Mixed convective flow of an Oldroyd-B nanofluid impinging over an unsteady bidirectional stretching surface with the significances of double stratification and chemical reaction

Iftikhar Ahmad¹ · Iqra Khurshid¹ · Muhammad Faisal¹ · Tariq Javed² · Zaheer Abbas³

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Abstract

The numerous applications of non-Newtonian nanofluids in engineering, geothermal and industrial processes motivated us to formulate a comprehensive model for unsteady 3D flow of an Oldroyd-B nanomaterial. Brownian motion and thermophoresis characteristics are also accommodated through a Buongiorno nanofluid model. Moreover, assessments of thermal stratification, chemical reaction, solutal stratification, mixed convection, and prescribed heat source are also elucidated in the combinations of momentum, energy, and concentration equations. The transformed system of ODEs is tackled by homotopy analysis method. A comparison benchmark for limited cases is also constructed to confirm the correctness of whole parametric exploration. It is scrutinized through present communication that intensifying values of Prandtl factor, unsteady factor, heat distribution indices, thermal stratification constraint, mixed convective parameter, and Deborah number for time retardation diminish the temperature of the nanomaterial. Also, escalating amounts of Lewis number, chemically reactive species, Brownian motion parameter, and solutal stratification constraint reduce the concentration profile.

Keywords Chemical reaction · HAM solution · Mixed convection · Oldroyd-B nanofluid · Prescribed heat distribution · Solutal stratification · Thermal stratification · Unsteady bidirectional stretching

List of symbols

a, b, c	Rates of stretching (s^{-1})	K_r	Reaction rate of species (s^{-1})
C	Nanoparticle concentration (–)	Le	Lewis factor (–)
C_w	Nanoparticle concentration at the wall (–)	N	Buoyancy factor (–)
C_∞	Surrounding nanoparticles concentration (–)	N_b	Brownian movement constraint (–)
$C_{\infty,0}$	Positive constant (–)	N_t	Thermophoresis constraint (–)
c_f	Operative heat ability of base liquid ($m^2 s^{-2} K^{-1}$)	Nu_x	Local Nusselt number (–)
c_p	Heat capability of nanoparticles ($m^2 s^{-2} K^{-1}$)	Pr	Prandtl factor (–)
D_B	Brownian diffusion coefficient ($m^2 s^{-1}$)	Re_x	Local Reynolds number (–)
D_T	Thermophoresis influence ($m^2 s^{-1}$)	r, s	Temperature controlled indices (–)
Gr	Grashof number (–)	S	Unsteady stretching constraint (–)
f, g	Dimensionless velocities (–)	Sh_x	Local Sherwood number (–)
g_e	Gravitational acceleration ($m s^{-2}$)	T	Temperature of the liquid (K)
k	Thermal conductivity ($W m^{-1} K^{-1}$)	T_w	Wall temperature (K)
		T_∞	Far away temperature field (K)

✉ Muhammad Faisal, muhammad.faisal@ajku.edu.pk | ¹Department of Mathematics, Azad Jammu & Kashmir University, Muzaffarabad 13100, Pakistan. ²Department of Mathematics and Statistics, International Islamic University, Islamabad 44000, Pakistan. ³Department of Mathematics, Islamia University of Bahawalpur, Bahawalpur 63100, Pakistan.

$T_{\infty,0}$	Positive constant (K)
t	Time (s)
U_w	Stretching velocity along x -direction (m s^{-1})
V_w	Stretching velocity along y -direction (m s^{-1})
u, v, w	Velocity components (m s^{-1})
x, y, z	Space coordinates (m)

Greek letters

α	Ratio of stretching rates (–)
α_m	Thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)
β_T	Thermal growth coefficient (K^{-1})
β_C	Concentration growth coefficient (–)
β_1	Deborah relation in the sense of relaxation (–)
β_2	Deborah relation in the sense of retardation (–)
γ	Chemical reaction constraint (–)
ε_1	Thermal stratification constraint (–)
ε_2	Solutal stratification constraint (–)
η	Similarity variable (–)
θ	Dimensionless temperature (–)
λ	Mixed convection constraint (–)
λ_1	Relaxation time (s)
λ_2	Retardation time (s)
μ	Liquid dynamic viscosity ($\text{kg m}^{-1} \text{s}^{-1}$)
ϑ	Kinematic viscosity of the liquid ($\text{m}^2 \text{s}^{-1}$)
ρ	Density of the liquid (kg m^{-3})
ϕ	Similarity function for concentration (–)

