Kalman filter based method for tracking dynamic transmission line parameters

Yiqi Zhang¹ · Yuan Liao¹

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Abstract

Transmission line parameters are required by various power system applications. Thus, it is important to develop accurate transmission line parameter estimation methods. This paper puts forward Kalman filter based methods to track the dynamically changing line parameters. The proposed methods can overcome the estimation inaccuracy problem caused by the effect of the noise in the measurements. The proposed methods are based upon the equivalent-circuit of long line and utilize voltage and current data recorded by the Phasor Measurement Units at both ends of the line. Non-compensated and differently configured series compensated lines have been considered in this paper. The case studies demonstrate that the proposed methods have the capability of dynamically tracking changing line parameters and yield accurate estimates with the presence of measurement noises.

Keywords Kalman filters · Line parameter estimation · Phasor measurement units · Transmission lines

1 Introduction

Transmission line parameters are critical information for various power system applications including fault location and classification [1–3], relay protection [4–6], and other applications [7, 8]. Thus, the accuracy of line parameters is important, and accurate line parameter estimation methods are demanded. Basically, the line parameter estimation methods can be divided into two categories: online methods and offline methods. Offline estimation methods are based on factors such as conductor parameters and tower configurations [9, 10]. However, those parameters may not be the same as the assumed condition due to uncertain weather and loading conditions. Thus, the line parameter estimation results of offline methods may deviate from the actual values. Online estimation methods became more favored recently, which are based on the equivalent circuit of transmission lines and utilize the measurement data from phasor measurement units. Online methods are

more likely to give the trustable estimation results since the measurements inherently reflect the prevailing operating and weather conditions.

Different online methods have been proposed in [11–27] to estimate line parameters considering the line length, specific line configurations, data availability, etc. The methods for estimating zero-sequence line parameters are proposed in [11–13]. The author of [14] proposed a method for identifying and estimating erroneous transmission line parameters. A non-linear weighted least-square error algorithm for three-terminal lines was proposed in [15]. Paper [16] proposed a method to estimate line parameters using a robust estimator to minimize the influence of outliers. Paper [17] presented a method to estimate the parameters of double circuit transmission lines. In [18], a Kalman filter based recursive regression method was proposed to estimate three-phase line parameters of non-transposed and non-symmetric medium length lines. A moving-window total least squares algorithm

✉ Yuan Liao, yuan.liao@uky.edu; Yiqi Zhang, yzh375@uky.edu | ¹Department of Electrical and Computer Engineering, University of Kentucky, Lexington, KY 40506, USA.

was proposed in [19] to estimate the parameters of a long transmission line. The method treats parameters as constant during a time window. In [20], the author proposed a method based on the Clarke's transformation matrix using faulted voltage and current measurements obtained at the line terminals. The authors of [21] proposed a method for estimating line parameters using data during a grounding fault. In [22], an estimation method was proposed considering dynamic operating states.

Series compensation has been used in long transmission lines for improved operation. The estimation methods for series compensated lines have been proposed in [23, 24]. The author of [23] proposed methods for long lines where the series compensators are installed in the middle of the line. In [24], a method using traveling waves generated by disturbance was proposed to estimate the parameters for series compensated lines.

Most estimation methods are proposed under the assumption that line parameters are static. However, line parameters are continuously changing over time with varying loading and weather conditions. Thus, it is desirable to have an effective dynamic line parameter tracking method. The authors of [25] proposed a method that essentially counted on the regular Kalman filter for dynamic tracking, which might not be highly efficient. In [26], the author discussed dynamic tracking but required additional information (e.g., weather forecast) to build the prediction model. In [27], the author used the sliding window method to track the moving tendency of line parameters. However, this method requires complicated tuning of several parameters such as the window size.

This paper proposes Kalman filtering based methods that employ voltage and current measurements from line terminals and require no extra information to track parameters of long transmission lines dynamically. This paper proposes improvements over commonly used Kalman filtering to better track the parameter variations. In addition, the proposed methods are applicable to transmission lines with different series compensation configurations.

