Wall reabsorption effects on heat and mass transfer of viscous fluid in a narrow leaky tube

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Abstract

The steady heat and mass transfer problem of flow of an incompressible Newtonian fluid is investigated in this study. The axisymmetric flow of a non-isothermal and highly viscous fluid is considered through a narrow leaky tube of uniform cross-section with linear reabsorption across the wall. The Navier–Stokes equations are solved for axially symmetric and slow flow in the tube. The effects of linear reabsorption parameter on the dimensionless radial and axial velocities, temperature, concentration, shear stress, leakage flux, fractional reabsorption, pressure drop across the tube as well as the Nusselt and Sherwood numbers have been investigated analytically. For 50% fractional reabsorption, inlet radial velocity is half of the reabsorption velocity along the tube. It has been shown that increase in the strength of linear reabsorption parameter considerably reduces the radial velocity, while, axial velocity shows opposite behaviour. At the exit region ($z = 0.9$), reverse flow phenomenon is observed for higher values of reabsorption parameter. The pressure, flow rate and wall shear stress decreases after entrance region ($z = 0.1$) of the tube. Nusselt number and Sherwood number show opposite behaviour inside the tube. At $z = 0.1$ the magnitude of Nusselt number is maximum, while the Sherwood number is minimum at the exit region with higher value of reabsorption parameter.

Keywords Leaky tube · Incompressible flow · Reabsorption · Exact solutions · Leakage flux · Fractional reabsorption · Heat and mass transfer

List of symbols

B_r	Brinkman number	Re	Reynolds number
C_p	Specific heat at constant pressure (J/kg k)	R	Radius of the tube (m)
C_1, C_2	Constants of integration	Q_0	Entrance flow rate (m ³ /s)
D	Coefficient of mass diffusivity (m ² /s)	T	Temperature (k)
Δp	Pressure drop	u	Radial velocity (m/s)
L	Length of the tube (m)	U_0	Normalized inlet fluid velocity (m/s)
k	Thermal conductivity coefficient (W/m k)	S_c	Schmidt number
K_T	Thermal–diffusion ratio	S_r	Soret number
p	Pressure (Pa)	N_u	Nusselt number
Pr	Prandtl number	S_h	Sherwood number
p_0	Inlet fluid pressure (Pa)	$Q(z)$	Normalized reduced flow rate
P_e	Peclet number	w	Axial velocity (m/s)
$q(z)$	Leakage flux (m ² /s)		
$\dot{q}$	Heat transfer rate		

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Greek symbols

α_L	Normalized reabsorption parameter
θ	Normalized temperature
ϕ	Normalized concentration
μ	Fluid viscosity (N s/m ²)
δ	Ratio of length to height
τ_w	Normalized wall shear stress (Pa)

1 Introduction

The flow of viscous fluids in a leaky wall tube has several applications in medical and engineering sciences, for example, separation processes in multi-component mixtures, inflow in the nephron tubules, the flow of impure blood through a dialyzer (artificial kidney), reverse osmosis desalination, ultrafiltration, membrane separation, diffusion and more recently cross flow microfiltration. The application of Poiseuille's law is not appropriate due to fluid leakage at the walls to study the heat and mass transfer in such situations. Berman [1] presented the hydrodynamical study of flow in a permeable tube. He obtained the expressions for velocity and pressure with uniform seepage across the walls. His work was extended by Sellars [2], and obtained series solution for large Reynolds number. Later on, approximate solution of flow of laminar fluid flow in permeable channel with uniform suction/injection was obtained by Yuan [3], who investigated the flow in case of moderate to high suction or injection velocities across the walls. Terrill [4] have studied the flow through permeable channel with different permeability for small Reynolds number to get the velocity components and pressure drop. Macey [5] studied the flow and pressure in a permeable channel with variable reabsorption across the walls. Also, Macey [6] obtained the expressions for flow and pressure in a porous tube with decaying flow rate. Later, Kozinski et al. [7] extended the work of Macey's and presented exact solution for the flow of linear viscous fluid through a porous walled duct. Marshall et al. [8] considered the creeping flow of a Newtonian fluid through a rigid permeable tube and obtained closed-form solutions for the pressure and velocity fields. They also obtained approximate solutions for the small seepage rates. Further, Radhakrishnamcharya et al. [9] also presented the solutions for creeping flow of Newtonian fluid in a permeable non uniform tube. Recently, Muthu et al. [10] studied the hydrodynamics of viscous fluid in a nonuniform porous channel using regular perturbation method. They found the velocity components at different locations inside the channel, also pressure drop along the channel/tube has been presented. Later, they also investigated the flow and pressure inside the porous tube with flow rate model [11]. Later, this work was extensively studied by Siddiqui et al. [12], they found the numeric values of reabsorption parameters for

