Effect of load distribution on longitudinal and lateral forces acting on each wheel of a compact electric vehicle

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Abstract

Autonomous vehicle is gaining popularity in the market worldwide. Most autonomous vehicle are based on electric vehicles since they are easy to control. The torque control of electric vehicles is precise and easy since electric motor torque can be manipulated by controlling the motor current. Furthermore, the load of the vehicle affects the motor torque of an electric vehicle. A higher vehicle load requires high motor torque to propel the vehicle. Often in autonomous vehicle, the vehicle parameters and stability measure are set within a limit based on standard vehicle settings. However, a vehicle loaded with extra mass on either side can offset these parameters affecting the efficiency of the controller. Thus, in this paper the effect of load on vehicle longitudinal and lateral forces is identified. A simulation model of a two rear in-wheel motored electric vehicle is developed. The model is used to analyze the effect of load on vehicle longitudinal and lateral forces. Based on the result, increasing the load on the side of direction of lateral motion increases the lateral force generated. The high lateral force causes the tires to approach the tire friction circle limit. This can affect the automated vehicle performance since the tires are in unstable region.

Keywords Tire forces · Vehicle stability · Nonlinear vehicle model

List of symbols

m	Total mass of the vehicle	l_f	Length of the front wheels from the center of gravity
h_{cg}	Height of the center of gravity of the vehicle from the ground	load _{ij}	Additional load at each wheel with i = front/rear and j = right/left
$\ddot{x}$	Longitudinal acceleration	l_r	Length of rear wheel from the center of the gravity
$C_{\alpha f}$	Front tire cornering stiffness	l_w	Track width
$C_{\alpha r}$	Rear tire cornering stiffness	m_{sij}	Static mass at each wheel with i = front/rear and j = left/right
C_{σ}	Longitudinal tire stiffness	$\ddot{y}$	Lateral acceleration
F_{xij}	Longitudinal force with i = front/rear and j = left/right	$\ddot{\phi}$	Yaw acceleration
F_{yij}	Lateral force with i = front/rear and j = left/right	K	Road constant
I_z	Yaw moment of inertia	g	Gravitational acceleration
N_{zij}	Vertical force acting on each tire with i = front/rear and j = right/left	l	Sum of l_f and l_r
		α	Tire slip angle

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δ	Wheel steering angle
μ	Tire–road friction coefficient
ρ	Tire slip ratio

1 Introduction

Development of autonomous vehicle depends on the information on the vehicle parameters and forces acting on it. A working autonomous vehicle system requires information on various vehicle parameters to drive the vehicle safely. The expansion of the autonomous vehicle industry is influenced by the development of electric vehicles due to its capability to accommodate precise control strategy [1, 2]. An autonomous vehicle control is designed with active elements to ensure the stability of the vehicle [3]. Few of such active safety elements are anti-lock braking system (ABS), adaptive cruise control (ACC), dynamic stability program (DSC) and electronic stability program (ESP) [4–6].

Most of the active safety controls are developed by using vehicle dynamic model which requires the vehicle lateral and longitudinal forces as an input in determining the vehicle states such as longitudinal acceleration, lateral acceleration, yaw rate and body slip angle. Most of the aforementioned vehicle states can be measured using instrumental sensors such as GPS and accelerometer [3]. However, such sensors are also unreliable due to their limitations such as loss of signals, effect of weather and high cost. Thus, Youssfi et al., 2018 suggested estimation of vehicle parameters using vehicle dynamic models. Moreover, estimation of vehicle parameters is widely used by many researchers in their quest to develop a robust controller [4] or estimating vehicle parameters and stability [7]. A vehicle dynamic model of the respective vehicle is required to implement the vehicle parameter estimation model.

