Peristaltic flow of a dusty electrically conducting fluid through a porous medium in an endoscope

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Abstract

This study presents the impact of MHD on peristaltic flow of a dusty fluid in an endoscope. The concentric tube is filled in with an incompressible fluid which is flowing through a porous medium. The outer tube wall is flexible while the inner tube is rigid. Coupled equations for both the solid granules and the fluid have been modelled by supposing long wave-length and low Reynolds number assumptions. Analytically results attained are presented through graphs. The impact of different parameters on velocity, pressure rise and streamlines have been included. It has been concluded that the trapped bolus reduces in the case of α and increases in the case of M .

Keywords Peristaltic flow · MHD · Porous medium · Dusty fluid · Endoscope

1 Introduction

A mechanism that occurs due to tightening and relaxing of muscles, channels, tube or passages is termed as peristalsis. This mechanism occurs in living organisms naturally like movement of food in esophagus, ureters, uterine tubes etc. Development of peristaltic pumps is motivated by this mechanism. These pumps have significant role in many industries as well as in life saving machines. Industries like plastic manufacturing plants, paint industries, paper making factories, petrol refineries etc., utilize such pumps. Peristaltic pumps are installed in many machines like heart-lung machine, dialysis machine to pump the blood without being contaminated. Due to extensive use of this mechanism, many research articles are available to understand it in different situations [1–9].

Dusty fluid can be referred as the fluid with dust particles concentrated in it. Few examples of such fluid can be juices with pulp, paints with glitter, infected urine, raw petrol etc. Dusty fluids under the peristaltic mechanism is useful in paper making factory, paint manufacturing,

dialysis procedure and petroleum purification procedures etc. Few early articles related to this mechanism are [10–12]. Parthasarathy et al. [13] investigated the wall properties effect on the dusty fluid that was induced by magnetic field. Viscoelastic fluid containing dust particles flowing past an inclined passage whose walls were porous as well, was studied by Chakraborty [14]. Khan and Tariq [15] presented a study on dusty Walter's B fluid and examined the wall properties effect on it. By considering an inclined magnetic field and heat transfer, this work was extended in the article [16]. Few recent studies dealing with dusty fluid are [17–19]. Flow through/over a porous medium has great importance due to its immense applications. In living organism lung, small blood vessels and gall bladder have porous surfaces. In pathological conditions, the supply of fatty cholesterol and bulged arteries are examples of porous medium. Elshehawey et al. [20] were the first who started work in porous medium with peristalsis. The peristaltic behavior of elastic porous tubes was examined by Choudhari et al. [21]. Hayat et al. [22] examined the impression of

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slip at the boundaries of a porous passage. Mekheimer and Abd Elmaboud [23] considered as porous annulus to study its impact on the fluid. An endoscope is a useful tool to observe the problems occurring in human organs like small intestine, ureters, stomach etc. The peristaltic movement of fluid under the effect of endoscope has a significant role for medical diagnosis. The interior of human organs can be seen by using this tool. Afsar et al. [21] studied the impact of endoscope on the flow of electrically conducting fluid. Few studies that deal with the endoscope are in references [24–29]. Mekheimer and Elmagboud [30] studied the flow of particle-fluid suspension flowing past an annulus.

a_1 is the radius of the inner tube and outer tube's radius a_2 , c is the wave speed, the time is signified by t , b is the amplitude, wavelength is denoted by λ . To avoid unsteady flow the transformation given below has been taken into account.

$$V = v + c, U = u, R = r, Z = z + ct, \quad (3)$$

where (U, V) and (u, v) represents the velocities in the laboratory and wave frame. The ruling equations of the dusty MHD fluid and the dust granules flowing past a porous tube are given as

$$\frac{\partial v}{\partial z} + \frac{u}{r^2} + \frac{\partial u}{\partial r} = 0, \quad (4)$$

$$\rho \left(u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial z} \right) = \mu \left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) - \frac{\partial p}{\partial r} - \frac{\mu \phi}{k} u + Lr^* (u_p - u), \quad (5)$$

$$\rho \left(u \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial z} \right) = \mu \left(\frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial z^2} + \frac{1}{r} \frac{\partial v}{\partial r} \right) - \frac{\partial p}{\partial z} + Lr^* (v_p - v) - \left(\sigma B_0^2 + \frac{\mu \phi}{k} \right) (v + c), \quad (6)$$

The available literature shows that no effort has been made to examine the effect of endoscope on the peristaltic dusty fluid flowing in porous medium under MHD effect. An endoscope with inner tube rigid and outer tube with peristaltic manner has been considered along with dusty fluid filled in between the gap. Long wavelength approximation has been imposed on the equations to obtain an initial insight of the situation.