1 Introduction

In the current century, the development of human society extremely depends upon nanotechnology. Nanofluid plays a fundamental role in the development of industrial sector. A new variety of energy transport fluid is well-known as nanomaterial, which is basically the mixture of base fluid and nanoparticles. Higher thermal conductivity is the basic requirement of many industrial and engineering applications. Thermal enactment of heat carrying liquid can be increased by submerging the nanoparticles into the base fluids. These supremacies of nanomaterials make them applicable in several progressions of heat transport like microelectronics, fuel cells, heat exchangers system, hybrid powered engines, refrigeration, cooling chambers, etc. In past few decades, various nanofluid models have been utilized to upturn the thermal conductivity of the materials but special preference is given to Buongiorno model [1]. In Buongiorno's model, the special characteristics of Brownian motion and thermophoresis are utilized to meet the requirements of industrial and engineering sectors. Applications of nanofluids in solar water heating, engine cooling, coolings of transformer oil and electronics, improving heat transport efficiency of chillers, in nuclear reactor, and defense, etc. are explained by Saidur et al.

[2]. Gireesha et al. [3] presented a numerical exploration on influence of suspended nanoparticles on boundary regime flow and heat transport aspects of a nanoliquid immersed in a porous space over a stretchable surface. The influences of Brownian motion and thermophoresis characteristics are scrutinized by Khan et al. [4] in the flows of nanomaterial impinging on an expandable surface, numerically. Khan et al. [5] also scrutinized the nanofluid flow upon a linear porous bi-directionally stretchable surface. Hayat et al. [6] explored the study of nanofluid flow due to bi-directionally expandable surface depending on heat and mass fluxes restrictions. Recently, Ahmad et al. [7] illustrated the time-dependent nanofluid flow impinging on a bi-directionally extendable surface with adjustable heat mechanism. Ahmad et al. [8] also scrutinized the nanofluid flow impinging on an unsteady bi-directionally extendable surface with predefined surface temperature and predefined surface heat flux in the consequence of Cattaneo–Christov double diffusion. Some latest contributions in this direction are also made by many investigators [9–15] under various aspects of nanofluid flow phenomena.

The energy conversion problems of Oldroyd-B nanoliquid flow due to stretchable surface have dominant role in the engineering, and industrial processes. Contribution of these materials arises in extrusion of polymer sheet, condensation processes, resonance imaging, heat exchanger devices, and many more. Khan et al. [16] illustrated the Oldroyd-B nanomaterial flow impinging on a bi-directionally stretchable obstacle with internal heat generator/absorber. After that, Hayat et al. [17] scrutinized the flow of an Oldroyd-B nanomaterial over a linearly bi-directionally extendable surface with the significance of magnetic coating, analytically. The influence of thermophoresis mechanism on an unsteady Oldroyd-B nanomaterial flow impinging on a linearly stretchable obstacle is critically analyzed by Awad et al. [18]. The influence of variable thermal conductivity on the flow of an Oldroyd-B nanoliquid impinging on an unsteady stretchable obstacle is investigated by Motsa and Ansari [19]. Radiative flow of an Oldroyd-B nanoliquid impinging on a stretchable obstacle is analytically analyzed by Hayat et al. [20]. Flow plus heat transport of an Oldroyd-B nanoliquid impinging on an unsteady stretchable obstacle are investigated by Zhang et al. [21]. Stagnated flow of an Oldroyd-B nanoliquid towards an unsteady stretchable obstacle with the performances of mixed convection, convective conditions, and thermal radiation is explained by Hayat et al. [22]. Shehzad et al. [23] explored the bi-directional stretched Oldroyd-B nanomaterial impinging on a radiative obstacle with the characteristic of magnetite field. An analytical attempt is made by Hayat et al. [24] to explore the Oldroyd-B nanoliquid flow induced by a stretchable surface with the performance of

internal heat generator/absorber. DTM-Padé solution for bi-directional flow of an Oldroyd-B nanoliquid due to a linearly stretchable obstacle is scrutinized by Gupta et al. [25].

Stratification is a conventional phenomenon which arises due to the presence of different liquids with different densities, variation in temperatures differences, or changes in concentration differences. This type of phenomenon is extremely used in geophysical flows and attains a considerably height in recent years. Such flows can be seen in ground water reservoirs system, thermal energy storage devices, rivers and seas, and lakes, etc. Husain et al. [26] studied the performance of double stratification in mixed convective flow of Maxwell nanoliquid impinging on a radiative stretchable obstacle. The enactments of mixed convection and internal heat generator/absorber on double stratified flow of Maxwell nanomaterial impinging on a linearly stretchable obstacle are analyzed by Abbasi et al. [27]. Waqas et al. [28] described the flow of an Oldroyd-B nanoliquid with the characteristics of stratification and internal heat generator over an expandable surface. Hayat et al. [29] also analyzed the influences of thermal and solutal stratifications in mixed convective Oldroyd-B nanomaterial flow impinging on a steady bi-directionally expandable obstacle with chemical reaction. Hashmi et al. [30] illustrated the mixed convection flow of an Oldroyd-B nanoliquid generated by stretchable disk with the influence of magnetic environment. The study of an Oldroyd-B nanoliquid flow impinging on a stretchable obstacle is carried out by many investigators [31–35] under the influences of Brownian motion, thermophoresis, heat generator/absorber, and chemically reacted species. Some more extraordinary explorations about the advancements in the flows of nanofluid are explored by the investigators [36–45].

