



Comprehensive analysis and synthesis of time delay compensated deadbeat control strategy

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Abstract

This research comprehensively investigates the effect of uncertain time delay on performance of the deadbeat control. A solution based on a non-integer order Smith predictor has been proposed and analyzed to compensate uncertain time delays occurring in a stable closed loop system. Deadbeat control has been widely and conveniently used in the control of linear and non-linear systems. However, this control technique is severely sensitive to unmodeled dynamics and parameter uncertainties. Thus the performance of system may deviate significantly and hence lead to instability in practical cases. Time delay in a closed loop control system is a common source of unmodeled dynamics, which severely deteriorates control accuracy and results in reduced stability margin of the deadbeat control system. Physical system considered in this research work is a single-phase inverter. The simulation results indicate that the Smith predictor based deadbeat control scheme offers accurate time delay compensations and thus enhances the robustness and stability margin. Application example comprehensively illustrates effectiveness of the proposed control technique.

Keywords Deadbeat control · Smith predictor · Uncertain time delay · Single-phase converter

1 Introduction

Good dynamic properties of deadbeat control, like low steady-state error, quick rise and settling time, make it a convenient and simple choice in various process controls [1–6]. Deadbeat control technique in comparison to other modern control techniques (*e.g.* resonant control, repetitive control) is simple, quick and achieves the steady state in minimum number of time steps. However, performance of deadbeat control is highly sensitive to parameter uncertainties and unmodelled dynamics of the system [7–10]. Unmodelled dynamics may cause unexpected delay in the systems. Uncertain time delays occur in control systems due to many reasons including processing delay of measuring and control circuits, unmodelled dynamics

and non-linearities. Uncertain time delay in deadbeat controlled systems is a critical issue resulting in degraded performance [9, 11].

The issue of time delay compensation in deadbeat controlled converters has been considered but not thoroughly addressed. One way of compensating uncertain time delay is to include repetitive controller along with the deadbeat and then employ phase lead compensation [12]. However, phase lead compensation cannot be directly used with the deadbeat control alone due to the non-causal delay item z^n where n is the number of phase lead steps. Another simple compensation method is two step forward prediction approach proposed in the frame of model predictive control [13, 14]. This approach only compensates for the computational delay of the controller whereas the uncertain delay

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due to other factors cannot be effectively compensated. Therefore, there is a need for a unified approach to effectively compensate overall uncertain delays occurring in the control system.

The Smith Predictor is a model based controller design technique, where an estimate of the plant/process along with uncertain delay is included in the controller and delay is effectively compensated [11, 15–20]. The conventional digital Smith predictor is an integer order controller employing a time delay element z^{-N} where $N = T_d/T_s$ with T_d being the uncertain time delay of the plant/process and T_s being the sampling frequency of the digital Smith predictor. T_s is usually fixed and it is impractical to vary the sampling frequency of the controller as it requires the redesigning and tuning of the controller parameters. Therefore, variable T_d may result in non-integer order N of the Smith predictor. The conventional Smith predictor can only realize integer delays so in case of non-integer N order is rounded off to the nearest integer value which results in approximate delay compensation and hence the performance degrades. In this research work, a fractional-order Smith Predictor delay compensation approach has been proposed and used for a deadbeat controlled single phase inverter. Fractional-order Smith predictor proposed and implemented in this research paper is able to achieve better control accuracy and robustness performance, which effectively suggests its widespread applications in practical systems. This control technique overcomes the uncertain delays due to controller, plant, measurement and computational time occurring a feedback control systems. Detailed analysis and synthesis of the proposed control scheme have also been carried out and provided. Various time delay scenarios considered, demonstrate the feasibility and effectiveness of the proposed scheme.

2 Smith predictor for an arbitrary time delay

2.1 Conventional Smith Predictor

Smith predictor has proven to be an effective delay compensation solution. A conventional Smith predictor is shown in Fig. 1, where $G_p(z)$ is the plant, z^{-N} is the delay, $G_p\hat{(z)}$ is the estimated plant model, $G_c(z)$ represents the deadbeat controller, $R(z)$ is input and $C(z)$ is output of the system.