An insight on the vehicle dynamic models highlights the importance of lateral and longitudinal forces acting on the wheels. These two forces are prominent in determining the behavior of the car based on the vehicle dynamic models such as linear and nonlinear models. The linear model highlights the longitudinal and lateral vehicle motion independently. On the other hand, the nonlinear model shows the dependence of the longitudinal and lateral motion. Furthermore, the development of vehicle dynamic model requires the inclusion of the tire model since the tires play an important role in determining the vehicle characteristic. However, there are few different tire

models available to describe the tire characteristic. The tire model is used to determine the lateral and longitudinal forces acting on each wheel. Tire models such as magic formula [8], LuGre [9] and Dugoff's tire model [2] are few of the commonly used models. Furthermore, most of the tire models are dependent on the vehicle vertical force acting (N_z) on each tire. The vertical force is often calculated using the load transfer equation [10]. The load transfer equation requires the mass of the vehicle to estimate the vertical force. However, increment of mass on either side of the vehicle can affect the vehicle stability. Often, only vehicle mass is considered in the study by researchers; however, this is not true since passengers or goods are loaded into the vehicle. The distribution of the additional load on the vehicle can affect the forces acting on each wheel, which in turn affect the vehicle parameters and stability.

This paper focuses on identifying the effect of load distribution for two in-wheel motored electric vehicles toward vehicle lateral and longitudinal forces.

2 Model development

The two in-wheel motored electric vehicle is modeled by integrating the DC motor model, vehicle dynamic nonlinear model, load transfer model and the Dugoff's tire model. The vehicle model is designed based on rear wheel motored electric vehicle. In this research, Dugoff's tire model is implemented since it is a pure empirical model which does not require parameters determined experimentally.

2.1 Vehicle dynamic model

The vehicle motion is determined through the vehicle dynamic model which comprises of longitudinal and lateral motion. The vehicle dynamic motion equation is determined based on Fig. 1.

The vehicle longitudinal motion is represented by Eq. (1), whereas the lateral motion is represented by Eq. (2). On the other hand, Eq. (3) is used to calculate the vehicle yaw rate.

$$m\ddot{x} = (F_{xfl} + F_{xfr}) \cos(\delta) + F_{xrl} + F_{xrr} - (F_{yfl} + F_{yfr}) \sin(\delta) + m\dot{\varphi}\dot{y} \quad (1)$$

$$m\ddot{y} = F_{yrl} + F_{yrr} + (F_{xfl} + F_{xfr}) \sin(\delta) + (F_{yfl} + F_{yfr}) \cos(\delta) - m\dot{\varphi}\dot{x} \quad (2)$$

Fig. 1 Vehicle dynamic model representation

$$I_z \ddot{\phi} = l_f (F_{xfl} + F_{xfr}) \sin(\delta) + l_f (F_{yfl} + F_{yfr}) \cos(\delta) - l_r (F_{yrl} + F_{yrr}) + \frac{l_w}{2} (F_{xfr} - F_{xfl}) \cos(\delta) + \frac{l_w}{2} (F_{xrr} - F_{xrl}) + \frac{l_w}{2} (F_{yfl} - F_{yfr}) \sin(\delta) \quad (3)$$

Based on Eqs. (1) to (3), it is evident that the lateral and longitudinal forces play an important role in the vehicle motion estimation.

2.2 Dugoff's tire model

In order to determine the vehicle lateral and longitudinal forces, the Dugoff's tire model is adopted due to its simple nature and relationship between tire forces, tire-road friction and vertical force.

$$\tau = \frac{\mu N_{zij}(1 + \rho)}{2 \left[(C_\sigma \rho)^2 + (C_\alpha \tan(\alpha))^2 \right]^{1/2}} \quad (4)$$

$$f(\tau) = (2 - \tau)\tau \text{ if } \tau < 1 \quad (5)$$

$$f(\tau) = 1 \text{ if } \tau > 1 \quad (6)$$

The longitudinal and lateral forces are calculated using Eq. (7) and (8).