The rest of the paper is organized as follows. Section 2 presents the proposed line parameter estimation methods. Section 3 presents the evaluation studies and results, followed by the conclusion.

2 Proposed methods for line parameter estimation

This section presents the proposed line parameter estimation methods for non-compensated lines and series compensated lines. According to [28], the series compensator is typically installed at the middle or the end of a transmission line. The schematic diagram of the non-compensated and

different series compensated transmission lines are shown in Fig. 1, where S is the sending end, and T is the receiving end. PMUs are installed at both ends for measuring the voltages and currents. V_S and I_S denote the sending end voltage and current and V_T and I_T represent the receiving end voltage and current.

The overall line parameter estimation method based on the Kalman filter technique is presented in Sect. 2.1, followed by specific methods for different line configurations.

2.1 Proposed methods based on Kalman filter

The Kalman filter method is defined by two steps: prediction and update. Define the state to be estimated as x . In the first step, the priori estimate for the state at time k is predicted based on the state at time $k - 1$ by using (1), and the priori error covariance at time k is calculated based on (2):

$$\hat{x}_k^- = \hat{x}_{k-1} \quad (1)$$

$$P_k^- = P_{k-1} + Q \quad (2)$$

where the subscript k is the instant, the superscript $-$ means priori, $\hat{x}$ means the estimate for x , P is the

Fig. 1 Studied line configurations with and without series compensators

error covariance, and Q is the process noise covariance. Then the Kalman gain K , the posteriori estimate $\hat{x}$ and the posteriori error covariance P at instant k are calculated by (3)–(5),

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R)^{-1} \quad (3)$$

$$\hat{x}_k = \hat{x}_k^- + K_k (w_k - H_k \hat{x}_k^-) \quad (4)$$

$$P_k = (I - K_k H_k) P_k^- \quad (5)$$

where R is the observation noise covariance, I is the identity matrix, H is the observation matrix, and w is the observation vector.

To dynamically track the state without knowing any information between the states at instant k and $k - 1$, it is critical to make the most of measurements. Regular Kalman filtering technique yields slow tracking performance as shown in case studies to be presented in Sect. 3. To improve tracking performance of the Kalman filter, the following two adjustment methods are proposed.

The first solution is to use the adjusted Kalman gain method, which increases the Kalman gain K . While the priori estimate $\hat{x}_k^-$ gets pulled away from the actual value by the previous estimate $\hat{x}_{k-1}$, it is reasonable to add more weight to the current measurements. The concept of adjusting the Kalman gain has already been considered such as [32]. In our work, a factor D ($D > 1$) is introduced as follows:

$$\hat{x}_k = \hat{x}_k^- + DK_k (w_k - H_k \hat{x}_k^-) \quad (6)$$

The second solution is called the fast Kalman method, which is novel to the best knowledge of the authors. This new method considers the direction in which the line parameters are moving toward, i.e., taking $(\hat{x}_k^- - \hat{x}_{k-n})$ into account, where $n \geq 1$. Thus,

$$\hat{x}_k = \hat{x}_k^- + K_k (w_k - H_k \hat{x}_k^-) + U (\hat{x}_k^- - \hat{x}_{k-n}) \quad (7)$$

where the variable U is used to adjust the weight of $(\hat{x}_k^- - \hat{x}_{k-n})$.

By rearranging the terms, $\hat{x}_k$ can be expressed as

$$\hat{x}_k = \frac{1}{1-U} \hat{x}_k^- + \frac{1}{1-U} K_k (w_k - H_k \hat{x}_k^-) - \frac{U}{1-U} \hat{x}_{k-n} \quad (8)$$

For the first n estimations (when $k - n < 0$), $\hat{x}_{k-n}$ is set to zero. Note that while increasing the value of D and U can increase the ability of dynamic tracking, it also brings instability to the algorithms. In the case studies, the value of D and U are chosen as to balance the performance of dynamic tracking and stability. The choice of optimal process noise

covariance Q and measurement noise covariance R are referred to [29–31].