80% fractional reabsorption and showed that the study has relevance with flow of glomerular filtrate with in the kidney. Also, this work was extended by Haroon et al. [13] for periodic wall reabsorption. They found the expressions for velocity components, flow rate and pressure drop inside the porous channel. Farooq et al. [14] also extended the work of Muthu et al. [10] to slip flow condition considering Newtonian fluid flow inside the wavy channel.

In preceding studies, authors considered the hydrodynamics of viscous fluid flow in porous channels/tubes with application to flow in the kidney. But in general, heat and mass transfer mechanism has also several applications in biosciences and in industries, for example, the air movement in the pores of fibrous material due to heat convection, conduction in tissues and radiation between environment and its surface, food processing mass and diffusion in reverse osmosis. Terrill et al. [15] presented the analytical solution for heat and mass transfer of the two dimensional laminar and non-isothermal flow of linearly viscous fluid in a porous channel. Walker et al. [16] found the exact solution of the diffusion equation, while numerical solutions of the energy equation were discussed for small injection rates. Raithby [17] used the similarity solutions to evaluate the pressure gradient, Nusselt number, and Sherwood number in the two dimensional flow in a permeable channel. Ross [18] developed a mathematical model of mass transfer in a long permeable tube and demonstrated that the exit concentration decreases markedly as the diameter, membrane permeability and tube length increase. Sreedevi et al. [19] studied the impacts of the governing parameters on the heat and mass transfer of nanofluid flow over stretching surface with chemical reaction, suction, slip effects and thermal radiation.

In the literature presented above, this study is new in different aspects. First, there is a lack of information regarding the heat and mass transfer through a permeable tube with reabsorption across the walls. Secondly, the model presented by researcher is improved to heat and mass transfer in a permeable tube with linear reabsorption across the walls. Also, the components of velocity, pressure drop, wall shear stress, temperature profile and mass concentration distribution as well as the heat transfer coefficient (Nusselt number) and the mass transfer coefficient (Sherwood number) at the porous wall of the tube with linear wall reabsorption are also presented.

2 Mathematical formulation of the problem

Consider the flow of a steady, non-isothermal, laminar and incompressible Newtonian fluid in a permeable tube of radius R and length L . A cylindrical coordinate system (r, θ, z) is chosen with the z -axis aligned with the midway

and r -axis being taken perpendicular to the walls of the tube, Fig. 1. At $z = 0$ fluid enters the tube having axial flow rate Q_0 , then some of the fluid at $r = R$ is filtered across the leaky walls of the tube and remaining Q_L leaves at $z = L$. The wall of the tube is maintained at temperature T_0 and the average temperature of the fluid is assumed to be T_a . The tube is assumed long ($R \ll L$) enough to neglect the end effects. The flow in this situation becomes two dimensional and the velocity profile is given by the following equation:

$$\mathbf{V} = [u(r, z), 0, w(r, z)], \quad (1)$$

where u and w are the velocities in the radial and axial directions, respectively.