Overall transfer function of the conventional Smith predictor shown in Fig. 1 is as follow.

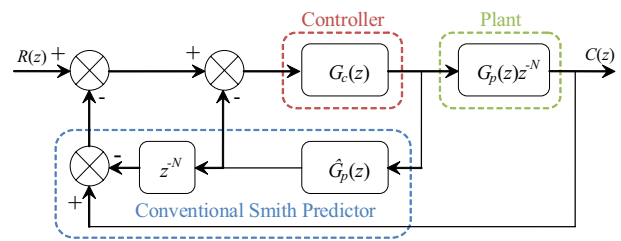


Fig. 1 Conventional Smith predictor based deadbeat control scheme

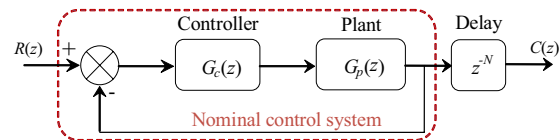


Fig. 2 A simplified block diagram of the control system shown in Fig 1

$$\frac{C(z)}{R(z)} = \frac{G_p(z)G_c(z)z^{-N}}{1 + G_x(z)G_c(z) + G_p(z)G_c(z)z^{-N}} \tag{1}$$

where,

$$G_x(z) = G_p\hat{(z)}(1 - z^{-N}) \tag{2}$$

If the predicted/estimated model of the plant $G_p\hat{(z)}$ is very accurate then $G_p\hat{(z)} = G_p(z)$ otherwise $G_p\hat{(z)} \approx G_p(z)$ then (1) becomes,

$$\frac{C(z)}{R(z)} \approx \frac{G_p(z)G_c(z)z^{-N}}{1 + G_p(z)G_c(z)} \tag{3}$$

Equation (3) represents that by the application of a Smith predictor, a plant $G_p(z)$ subjected to an uncertain time delay z^{-N} in a closed loop, and controlled by a controller $G_c(z)$ becomes equivalent to a nominal closed loop system $\frac{G_p(z)G_c(z)}{1+G_p(z)G_c(z)}$ separated from the same time delay as shown in Fig. 2. By applying a Smith predictor, stability of the overall system is guaranteed if and only if, poles of the nominal closed loop system followed by time delay element exists inside a unit circle. Poles of the pure delay element always exist at the center of the unit circle thus the stability criteria is based on poles of the nominal closed loop transfer function only.

In digital implementations, the time delay element z^{-N} can only be realized with integer value N . However, in practical implementation $N = T_d/T_s$, where T_d is the uncertain time delay of the system and T_s is fixed sampling period, may result in non-integer value. Therefore, implementation of integer period, N , based Smith predictor in some applications restricts performance of the control system. In order to deal with this issue a Fractional Order Smith Predictor has been proposed, implemented and analyzed.

2.2 Fractional order Smith predictor

In the events of mismatch between T_d and T_s , their ratio T_d/T_s becomes non-integer which leads to sever frequency robustness issues in power converter applications. In such cases, there is a need to implement Smith predictor based controller with non-integer order N . otherwise, performance of overall system degrades.

When the ratio of T_d and T_s is non-integer, assuming that the non-integer ratio N can be expressed as $N = N_i + F$ where $N_i = \lfloor N \rfloor$ and $F = N - N_i$, the required delay can be realized as $z^{-N} = z^{-N_i - F}$. In other words, N is divided into two components; an integer part N_i and a fractional part F . The delay based on the fractional part z^{-F} cannot be realized directly in digital implementations. Therefore, the fractional delay z^{-F} can be approximated by employing a Lagrange interpolation based fractional delay filter as follow [21]:

$$z^{-F} \approx \sum_{i=1}^n a_i z^{-i} \tag{4}$$

where a_i is the coefficients of each delay terms and approximation in the above equation can be converted to equality if n is equal to infinity. Practically $n = \inf$ is not possible therefore n is limited to some high value. However, value of n directly effects computational cost and complexity. The coefficients a_i of delay terms can be calculated as:

$$a_i = \prod_{j=1, j \neq i}^n \frac{F-j}{i-j}, \quad i, j = 1, 2, 3 \dots n \tag{5}$$