$$F_{xij} = C_\sigma \frac{\rho}{1 + \rho} f(\tau) \quad (7)$$

$$F_{yij} = C_\alpha \frac{\tan \alpha}{1 + \rho} f(\tau) \quad (8)$$

2.3 Load transfer model

Both longitudinal and lateral tire forces depend on the vertical force acting on each tire. The vehicle vertical force is determined from the load transfer model which is adapted from [11]. The load transfer model considers both longitudinal and lateral load transfer which requires longitudinal and lateral acceleration of the vehicle, as shown in Eqs. (9) to (12). The load transfer model is divided into three parts: static force, longitudinal load transfer and lateral load transfer. The additional load introduced on the vehicle is added to the static force of the vehicle. In this study, the initial mass on each tire is considered equal. However, a set of loads is added on each tire to simulate an instance of unbalanced overloading.

$$N_{zfr} = (m_{sij} + \text{load}_{ij}) g \frac{l_r}{l} + \frac{m a_y h_{cg}}{l_w} - m a_x \frac{h_{cg}}{l} \quad (9)$$

$$N_{zfl} = (m_{sij} + \text{load}_{ij}) g \frac{l_r}{l} - \frac{m a_y h_{cg}}{l_w} - m a_x \frac{h_{cg}}{l} \quad (10)$$

$$N_{zrr} = (m_{sij} + \text{load}_{ij}) g \frac{l_f}{l} + \frac{m a_y h_{cg}}{l_w} + m a_x \frac{h_{cg}}{l} \quad (11)$$

Fig. 2 Simulation model representation

Fig. 3 Wheel steering input

$$N_{zrl} = (m_{sij} + \text{load}_{ij})g \frac{l_f}{l} - \frac{ma_y h_{cg}}{l_w} + ma_x \frac{h_{cg}}{l} \quad (12)$$

The inertial acceleration of the vehicle along the y axis is given by a_y and is measured using Eq. (13). On the other hand, the inertial acceleration of the vehicle along the x -axis is given by a_x and is measured using Eq. (14).

$$a_y = \ddot{y} + V_x \dot{\phi} \quad (13)$$

$$a_x = \ddot{x} - V_y \dot{\phi} \quad (14)$$

Table 1 Percentage of load distribution on the right

Percent-age (%)	Total load (kg)	Front load (kg)	Rear load (kg)
5	12.97	6.48	6.48
10	25.93	12.97	12.97
15	38.90	19.45	19.45
20	51.86	25.93	25.93
25	64.83	32.41	32.41
30	77.80	38.89	38.89
35	90.76	45.38	45.38
40	103.72	51.86	51.86

2.4 Simulation setup

The simulation is designed using MATLAB Simulink. The simulation model representation is as shown in Fig. 2. The simulation model includes the DC motor, Dugoff's tire model, vehicle dynamic model and load transfer model. Furthermore, the model is tested at two different velocities of 10 km/h (low) and 25 km/h (high) since the vehicle used is a compact electric vehicle (EV) with max velocity of 30 km/h. The EV is given a steering input to turn around

a constant radius as shown in Fig. 3. Furthermore, Table 1 shows the value of additional load distributed at the right side of the vehicle. Only the effect of additional load on the right side is discussed to highlight the extent of variation in force that occurs during a right cornering motion. The additional load is measured as the percentage of the

Table 2 Vehicle parameter and physical measurement

Symbol	Value	Units
I_f	0.53	m
I_r	0.62	m
C_{af}	2000	N/rad
C_{ar}	2612.62	N/rad
C_σ	3000	N/m ³
k	1 = dry asphalt	–
g	9.81	m/s ²
m	322.1	kg
h_{cg}	0.105	m
I_w	1.43	m
I_z	1470	kgm ²

Table 3 Static mass of each tire

M_{fr}	M_{fl}	M_{rr}	M_{rl}
Mass with driver			
88	71	70.5	92.6

vehicle mass. In this simulation testing, the mass of the vehicle is measured from a compact electric vehicle.