The equations governing the motion of an incompressible and non-isothermal Newtonian fluid through a tube are given by [18]:

$$\frac{1}{r} \frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0, \quad (2)$$

$$\rho \left[u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right] = -\frac{\partial p}{\partial r} + \mu \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right], \quad (3)$$

$$\rho \left[u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right] = -\frac{\partial p}{\partial z} + \mu \left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right], \quad (4)$$

$$\rho C_p \left[u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right] = k \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right] + \mu \left[2 \left\{ \left(\frac{\partial u}{\partial r} \right)^2 + \left(\frac{u}{r} \right)^2 + \left(\frac{\partial w}{\partial r} \right)^2 \right\} + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right)^2 \right], \quad (5)$$

$$\left[u \frac{\partial C}{\partial r} + w \frac{\partial C}{\partial z} \right] = D \left[\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} \right] + \frac{DK_T}{T_a} \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right], \quad (6)$$

where p is the hydrodynamic pressure, ρ is the density, k is thermal conductivity, C_p is the specific heat at constant pressure, D is the coefficient of mass diffusivity, K_T is the thermal-diffusion ratio, and T_a is the mean temperature and μ is the viscosity of the fluid. The boundary conditions are taken as

$$u(R, z) = V_0 - \alpha_L z, \quad (7)$$

$$w(R, z) = 0, \quad (8)$$

$$T(R, z) = T_0, \quad (9)$$

$$C(R, z) = C_0, \quad (10)$$

$$u(0, z) = 0, \quad (11)$$

$$\frac{\partial w(0, z)}{\partial r} = 0, \quad (12)$$

$$\frac{\partial T(0, z)}{\partial r} = 0, \quad (13)$$

$$\frac{\partial C(0, z)}{\partial r} = 0, \quad (14)$$

$$2\pi \int_0^R r w(r, 0) dr = Q_0, \quad (15)$$

$$p(r, 0) = p_0, \quad (16)$$

where V_0 is inlet upward velocity at $z = 0$ and α_L is linear reabsorption velocity parameter along the tube.

Using the dimensionless quantities

Fig. 1 Geometry of the problem

$$r^* = \frac{r}{R}, \quad z^* = \frac{z}{L}, \quad u^* = \frac{\pi R L u}{Q_0}, \quad w^* = \frac{\pi R^2 w}{Q_0}, \quad w(1, z) = 0, \quad (24)$$

$$p^* = \frac{\pi R^4 p}{\mu Q_0 L}, \quad \theta(1, z) = 1, \quad (25)$$

$$\theta = \frac{T - T_0}{T_0}, \quad P_e = \frac{\rho C_p Q_0}{\pi k R}, \quad B_r = \frac{\mu Q_0^2}{\pi^2 k T_0 R^4}, \quad \phi(1, z) = 1, \quad (26)$$

$$\delta = \frac{R}{L}, \quad u(0, z) = 0, \quad (27)$$

$$R_e = \frac{Q_0}{\pi \nu L}, \quad \phi = \frac{C - C_0}{C_0}, \quad S_c = \frac{\mu}{\rho D}, \quad \frac{\partial w(0, z)}{\partial r} = 0, \quad (28)$$

$$S_r = \frac{\mu K_T T_0}{\rho D T_m C_0}, \quad \frac{\partial \theta(0, z)}{\partial r} = 0, \quad (29)$$

where P_e , B_r , R_e , S_c and S_r are Peclet, Brinkman, Reynolds, Schmidt and Soret numbers. Equations (2–5), after removing * becomes

$$\frac{1}{r} \frac{\partial(ru)}{\partial r} + \frac{\partial w}{\partial z} = 0, \quad (18)$$

$$R_e \delta^2 \left[u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right] = -\frac{\partial p}{\partial r} - \delta^2 \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{\nu}{r^2} + \delta^2 \frac{\partial^2 u}{\partial z^2} \right], \quad (19)$$

$$R_e \left[u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right] = -\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \delta^2 \frac{\partial^2 w}{\partial z^2}. \quad (20)$$