2 Mathematical formulation

Considering that viscous dusty incompressible fluid is flowing past a co-axial tube. The space between the inner and outer tube is filled up with the dusty fluid. The inner tube is rigid while the outer tube exhibits the peristaltic behavior. An even applied magnetic field B_0 makes the fluid to be electrically conducting. The induced magnetic field can be ignored for smaller magnetic Reynolds number. Cylindrical polar coordinates $(\bar{R}, \bar{Z})$ represent the radial and the centerline of the tubes. Mathematically two walls can be described as

$$r_1 = a_1, \quad (1)$$

$$r_2 = a_2 + b \sin \frac{2\pi}{\lambda} (z - ct), \quad (2)$$

$$\frac{\partial v_p}{\partial z} + \frac{u_p}{r^2} + \frac{\partial u_p}{\partial r} = 0, \quad (7)$$

$$\rho \left(u_p \frac{\partial u_p}{\partial r} + v_p \frac{\partial u_p}{\partial z} \right) = \frac{r^*}{m} (u - u_p) - \frac{\partial p}{\partial r}, \quad (8)$$

$$\rho \left(u_p \frac{\partial v_p}{\partial r} + v_p \frac{\partial v_p}{\partial z} \right) = \frac{r^*}{m} (v - v_p) - \frac{\partial p}{\partial z}, \quad (9)$$

where fluids density is ρ , the pressure is denoted by p , the strength of the magnetic field is B_0 , r^* is the resistance coefficient for the solid particles, the electric conductivity of the fluid is σ , porosity and permeability are ϕ and k . The number density of the dust granules is considered to be fixed and is denoted by L . The relevant boundary conditions are

$$v = -c, u = 0 \text{ at } r = r_1, \quad (10)$$

$$v = -c \text{ at } r = r_2 = a_2 + b \sin \frac{2\pi}{\lambda} z. \quad (11)$$

The non-dimensional quantities that have been employed are

$$\bar{r} = \frac{r}{a_2}, \bar{r}_1 = \frac{r_1}{a_2} = \frac{a_1}{a_2} = \varepsilon < 1, \bar{z} = \frac{z}{\lambda}, \bar{p} = \frac{a^2 p}{\lambda \mu c}, \bar{v} = \frac{v}{c}, \frac{1}{\alpha} = \frac{\phi a_2^2}{k}$$

$$\bar{u} = \frac{u}{\delta c}, \delta = \frac{a_2}{\lambda}, \theta = \frac{b}{a_2}, \bar{r}_2 = \frac{r_2}{a_2} = 1 + \theta \sin 2\pi z, M^2 = \frac{\sigma B_0^2 a_2^2}{\mu}.$$

Under the long wavelength and low Reynolds number assumption, the system of equations after dropping the bar is given as

$$\frac{\partial v}{\partial z} + \frac{u}{r^2} + \frac{\partial u}{\partial r} = 0, \quad (12)$$

$$\frac{dp}{dr} = 0, \quad (13)$$

$$\frac{dp}{dz} = A(v_p - v) + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2} - \eta^2(v + 1), \quad (14)$$

$$\frac{dp}{dz} = B(v - v_p), \quad (15)$$

with

$$v = -1, u = 0 \text{ at } r = r_1, \quad (16)$$

$$v = -1 \text{ at } r = r_2 = 1 + \theta \sin 2\pi z. \quad (17)$$

where $A = \frac{Lr^* a_2^2}{\mu}$, $B = \frac{r^* a_2}{mc}$ and $\eta^2 = \frac{1}{\alpha} + M^2$.