The vehicle parameter and physical measurement are represented in Table 2. The vehicle physical parameters including I_f , I_r , h_{cg} and m are measured from a compact

electric vehicle. Other parameters are adopted from the following paper [12].

The static mass of the vehicle is measured by placing a weighing scale below each tire, and the corresponding mass is listed in Table 3. Furthermore, the static mass measured is inclusive of the mass of the driver.

The wheel steering input for the vehicle at 10 km/h and 20 km/h is given at two different times in order to facilitate the same turn radius as depicted in Fig. 4.

3 Results and discussion

The vehicle is simulated at two velocities: 10 and 25 km/h. For each vehicle velocity, the vehicle is added with load at the right side in an increment of 5%. The vehicle tire forces are observed and displayed in Figs. 5, 6, 7 and 8. The vehicle tire forces with respect to the planar motion are divided into longitudinal and lateral forces. For the first part, the vehicle longitudinal force is studied as shown in Figs. 5 and 6. Based on Fig. 5, the longitudinal force on each tire varies as the load is increased on the right side. As observed, the right tire experiences more longitudinal force compared to the left tire. Furthermore, the right tire experiences highest longitudinal force at 40% increased load compared to the no-load condition (0%).

Fig. 4 Direction of motion of the vehicle at 10 km/h and 25 km/h

Fig. 5 Longitudinal force at each tire for $V = 10$ km/h

Fig. 6 Longitudinal force at each tire for $V = 25$ km/h

Similarly, the vehicle traveling at 25 km/h experiences increased longitudinal force at each tire with increase in additional right load. However, at higher velocity (25 km/h), the longitudinal force generated at each tire is far greater than the vehicle traveling at 10 km/h. Based on Fig. 6, the longitudinal force at each tire increases with increasing load. Furthermore, at 40% increased load, the

tire force increases continuously. This condition occurs when the tire starts slipping.

The lateral force acting on each tire for vehicle traveling at 10 km/h and 25 km/h is depicted in Figs. 7 and 8. Based on Fig. 7, the lateral force at each tire increases with the additional load. The lateral force is generated when the vehicle is taking a right corner. Thus, it can be observed

Fig. 7 Lateral force at each tire for $V=10$ km/h

Fig. 8 Lateral force at each tire for $V=25$ km/h

that the tire force at the right is higher than that at the left. This is because the inner tire experiences more force during cornering. Furthermore, the right tires have the highest lateral force when an additional load of 40% is added. This shows that increasing the additional load at the tires can increase the tire's lateral force.

Similarly, Fig. 8 depicts the lateral force at each tire for a vehicle traveling at 25 km/h. The lateral force at the right tires increases with increasing load. The front right tire is observed to have a larger effect when an additional load of 40% is added. The lateral force at the front right tire is almost 500 N. Furthermore, comparing Figs. 7 and 8 shows that the lateral force experienced by the right tires at high

velocity is approximately 10 times higher than that at low velocity. This shows that at higher velocity, the tires experience higher lateral force during cornering. The lateral force is also affected by the additional load on the tires. Increasing more load on the tires increases the lateral force.

Based on the observation on Figs. 5, 6, 7 and 8, the force acting on each tire increases with the increase in load at respective tires. The correlation between vertical force and longitudinal and lateral force can be observed from Eqs. (4), (7) and (8). Increased load on each tire increases the vertical force acting on the tires toward the road. The increased vertical force causes the vehicle to generate higher longitudinal force to move the vehicle forward. Furthermore, a higher lateral force is generated to facilitate the required motion due to the high vertical force acting on the respective tires. Moreover, the longitudinal and lateral forces acting on the right tire increase drastically at high velocity (25 km/h). At the highest load (40%), both right tires experience continuously increasing longitudinal force compared to other load increment setting. This shows that there is a limit for the amount of load that can be sustained by a tire at high velocity before experiencing instability. This scenario is also observable for the lateral force at the front right tire for the vehicle traveling at 25 km/h.