In the following subsections, Kalman filter formulation is presented for different line configurations with and without series compensators, based on which the states and then the line parameters can be derived.

2.2 Non-compensated Line

The equivalent- π circuit of the positive-sequence network of the non-compensated transmission line is shown in Fig. 2. The following sections for different lines are all based on this model. Z is the series impedance and $\frac{Y}{2}$ is the shunt admittance of this circuit at one end. The relationship between Z and Y and transmission line parameters is defined by the following equations,

$$Z = Z_c \sinh(\gamma l) \quad (9)$$

$$\frac{\gamma}{2} = \frac{\tanh(\gamma l/2)}{Z_c} \quad (10)$$

$$Z_c = \sqrt{z/y} \quad (11)$$

$$\gamma = \sqrt{zy} \quad (12)$$

where l is the length of the line, Z_c is the characteristic impedance of the line, γ is the propagation constant. z and y are the positive sequence series impedance and shunt admittance per unit length.

The voltages and currents in Fig. 2 can be related by the admittance matrix

$$\begin{bmatrix} I_S \\ I_T \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} V_S \\ V_T \end{bmatrix} \quad (13)$$

where $a = \frac{\gamma}{2} + \frac{1}{Z}$ and $b = -\frac{1}{Z}$. Rewrite (13),

Fig. 2 The equivalent circuit of the non-compensated line

$$\begin{bmatrix} I_S \\ I_T \end{bmatrix} = \begin{bmatrix} V_S & V_T \\ V_T & V_S \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \quad (14)$$

To use the Kalman filtering techniques presented in Sect. 2.1, define the state to be estimated as $x = \begin{bmatrix} a \\ b \end{bmatrix}$. The observation vector $w = \begin{bmatrix} I_S \\ I_T \end{bmatrix}$ and the observation matrix $H = \begin{bmatrix} V_S & V_T \\ V_T & V_S \end{bmatrix}$.

The state x can be estimated following the method shown in Sect. 2.1. After x is estimated, Z and Y can be calculated as

$$Z = -\frac{1}{b} \quad (15)$$

$$Y = 2(a + b) \quad (16)$$

Multiplying Z and Y

$$ZY = 2 \sinh(\gamma l) \tanh\left(\frac{\gamma l}{2}\right) \quad (17)$$

Let $d = e^{\gamma l}$, and (17) can be written as

$$d^2 - (2 + ZY)d + 1 = 0 \quad (18)$$

Solving for d , γ can be calculated as

$$\gamma = \frac{\ln(d)}{l} \quad (19)$$

At last, the line parameters are determined as

$$z = Z_c \gamma \quad (20)$$

$$y = \frac{\gamma}{Z_c} \quad (21)$$

2.3 One-end compensated line

The configuration where the series compensator is installed at one end of a transmission line is discussed in this section. Assume that the impedance of the series compensator Z_{cap} is unknown. The equivalent- π circuit of this line configuration is shown in Fig. 3.

The voltage and current measured at each bus can be related by the admittance matrix

$$\begin{bmatrix} I_S \\ I_T \end{bmatrix} = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} V_S \\ V_T - Z_{cap} I_T \end{bmatrix} \quad (22)$$

where $a = \frac{Y}{2} + \frac{1}{Z}$ and $b = -\frac{1}{Z}$. The unknown variables can be separated from measurements,