$$P_e \delta \left[u \frac{\partial \theta}{\partial r} + w \frac{\partial \theta}{\partial z} \right] = \left[\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \delta^2 \frac{\partial^2 \theta}{\partial z^2} \right] + B_r \left[2\delta^2 \left\{ \left(\frac{\partial u}{\partial r} \right)^2 + \left(\frac{u}{r} \right)^2 + \left(\frac{\partial w}{\partial r} \right)^2 \right\} + \left(\delta^2 \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right)^2 \right], \quad (21)$$

$$R_e \left[u \frac{\partial \phi}{\partial r} + w \frac{\partial \phi}{\partial z} \right] = \frac{1}{S_c} \left[\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \delta^2 \frac{\partial^2 \phi}{\partial z^2} \right] + S_r \left[\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \delta^2 \frac{\partial^2 \theta}{\partial z^2} \right], \quad (22)$$

The boundary conditions in dimensionless form are

$$u(1, z) = U_0 - \alpha z, \quad (23)$$

$$w(1, z) = 0, \quad (24)$$

$$\theta(1, z) = 1, \quad (25)$$

$$\phi(1, z) = 1, \quad (26)$$

$$u(0, z) = 0, \quad (27)$$

$$\frac{\partial w(0, z)}{\partial r} = 0, \quad (28)$$

$$\frac{\partial \theta(0, z)}{\partial r} = 0, \quad (29)$$

$$\frac{\partial \phi(0, z)}{\partial r} = 0, \quad (30)$$

$$2 \int_0^1 r w(r, 0) dr = 1, \quad (31)$$

$$p(r, 0) = P_0, \quad (32)$$

where $U_0 = \frac{\pi R L V_0}{Q_0}$, $\alpha = \frac{\pi R L^2 \alpha_i}{Q_0}$ and $p_0 = \frac{\pi R^4 p_0}{\mu Q_0 L}$ are inlet, linear reabsorption velocities and entrance pressure parameters. Equations (18–32) is the developed pressure model capable of describing the heat and mass transfer of Newtonian fluid through a permeable tube with reabsorption across the walls. The solution of the model is explained in the next section.

3 Solution of the problem

For the solution of the problem, δ and R_e are assumed small [20], the above equations can be written as

$$\frac{\partial p}{\partial r} = 0, \quad (33)$$

$$\frac{\partial p}{\partial z} - \frac{\partial^2 w}{\partial r^2} - \frac{1}{r} \frac{\partial w}{\partial r} = 0. \quad (34)$$

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + B_r \left(\frac{\partial w}{\partial r} \right)^2 = 0, \quad (35)$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + S_c S_r \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) = 0, \quad (36)$$

From Eq. (33), it is observed that $p = p(z)$ only. Therefore, the solution of Eq. (34) using appropriate boundary conditions (24) and (28) is obtained as

$$w(r, z) = \frac{1}{4} \frac{dp}{dz} (r^2 - 1). \quad (37)$$

Substituting above Eq. (37) in Eq. (18), the expression for radial velocity with the help of boundary conditions (27) reduces to

$$u(r, z) = \frac{1}{16} \frac{d^2 p}{dz^2} (2r - r^3). \quad (38)$$

Equations (37–38) cannot describe the motion of the fluid particles inside the tube because of the unknown quantity $\frac{dp}{dz}$. To evaluate $\frac{dp}{dz}$, the boundary condition (23) is used, which transforms Eq. (38) into the following differential equation in terms of pressure

$$\frac{d^2 p}{dz^2} = 16(U_0 - \alpha z). \quad (39)$$

Integrating Eq. (39) twice, pressure becomes

$$p(z) = -\frac{8}{3} \alpha z^3 + 8U_0 z^2 + C_1 z + C_2. \quad (40)$$

After using condition (32) above equation reduces to

$$p(z) = -\frac{8}{3} \alpha z^3 + 8U_0 z^2 + C_1 z + P_0. \quad (41)$$

Now, C_1 is the only constant to determine. Combining Eqs. (42) and (37) in Eq. (31), the value of constant is