To demonstrate the accuracy of approximation of fractional delay z^{-F} using Lagrange interpolations, the magnitude responses of the proposed scheme are shown in Fig. 3. It is noticed that increasing n results in higher bandwidth at the cost of computational burden. Even a first order filter achieves a good approximation within 50% of the Nyquist frequency. That is to say that the designed filter accurate;y compensates any arbitrary delay within this bandwidth. In practical applications, in order to keep a

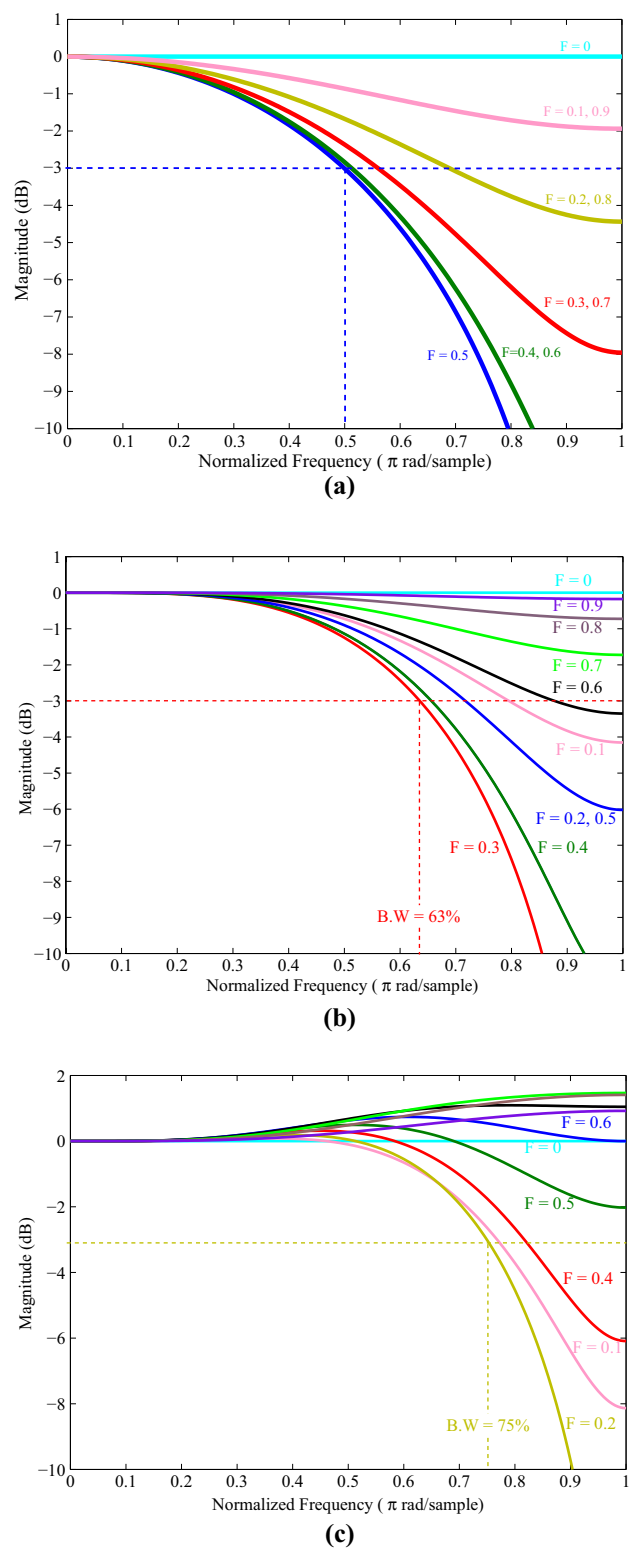


Fig. 3 Magnitude responses of the fractional delay filters **a** $n = 1$, **b** $n = 2$, **c** $n = 3$

reasonable balance between simplicity and approximation accuracy, the order n is usually chosen as 1 or 2.