Fig. 3 The equivalent circuit of the one-end compensated line

$$\begin{bmatrix} I_S \\ I_T \end{bmatrix} = \begin{bmatrix} V_S & V_T & -I_T & 0 \\ V_T & V_S & 0 & -I_T \end{bmatrix} \begin{bmatrix} a \\ b \\ g \\ h \end{bmatrix} \quad (23)$$

where $g = bZ_{cap}$ and $h = aZ_{cap}$. Since there are two additional unknown variables g and h , one more set of measurements is needed to solve for the four variables. Thus, with two sets of measurements, (23) becomes

$$\begin{bmatrix} I_{S_1} \\ I_{T_1} \\ I_{S_2} \\ I_{T_2} \end{bmatrix} = \begin{bmatrix} V_{S_1} & V_{T_1} & -I_{T_1} & 0 \\ V_{T_1} & V_{S_1} & 0 & -I_{T_1} \\ V_{S_2} & V_{T_2} & -I_{T_2} & 0 \\ V_{T_2} & V_{S_2} & 0 & -I_{T_2} \end{bmatrix} \begin{bmatrix} a \\ b \\ g \\ h \end{bmatrix} \quad (24)$$

To use the Kalman filtering techniques presented in Sect. 2.1, define the state to be estimated as

$$x = [a \ b \ g \ h]^T. \text{ The observation vector is } w = \begin{bmatrix} I_{S_1} \\ I_{T_1} \\ I_{S_2} \\ I_{T_2} \end{bmatrix} \text{ and}$$

$$\text{the observation matrix is } H = \begin{bmatrix} V_{S_1} & V_{T_1} & -I_{T_1} & 0 \\ V_{T_1} & V_{S_1} & 0 & -I_{T_1} \\ V_{S_2} & V_{T_2} & -I_{T_2} & 0 \\ V_{T_2} & V_{S_2} & 0 & -I_{T_2} \end{bmatrix}. \text{ Once } x$$

is known, the line parameters can be solved using (15)–(21).

2.4 Mid-compensated line with known series compensator

The series compensator can also be installed at the middle of transmission line. The equivalent circuit is shown in Fig. 4. V_R and V_G are the voltage at each side of the series compensator. Assume that the admittance of the series compensator Y_{cap} is unknown.

Similarly, the relationship between the voltages and currents at each node can be built using the admittance matrix M .

Fig. 4 The equivalent circuit of the mid-compensated line with known Y_{cap}

Fig. 5 The equivalent circuit of the mid-compensated line with known compensator current

$$\begin{bmatrix} I_S \\ I_T \\ 0 \\ 0 \end{bmatrix} = M \begin{bmatrix} V_S \\ V_T \\ V_R \\ V_G \end{bmatrix} \quad (25)$$

$$\text{where } M = \begin{bmatrix} a & 0 & b & 0 \\ 0 & a & 0 & b \\ b & 0 & a + Y_{cap} & -Y_{cap} \\ 0 & b & -Y_{cap} & a + Y_{cap} \end{bmatrix}, \quad a = \frac{Y}{2} + \frac{1}{Z} \text{ and}$$

$b = -\frac{1}{Z}$. The matrix can be reduced to (26) using node elimination technique

$$\begin{bmatrix} I_S \\ I_T \end{bmatrix} = \begin{bmatrix} A & B \\ B & A \end{bmatrix} \begin{bmatrix} V_S \\ V_T \end{bmatrix} \quad (26)$$

where $A = a - \frac{b^2}{a} + \frac{Y_{cap}b^2}{a^2 + 2aY_{cap}}$ and $B = -\frac{b^2Y_{cap}}{a^2 + 2aY_{cap}}$. Equation (26) can be rewritten as

$$\begin{bmatrix} I_S \\ I_T \end{bmatrix} = \begin{bmatrix} V_S & V_T \\ V_T & V_S \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} \quad (27)$$

To use the Kalman filtering techniques presented in Sect. 2.1, define the state to be estimated as $x = \begin{bmatrix} A \\ B \end{bmatrix}$.

The observation vector $w = \begin{bmatrix} I_S \\ I_T \end{bmatrix}$ and the observation matrix is $H = \begin{bmatrix} V_S & V_T \\ V_T & V_S \end{bmatrix}$.

Once A and B are obtained, a and b can be calculated as

$$a = \frac{A - B}{1 + B/Y_{cap}} \quad (28)$$

$$b = \sqrt{-B(a^2 + 2aY_{cap})/Y_{cap}} \quad (29)$$

Then the line parameter can be derived using the method described in Sect. 2.2.