$$C_1 = -8. \quad (42)$$

So, the final expression for pressure becomes

$$p(z) = -\frac{8}{3} \alpha z^3 + 8U_0 z^2 - 8z + p_0, \quad (43)$$

therefore, the expressions for velocity components are

$$u(r, z) = -\frac{1}{2} (2\alpha z - 2U_0) (2r - r^3), \quad (44)$$

$$w(r, z) = -2(\alpha z^2 - 2U_0 z + 1)(r^2 - 1). \quad (45)$$

After using Eq. (45) in Eq. (35), the expression for temperature profile inside the permeable tube is obtained as

$$\theta(r, z) = 1 - B_r (\alpha z^2 - 2U_0 z + 1)^2 (r^4 - 1). \quad (46)$$

Using Eq. (46) in Eq. (35), the resultant equation is solved with conditions (26) and (30) and expression for concentration profile inside the permeable tube becomes

$$\phi(r, z) = 1 + S_c S_r B_r (\alpha z^2 - 2U_0 z + 1)^2 (r^4 - 1), \quad (47)$$

which depends on S_c , S_r and B_r .

3.1 Flow rate

The flow rate inside the tube can be calculated by using the formula

$$Q(z) = \frac{\pi \bar{Q}(z)}{Q_0} = 2 \int_0^1 r w(r, z) dr, \quad (48)$$

where $\bar{Q}(z)$ is dimensional flow rate. Using Eq. (45) in Eq. (48), the flow rate becomes

$$Q(z) = 1 + \alpha z^2 - 2U_0 z. \quad (49)$$

From above equation flow behaviour in axial direction can be analyzed.

3.2 Average pressure

The average pressure inside the tube can be calculated by using the formula

$$p(\bar{z}) = 2 \int_0^1 r p(z) dr = p(z), \quad (50)$$

The pressure drop inside the tube can now be calculated using the following formula

$$\Delta p = \bar{p}(0) - \bar{p}(z), \quad (51)$$

using Eq. (43) in above formula, pressure drop takes the following form

$$\Delta p = \frac{8}{3} \alpha z^3 - 8U_0 z^2 + 8z. \quad (52)$$

From Eq. (52), the amount of pressure that is permanently lost as the fluid passes inside tube can be observed.

3.3 Wall shear stress

The wall shear stress is calculated by

$$\begin{aligned} \tau_w &= -\tau_{rz} \Big|_{r=1} \\ &= -\frac{\partial u}{\partial r} \Big|_{r=1} \end{aligned} \quad (53)$$

Using Eq. (44) in above relation we find

$$\tau_w = U_0 - \alpha z. \quad (54)$$

From Eq. (54), it is noted that wall shear stress decreases inside the tube.

3.4 Leakage flux

The leakage at every location inside the tube is obtained as

$$q(z) = \frac{dQ(z)}{dz} = 2(U_0 - \alpha z), \quad (55)$$

Above equation describes the quantity of fluid reabsorbed at any axial location inside the tube.

3.5 Fractional reabsorption

The amount of fluid filtered from the leaky walls is calculated by using the relation

$$FR = \frac{Q(0) - Q(1)}{Q(0)} = 2U_0 - \alpha. \quad (56)$$

The fractional reabsorption is important quantity that describes the quantity of fluid reabsorbed across the tube walls. For 50% fractional reabsorption, inlet radial velocity is half of the reabsorption velocity along the tube. Also, amount of mass flow rate can also be studied from Eq. (56).

3.6 Heat transfer

Heat is transported mainly by conduction near the walls, so

$$\begin{aligned} \ddot{q} &= -k \left(\frac{1}{r} \frac{\partial(rT)}{\partial r} + \frac{\partial T}{\partial z} \right)_{r=H} \\ \dot{q} &= - \left(\frac{1}{r} \frac{\partial(r\theta)}{\partial r} + \delta \frac{\partial \theta}{\partial z} \right)_{r=1}, \end{aligned} \quad (57)$$