In the view of over-head literature cited, it is noted that there exist a slab for the investigation of mixed convective flow of an Oldroyd-B nanofluid impinging on an unsteady bi-directionally expandable surface. Moreover, the performances of chemical reaction, solutal stratification, thermal stratification, Brownian motion, and thermophoresis are also incorporated. The novel characteristic of this exploration is that we considered the unsteady flow of an Oldroyd-B nanomaterial with the prescribed distribution of heat, which is not reported previously to the best of our information. Boundary layer approximations are used to reshape the modeled equations of bidirectional flow of an Oldroyd-B nanofluid before transforming into dimensionless forms. Furthermore, analytical solution of dimensionless system of non-linear ODEs is made by employing homotopy analysis method (HAM) [46–49]. Impressions of

sundry parameters on temperature and nanoparticles concentration distributions are plotted and discussed carefully. It is expected that this marvelous addition in the field of non-Newtonian nanofluid flow will not only attract its readers but also will be a wonderful spectrum for future endeavors.

2 Modeling

Consider the mixed convective Oldroyd-B nanomaterial flow impinging on an unsteady bi-axially stretchable sheet. The surface moves along x -direction with stretching velocity $U_w = \frac{ax}{1-ct}$. It moves with the stretching velocity $V_w = \frac{by}{1-ct}$ along y -direction, whereas the region $0 < z < \infty$ is engaged by an Oldroyd-B nanomaterial as portrayed in Fig. 1. The effectiveness's of thermal and solutal stratifications are also added through thermal and concentration surface conditions, respectively. Moreover, the effects of chemical reactive species are studied jointly with the concentration equation. The uniqueness of this investigation is that the surface temperature is maintained through a prescribed heat mechanism rather than any arbitrary temperature of the surface. The novel two phase nanomaterial relation has been adopted for the inspection of heat plus mass transportations. Moreover, the gravitational force g_e is acted parallel to x -axis with opposite direction. As a result, only x -component has been appeared in the momentum equation. Boundary layer estimations are used to reshape the mass, momentum, energy, and concentration equations under the view of over-head assumptions and are described as follows (see Refs [16, 17, 19, 25, 29]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

Fig. 1 Visualization of Oldroyd-B nanofluid flow model

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \lambda_1 \left(\frac{\partial^2 u}{\partial t^2} + 2u \frac{\partial^2 u}{\partial x \partial t} + 2v \frac{\partial^2 u}{\partial y \partial t} + 2w \frac{\partial^2 u}{\partial z \partial t} + u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + w^2 \frac{\partial^2 u}{\partial z^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} + 2vw \frac{\partial^2 u}{\partial y \partial z} + 2uw \frac{\partial^2 u}{\partial x \partial z} \right) \\ = \vartheta \frac{\partial^2 u}{\partial z^2} + \vartheta \lambda_2 \left(\frac{\partial^3 u}{\partial t \partial z^2} + u \frac{\partial^3 u}{\partial x \partial z^2} + v \frac{\partial^3 u}{\partial y \partial z^2} + w \frac{\partial^3 u}{\partial z^3} - \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial z^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial z^2} - \frac{\partial u}{\partial z} \frac{\partial^2 u}{\partial z^2} \right) + g_e (\beta_T (T - T_\infty) + \beta_C (C - C_\infty)), \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \lambda_1 \left(\frac{\partial^2 v}{\partial t^2} + 2u \frac{\partial^2 v}{\partial x \partial t} + 2v \frac{\partial^2 v}{\partial y \partial t} + 2w \frac{\partial^2 v}{\partial z \partial t} + u^2 \frac{\partial^2 v}{\partial x^2} + v^2 \frac{\partial^2 v}{\partial y^2} + w^2 \frac{\partial^2 v}{\partial z^2} + 2uv \frac{\partial^2 v}{\partial x \partial y} + 2vw \frac{\partial^2 v}{\partial y \partial z} + 2uw \frac{\partial^2 v}{\partial x \partial z} \right) \\ = \vartheta \frac{\partial^2 v}{\partial z^2} + \vartheta \lambda_2 \left(\frac{\partial^3 v}{\partial t \partial z^2} + u \frac{\partial^3 v}{\partial x \partial z^2} + v \frac{\partial^3 v}{\partial y \partial z^2} + w \frac{\partial^3 v}{\partial z^3} - \frac{\partial v}{\partial x} \frac{\partial^2 v}{\partial z^2} - \frac{\partial v}{\partial y} \frac{\partial^2 v}{\partial z^2} - \frac{\partial v}{\partial z} \frac{\partial^2 v}{\partial z^2} \right), \end{aligned} \quad (3)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_m \frac{\partial^2 T}{\partial z^2} + \frac{(\rho C)_p}{(\rho C)_f} \left(D_B \left(\frac{\partial T}{\partial z} \frac{\partial C}{\partial z} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial z} \right)^2 \right), \quad (4)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = D_B \frac{\partial^2 C}{\partial z^2} + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial z^2} \right) - K_r (C - C_\infty). \quad (5)$$

