$P = \begin{bmatrix} 2.1663 \times 10^{12} & 2.3738 \times 10^{-10} \\ 2.3738 \times 10^{-10} & 0.0031 \end{bmatrix}. \quad (119)$$

With regard to Eq. (39), $W = G = 2.8153 \times 10^4$.

The results of the designed ASMC-SMLO simulation are given on Fig. 5. It is so convincing that one may accept the new control system performance as a terrific outcome because the goal of the desired trajectory tracking is achieved in a precise manner. As it is observed on Fig. 5a, the plate displacement is exactly $0.667 \text{ } \mu\text{m}$ far from its initial distance. The gold, green and orange

Fig. 5 Results of the ASMC-SMLO design simulation-tracking performance; **a** Convergence of $x(t)$ towards $x_r(t)$. **b** Control effort signal as well as the sliding and tracking errors which are dramatically close to zero

curves, on Fig. 5a, are respectively the real, desired and estimated trajectories of the movable plate displacement designated with $x(t)$, $x_r(t)$ and $\hat{x}(t)$. The first and second plots on Fig. 5b demonstrate zero-convergence of tracking errors, representative of a perfect tracking performance in our design. The blue and red curves shown in another plot, as the third diagram on Fig. 5b, are the control effort signal and disturbance applied over the system. The bounded-ness of entries of the matrix of $\hat{\Phi}_1$ and scalar of $\hat{\Phi}_2$, which promote mathematically proven system stability, is displayed on Fig. 6.

Adopting $\tanh(1000(WC\hat{X} - WY))$ and $\tanh(1000\hat{s})$ instead of $\text{sgn}(WC\hat{X}) = \text{sgn}(1000(WC\hat{X} - WY))$ and $\text{sgn}(\frac{\hat{s}}{\|\hat{s}\|})$ in the Leunberger observer and control law

equations respectively, reduces the chattering effect. The simulation results are depicted on Fig 7. For the smoothed simulation, most of the parameters are as the same as the parameters of the first simulation except for $p_{1,2} = -8000 \pm i$, and $L = [0.6962 \quad 6.9910 \times 10^4]^T$. Some control law parameters are also different: $W = G = 281.5315$, and $\eta_{ml} = \eta_{ul} = 10^{-6}$.

5 Conclusion

The dominant component in an electronic AC VRS is the MEMS tunable capacitor which one of its capacitive plates is movable and another is stationary. The movable plate

Fig. 6 Estimation of the values of the defined parameters in the ASMC-SMLO system

Fig. 7 Results of ASMC design simulation-tracking performance; **a** Convergence of $x(t)$ towards $x_r(t)$. **b** Control effort signal as well as the sliding and tracking errors which are dramatically close to zero. The simulation is done using smooth transient functions to lessen chattering effect

should be guided and moved towards the pull-in region in order to access the desired output. The potential matched and unmatched disturbance and uncertainties, seen in the plate dynamics, source serious problems while trying to accomplish movable plate perfect displacement tracking performance, and settle it at the pull-in point. It is more challenging when one intends to do it by employing just a conventional Model Reference Adaptive Control (MRAC) or an uncomplicated pole placement controller. Even though the manufacturers endeavour to address problematic issues in association with fabrication procedures to lessen the uncertainty effect sources, uncertain damping and stiffness coefficients remain in practice. Moreover, the matched and unmatched disturbances such as electrostatic forces, cross stiffness and damping forces, external inertial and weight force may all wreck the desired dynamics while bringing the device under load at actual implementation. The recommended designed PI ASMC-SMLO is efficient in impeccable tracking of desired position trajectory of the movable plate, being prone to separate lumped nonlinearities, matched and unmatched parametric uncertainties as well as matched and unmatched exogenous disturbance. Thenceforth, the AC VRS engenders the desired output voltage. The major contribution in this work is embedding a sliding mode Luenberger observer in the system for estimation of plate position instead of its direct sensing and measurement. By and large, two parameter update laws in coordination with a state estimator as well as a suitable sliding surface are all recruited with the appropriate control law so that the overall system gets stable. The stability consequences are firstly bounded-ness of parameter as well as state estimation errors, and secondly zero convergence of the sliding surface error.

Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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