In the absence of time delay $N = 0, Ni = 0, F = 0$ the $G_x(z) = G_p(z)$ and this (1) would reduce to the following:

$$\frac{C(z)}{R(z)} \approx \frac{G_c(z)G_p(z)}{1 + G_c(z)G_p(z)} \tag{6}$$

Above equation implies that the proposed universal fractional-order Smith Predictor works as a unity subsystem and performance of the controller $G_c(z)$ would be no less than the conventional deadbeat control scheme.

3 Fractional order Smith predictor based single-phase inverter

3.1 Modeling of single-phase PWM inverter

The dynamics of a single-phase PWM inverter connected to a linear resistive load as shown in Fig. 4 can be represented as [22]:

$$\begin{bmatrix} \dot{v}_c \\ \dot{i}_c \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{L_n C_n} & -\frac{1}{R_n C_n} \end{bmatrix} \begin{bmatrix} v_c \\ i_c \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_n C_n} \end{bmatrix} v_{ab} \tag{7}$$

where, v_c is output voltage, v_{ab} is PWM modulated voltage available at the input of low pass LC filter; L_n, C_n and R_n are nominal values of passive components. The system given by (7) is a linear system of the form $\dot{x} = Ax + Bu$. Its sampled data form can be achieved by applying a PWM pulse to the converter and thus $x((k+1)T) \approx e^{AT}x(kT) + e^{AT/2}BE_nAT$ which after few manipulations becomes [21].

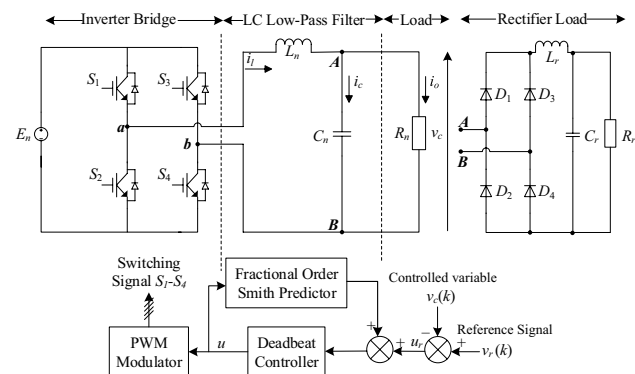


Fig. 4 Delay compensated deadbeat control of single-phase PWM inverter

$$\begin{bmatrix} v_c(k+1) \\ \dot{v}_c(k+1) \end{bmatrix} = \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{bmatrix} \begin{bmatrix} v_c(k) \\ \dot{v}_c(k) \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \Delta T(k) \tag{8}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v_c(k) \\ \dot{v}_c(k) \end{bmatrix}$$

where

$$\begin{aligned} g_1 &= T^2/2L_n C_n \\ g_2 &= T/L_n C_n - T^2/2L_n C_n^2 R_n \\ \varphi_{11} &= 1 - T^2/2C_n L_n \\ \varphi_{12} &= T/3C_n - T^2/6R_n C_n^2 \\ \varphi_{21} &= -T/L_n C_n + T^2/2L_n C_n^2 R_n \\ \varphi_{22} &= 1 - T/R_n C_n - T^2/2L_n C_n + T^2/2R_n^2 C_n^2 \end{aligned}$$

$v_c(k), \dot{v}_c(k)$ and $\Delta T(k)$ represent values of variables at sampling instant $t = kT$. The main control objective is to enable the output voltage (v_c) to precisely and accurately track the ac reference signal (v_r) in the presence of uncertain time delays.

3.2 Controller design

Controller for the single-phase PWM inverter shown in Fig. 4 is composed of two components: a plug-in fractional order Smith predictor and a conventional deadbeat controller.

3.2.1 Conventional deadbeat controller

The conventional deadbeat controller for a plant described by (8) can be achieved by calculating its Auto Regressive Moving Average (ARMA) equation and then replacing $u(k+1)$ by $v_r(k)$ as the deadbeat control achieves tracking of the signal in next time step, that is to say $H(z) = z^{-1}$. Thus the control signal $u(k)$ is:

$$u(k) = \frac{1}{m_1} (v_r(k) + p_1 y(k) - m_2 u(k-1) + p_2 y(k-1)) \tag{9}$$

where, $u(k) = \pm \Delta T(k)$, $m_1 = g_1$, $m_2 = \varphi_{12}g_2 - \varphi_{22}g_1$, $p_1 = -\varphi_{22} - \varphi_{11}$, $p_2 = \varphi_{22}\varphi_{11} - \varphi_{21}\varphi_{12}$, and $v_r(k) = y(k+1)$.