The boundary conditions related to the system of Eqs. (1)–(5) are [7, 8, 29]:

$$\begin{cases} u = \frac{ax}{1-ct}, v = \frac{by}{1-ct}, w = 0, T = T_w = T_{\infty,0} + \frac{M_1 x^r y^s}{1-ct}, & \text{at } z = 0, \\ C = C_w = C_{\infty,0} + \frac{N_1 x^r y^s}{1-ct}, & \text{at } z = 0, u \rightarrow 0, v \rightarrow 0, \text{ as } z \rightarrow \infty, \\ T \rightarrow T_\infty = T_{\infty,0} + A_1 \frac{x^r y^s}{1-ct}, C \rightarrow C_\infty = C_{\infty,0} + B_1 \frac{x^r y^s}{1-ct}, \frac{\partial u}{\partial z} \rightarrow 0, \frac{\partial v}{\partial z} \rightarrow 0, & \text{as } z \rightarrow \infty, \end{cases} \quad (6)$$

where x, y and z symbolize for the space coordinates, respectively; $(u, v$ and $w)$ epitomize the velocity components along $(x-, y-$ and $z-)$ directions, respectively; t represents the time; λ_1 symbolizes for the relaxation time; λ_2 symbolizes for the retardation time; $\vartheta = \mu/\rho$ symbolizes for the kinematic viscosity; μ symbolizes for the dynamic viscosity; ρ represents for the density; g_e stands for gravitational attraction; β_T symbolizes for the thermal growth coefficient; β_C symbolizes for the concentration growth coefficient; T stands for temperature; C stands for concentration; α_m describes the thermal diffusivity; D_B epitomizes

for the Brownian coefficient; D_T epitomizes for the thermophoresis coefficient; $(\rho C)_p$ represents the operative heat distribution of nanoliquid; $(\rho C)_f$ epitomizes the operative heat distribution of the Oldroyd-B liquid; K_r epitomizes the response rate of species; (r, s) are temperature controlled

indices; (a, b, c) are the stretching rates; T_∞ epitomizes for the faraway temperature with faraway concentration C_∞ , T_w epitomizes for the surface temperature with suggested concentration C_w , and $(N_1, M_1, A_1, B_1, T_{\infty,0}, C_{\infty,0})$ are all positive constants. The locally similar transformations for Eqs. (1)–(6) are described as follows (see Refs. [7, 8]):

$$\left. \begin{aligned} u &= \frac{ax}{1-ct} f'(\eta), v = \frac{ay}{1-ct} g'(\eta), w = -\left(\frac{a\vartheta}{1-ct} \right)^{\frac{1}{2}} (f + g), \\ \theta(\eta) &= \frac{T - T_\infty}{T_w - T_{\infty,0}}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_{\infty,0}}, \eta = \left(\frac{a}{\vartheta(1-ct)} \right)^{\frac{1}{2}} z. \end{aligned} \right\} \quad (7)$$

Mass-conservation Eq. (1) is robotically contented and the continuing Eqs. (2)–(5) are distorted into the following arrangements as:

$$Nu_x = -(Re_x)^{1/2} \theta'(0), Sh_x = -(Re_x)^{1/2} \phi'(0). \quad (13)$$

$$f''' - S\left(f' + \frac{\eta}{2}f''\right) - (f')^2 + (f+g)f'' - \beta_1\left((f+g)^2f''' - 2(f+g)f'f'' + S^2\left(2f' + \frac{7\eta}{4}f'' + \frac{\eta^2}{4}f''' \right) + S\{2f'^2 + \eta f'f'' - (f+g)(3f'' + \eta f''')\}\right) + \beta_2((f'' + g'')f'' - (f+g)f'''' + S(\eta f'''' + 2f''')) + \lambda(\theta + N\phi) = 0, \quad (8)$$

$$g''' - S\left(g' + \frac{\eta}{2}g''\right) - (g')^2 + (f+g)g'' - \beta_1\left((f+g)^2g''' - 2(f+g)g'g'' + S^2\left(2g' + \frac{7\eta}{4}g'' + \frac{\eta^2}{4}g''' \right) + S\{2g'^2 + \eta g'g'' - (f+g)(3g'' + \eta g''')\}\right) + \beta_2((f'' + g'')g'' - (f+g)g'''' + S(\eta g'''' + 2g''')) = 0, \quad (9)$$

