Experimental and numerical study on the performance of externally prestressed reinforced high strength concrete beams with openings

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Abstract

This study investigates experimentally and numerically the performance of externally prestressed reinforced high strength concrete (HSC) beams with central openings. Seven externally prestressed rectangular HSC beams (six with central openings and a reference solid beam) are loaded incrementally to failure. All the beams have the same dimensions, reinforcement ratio and openings of variable size. Experimentally, the results show that, the appearance of the first flexural crack and the flexural stiffness reduction are largely governed by opening height. In contrast, the opening length greatly affects the presence of the first shear crack and the obtained values of strains in stirrups. Additionally, the opening length and height when combined can affect the strains in top- and bottom-bars and the failure load of the teste beams. Numerically, a three-dimensional nonlinear finite element analysis using ANSYS has been carried-out to analyze seventy (70) externally prestressed HSC beams with central openings. Based on the numerical results, a general formula to predict the ultimate moment is generated and verified. It can be used to predict the load carrying capacity of aging concrete elements with openings retrofitted using external prestressing techniques.

Keywords Openings · Externally prestressed beams · High strength concrete · Finite element analysis

1 Introduction

Recently, a strong need around the world to retrofit ageing bridges has increased. External prestressing is not only one of the preferred methods to retrofit bridges, but also a perfect method to construct new ones. External prestressing technique enhances durability, serviceability, and stiffness of constructions. Also, it increases the flexural strength and decrease the deflection and crack widths of the concrete members [1].

Due to its advantages; mechanical properties and durability performance a great demand for high strength concrete (HSC) has increased in the last decades [2].

Openings in reinforced concrete (RC) beams are frequently provided for major services; ventilating ducts, network of pipes, door openings, and widows. So,

investigating the influence of openings on the performance of RC beams has become a necessity.

A lot of previous studies have investigated the influence of openings presence on behavior of RC beams [3–7] or HSC beams [8, 9]. Fewer studies have explored the effect of external prestressing on behavior of RC beams [10–14]. Rare ones have studied the effect of external prestressing on HSC elements [15]. So, rationally, it is difficult to find previous researches that investigate the performance of externally prestressed HSC beams with central openings. Actually, data in this field are suffering from a lack.

Ghallab and Beeby [16] tested thirteen beams; one with internal prestressing steel only and the other beams with internal steel tendons and external prestressing using Parafil rope. Ghallab and Beeby [16] suggested a method for calculating the ultimate strength of internal

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bonded prestressed beams, internal unbonded prestressed beams, external unbonded prestressed beams or external-internal prestressed concrete beams.

Aparicio et al. [10] performed a test program on five monolithic and three segmental externally prestressed concrete beams. The beams were tested in bending and in combined bending and shear to study their behavior. The main conclusion was; the tendon's length had a great effect on the ultimate load capacity of externally prestressed concrete beams.

Ng et al. [17] performed an exploratory study on eight RC beams and seven prestressed concrete beams. The prestressed beams smaller shear span was externally prestressed using Carbon fiber reinforced polymers (CFRP) rods. The test considered parameters were; span to depth ratio, external prestressing level, and shear reinforcement. The results indicated that; shear enhancement can't be predicted with the shear design equations in ACI 318-08 [18].

Ghallab et al. [19] performed an experimental investigation in order to study shear behavior of externally prestressed beams. Test results demonstrated that; the shear span to depth ratio had a great effect on the failure mode and shear strength the tested beams. Also, the presence of external prestressing force increased the ultimate load of the tested beams by 75%.

Classen et al. [20] tested six T-shaped prestressed concrete beams with openings to study their behavior. Test results demonstrated that; the load carrying capacity of concrete beams with openings can attain approximately the same load carrying capacity as beams without openings.

Abdalla et al. [21] presented nonlinear algorithm and design of simply supported prestressed concrete beams with rectangular openings. A design method for each failure mode of prestressed concrete beams with openings was presented in this study.

Salem et al. [22] presented the results of testing partially prestressed HSC beams in flexure. Their test's results showed that; an increase in the concrete nominal strength of the tested beams from 42 to 114 MPa increased the cracking load by 10% while, corresponding deflection decreased by 45%. The ultimate load increased by 9.2%, whereas its corresponding deflection increased by about 16%.

Shehabeldin et al. [23], tested ten prestressed concrete beams with and without openings. The results showed that; an increase in opening dimensions led to an increase in the toughness and a decrease in the strength of the tested beams. Also, Increasing the shear span-to-depth ratio increased the strength and decreased the toughness of the tested beams.

None of the above investigations studied the performance of externally prestressed HSC beams with openings. The current work contribution is to provide some data in this field, that suffers from a scarcity, and propose a general formula that can predict ultimate moments of externally prestressed reinforced HSC beams with central rectangular openings. This formula importance is to predict the load carrying capacity of concrete elements that retrofitted using external prestressing techniques.

To achieve the aforementioned goals, the present investigation is divided into four parts. The first part is to explain in detail the experimental program and input data of considered beams, the second part aims to present test results and showing the effect of openings on the performance of the considered beams. In the third part, using ANSYS [24], a three-dimensional nonlinear finite element analysis has been carried-out on the experimentally tested beams and others tested by previous researchers. The obtained numerical and experimental ultimate moments and central deflection values are compared. Finally, a general formula was generated to predict the ultimate moment of externally prestressed HSC beams with central rectangular openings.