4 Conclusion and recommendations

There is a significant effect of load distribution on the longitudinal and lateral forces acting on each tire. The additional load added on the side of the vehicle lateral motion induces increased longitudinal and lateral tire force. Based on the result, increasing load on the right side of the vehicle above a certain limit causes the vehicle to crash. This can be observed at the vehicle traveling at 25 km/h with a right load of 40%. The longitudinal force generated at the tires increases continuously, depicting the tire slip ratio exceeded one. The tire slip ratio for a stable driving condition is limited below one. Therefore, it can be concluded that increasing load on the same side of the vehicle lateral motion can cause the tire longitudinal and lateral forces to increase. Increasing the load above a limit causes the vehicle to experience instability.

For future work, the study can be enhanced by including the effect of load distribution at the opposing side of the lateral motion. For example, the load can be added to the left side of vehicle taking a right turn. Furthermore, the vehicle stability can be analyzed using the stability criteria such as rollover index, friction circle coefficient and lateral acceleration limit.

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Compliance with ethical standard

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Wu D, Li Y, Zhang J, Du C (2018) Torque distribution of a four in-wheel motors electric vehicle based on a PMSM system model. *Proc Inst Mech Eng Part D J Automob Eng* 232(13):1828–1845
2. Jia F, Liu Z, Zhou H, Chen W (2018) A novel design of traction control based on a piecewise-linear parameter-varying technique for electric vehicles with in-wheel motors. *IEEE Trans Veh Technol* 67(10):9324–9336
3. El Yousfi N, Oudghiri M, Aitouche A, El Bachtiri R (2018) Fuzzy sliding-mode observer for lateral dynamics of vehicles with consideration of roll motion. In: 2018 26th mediterranean conference on control and automation (MED) pp. 861–866
4. Zhang L, Ding H, Guo K, Zhang J, Pan W, Jiang Z (2019) Cooperative chassis control system of electric vehicles for agility and stability improvements. *IET Intell Transp Syst* 13(1):134–140
5. Wang Y, Wang Z, Zhang L, Liu M, Zhu J (2019) Lateral stability enhancement based on a novel sliding mode prediction control for a four-wheel-independently actuated electric vehicle. *IET Intell Transp Syst* 13(1):124–133
6. Wang J, Luo Z, Wang Y, Yang B, Assadian F (2018) Coordination control of differential drive assist steering and vehicle stability control for four-wheel-independent-drive EV. *IEEE Trans Veh Technol* 67(12):11453–11467
7. Zhao YQ, Li HQ, Lin F, Wang J, Ji XW (2017) Estimation of road friction coefficient in different road conditions based on vehicle braking dynamics. *Chin J Mech Eng* 30(4):982–990
8. Kunnappillil Madhusudhanan A, Corno M, Arat MA, Holweg E (2016) Load sensing bearing based road-tyre friction estimation considering combined tyre slip. *Mechatronics* 39:136–146
9. Hashemi E, Pirani M, Khajepour A, Kasaiezadeh A (2016) A comprehensive study on the stability analysis of vehicle dynamics with pure/combined-slip tyre models. *Veh Syst Dyn* 54(12):1736–1761
10. Mashadi B, Mostaghimi H (2017) Vehicle lift-off modelling and a new rollover detection criterion. *Veh Syst Dyn* 55(5):704–724
11. K. Baarath, M. A. Zakaria, N. A. Zainal (2018) An investigation on the effect of lateral motion on normal forces acting on each tires for nonholonomic vehicle, In: *Intelligent manufacturing & mechatronics*, Springer, Singapore, pp 611–621
12. Ishak MI, Ogino H, Yamamoto Y (2016) Numerical simulation analysis of an oversteer in-wheel small electric vehicle integrated with four-wheel drive and independent steering. *Int J Veh Technol* 2016:1–12

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