3.2.2 Fractional Order Smith Predictor

Practically, the performance of conventional deadbeat controller undergoes severe fluctuations due to unmodeled dynamics and parameter uncertainties including uncertain time delay. Therefore, a fractional order Smith predictor can be plugged into the deadbeat controlled

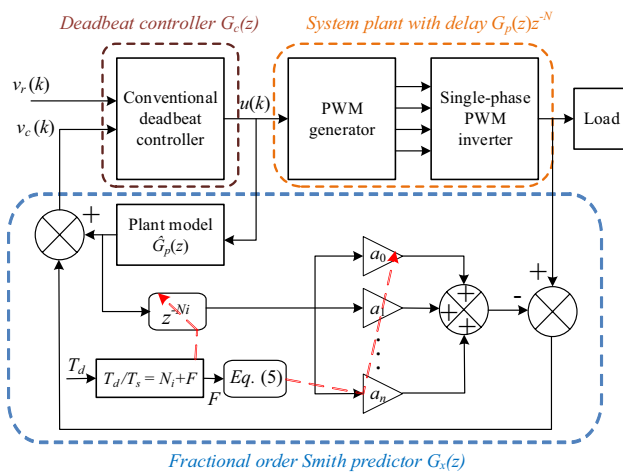


Fig. 5 Fractional-order Smith predictor based deadbeat control scheme for single-phase PWM inverter

inverter to overcome performance degradation issues due to uncertain time delay, as shown in Fig. 5.

3.3 Simulation Results and Discussion

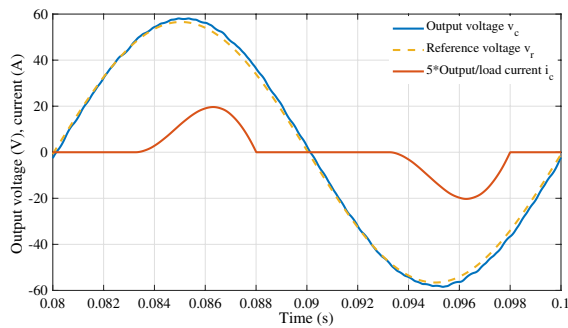
Performance of the proposed control scheme is tested and evaluated on a single-phase PWM inverter system. Nominal values of the converter and control parameters are given in Table 1. Figure 6 shows response of only deadbeat controlled single-phase inverter with rectifier load under three different time delays $T_d = 0, 1e^{-4}, 2e^{-4}$ in (a), (b) and (d) respectively. It is clear that deadbeat control can only track the reference signal in the absence of time delay. Thus, it can be concluded that the performance of the deadbeat control degrades drastically in the presence of uncertain time delay. Figure 6c, d represent very high tracking error in the presence of time delay. Long time delay leads to higher error and thus the system may

Table 1 Inverter system parameters

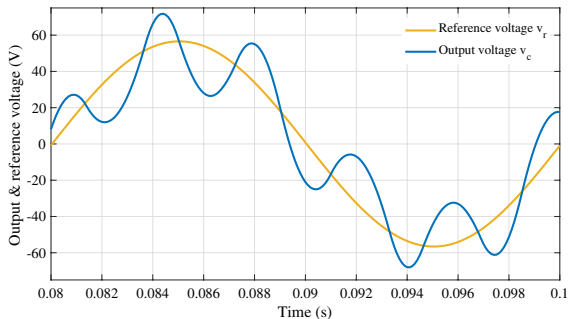
Parameters	Symbol	Value
Input dc voltage	E_n	120 V
Load resistance	R_r	30 Ω
Load inductance	L_r	5 mH
Load capacitance	C_r	500 μ F
Output voltage (rms)	v_c	40 V
Sampling frequency	f_s	10 kHz
Sampling time	T_s	$1e^{-4}$ s
Filter inductance (Low-pass)	L_n	5 mH
Filter capacitance (Low-pass)	C_n	60 μ F
Fractional delay filter order	n	2
Nominal fundamental frequency	f	50 Hz

become unstable in the absence of time delay compensation techniques.