2 Experimental program

Seven externally prestressed rectangular HSC beams with central web openings were loaded to failure in order to investigate their behavior. All the beams were externally prestressed with constant force of 50kN. They all had similar dimensions, concrete compressive strength, and reinforcement ratio, but not reinforcement amount.

2.1 Concrete and reinforcement

The target compressive strength of concrete was to be 80 MPa. The concrete mix-design is shown in Table 1. The mix was used in casting six standard cubes $150 \times 150 \times 150$ mm and three standard cylinders 150×300 mm. All the specimens were left in water for curing. After seven days,

Table 1 Concrete mix-design

Cement, kg/m ³	Silica fume, kg/m ³	Water content, kg/m ³	Plasticizers (viscocrete), kg/m ³	Sand, kg/m ³	Gravel, kg/m ³	Sand-to-gravel ratio
500	75	175	20	447	1342	1-to-3

compressive strength test was applied on three cubes, while the other cubes and cylinders were tested after 28 days. The tests were; compressive strength test for three cubes, splitting test for cylinders, and modulus of elasticity test for cylinders, Table 2.

Steel bars (16, 12, and 10 mm Dia.) were used as longitudinal reinforcement and (8 mm Dia.) were used as stirrups. Mechanical properties of reinforcement are shown in Table 3.

2.2 Specimens

Based on Table 1, seven HSC beams with varied openings dimensions were cast. All beams had the same width of 150 mm, total height of 500 mm, and the same length of 2100 mm, Table 4.

2.3 Classification

The tested beams were classified into three groups according to their openings heights; H_{100} , H_{150} , and H_{200} . Each group consisted of two beams having the same openings height but different openings lengths. Group H_{100} included Beams B2 and B5, Group H_{150} included Beams B3 and B6, and Group H_{200} included Beams B4 and B7. Symbol H stands for opening height and numbers (100, 150, and 200) refer to openings heights in mm units.

Additionally, the tested beams were classified into two groups according to their openings lengths; L_{400} and L_{800} . Each group consisted of three beams having the same openings length but varied openings heights. Group L_{400} included Beams B2, B3, and B4, and Group L_{800} included Beams B5, B6, and B7. Symbol L stands for opening length

and numbers (400 and 800) refer to openings lengths in mm units.

2.4 Test instrumentations

Strain gauges were used to accurately measure strains in steel bars. Dial gauge was used to measure mid-span deflections of each beam. External prestressing force was applied using a hydraulic jack connected to a pump. The beams were loaded to failure using a hydraulic jack connected to a dial gauge with a capacity greater than 1500kN. Furthermore, two high-strength bolts and two steel rods (16 mm dia.) were used for the application of prestressing technique.

2.5 Preparing for testing

After 28 days all the beams were ready for conducting the experimental test. Firstly, three strain gauges for each beam with length of 10 mm were pasted to steel reinforcement cage to measure strains in steel bars. The first strain gauge was located at the mid-height point of the middle stirrup of the reference solid beam and to the mid-height point of first stirrup next to each opening, in the beams with openings. The second strain gauge was located at the midpoint of one of the bottom bars and the third was at the midpoint of one of the top bars. The gauges were connected to a strain meter to measure strains in bars. All the beams were painted with white color for photographing. After that, dial gauge was used at midpoint of the bottom face of each tested beam to measure mid-span deflections. A very small glassy square plate was pasted between the dial gauge and the bottom surface of beam to guarantee accurate readings.

2.6 Test-setup

The beams were tested in the Faculty of Engineering, Mansoura University. At first, a very rigid steel beam on two steel rods was used to transfer load from the hydraulic jack to the top of the tested beams as two-point loads.

External prestressing system consisted of two long steel bars (16 mm dia.) with two U-shape steel channels

Table 2 Concrete properties

Average concrete compressive strength after 7 days, MPa	Average concrete compressive strength after 28 days, MPa	Splitting strength of cylinders, MPa	Modulus of elasticity, MPa
58	79	5.5	37,000

Table 3 Properties of steel reinforcement

Diameter, mm	Cross-sectional area, mm ²	Weight/unit length, N/mm	Yield strength, MPa	Ultimate strength, MPa	Modulus of elasticity, MPa
16	201	0.01578	372	535	200,000
12	113	0.00888	366	527	
10	78.5	0.00617	370	530	
8	50.3	0.00503	250	360	

Table 4 Characteristics of considered beams

Beam	Length, mm	Width, mm	Height, mm	Central openings		f_{cu} , MPa	Longitudinal RFT		RFT around openings		RFT ratio (p)	Shear RFT
				Length, mm	Height, mm		Main	Second-dary	above	below		
B1	2100	150	500	without opening		79	4D16	2D12	2D10		0.013	7D8 @ 100 mm vertical stirrups
B2				400	100		3D12	2D10				
B3				400	150		2D12	2D10				
B4				400	200		2D12	2D10	2D8	2D8		
B5				800	100		3D12	2D10	2D10			
B6				800	150		2D12	2D10				
B7				800	200		2D12	2D10	2D8	2D8		