2.5 Mid-compensated line with known current through series compensator

For the mid-compensated line, another case is that the impedance of the series compensator is unknown, but the current through it is available. Figure 5 shows the equivalent circuit of the situation. I_R is the current flowing through the series compensator.

Define the unknown variables $a = \frac{Y}{2} + \frac{1}{Z}$ and $b = -\frac{1}{Z}$. Equations (30)–(33) can be written using Kirchhoff's Current Law (KCL) for each node

$$I_S = aV_S + bV_R \quad (30)$$

$$I_R = aV_R + bV_S \quad (31)$$

$$I_T = aV_T + bV_G \quad (32)$$

$$-I_R = aV_G + bV_T \quad (33)$$

Based on (30) and (31), V_R can be eliminated,

$$aI_S - bI_R = (a^2 - b^2)V_S \quad (34)$$

Based on (32) and (33), V_G can be eliminated,

$$aI_T + bI_R = (a^2 - b^2)V_T \quad (35)$$

Multiply V_T to (34) and V_S to (35) and then subtract the resultant equations. Then a can be expressed in terms of b ,

$$a = Cb \quad (36)$$

where $C = \frac{I_R(V_S + V_T)}{I_S V_T - I_T V_S}$. The expression of b can be obtained by substituting (36) for a in (34),

$$b = \frac{CI_S - I_R}{(C^2 - 1)V_S} \quad (37)$$

Then based on (36) and (37), we can write

$$\begin{bmatrix} C(Cl_S - I_R) \\ Cl_S - I_R \end{bmatrix} = \begin{bmatrix} (C^2 - 1)V_S & 0 \\ 0 & (C^2 - 1)V_S \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \quad (38)$$

In this case, to use the Kalman filtering techniques presented in Sect. 2.1, define the state $x = \begin{bmatrix} a \\ b \end{bmatrix}$, the measurement vector $w = \begin{bmatrix} C(Cl_S - I_R) \\ Cl_S - I_R \end{bmatrix}$, and the measurement matrix $H = \begin{bmatrix} (C^2 - 1)V_S & 0 \\ 0 & (C^2 - 1)V_S \end{bmatrix}$. Then x can be used to solve for the line parameters.

2.6 Two-ends compensated line

The two-ends compensated line configuration is the case where two equivalent series compensators are installed at both ends of the line. The equivalent circuit is shown in Fig. 6. Y_{cap} represents the admittance of the series compensators.

The voltages and currents injected at each node can be related using the admittance matrix M .

$$\begin{bmatrix} I_S \\ I_T \\ 0 \\ 0 \end{bmatrix} = M \begin{bmatrix} V_S \\ V_T \\ V_R \\ V_G \end{bmatrix} \quad (39)$$

$$\text{where } M = \begin{bmatrix} Y_{cap} & 0 & -Y_{cap} & 0 \\ 0 & Y_{cap} & 0 & -Y_{cap} \\ -Y_{cap} & 0 & a + Y_{cap} & b \\ 0 & -Y_{cap} & b & a + Y_{cap} \end{bmatrix}, \quad a = \frac{Y}{2} + \frac{1}{Z}$$

$$\text{and } b = -\frac{1}{Z}.$$

Equation (39) can be reduced to (40)

Fig. 6 The equivalent circuit of the two-ends compensated line

Table 1 The transmission line parameters used in the study

Total line length	217.48 miles		
Base voltage	500 kV		
Base power	1000 MVA		
System frequency	60 Hz		
Positive sequence parameters	Series resistance (p.u./mile)	Series reactance (p.u./mile)	Shunt susceptance (p.u./mile)
	9.9667×10^{-4}	2.3566×10^{-3}	1.8349×10^{-3}

$$\begin{bmatrix} I_S \\ I_T \end{bmatrix} = \begin{bmatrix} A & B \\ B & A \end{bmatrix} \begin{bmatrix} V_S \\ V_T \end{bmatrix} \quad (40)$$