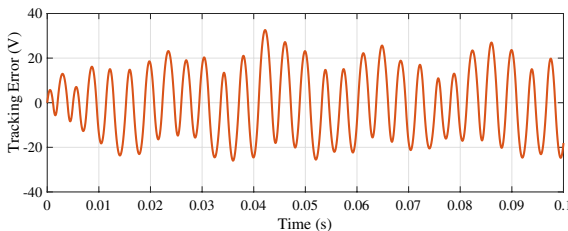
Figure 7 shows the response of fractional order Smith predictor based deadbeat controlled inverter for two different non-integer time delays ($T_d = 1.2e^{-4}, 2.3e^{-4}$). A fractional order Smith predictor having $n = 2$ has been employed. It is clear from Fig. 7a, b that the output of inverter tracks the reference voltage with acceptable/low steady-state error and Total Harmonic Distortion (THD) much less than 1%. In order to further emphasize the significance of fractional order Smith predictor based deadbeat control it has been compared with the conventional Smith predictor based control technique. Figure 8 shows the response of conventional Smith predictor based compensation technique when delay time is $T_d = 3.5e^{-4}$. Since, the conventional Smith predictor can only compensate integer delay therefore a delay of $3.5e^{-4}$ is rounded off to the nearest integers 3, 4 and an increase in steady-state error and THD has been observed in both cases. THD is greater than 1% because $N = 3, 4$ cannot compensate the



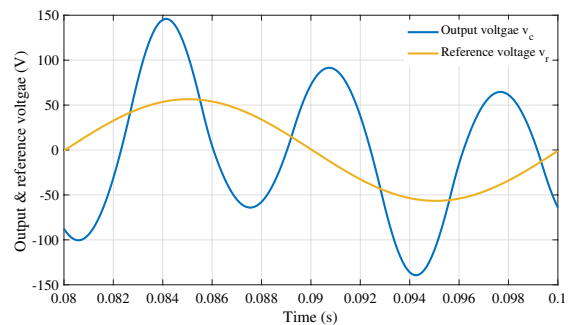
(a) Steady-state response of deadbeat controlled inverter.



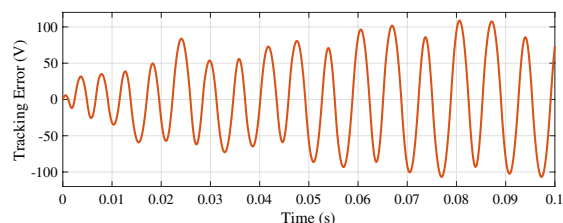
(b) Deadbeat controlled inverter with time delay $T_d = 1e^{-4}$.



(c) Tracking error with time delay $T_d = 1e^{-4}$.

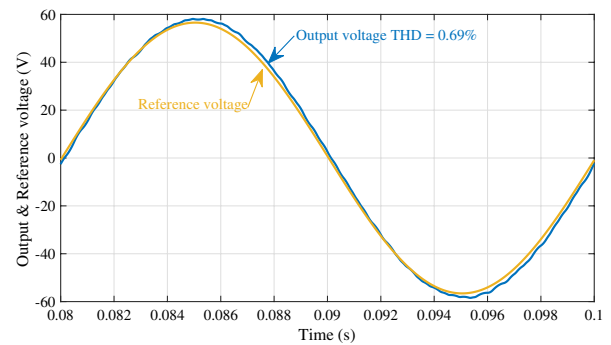


(d) Deadbeat controlled inverter with time delay $T_d = 2e^{-4}$.