where $\ddot{q}$ is dimensional heat transfer rate, while $\dot{q} = \frac{\ddot{q}R}{T_0 k}$ is dimensionless heat transfer rate. Since δ is small so, by using Eq. (46) in Eq. (57), the expression for heat transfer becomes

$$\dot{q} = 4(\alpha z^2 - 2U_0 z + 1)^2 B_r - 1. \quad (58)$$

3.7 Nusselt number

The Nusselt number may be obtained by a non dimensional analysis of Fourier's law since it is equal to the dimensionless temperature gradient of the form

$$\begin{aligned} Nu &= \frac{\dot{q}L}{k(T - T_0)} \\ \frac{Nu k T_0}{L} &= \dot{q}, \end{aligned} \quad (59)$$

after using Eqs. (57) and (46), the expression for Nusselt number becomes

$$Nu = \frac{4(\alpha z^2 - 2U_0 z + 1)^2 B_r - 1}{1 - B_r(\alpha z^2 - 2U_0 z + 1)^2 (r^4 - 1)}. \quad (60)$$

At the wall the above expression is same with Eq. (58).

3.8 Mass transfer

The mass transfer coefficient at the wall is defined as

$$\begin{aligned} J &= -D \left(\frac{1}{r} \frac{\partial(r\phi)}{\partial r} + \frac{\partial \phi}{\partial z} \right)_{r=R} \\ S_h &= \frac{JR}{C_0 D} = - \left(\frac{1}{r} \frac{\partial(r\phi)}{\partial r} + \delta \frac{\partial \phi}{\partial z} \right)_{r=1} \end{aligned} \quad (61)$$

where J is dimensional mass transfer rate, while $S_h = \frac{JR}{C_0 D}$ is a Sherwood number (dimensionless mass transfer rate). Since, δ is small so, by using Eq. (47) in Eq. (61), above equation becomes

$$S_h = -1 - 4 S_c S_r B_r (\alpha z^2 - 2U_0 z + 1)^2, \quad (62)$$

which is strongly dependent on the involved parameters.

4 Results and discussion

The current research has been carried out to study the wall reabsorption effect on heat and mass transfer of non isothermal viscous fluid in a permeable tube. The mathematical model is exactly solved for the dimensionless radial and axial velocities ($u(r, z)/w(r, z)$), temperature ($\theta(r, z)$) and concentration ($\phi(r, z)$). For the validation of the results, it is pointed out here that all the mathematical results given in Eqs. (33–36) along with boundary conditions (23–32) are satisfied.

The influence of linear reabsorption parameter α on $u(r, z)/w(r, z)$, $\theta(r, z)$, $\phi(r, z)$, flow rate $Q(z)$, pressure drop $\Delta p(z)$, wall shear stress τ_w , Nusselt number Nu and Sherwood number S_h are illustrated in Figs. 2, 3, 4, 5, 6, 7, 8, 9 and 10. In Figs. 2 and 3, the impact of α on the dimensionless radial and axial velocities at different locations, such as, entrance ($z = 0.1$), middle position ($z = 0.5$) and exit region ($z = 0.9$) is shown, respectively. It is perceived that the $u(r, z)$ is higher at ($z = 0.1$) as compared to ($z = 0.5$) and ($z = 0.9$) due to fluid leakage across the walls of the tube. Also, the magnitude of $u(r, z)$ reduces along the tube with an increase in α , while, opposite behavior is observed for the $w(r, z)$ at the same locations, see Fig. 3) At $z = 0.5$, the velocity $w(r, z)$ profile is found to be higher as compared to other location. At $z = 0.9$, reverse flow phenomenon is

Fig. 2 Effect of α on radial velocity at at different location ($z = 0.1, 0.5, 0.9$), when $U_0 = 0.5$

Fig. 4 Effect of α on flow rate along the tube, when $U_0 = 0.5$

Fig. 3 Effect of α on axial velocity at at different location ($z = 0.1, 0.5, 0.9$), when $U_0 = 0.5$

Fig. 5 Effect of α on pressure along the tube, when $U_0 = 0.5$

observed that indicates the leakage of the fluid across the walls before reaching the end region. Also, it is observed that near the centerline of the tube, flow is maximum due to pressure gradient and near the walls the fluid flow becomes stationary due to wall friction and fluid viscosity.