D = Rebar diameter (mm units); RFT = Reinforcement

at their two ends. The first end of each bar was welded to a square steel so as not to leave the first U-channel during applying the external prestressing force. For restraining purpose, the second end was welded to high tensile bolt with a nut. Table 3 shows the two bars properties. The first U-channel was located at the first end of each tested beam and the second one was located next to the second end of each beam; enough distance was left between them to put a hydraulic jack. The hydraulic jack is connected to a pump and a dial gauge. The jack was placed between the second U-channel and each tested beam. It was located vertically at the center of the lower chord of each tested beam and connected to a dial gauge to measure the prestressing force. Loading the pump of the hydraulic jack urged the inner cylinder of the jack to push the U-channel and generate compression force on the first side of each beam, simulating the first source of the external prestressing force, and tension force in the two steel bars. Tension force generated in the two bars will be transmitted to the opposite U-channel and resulted in compression force on the second side of each tested beam; simulating the second source of the external prestressing force.

After that, each beam was loaded to failure under two concentrated loads, the distance between them was 600 mm. During loading, all precautions were taken to obtain accurate results. All measurements were recorded for gradual load increments of 50kN, Figs. 1, 2, and 3.

3 Test results and discussion

All recorded results of the tested beams will be presented in this section. That include crack patterns, failure and cracking loads, strains (bottom bars, stirrups, and top bars), and mid-span deflections for the tested beams.

3.1 Cracking load

During the experimental test, cracks propagation was not similar in all specimens. For some beams, after applying the load, flexural cracks initiated first between the two concentrated loads within the middle third of each beam's span. The cracks formed adjacent to each other and perpendicular to the center-line of the beam. Then, shear cracks appeared with increasing the applied load. For other beams, diagonal shear cracks appeared first, within shear span before the appearance of flexural cracks which were observed at a higher load level, Fig. 4.

Beams B2 and B5, in Group H_{100} , showed delayed flexural cracks in comparison with the other beams with openings. While, Beams B2, B3, and B4, in Group L_{400} , showed delayed shear cracks relative to the other beams with openings, Tables 5 and 6.

Fig. 1 Test-setup and used machines

Fig. 2 Typical concrete dimensions and reinforcement details of Beam B4

Fig. 3 Testing of Beam B7

(a) Beam B1

(b) Beam B2

(c) Beam B3

(d) Beam B4

(e) Beam B5

(f) Beam B6

(g) Beam B7

Fig. 4 Crack patterns of the tested beams. **a** Beam B1, **b** Beam B2, **c** Beam B3, **d** Beam B4, **e** Beam B5, **f** Beam B6, **g** Beam B7

From the aforementioned observations, it could be concluded that; the formation of the first flexural crack was largely governed by opening's height, whereas opening's length was the predominant parameter on the formation of the first shear crack.

3.2 Failure load

The significant effect of openings on beams failure loads was very obvious, Table 7. The failure load of Beam B2 to that of the solid beam was 0.785. The ratios of the

Table 5 First cracking shear load

Beam	Group	First cracking shear load, kN
B2	L ₄₀₀	180
B3		170
B4		
B5	L ₈₀₀	130
B6		
B7		120

Table 6 First cracking flexural load

Beam	Group	First cracking flexural load, kN
B2	H ₁₀₀	180
B5		175
B3	H ₁₅₀	130
B6		
B4	H ₂₀₀	125
B7		120

Table 7 Failure loads

Beam	Failure load, kN
B1	420
B2	330
B3	290
B4	255
B5	285
B6	240
B7	215

failure loads of Beams B3, B4, B5, B6, and B7 to that of the solid beam were 0.69, 0.607, 0.678, 0.571, and 0.511, respectively.

3.3 Load–deflection characteristics

Figure 5 shows the load–central deflection curves for the tested beams. In Group L₄₀₀, Beam B2 had higher stiffness compared to Beam B3 and Beam B3 had higher stiffness compared to Beam B4. That is closely related to the opening's height in this group. Thus, as Fig. 5a illustrates and compared to the solid beam, the central deflection of the beams in Group L₄₀₀ increased with increasing the opening's height. The same trend was observed for the beams in Group L₈₀₀, Fig. 5b, where Beam B7 showed higher deflection values in comparison with Beam B6. Also, Beam B6 showed higher deflection values compared with Beam B5. Obviously, because the opening's height is inversely

proportional to beams stiffness, the increase in the opening's height increases the central deflection values.

Following the behavior of the beams in Groups H₁₀₀, H₁₅₀, and H₂₀₀, Fig. 5c, d, and e respectively, it can be observed that; each two beams in the same group showed approximately the same load–deflection behavior, especially at early loading stages; Beams B2 and B5 in Group H₁₀₀, B3 and B6 in Group H₁₅₀, and B4 and B7 in Group H₂₀₀. From this finding, it could be concluded that; the opening's length had an insignificant effect on central deflection compared with the effect of the opening's height, especially at early loading stages.

3.4 Steel strains

3.4.1 Bottom and top bars

The load–bottom and–top bars strains relations are shown in Fig. 6. For the beams in Group L₄₀₀, Fig. 6a, the opening's area had the major effect on strains values of both top and bottom bars; direct proportional. In group L₄₀₀, Beam B4 (having the largest opening area) showed the highest strains values in its bottom and top bars relative to the other beams in this group. Also, Beam B3 showed higher strains values in its bottom and top bars than those in Beam B2. Moreover, the solid Beam B1 showed the minimum strains values among all the other beams in Group L₄₀₀.