$$\theta'' + Pr\left((f+g)\theta' - (rf' + sg')\theta - S\left(\theta + \frac{\eta}{2}\theta'\right) + N_b\theta'\phi' + N_t\theta'^2\right) = 0, \quad (10)$$

$$\phi'' + LePr\left((f+g)\phi' - (rf' + sg')\phi - S\left(\phi + \frac{\eta}{2}\phi'\right) - \gamma\phi\right) + \frac{N_t}{N_b}\theta'' = 0, \quad (11)$$

and the relevant conditions are

$$\left. \begin{aligned} f(0) = 0, g(0) = 0, f'(0) = 1, g'(0) = \alpha, \theta(0) = 1 - \varepsilon_1, \phi(0) = 1 - \varepsilon_2, \\ f'(\infty) \rightarrow 0, g'(\infty) \rightarrow 0, f''(\infty) \rightarrow 0, g''(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0, \end{aligned} \right\} \quad (12)$$

where f, g, θ and ϕ epitomize the similarity functions for flow fields, temperature spreading, and concentration spreading, respectively; η epitomizes the independent variable; $S = c/a$ represents the unsteady stretching constraint; $\beta_1 = \lambda_1 a$ relates for the Deborah expression in term of relaxation; $\beta_2 = \lambda_2 a$ communicates for the Deborah expression in term of retardation; $\lambda = Gr/Re_x^2$ leads to mixed convection constraint where $Gr = g_e \beta_T (T_w - T_{\infty,0}) x^3 / \vartheta^2$ symbolizes for the Grashof number and $Re_x = x U_w / \vartheta$ represents the Reynolds number; $Pr = \vartheta / \alpha_m$ leads to Prandtl number; $N_b = (\rho c)_p D_b (C_w - C_{\infty,0}) / \vartheta (\rho c)_f$ states the Brownian motion constraint; $N_t = (\rho c)_p D_T (T_w - T_{\infty,0}) / T_{\infty} \vartheta (\rho c)_f$ states the thermophoretic constraint; $Le = \alpha_m / D_b$ leads to Lewis number; $\gamma = K_r / a$ describes the chemically reacted constraint; $\alpha = b/a$ represent the stretching ratio constraint; $N = \beta_c (C_w - C_{\infty,0}) / \beta_T (T_w - T_{\infty,0})$ leads to buoyancy ratio constraint; $\varepsilon_1 = A_1 / M_1$ symbolizes for the thermal stratification constraint; $\varepsilon_2 = B_1 / N_1$ symbolizes for the solutal stratification constraint, and differentiation with respect to η is described through Lagrange's notation. The reduced Nusselt number Nu_x is calculated to scrutinize the rate of heat transferal, whereas the reduced Sherwood number Sh_x is calculated to scrutinize the rate of mass transferal and can be expressed as [9–11]:

Fig. 2 Flow chart of homotopy analysis method (HAM) procedure

Fig. 3 Profile of h -curves at 16th order of approximations

Table 1 Convergence of HAM solution for various orders of approximations for suitable amounts of involved parameters $\lambda = \varepsilon_1 = \varepsilon_2 = 0.3, \alpha = 0.1, \gamma = \beta_1 = \beta_2 = N = N_t = 0.2, Pr = Le = r = s = 1.0, N_b = S = 0.5$ and $h = -0.7$

Order of approximations	$-f''(0)$	$-g''(0)$	$-\theta'(0)$	$-\phi'(0)$
5	1.05352	0.08487	0.73916	0.77012
10	1.05341	0.08489	0.73882	0.77613
11	1.05340	0.08489	0.73882	0.77638
15	1.05340	0.08489	0.73882	0.77674
19	1.05340	0.08489	0.73882	0.77679
20	1.05340	0.08489	0.73882	0.77679
40	1.05340	0.08489	0.73882	0.77679

3 Method for solution

It is noted that the Eqs. (8)–(11) are extremely non-linear and coupled ODEs. The exact/closed form solution of this system is difficult to obtain yet, therefore, we preferred for the analytical solution. The well-known analytical method, specifically, homotopy analysis method (HAM) is used to find the solution of the present boundary value problem. This approach is highly convergent, easy to understand, totally analytic, free from small/large parameters, and easy to program. This method has advantages over routine numerical methods in the sense that it is free of rounding of errors which arise due to discretization procedure. Additionally, it does not require large computer memory and time. The comprehensive procedure to implement this method is described by Liao [47] and here it is presented in the form of flow chart (see Fig. 2). This method is successfully applied in various non-linear engineering and industrial problems. Following initial guesses for velocity