The effect of α on flow rate, $Q(z)$ is observed in Fig. 4, which decreases from entrance region to exit region. Also with increasing α , the profile of $Q(z)$ increases and maximum magnitude is noticed at the exit of the tube. For the high value of α , the fluid inside the tube reduces that confirms high flow rate near the exit region. The pressure inside the tube also decays from entrance region to the exit region and shows opposite behaviour as compared to flow rate. With increasing the strength of α , pressure

reduces near the exit region. The shear stress at the wall τ_w has same nature as pressure inside the tube, see Figs. 4 and 5.

The influence of α on the dimensionless $\theta(r, z)$ at different locations are explained in Fig. 6. The effects of α on the dimensionless $\theta(r, z)$ are found to be highest at the center and then reduces to minimum at the wall surface. At the entrance region $\theta(r, z)$ is found to be higher than middle and exit region of the tube. As α increases, the dimensionless $\theta(r, z)$ increases at each axial location.

The variation of dimensionless $\phi(r, z)$ in radial direction with α can be visualized in Fig. 8 for $U_0 = 0.5$ at different locations inside the tube. It is inferred that the dimensionless concentration at the entrance is lower than middle and exit region of the tube. Also it

Fig. 6 Effect of α on wall shear stress along the tube, when $U_0 = 0.5$

Fig. 8 Effect of α on concentration at different location ($z = 0.1, 0.5, 0.9$), when $U_0 = 0.5, B_r = 1, S_c = 1, S_r = 1$

Fig. 7 Effect of α on temperature at different location ($z = 0.1, 0.5, 0.9$), when $U_0 = 0.5, B_r = 1$

Fig. 9 Effect of α on Nusselt number inside the tube, when $U_0 = 0.5, B_r = 1$

increases from center of the tube to the surface of the tube for inlet velocity $U_0 = 0.5$. In fact, higher values of α reduces the dimensionless $\phi(r, z)$ in radial direction. When α increases, the dimensionless $\phi(r, z)$ decreases in the axial direction.

The variation of Nu in axial direction with α is illustrated in Fig. 9. For same inlet velocity, there is a maximum Nu at the entrance region that decreases in the axial direction. Also α enhances the Nu in the exit region. Figure 10 displays the variation of S_h in axial direction with α , for the same inlet velocities. In Fig. 10, it is noticed that S_h is found to be lower at the exit region and then increases in the axial direction with an increase in the α . At the exit region, S_h is maximum for lower value of α .

5 Concluding remarks

The steady flow of a non-isothermal, laminar and incompressible Newtonian fluid is considered in a leaky tube with linear reabsorption across the walls. From this study, the following conclusions can be drawn.

1. Radial velocity component is higher in entrance region as compared to the middle position and the exit region of the tube.
2. Reverse flow phenomenon is noticed for axial velocity at the exit region of the tube.

Fig. 10 Effect of α on Sherwood number inside the tube, when $U_0 = 0.5$, $B_r = 1$

- For 50% fractional reabsorption, inlet radial velocity is half of the reabsorption velocity along the tube.
- With increasing the strength of linear reabsorption parameter radial velocity decreases while axial velocity increases at different locations inside the tube.
- Temperature field increases at the centerline of the tube with decreasing linear reabsorption parameter.
- With increasing linear reabsorption parameter, concentration decreases from center to the surface for same inlet velocity while increases for higher inlet velocity.
- Flow rate, pressure and wall shear stress decreases along the tube.
- With increasing linear reabsorption pressure decreases while flow rate and wall shear stress increases along the tube.
- Nusselt number at the wall decreases along the tube its magnitude is maximum at the exit region of the tube with increases linear reabsorption parameter.
- Sherwood number increases along the tube and with increasing linear reabsorption parameter its profile decreases near the exit region of the tube.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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