The beams in Group L₈₀₀, Fig. 6b, did not follow the same trend as those in Group L₄₀₀. Though, Beams B6 and B7 have larger opening areas than that of Beam B5, they showed lower strains values in their bottom bars than Beam B5, at the same load level. Also Beams B6 and B7 showed the same top bars strains values at the same load level. That is meaning; proportionality is not valid in Group L₈₀₀.

Although Beams B4 and B5 (from two different groups) have the same opening's area, Beam B4 showed higher strains values in its bottom and top bars than those in Beam B5, at the same load level, Fig. 6c. That is because, the opening's height of Beam B4 is higher than that of Beam B5.

Based on all reported data, it could be concluded that; increasing the opening's area increased the values of strains in both bottom and top bars. This was not general, but (based on this study) restricted to beams having opening's length and height up to 0.40 beam's length and, at the same time, 0.20 its height, respectively.

3.4.2 Stirrups

Following the behavior of the beams in Group L₄₀₀, Fig. 7a, Beams B3 and B4 showed almost the same values of strains

Fig. 5 Load–deflection curve of the tested beams

in stirrups at the same load level, in spite of the higher opening's height in Beam B4 than that in Beam B3. A similar result was observed in Group L_{800} , Fig. 7b, where values of stirrups strains in Beams B5 and B6 were almost the same at the same load level. So, it could be concluded that; stirrups strains in the tested beams were not governed by the opening's height.

For the beams in Groups H_{100} and H_{150} , Fig. 7c and d, the stirrups strains values in Beams B5 and B6 were higher than those in Beams B2 and B3, respectively, at the same loading stage. Thus, the opening's length had the predominant effect on the stirrups strains.

The stirrups strains in the beams were negative until the first cracking load which was 200kN in Beam B2 and 150kN in all the other beams, Fig. 7.

Figure 7b illustrates that; Beam B7 failed with negative values of strains in its stirrups. This is due to its failing at an early loading stage; local failure under loading plates.

Based on the above data, it could be concluded that; the opening's length was directly proportional to the stirrups strains, while the opening's height had an insignificant effect on the stirrups strains for the tested beams.

4 Finite element analysis

In order to verify the obtained experimental results, a three-dimensional nonlinear finite element analysis (FEA) based on ANSYS was carried-out. In which, constitutive

Fig. 6 Loads-bottom and-top bars strains curves

models were implemented to represent concrete, rigid plates, and steel bars.

Furthermore, ANSYS was utilized in predicting ultimate moments for more than seventy externally prestressed high strength rectangular concrete beams with central web openings. The obtained results were considered enough to formulate a general equation that can be used in calculating ultimate moment for externally prestressed HSC beams with central web openings.

To analyze the considered beams using ANSYS, the following commands were followed:

- Define Materials,
- Create, Divide, Merge, and Mesh Volumes, subsequently,
- Apply Loads and Boundary conditions, and
- Solve.

In the first step, the used “Element Type”, “Element Real Constants”, and “Materials Properties” must be defined. The

second step is modeling the considered beams, whereas in third step, the applied loads, load steps options, and analysis type are defined. Finally, “Solve” command can be used.

4.1 Element type

Concrete was modeled using a three dimensional element called SOLID65 which is capable of crushing in compression and cracking in tension. It has eight nodes with three translational degrees of freedom (x, y, and z) at each node, Fig. 8. The following basic assumptions should be considered in the element formulation El-Demerdash [25]:

- Cracking is permitted in three orthogonal directions at each integration point.
- If cracking occurs at an integration point, the cracking is modeled through an adjustment of material properties which effectively treats cracking as a smeared band of cracks, rather than discrete cracks.

Fig. 7 Strains in stirrups

- The concrete material is assumed to be initially isotropic.
- All elements must have eight nodes.

Fig. 8 SOLID65 three dimensional element

- The element is nonlinear and requires an iterative solution.
- The Load must be applied slowly to prevent possible fictitious crushing of the concrete before proper load transfer can occur through a closed crack.

Main reinforcement and stirrups were modeled using LINK180. LINK180 is a 3-D spar that is useful in a variety of engineering applications. The element can be used to model trusses, sagging cables, links, springs, and so on. The element is a uniaxial tension-compression element with three degrees of freedom at each node; translations in the nodal x, y, and z directions.

The steel plates at the supports and at loading points of the beam were modeled using SOLID185 to avoid stress concentration problems. This element has eight nodes with three translational degrees of freedom at each node.

4.2 Input data

In order to obtain accurately represent concrete, the SOLID65 element required linear isotropic and multilinear isotropic material properties. Mathematical representation of the stress-strain curve of concrete in compression was used. Based on El-Demerdash [25], material models were defined as shown in Table 8.