For the laboratory frame, the rate of flow is specified by

$$\Theta = 2\pi \int_{r_1}^{r_2} V R dR. \quad (18)$$

By inducing Eq. (3) and after integration, the above equation becomes

$$\Theta = q + \pi c(r_2^2 - r_1^2), \quad (19)$$

where the flow rate is q in wave frame and it is described

$$\frac{dp}{dz} = \left(\frac{A}{B} - 1\right)^{-1} \left(\eta^4 (r_2^2 + 2F - \varepsilon^2) (I_0(\eta r_2) K_0(\eta \varepsilon) - I_0(\eta \varepsilon) K_0(\eta r_2)) \right. \\ \left. (-\eta^2 r_2^2 I_2(\varepsilon r_2) K_0(\varepsilon \eta) - \varepsilon^2 \eta^2 I_2(\varepsilon \eta) K_0(\eta r_2) + \eta^2 r_2^2 I_0(\varepsilon \eta) K_2(r_2 \eta) + \varepsilon^2 \eta^2 I_0(r_2 \eta) K_2(\eta \varepsilon) - 4) \right)^{-1}. \quad (28)$$

by

$$q = 2\pi \int_{r_1}^{r_2} v r dr. \quad (20)$$

The time-averaged flow rate is

$$\bar{Q} = \frac{1}{T} \int_0^T \Theta dt. \quad (21)$$

By inserting Eq. (19) into Eq. (21) and integration yields

$$\bar{Q} = q + \pi c \left(a_2^2 - a_1^2 + \frac{b^2}{2} \right). \quad (22)$$

The above equation becomes

$$Q = F + \frac{1}{2} \left(1 - \varepsilon^2 + \frac{\theta^2}{2} \right), \quad (23)$$

where $Q = \frac{\bar{Q}}{2\pi a_2^2 c}$, $F = \frac{q}{2\pi a_2^2 c}$ are the non-dimensional mean flows in laboratory and wave frames respectively.

The dimensionless volume flow rate is

$$F = \int_{r_1}^{r_2} \bar{r} \bar{v} dr. \quad (24)$$

Equations (14) and (15) are solved w.r.t boundary conditions Eqs. (16) and (17) as a modified Bessel equation, to obtain the solution as

$$v(r, z) = C_1 I_0(\eta r) + C_2 K_0(\eta r) - \frac{1}{\eta^2} \left(\left(\frac{A}{B} - 1 \right) \frac{dp}{dz} + \eta^2 \right), \quad (25)$$

where modified Bessel functions of the first and second kinds are I_0 and K_0 . Also,

$$C_1 = \frac{(K_0(\eta \varepsilon) - K_0(\eta r_2)) \left(\frac{A}{B} - 1 \right) \frac{dp}{dz}}{\eta^2 (I_0(\eta r_2) K_0(\eta \varepsilon) - I_0(\eta \varepsilon) K_0(\eta r_2))}, \quad (26)$$

$$C_2 = \frac{(I_0(\eta r_2) - I_0(\eta \varepsilon)) \left(\frac{A}{B} - 1 \right) \frac{dp}{dz}}{\eta^2 (I_0(\eta r_2) K_0(\eta \varepsilon) - I_0(\eta \varepsilon) K_0(\eta r_2))}. \quad (27)$$

The expression for $\frac{dp}{dz}$ is

$$\quad (28)$$

If we consider $A=4$ and $B=2$ then similar results can be obtained as given by Afsar et al. [21].

The expression for the velocity of solid particles can be obtained using

$$v_p(r, z) = v(r, z) - \frac{1}{B} \frac{dp}{dz}. \quad (29)$$

Fig. 1 Velocity profile of the fluid for various values of α with $M=0.4$, $\theta=0.8$, $A=2$, $B=2$, $\varepsilon=0.5$, $Q=0.9$

Fig. 4 The velocity profile of the solid particles for various values of M with $\alpha=0.15$, $\theta=0.8$, $A=2$, $B=2.5$, $\varepsilon=0.5$, $Q=0.3$

Fig. 2 The velocity profile of the fluid for various values of M with $\alpha=0.4$, $\theta=0.4$, $A=2$, $B=1.5$, $\varepsilon=0.2$, $Q=0.9$