Fig. 4 **a** Performance of Deborah number β_1 on dimensionless temperature $\theta(\eta)$ distribution, **b** performance of Deborah number β_1 on dimensionless concentration $\phi(\eta)$ distribution

fields $f'(\eta)$ and $g'(\eta)$; temperature field $\theta(\eta)$ and concentration field $\phi(\eta)$ are suggested through boundary conditions via Eq. (12) as:

$$\left. \begin{aligned} f_0(\eta) &= 1 - \exp(-\eta), g_0(\eta) = \alpha(1 - \exp(-\eta)), \\ \theta_0(\eta) &= (1 - \varepsilon_1)\exp(-\eta), \phi_0(\eta) = (1 - \varepsilon_2)\exp(-\eta), \end{aligned} \right\} \quad (14)$$

and the auxiliary linear operators for Eqs. (8)–(11) are

$$\mathcal{L}_f = f''' - f', \mathcal{L}_g = g''' - g', \mathcal{L}_\theta = \theta'' - \theta, \mathcal{L}_\phi = \phi'' - \phi. \quad (15)$$

3.1 Convergence analysis

After modeling of the present model and implementation of flow chart (see Fig. 2), our first aim was to predict the missing boundary conditions which are recovered through

Fig. 5 **a** Performance of Deborah number β_2 on dimensionless temperature $\theta(\eta)$ distribution, **b** performance of Deborah number β_2 on dimensionless concentration $\phi(\eta)$ distribution

Fig. 6 **a** Performance of mixed convection constraint λ on temperature $\theta(\eta)$ distribution, **b** performance of mixed convection constraint λ on concentration $\phi(\eta)$ distribution

$\hbar$ -curves as shown in Fig. 3. Through these $\hbar$ -curves, we can easily predict the convergence region of the final solution. Through Fig. 3, it is detected that the admissible region for the missing boundary conditions can be seen where the section of curves are parallel to the horizontal axis. The top curve describes the plot of $g''(0)$ and its admissible region is $-1.5 \leq \hbar_g \leq -0.2$. The admissible region for the plot of $f''(0)$ is $-1.4 \leq \hbar_f \leq -0.1$. It is detected that the left corner of this curve is bending downward, whereas the right corner of this curve is bending up-ward. The admissible regions for $\theta'(0)$ and $\phi'(0)$ are the same and the curve bending is also identical. It is detected that the convergence region for $\theta'(0)$ and $\phi'(0)$ are in the range $-1.5 \leq (\hbar_\theta, \hbar_\phi) \leq -0.1$. So, throughout present analysis we choose $\hbar_g = \hbar_f = \hbar_\theta = \hbar_\phi = -0.7$. Furthermore, the convergence of the solution is generated and the outcomes are presented through Table 1 for apposite amounts of engaging parameters. It is detected via Table 1 that less number of iterations are required to obtain the missing

boundary conditions $f''(0)$ and $g''(0)$, while some more iterations are required to attain the missing boundary conditions $\theta'(0)$ and $\phi'(0)$. For the stability and consistency of the solution number of iterations is increased up-to forty, and it is noted that the results are stable and consistent.

4 Results and discussion

In this subdivision of the article, the performances of the involved constraints on temperature distribution, concentration distribution, amount of heat transportation and amount of mass transportation are briefly analyzed. Firstly, graphical results for temperature and concentration spreadings are arranged through various plots (see Figs. 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14) against various choices of involved parameters. After that the tabular results for heat and mass transports are discussed in detail. Lastly, a comparison for amount of heat and mass transports is

Fig. 7 **a** Performance of thermal stratification constraint ε_1 on temperature $\theta(\eta)$ distribution, **b** performance of solutal stratification constraint ε_2 on concentration $\phi(\eta)$ distribution

Fig. 8 **a** Performance of temperature controlled index r on temperature $\theta(\eta)$ distribution, **b** performance of temperature controlled index r on concentration $\phi(\eta)$ distribution

established to validate the mathematical formulation and applied methodology.

4.1 Thermal and concentration profiles

In this portion, the performances of Deborah numbers (β_1, β_2), temperature controlled indices (r, s), mixed convection constraint λ , chemical reaction parameter γ , unsteadiness constraint S , thermal stratification constraint ε_1 , solutal stratification constraint ε_2 , Prandtl number Pr , Lewis number Le , Brownian factor N_b , and thermophoresis constraint N_t on temperature $\theta(\eta)$ and concentration $\phi(\eta)$ spreadings are analyzed through various plots.