Some input data in Table 8 can be defined as follows:

- Uniaxial cracking stress = ultimate uniaxial tensile strength = 5.5 MPa; experimental tests.
- Uniaxial crushing stress = the equivalent compressive strength for the standard cylinders = 79×0.8 equals about 64 MPa.
- According to Kachlakev et al. [26], the shear transfer coefficient represents conditions of the crack face. Its value ranges from 0.0 to 1.0, with 0.0 representing a smooth crack (complete loss of shear transfer) and 1.0 representing a rough crack (no loss of shear transfer).

Table 8 Material models required in ANSYS

Element type	Material-models
SOLID65	Linear isotropic
	Modulus of elasticity 37000 MPa
	Poisson's ratio 0.20 [26]
	Multi Linear Isotropic
	Five coordinates represented the stress–strain curve [24]
	Concrete
	Open shear transfer coefficient 0.20
	Closed shear transfer coefficient 0.80
	Uniaxial cracking stress 5.5 MPa
	Uniaxial crushing Stress 64.0 MPa
SOLID185	Linear Isotropic
	Modulus of elasticity 200000 MPa [26]
	Poisson's ratio 0.30 [25]
LINK180	Linear isotropic
	Modulus of elasticity 200000 MPa
	Poisson's ratio 0.30
	Bilinear isotropic
	Yield stress 360 MPa for all reinforcement and 240 MPa for stirrups
	Tangent modulus 20 MPa

Convergence problems occurred when the shear transfer coefficient for the open crack dropped below 0.2.

- As no deviation of the response occurs with the change of the coefficient, the coefficient for the open crack was set to 0.2 and the coefficient for the closed crack was set to 0.8.
- Tangent modulus can be defined as the slope of a line tangent to the stress–strain curve at a point of interest. Usually its value ranges from 10 to 20 MPa to avoid loss of stability upon yielding.

4.3 Discretization

ANSYS models were the same as experimental specimens, Table 2. All precautions were taken to ensure that the models would behave similar to the experimental beams and in the proper way.

The beam cross section was discretized into vertical strips according to the main bottom and top RFT. Horizontally, the beam volume was discretized into strips according to top and bottom reinforcement, openings locations, reinforcement above and below openings and the place of the two lateral rigid plates, Fig. 9.

To achieve complete bonding at the interface between the concrete and steel bars and to prevent slippage, the "Merge Items" command was used in merging separate entities that have the same location. These items would then be merged into a single entity.

Fig. 9 Discretization of the Solid Beam B1

4.4 Meshing

To obtain accurate results from the SOLID65 element, the use of a small rectangular mesh was recommended. Therefore, the "Volume Sweep" command using hexagonal shape was used to mesh the steel rigid plates, concrete portion, and supports. This properly sets the dimensions of elements in the rigid plates and supports to be consistent with the elements and nodes in the concrete parts of the model. Regarding the steel bars and stirrups, the "Lines Mesh" command was used to mesh the steel reinforcement.

4.5 Loads and boundary conditions

The tested beams and the finite element models had to act similarly. Therefore, boundary conditions were needed. In order to represent the two supports and the four loading plates for each beam, six stiff concrete solid elements SOLID185 were used. Four of them were used as loading plates and the two others were placed at the lower parts of the concrete portion as supports. Hinges and rollers were placed across the centerline of the lower rigid plates to represent the supports.

Horizontal loads that represent external prestressing force acted at five nodes across the centerlines of two rigid plates at the two sides of the beam. On the other hand, the vertical loads were applied across the center lines of two rigid plates placing above the concrete portion. The point load was acting at five nodes on the plate. Meshing, boundary conditions, and loads are as illustrated in Fig. 10.

During the experimental tests in some beams, it was observed a loss occurred in external prestressing force, due to their deflection. This loss was about 15% of external prestressing force. Such a reduction in the external prestressing force was numerically considered in the finite element models.

4.6 Solution strategy

To properly simulate applying the vertical load, external prestressing force, and reduction in prestressing force, Load Step Files, a command in ANSYS-19.2, has been utilized as follows:

1. 50kN horizontal force was applied at the two ends of each modeled beam at the center of its lower chord and stored in Load Step File 1 (LS1) to simulate the external prestressing force.
2. The incremental vertical load accompanied with about 7.5kN horizontal force, as 15% losses, were applied to each modeled beam. The horizontal force was placed at the two ends of each modeled beam at the center of its lower chord but in the opposite direction of the external prestressing force.

Fig. 10 Loads and boundary conditions for beam B7

3. The incremental vertical load and 7.5kN horizontal force were stored together in Load Step File 2 (LS2).
4. "Solve From LS Files" command was used; "Starting LS File Number" and "Ending LS File Number", were chosen (1), and (2), respectively. This procedure forced ANSYS-19.2 to completely apply 50kN external prestressing force first and afterwards, apply the vertical load and the opposite horizontal load in an incremental manner, simultaneously up to failure.

4.7 Results

Based on the FEA, the predicted failure loads for the considered beams P_u^{FEA} were compared with the experimental failure loads $P_u^{Exp.}$, Table 9 and Fig. 11. Also, a reference solid beam having the same characteristics but not prestressed was modeled using ANSYS. The reference Beam B failure load is shown in Table 9. Load–deflection behavior of three tested beams that numerically obtained was compared to that experimentally obtained, Fig. 12.