where

$$A = Y_{cap} - \frac{Y_{cap}^2(Y_{cap} + a)}{(Y_{cap} + a)^2 - b^2} \quad (41)$$

and

$$B = \frac{Y_{cap}^2 b}{(Y_{cap} + a)^2 - b^2} \quad (42)$$

Equation (40) can be rewritten as

$$\begin{bmatrix} I_S \\ I_T \end{bmatrix} = \begin{bmatrix} V_S & V_T \\ V_T & V_S \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} \quad (43)$$

To use the Kalman filtering techniques presented in Sect. 2.1, define the state to be estimated as $x = \begin{bmatrix} A \\ B \end{bmatrix}$. The

observation vector $w = \begin{bmatrix} I_P \\ I_Q \end{bmatrix}$ and the observation matrix

is $H = \begin{bmatrix} V_S & V_T \\ V_T & V_S \end{bmatrix}$. Then the line parameters can be solved based on (41) and (42) using non-linear equation solving techniques.

3 Results and discussion

This section presents results and discussion based on evaluation studies utilizing simulated data. Different line configurations were built using Matlab Simscape Electrical, and the proposed algorithms were implemented in Matlab. The actual parameters of the transmission line are shown in Table 1. Simulation studies were run to generate voltage and current phasor measurements during varying operating conditions. The measurements were added with normally distributed random noises. Each random noise consists of two parts: the magnitude noise and the angle noise. Both noises have their mean set to zero. For the magnitude noise, its mean $\mu_{mag} = 0$ and its standard

deviation σ_{mag} varies for each case; for the angle noise, its mean $\mu_{angle} = 0$ and its standard deviation $\sigma_{angle} = 0.2^\circ$.

Since the series resistance is more susceptible to the change of loading and weather conditions when compared to the series reactance and shunt susceptance, we only changed the series resistance dynamically while the series reactance and shunt susceptance were kept static. In all the simulations, the series resistance was first set to its normal value and then changed to 120%, 150%, 100%, and 80% of its normal value during different time periods.

For each case study, the estimation results of the normal Kalman filter method, adjusted Kalman filter method, and fast Kalman filter method (the parameter n is set to 2 for all cases) were compared.

3.1 Case study for non-compensated line

The variables D , U_1 and U_2 are set to 20, 0.4 and 0.3, which correspond to the adjusted Kalman gain method, fast KF1 and fast KF2 method, respectively. σ_{mag} is set to 0.5%.

The estimation results for the parameters of the non-compensated line are shown in Table 2 and Fig. 7. The formula of the Mean Absolute Percentage Error (MAPE) is given by (44), where n is the number of estimations, $\hat{x}$ is the estimated value, and x is the actual value. It is shown that the adjusted Kalman gain method has the best performance among all the methods when tracking the dynamically changing series resistance.

$$MAPE = \frac{1}{n} \sum \left| \frac{x - \hat{x}}{x} \right| \times 100\% \quad (44)$$

3.2 Case study for one-end compensated line

The variables D , U_1 and U_2 are set to 10, 0.4 and 0.3, which correspond to the adjusted Kalman gain method, fast KF1 and fast KF2 method, respectively. σ_{mag} is set to 0.1%.

The estimation results for the positive sequence series resistance, series reactance, shunt susceptance and the reactance of the series compensator of the one-end compensated line are shown in Table 3 and Fig. 8. From the results, we can see that the proposed methods are able to track the dynamically changing line parameters accurately and quickly.

Table 2 The MAPE for the resistance estimate of each method for the non-compensated line

Methods	Normal KF	Adjusted Kalman gain	Fast KF1	Fast KF2
MAPE	16.47%	3.95%	5.70%	11.49%

(a) series resistance

(b) series reactance

(c) shunt susceptance

Fig. 7 The estimation results for the non-compensated line

3.3 Case study for mid-compensated line with known series compensator

The variables D , U_1 and U_2 are set to 10, 0.4 and 0.3, which correspond to the adjusted Kalman gain method, fast KF1 and fast KF2 method, respectively. σ_{mag} is set to 0.5%.