Figure 4a illustrates the inspiration of β_1 on temperature $\theta(\eta)$ spreading, whereas Fig. 4b describes the impact of β_1 on concentration $\phi(\eta)$ distribution. An improvement in the temperature as well as in the concentration distributions is observed for larger amounts of β_1 . The thicknesses

of thermal and concentration layers are identical. These changes in temperature and concentration distributions are obtained due to the increments in the relaxation time and rate of stretching along horizontal direction. Figure 5a demonstrates the consequence of β_2 on temperature $\theta(\eta)$ spreading, while Fig. 5b expresses the impact of β_2 on concentration $\phi(\eta)$ profile. A reduction in the temperature, as well as concentration distributions is perceived for superior amounts of β_2 . The widths of thermal and concentration layers are alike. These vicissitudes in temperature and concentration deliveries are attained due to growths in retardation time and rate of stretching along horizontal direction. Figure 6a describes the stimulation of mixed convective constraint λ on temperature $\theta(\eta)$ supply, whereas Fig. 6b demonstrates the influence of λ on concentration $\phi(\eta)$ distribution. A downfall in thermal and solutal distributions is occurred for higher amounts of λ . These extraordinary results are achieved due to larger gravitational pull and higher amount of Grashof number

Fig. 9 **a** Performance of temperature controlled index s on temperature $\theta(\eta)$ distribution, **b** performance of temperature controlled index s on concentration $\phi(\eta)$ distribution

Gr. The thicknesses of thermal and concentration layers are very small and identical to each other. Figure 7a clarifies the influence of thermal stratification constraint ε_1 on temperature $\theta(\eta)$ spreading, while Fig. 7b demonstrates the impact of solutal stratification ε_2 on concentration $\phi(\eta)$ field. Through Fig. 7a it clarified that temperature lessens for greater amounts of ε_1 . Moreover, the classical thermal results are achieved when $\varepsilon_1 = 0$. It is sensed via Fig. 7b that concentration of nanoparticles reduces due to enrichment in the amounts of ε_2 . Furthermore, the results of classical concentration boundary conditions are achieved by substituting $\varepsilon_2 = 0$. Figure 8a illuminates the performance of temperature controlled index r along x -direction for temperature $\theta(\eta)$ distribution, whereas Fig. 8b explains the performance of r on concentration distribution $\phi(\eta)$. It is detected through Fig. 8a that negative amounts of r produce more thermal heat than positive amounts of r . The results of arbitrary surface heat are achieved when $r = 0$. Besides, the width of thermal layer is dominant for higher amounts of r . It is observed through Fig. 8b that

Fig. 10 **a** Performance of unsteady constraint S on temperature $\theta(\eta)$ distribution, **b** performance of unsteady constraint S on concentration $\phi(\eta)$ distribution

higher amounts of r contracts the nanoparticles concentration. Furthermore, the concentration level is very higher for negative and small amounts of r . Also, the thickness of concentration boundary is similar to that of thickness of thermal layer. Figure 9a provides the enactment of temperature distribution index s along y -direction for temperature $\theta(\eta)$ profile, while Fig. 9b explicates the concert of s on concentration distribution $\phi(\eta)$. It is identified through Fig. 9a that deleterious amounts of s create more thermal heat than progressive amounts of s . The results of arbitrary surface heat are achieved when $s = 0$. Besides, the width of thermal layer is not dominant for higher amounts of s . It is observed through Fig. 9b that higher amounts of s contracts the nanoparticles concentration. Furthermore, the concentration level is very higher for negative and small amounts of s . Also, the thickness of concentration boundary is similar to that of thickness of thermal layer. Figure 10a explains the impact of unsteady constraint S on temperature $\theta(\eta)$ distribution, while Fig. 10b clarifies the inspiration of S on concentration $\phi(\eta)$ spreading. It

(a)

(a)

(b)

(b)

Fig. 11 **a** Performance of thermophoresis constraint N_t on temperature $\theta(\eta)$ distribution, **b** performance of thermophoresis constraint N_t on concentration $\phi(\eta)$ distribution

Fig. 12 **a** Performance of Brownian constraint N_b on temperature $\theta(\eta)$ distribution, **b** performance of Brownian constraint N_b on concentration $\phi(\eta)$ distribution

is deduced through Fig. 10a that temperature of the liquid reduces for larger amounts of S . As S is the ratio of expanding rates, therefore, an escalation in the amounts of S produces high rate of unsteady stretching c . Moreover, the results of steady state case are achieved when $c = 0$. It is noticed through Fig. 10b that nanoparticle concentration reduces against larger amounts of S . The widths of thermal and solutal layers are identical to each other for higher choices of S . Figure 11a styles the performance of thermophoresis constraint N_t on temperature profile whereas Fig. 11b illustrates the impact of N_t on concentration spreading $\phi(\eta)$. It is deduced via Fig. 11a that temperature augments for greater amounts of N_t , but the thickness of thermal layer is very small in this case. This type of result is achieved due to positive tendency in the thermophoretic coefficient. It is perceived via Fig. 11b that concentration spreading boosts for higher amounts of N_t . The thickness of concentration layer is relatively dominant to that of thermal layer thickness. These results are