The above comparison showed that, the mean value of the ratio $(P_u^{Exp.}/P_u^{FEA}) = 0.95$. This percentage demonstrated that; the three dimensional nonlinear FEA provided good prediction of the failure loads for the considered beams. Figure 12 shows a relation between deflection values of beams using FEA and that experimentally obtained.

4.8 Previous studies

To increase the level of assurance in FEA results, ANSYS was used to obtain failure loads for beams tested previously by other researchers. Five HSC deep beams with openings tested by Yoo et al. [9] were modeled using ANSYS. The experimentally obtained failure loads $P_u^{Exp.}$ were compared with the numerical failure loads P_u^{FEA} , Table 10.

Table 9 Numerical and experimental failure loads

Beam	Failure load, kN		$\frac{P_u^{Exp.}}{P_u^{FEA}}$
	Experimental $P_u^{Exp.}$	FEA failure load P_u^{FEA}	
B		370	
B1	420	440	0.95
B2	330	333	0.99
B3	290	290	1.0
B4	255	265	0.963
B5	285	300	0.95
B6	240	270	0.89
B7	215	240	0.895
Mean value			0.95
Standard deviation			0.0426
Coefficient of variation			0.0449

Fig. 11 Failure load comparison

EL-Azab [27] tested experimentally RC beams with openings. Some of the specimens were modeled using ANSYS to obtain the failure loads. The comparison between the experimental failure loads P_u^{Exp} and the predicted FEA failure loads P_u^{FEA} is shown in Table 11.

A three-dimensional nonlinear FEA based on ANSYS was performed by EL-Azab [27] to examine the accuracy of the experimental results. The obtained mean value of the ratio ($P_u^{\text{Exp}}/P_u^{\text{FEA}}$) was 87.3%.

Ghallab et al. [19] examined experimentally five beams to study their behavior. Four beams were externally prestressed, while the fifth one was not. All the beams had a T-section shape. In this paper, ANSYS was utilized to model four of them. The predicted numerical failure load was compared with the experimental failure load [19], Table 12, Fig. 13.

Clearly, the adopted three dimensional nonlinear FEA provided a helpful tool in predicting failure loads for normal strength concrete (NSC) beams, HSC beams, externally prestressed concrete beams, and beams with and without web openings. The FEA models for previously mentioned studies are shown in Fig. 14.

5 Proposed equation

A three dimensional nonlinear FEA based on ANSYS was performed for seventy beams, Fig. 14. The considered beams had the properties shown in Tables 13 and 14. The beams had the same reinforcement ratio (ρ) of 0.013 and some varied parameters such as:

Fig. 12 Experimental and numerical load–deflection curves of three tested beams. **a** Beam B2, **b** Beam B3, **c** Beam B5

Table 10 Failure load comparison [9]

Beam	Experimental failure load [14] $P_u^{\text{Exp.}}$, kN	FEA failure load P_u^{FEA} , kN	$\frac{P_u^{\text{FEA}}}{P_u^{\text{Exp.}}}$
Cat (5) Beam (1)	309	261	0.85
Cat (5) Beam (3)	112	100	0.90
Cat (8) Beam (3)	79.5	90	1.13
Cat (10) Beam (1)	657	615	0.945
Cat (10) Beam (4)	174	208	1.19
Mean value			1.003
Standard deviation			0.148
Coefficient of variation			0.147

Table 11 Failure load comparison [27]

Beam	Experimental failure load P_u^{Exp} , kN	FEA failure load P_u^{FEA} , kN	$\frac{P_u^{FEA}}{P_u^{Exp}}$
DSO4	100	80	0.80
DSO5	220	211	0.96
DSO6	290	226	0.78
DSO7	160	150	0.9375
DSO8	190	178	0.937
Mean value			0.883
Standard deviation			0.084
Coefficient of variation			0.095

Table 12 Failure load comparison [19]

Beam	Experimental failure load P_u^{Exp} , kN	FEA failure load P_u^{FEA} , kN	$\frac{P_u^{FEA}}{P_u^{Exp}}$
Rc	150	136	0.91
PS 1–1	171	150	0.878
PS 1–3	263	230	0.875
PS 2–1	269	237	0.881
Mean value			0.886
Standard deviation			0.016
Coefficient of variation			0.0182

- Opening height and length,
- Concrete compressive strength, and
- External prestressing force.

The obtained numerical results were the basis to propose a general equation, Eq. 1, that can be applied to externally prestressed rectangular HSC beams with web openings. Thus,

$$M_u = 100 \times m^{0.15} \times P^{0.2} \times \left(\frac{f_{cu}}{f_y}\right)^{0.25} \times (n\rho)^{0.3} \times (L - S) \quad (1)$$

where M_u = ultimate moment, kNm, 100 = constant. m = beam length-to-opening length, (L/X) , n = beam height-to-opening height, (H/Y) , P = prestressing force, kN, f_{cu} = concrete compressive strength, MPa, and f_y = main bars yield strength, MPa. ρ = reinforcement ratio. L = beam length, m. S = distance between two concentrated loads = 0.6 m.

The proposed formula, Eq. 1, can be utilized to calculate the ultimate moment of the considered beams, M_u^{Eq} . Ultimate moment obtained by Eq. 1 were compared to

that predicted by FEA, Table 15. The mean value of the ratio M_u^{FEA}/M_u^{Eq} ranged from 95 to 105%.