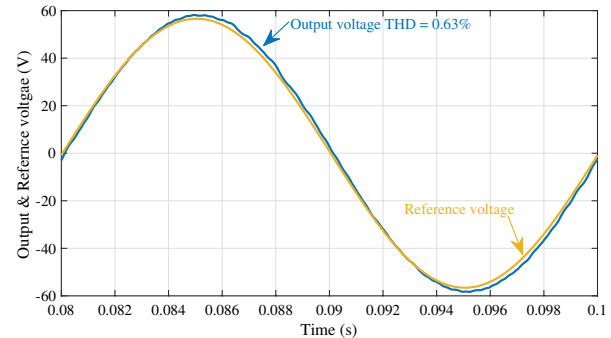


(e) Tracking error with time delay $T_d = 2e^{-4}$.

Fig. 6 Steady-state response of deadbeat controlled inverter with rectifier load under various time delays



(a) Steady-state response when time delay $T_d = 1.2e^{-4}$.



(b) Steady-state response when time delay $T_d = 2.3e^{-4}$.

Fig. 7 Steady-state response of deadbeat control along with fractional order Smith predictor under various time delays

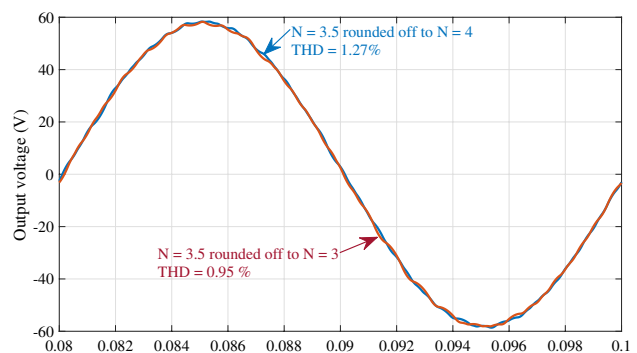


Fig. 8 Steady-state response of deadbeat control along with conventional Smith predictor when time delay $T_d = 3.5e^{-4}$

required delay accurately hence the need of a fractional delay compensation technique is clear.

Table 2 compares the performance of three different control techniques. Techniques compared are deadbeat control, conventional Smith predictor based deadbeat control (Deadbeat+SP) and fractional order Smith predictor based deadbeat control (Deadbeat+FOSP). Conventional deadbeat control can only be implemented with integer delays, therefore, performance isn't evaluated where fractional delay occurs. For conventional Smith predictor based deadbeat control the delays are approximated to

Table 2 Comparison of control techniques

Delay	Deadbeat		Deadbeat + SP		Deadbeat + FOSP	
	Error (V)	THD%	Error (V)	THD%	Error (V)	THD%
1	30	44	3.2	0.69	3.2	0.69
1.2	–	–	4.8	1.8	3.6	0.83
2	110	278.6	3	0.63	3	0.63
2.3	–	–	4.9	1.5	4.1	0.69
3	191.7	381.7	3.6	0.65	3.6	0.65
3.5	–	–	5.8	1.27	3.9	0.75
4	265	370.7	4	0.7	4	0.7
4.6	–	–	5.1	1.02	3.6	0.89
5	353.9	495.4	3.5	0.64	3.5	0.64

the nearest integer value and then performance is evaluated and it can be seen that where the required delay is non-integer approximation results in increased steady-state error and THD. For fractional order Smith predictor based deadbeat control the THD is always lower than 1% for the evaluated delay cases. When the required delay is integer Deadbeat+FOSP becomes exactly same as that of Deadbeat+SP and hence the performance is similar.

4 Conclusion

This paper develops and applies a fractional-order Smith Predictor based delay compensation technique, for output ac voltage control of a single-phase PWM inverter when arbitrary time delays occur in the control system. Using this technique any arbitrary time delay occurring in the deadbeat controlled system can be accurately approximated and compensated. A deadbeat control scheme which is capable of compensating unknown time delays accurately and effectively promotes its widespread applications in practical systems. Simulated inverter system demonstrates the feasibility of proposed technique. Results indicate high tracking accuracy, fast transient response, and good adaptation to uncertain delay time. The proposed simple and effective fractional-order time delay compensation solution can be applied with minor changes to other applications including unmanned aerial vehicles and predictive control.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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