Fig. 5 The pressure rise for various of α with $\varepsilon=0.4$, $\theta=0.8$, $A=2$, $B=0.2$, $M=0.4$

Fig. 3 The velocity profile of the fluid for various values of B with $\alpha=0.4$, $\theta=0.4$, $A=2$, $M=0.5$, $\varepsilon=0.2$, $Q=0.9$

Fig. 6 The pressure rise for various values of M with $\alpha=0.4$, $\theta=0.8$, $A=2$, $B=0.2$, $\varepsilon=0.5$

Fig. 7 The pressure rise for various values of B with $\alpha=0.4$, $\theta=0.8$, $A=2$, $M=0.5$, $\epsilon=0.2$

Fig. 8 The pressure rise for various values of ϵ with $\alpha=0.4$, $\theta=0.8$, $A=2$, $B=0.2$, $M=0.5$

3 Discussion

In order to check the validity of the analytical results, we have drawn them graphically as well, to analyze the impact of different parameters on velocity, pressure rise and streamline patterns of the dusty fluid. Figures 1 and 2 show the influence of porosity parameter α and the magnetic field M , on the velocity of the dusty fluid. Porous surface as the ability to absorb the fluid in it efficiently, thus it acts like a resistive force. Figure 1 shows that as the porous parameter is enhanced, the velocity of the dusty fluid declines. Increase in porous parameters means that the soaking in ability of the passage is getting more, thus causing decrease in velocity. Hartmann number is signified by M . Figure 2 shows decrease in the velocity as M is increased. Increase in electromagnetic force causes friction thus we observe drop in the velocity. Velocity decays as the concentration parameter B increases (Fig. 3). Velocity of the

solid particles descends as the magnetic force enhances as demonstrated in Fig. 4.

Impact of α on the pressure rise is shown in Fig. 5. The augmented pumping occurs for $0.1 < Q < 2$. At $Q=0.1$, no pumping occurs. The region of peristaltic pumping is wider for the porous parameter. As the porous parameter is increase, we see drop in the pressure rise. Pressure rise of the fluid under the effect of Hartman number (M) can be seen in Fig. 6. The pressure rises in the pumping region with increase in (M). It can be deduced that as Hartman number is raised an additional pressure will be necessary to pump fluid. The concentration parameter is denoted by B . Increase in concentration parameter indicates that more solid particles are suspended in the fluid thus increased pressure will be essential to pump the fluid (Fig. 7). The radius ratio of the tube is denoted by ϵ . As the radius ratio is enlarged, lesser pressure is exerted by the fluid. Figure 8 shows that impact of increase in ϵ as a reverse influence on the pressure rise.

The streamline patterns for the porous parameter and Hartmann number are given in Figs. 9 and 10. The streamlines in Fig. 9 show the influence of porous parameter. It can be concluded that with the rise in porosity the trapped bolus expands and is pushed outward. It shows that the motion of the fluid is getting even as the bolus is expanding. While for the magnetic parameter (M), the trapped bolus contracts. It means, the motion of the fluid is getting fraught as magnetic parameter causes resistance in the flow.

4 Conclusion

This current analysis describes the peristaltic flow of dusty fluid through a porous annulus. The exact solution for pressure gradient and axial velocity are found. The conclusions that are worth mentioning are as follow:

The porous parameter and magnetic field cause decrease in the velocity of the dusty fluid flowing through an endoscope. Growth in pressure rise is observed for the magnetic parameter M and the concentration parameter B . They stimulate the motor activity of the wall. The trapped bolus inflated by increasing porous parameter. The flow of the fluid gets smoother for the porous parameter. The trapped bolus compresses by increasing in M . Magnetic field parameter acts as a force of friction on the fluid flowing. Thus, the flow of the fluid gets effected and the trapped bolus contracts.

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Fig. 9 Contour graphs of the fluid for various values of α with $M=0.5$, $\theta=0.15$, $A=2$, $B=3$, $\epsilon=0.5$, $Q=1.5$

Fig. 10 Contour graphs of the fluid for various values of M with $\alpha=0.2$, $\theta=0.15$, $A=2$, $B=3$, $\varepsilon=0.5$, $Q=1.5$

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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