Table 16 shows a comparison between the experimentally obtained ultimate moment M_u^{Exp} and that predicted by the proposed equation M_u^{Eq} . The mean value of the ratio M_u^{Exp}/M_u^{Eq} is 93%.

Equation 2 can be used to calculate ultimate moment for all the tested beams using their failure load;

$$M_u = \frac{P_u}{2} \times \frac{L - S}{2} \quad (2)$$

where P_u = failure load.

5.1 Features

As shown in Tables 15 and 16 Eq. 1, illustrates high level of accuracy in calculating ultimate moment of the considered beams compared with those experimentally predicted and those obtained by the FEA. Equation 1 benefits and limitations were summarized as follows:

5.2 Benefits

- The proposed equation showed a good level of accuracy in predicting ultimate moment of externally prestressed HSC rectangular beams with central web openings ($\pm 5\%$) compared with those predicted by FEA.
- Eq. 1 showed excellent accuracy in calculating ultimate moment of the beams having concrete compressive strength of 50 and 80 MPa. So, it can be said that; Eq. 1 can be applied to all other beams having the same characteristics and concrete compressive strength varied from 50 to 80 MPa; such as, 55, 60, 65, and 70, etc.
- As the proposed equation succeed in predicting ultimate moments of externally prestressed beams with compression forces of 50, 75, and 100kN, it can be used to accurately predict ultimate moments of externally prestressed beams with any prestressing force ranging from 50 to 100kN.
- Eq. 1 could be applied to externally prestressed HSC beams having various dimensions of the central web opening, Table 13.

5.3 Limitations

- The proposed equation can only be applied to externally prestressed HSC rectangular beams with central web openings.
- Yield strength of the main steel bars in the considered beams was 360 MPa. So that, the change in yield strength may affect the accuracy of the results.

Fig. 13 Finite element models for previous studies. **a** Representation of Yoo et al. specimens [9]. **b** Representation of EL-Azab specimens [27]. **c** Representation of Ghallab specimens [19]

Fig. 14 Some of finite element models. **a** Opening length=400 mm. **b** Opening length=600 mm. **c** Opening length=800 mm. **d** Opening length=1000 mm. **e** Opening length=1200 mm

Table 13 Specimens properties

Central opening		f_{cu} , MPa	Prestressing force, kN
Length, mm	Height, mm		
400	50, 100, 150, 200, 250, and 300	50 and 80	50, 75, and 100
600	100, 150, and 200	50 and 80	50, 75, and 100
800	50, 100, 150, 200, 250, and 300		
1000	100, 150, and 200		
1200	50, 100, 150, 200, 250, and 300		

Table 14 Reinforcement properties

Opening height, mm	Longitudinal reinforcement		Reinforcement around openings		Shear reinforcement
	Bottom	Top	above	below	
50	4D12	2D10	2D10	2D10	7D8 @ 1000 mm vertical stirrups
100	3D12	2D10			
150	2D12	2D10			
200	2D12	2D10	2D8	2D8	
250	2D12	2D10	2D6	2D6	
300	4D8	2D8			

Table 15 Ultimate moment comparison

X/L	Y/H	Opening dimensions, mm		Prestressing force, kN	Concrete compressive strength f_{cu} , MPa					
					80			50		
		X	Y		Ultimate Moment, kNm		$\frac{M_u^{FEA}}{M_u^{Eq.}}$	Ultimate Moment, kNm		$\frac{M_u^{FEA}}{M_u^{Eq.}}$
M_u^{FEA}	$M_u^{eq.}$	M_u^{FEA}	$M_u^{eq.}$							
0.2	0.1	400	50	50	142.8	145.075	0.98	126	129.01	0.98
				75	151.55	157.325	0.96		139.895	
				100	161	166.67	0.97	143.5	148.19	0.97
			100	50	116.55	117.845	0.99	107.8	104.79	1.03
				75	125.3	127.82	0.98		113.645	
				100	132.3	135.38	0.98	119	120.365	0.99
			150	50	101.5	104.335	0.97	93.8	92.785	1.01
				75	108.5	113.19	0.96		100.625	
				100	114.8	119.875	0.96	105	106.575	0.99
	0.4	200	50	92.75	95.725	0.97	86.8	85.12	1.02	
			75	100.45	103.81	0.97		92.295		
			100	105.7	109.935	0.96	100.1	104.055	0.96	
	0.5	250	50	87.5	89.53	0.98	80.5	79.59	1.01	
			75	94.5	97.09	0.97		86.31		
			100	102.55	102.83	1.00	95.55	92.47	1.03	
	0.6	300	50	80.5	84.77	0.95	75.95	75.355	1.01	
			75	87.15	91.91	0.95		81.725		
			100	95.2	97.335	0.98	88.2	86.555	1.02	
0.4	0.1	800	50	50	134.75	130.76	1.03		116.27	
				75	143.5	141.82	1.01		126.07	
				100	150.5	150.22	1.00		133.56	
			100	50	105	106.225	0.99		94.43	
				75	114.8	115.185	1.00		102.41	
				100	121.45	122.01	1.00		108.465	
			150	50	94.5	94.045	1.00	87.5	83.65	1.05
				75	101.85	102.025	1.00		90.685	
				100	108.85	108.045	1.01	100.45	96.075	1.05
	0.4	200	50	84	86.275	0.97		76.685		
			75	93.8	93.555	1.00		83.195		
			100	98	99.085	0.99		88.1125		
	0.5	250	50	77	80.675	0.95	73.5	71.75	1.02	
			75	83.3	87.5	0.95		77.805		
			100	91	92.68	0.98	85.75	82.39	1.04	
	0.6	300	50	72.8	76.405	0.95	126	67.9		
			75	79.1	82.845	0.95		73.675		
			100	85.05	87.745	0.97	143.5	78.05		
0.6	0.1	1200	50	50	126	123.06	1.02	107.8	109.41	
				100	140	141.33	0.99		125.685	
	0.2	100	50	96.25	99.96	0.96	119	88.865		
			100	112.35	114.8	0.98	93.8	102.095		
	0.3	150	50	89.25	88.515	1.01		78.68		
			100	104.3	101.675	1.03	105	90.37		
	0.4	200	50	70	81.165	0.86	Equation results are not good			
			100	84	93.24	0.90				
	0.5	250	50	43.75	75.95	0.58				
	0.6	300	50	28	71.89	0.39				