The estimation results for the positive sequence series resistance, series reactance, and shunt susceptance of the mid-compensated line with known impedance of series compensator are shown in Table 4 and Fig. 9. The results show that all the proposed methods have the capability to

Table 3 The MAPE for the resistance estimate of each method for the one-end compensated line

Methods	Normal KF	Adjusted Kalman gain	Fast KF1	Fast KF2
MAPE	11.18%	5.52%	8.04%	7.40%

(a) series resistance

(b) series reactance

(c) shunt susceptance

(d) compensator reactance

Fig. 8 The estimation results for the one-end compensated line**Table 4** The MAPE for the resistance estimate of each method for the mid-compensated line with known impedance

Methods	Normal KF	Adjusted Kalman gain	Fast KF1	Fast KF2
MAPE	16.78%	1.73%	5.53%	11.83%

(a) series resistance

(b) series reactance

(c) shunt susceptance

Fig. 9 The estimation results for the mid-compensated line with known series compensator**Table 5** The MAPE for the resistance estimate of each method for the mid-compensated line with known current through series compensator

Methods	Normal KF	Adjusted Kalman gain	Fast KF1	Fast KF2
MAPE	17.62%	4.66%	15.14%	16.88%

accurately and quickly track the changing line parameters, and the adjusted Kalman gain method performs best.

(a) series resistance

(b) series reactance

(c) shunt susceptance

Fig. 10 The estimation results for mid-compensated line with known compensator current

3.4 Case study for mid-compensated line with known current through series compensator

The variables D, U_1 and U_2 are set to 10, 0.3 and 0.2, which correspond to the adjusted Kalman gain method, fast KF1 and fast KF2 method, respectively. σ_{mag} is set to 0.1%.

The estimation results for the positive sequence series resistance, series reactance, and shunt susceptance of the one-end compensated line are shown in Table 5 and Fig. 10.

The proposed methods can successfully track the dynamic line parameters. It is seen that the adjusted Kalman gain method performs best, while Fast KF1 and KF2 have difficulty in estimating the shunt susceptance parameter.

Table 6 The MAPE for the resistance estimate of each method for the two-ends compensated line

Methods	Normal KF	Adjusted Kalman gain	Fast KF1	Fast KF2
MAPE	16.76%	2.58%	12.24%	15.04%

3.5 Case study for two-ends compensated line

The variables D, U_1 and U_2 are set to 10, 0.3 and 0.2, which correspond to the adjusted Kalman gain method, fast KF1 and fast KF2 method, respectively. σ_{mag} is set to 0.5%.

The estimation results for the positive sequence series resistance, series reactance, shunt susceptance and susceptance of the series compensator of the one-end compensated line are shown in Table 6 and Fig. 11. It is manifested that all three line parameters can be tracked

(a) series resistance

(b) series reactance

(c) shunt susceptance

Fig. 11 The estimation results for the two-ends compensated line

successfully by the proposed methods and the adjusted Kalman method performs best.

4 Conclusion

Transmission line parameters are essential inputs to various power system applications. This paper puts forward several improved variants of Kalman filter based methods to dynamically and accurately track the positive transmission line parameters. The proposed methods utilize the voltage and current measurements obtained at both terminals of the line. Non-compensated and different types of series compensated lines have been discussed. The case study results have demonstrate that the proposed methods can accurately and quickly track the dynamically changing line parameters under the effect of measurement noises.

In general, the adjusted Kalman gain method outperforms other methods in the speed of tracking dynamically changing parameters and can be regarded as the first choice method, while other approaches including the Normal KF, Fast KF1 and Fast KF2 can be used for corroboration.

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Code availability Not applicable.

Compliance with ethical standards

Conflicts of interest The authors declare that they have no conflicts of interest.

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