Table 15 (continued)

X/L	Y/H	Opening dimensions, mm		Prestressing force, kN	Concrete compressive strength f_{cu} , MPa					
					80		50			
		X	Y		Ultimate Moment, kNm		$\frac{M_u^{FEA}}{M_u^{Eq.}}$	Ultimate Moment, kNm		$\frac{M_u^{FEA}}{M_u^{Eq.}}$
					M_u^{FEA}	$M_u^{Eq.}$		M_u^{FEA}	$M_u^{Eq.}$	
0.3	0.2	600	100	50	113.05	110.915	1.02		98.595	
	0.3		150	50	98.7	98.21	1.01		87.29	
	0.4		200	50	91	90.09	1.01		80.08	
0.5	0.2	100	100	50	102.2	102.725	0.99		91.315	
	0.3		150	50	92.4	90.965	1.02		80.85	
	0.4		200	50	82.95	83.44	0.99		74.2	

Table 16 Ultimate moment comparison

Beam	Ultimate moment, kNm		$\frac{M_u^{Exp.}}{M_u^{Eq.}}$
	Experimental $M_u^{Exp.}$	Equation $M_u^{Eq.}$	
B1	147		0.95
B2	115.5	117.6	98.21
B3	101.5	104.3	97.3
B4	89.25	95.72	93.2
B5	99.75	106.2	93.9
B6	84	94	89.3
B7	75.25	86.3	87.2
Mean value			0.93

- For all the considered specimens, the ratio was about 0.013.
- Although the equation could be applied to many beams having various dimensions of the central web opening, all the considered beams had the same overall dimensions
- As shown in Table 15, Eq. 1 dedicated less accuracy in predicting ultimate moments of the specimens having the ratio of opening length-to-beam length of 0.6 and opening height-to-beam height more than 0.3, simultaneously.
- Changing any of the parameters may change the accuracy of the obtained results.

5.4 Recommendations

Combining the effect of external prestressing and openings on the behavior of HSC beams is very important. However, data in this field suffered from a hard lack. So, proposing an equation to predict ultimate moments of externally prestressed HSC rectangular beams with central

web openings is a forward move in the field of structural engineering.

Eventually, to generalize the proposed equation, it is recommended to assimilate more variable characteristics of the tested beams; other reinforcement ratios and changeable main steel yield strengths.

6 Conclusions

Based on the obtained experimental and numerical results of externally prestressed rectangular HSC beams with central web openings, the following conclusions can be drawn:

- In an externally prestressed HSC beam, the presence of central web opening with length of 0.2 of beam's length and height of 0.2 of beam's height decreased its failure load by 21.5% compared with a reference solid beam with the same dimensions and reinforcement ratio. Moreover, the increase in opening's area to about 150%, 200%, 300%, and 400% led to a reduction in the failure load by about 12.2%, 22.7%, 27.3%, and 35%, respectively.
- For the tested beams, beam's stiffness was function of the opening's height. Increasing the opening's height increased the central deflection values.
- The opening's length had a little effect on central deflection values of the tested beams.
- For the tested beams, the opening's length was the parameter that mainly affected the strains values of stirrups, whereas the opening's height had a little effect.
- Increasing the opening's area resulted in an increase in the values of strains in bottom and top bars. This was not general, but depended on the opening's area.

- During applying vertical loads, the external prestressing force was observed to be decreased in some beams by about 15% due to beams deflection. Such a loss should be considered.
- The proposed equation predicted perfectly the ultimate moments of the considered seventy externally prestressed rectangular HSC beams with central web openings compared to those predicted numerically using ANSYS.
- The proposed equation can be applied to a wide domain in which:
 - Concrete strengths range from 50 to 80 MPa.
 - External prestressing forces range from 50 to 100kN.
 - Various opening lengths and heights.
- Enhancing the proposed equation by including more specimens having varied yield strengths and reinforcement ratios is a must.

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Code availability ACI318-08 Building code requirements for structural concrete and commentary. Michigan, USA: American Concrete Institute; 2008 was used in this paper.

Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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