achieved due to positive tendencies in the temperature difference, thermophoresis diffusion coefficient, and heat capacity of nanoparticles. Figure 12a designates the depiction of Brownian constraint N_b on temperature distribution, while Fig. 12b clarifies the influence of N_b on concentration distribution $\phi(\eta)$. It is comprehended through Fig. 12a that temperature augments for greater amounts of N_b , but the thickness of thermal layer is very small in this case. This type of result is achieved due to positive tendency in the Brownian diffusion coefficient. It is observed through Fig. 12b that concentration spreading declines for higher amounts of N_b . The thickness of concentration layer is relatively dominant to that of thermal layer thickness. These results are achieved due to positive tendencies in the nanoparticle concentration difference, Brownian diffusion coefficient, and heat capacity of nanoparticles. Figure 13a describes the impact of Pr on temperature $\theta(\eta)$ distribution, whereas Fig. 13b illustrates the role of Pr on concentration distribution $\phi(\eta)$. It is observed that both

Fig. 13 **a** Performance of Prandtl number Pr on dimensionless temperature $\theta(\eta)$ distribution, **b** performance of Prandtl number Pr on dimensionless concentration $\phi(\eta)$ distribution

Fig. 14 **a** Performance of Lewis number Le on dimensionless temperature $\theta(\eta)$ distribution, **b** performance of chemical reaction constraint γ on concentration $\phi(\eta)$ distribution

temperature and concentration profiles behave alike for greater amounts of Pr . Moreover, the thicknesses of both thermal and concentration layers are identical in magnitude. These results are achieved because thermal diffusivity decreases for larger amounts of Prandtl number. Due to the declination in the thermal diffusivity the amount Brownian constraint reduces, as a result temperature and concentration decline. Figure 14a elucidates the impact of Le on concentration $\phi(\eta)$ distribution, however Fig. 14b demonstrates the impact of γ on concentration distribution $\phi(\eta)$. It is observed through Fig. 14a that concentration of nanoparticles reduces for larger amounts of Lewis number Le . The width of concentration layer also reduced against higher amounts of Le . These results are achieved because Brownian constraint reduces for large amounts of Lewis number. As a result, the concentration of nanoparticles is reduced.

4.2 Heat and mass transports

In this particular section, the amount of heat transportation $-\theta'(0)$ as well as amount of mass transportation $-\phi'(0)$ are discussed, numerically, against various amounts of temperature controlled indices (r, s), unsteady constraint S , thermal stratification constraint ε_1 , solutal stratification constraint ε_2 , Deborah numbers (β_1, β_2), ratio of expanding rates α , and mixed convection constraint λ through Tables 2 and 3.

Table 2 describes that amount of heat as well as amount of mass transportation are boosted against larger amounts of unsteady stretching constraint S , temperature distribution index r along x -direction, and temperature distribution index s along y -direction. It is observed that larger amounts of S strengthens the impact of chemical reaction and reducing the amounts of Deborah numbers, as a result both heat and mass transports are enhanced. Moreover,

Table 2 Fallouts of the local Nusselt number Nu_x and local Sherwood number Sh_x for various amounts of thermal stratification constraint ε_1 , temperature behavior indices (r, s) , unsteady constraint S , and solutal stratification constraint ε_2 when $\alpha = 0.1, \gamma = \beta_1 = \beta_2 = N = N_t = 0.2, \lambda = 0.3, N_b = 0.5, Pr = Le = 1.0$ are being fixed

Parameters				$(Re_x)^{-1/2} Nu_x$		$(Re_x)^{-1/2} Sh_x$	
S	r	s	ε_1	$\varepsilon_2 = 0.3$	$\varepsilon_2 = 0.7$	$\varepsilon_2 = 0.3$	$\varepsilon_2 = 0.7$
0.1	1.0	1.0	0.3	0.66169	0.71320	0.71463	0.21028
0.2	–	–	–	0.68138	0.73334	0.73044	0.21533
0.3	–	–	–	0.70079	0.75325	0.74601	0.22025
0.1	– 1.0	1.0	0.3	0.05889	0.09778	0.32073	0.13861
–	0.0	–	–	0.41471	0.45681	0.53106	0.16896
–	2.0	–	–	0.85796	0.91927	0.87600	0.25177
0.1	1.0	– 1.0	0.3	0.61531	0.66448	0.67813	0.20125
–	–	0.0	–	0.63877	0.68912	0.69649	0.20576
–	–	2.0	–	0.68410	0.73677	0.73258	0.21481
0.1	1.0	1.0	0.0	0.93854	1.01140	0.66334	0.14957
–	–	–	0.5	0.47482	0.51186	0.74990	0.25195
–	–	–	0.7	0.28616	0.30853	0.78600	0.29459

Table 3 Comparison benchmark of heat transport $-\theta'(0)$ and mass transport $-\phi'(0)$ for various amounts of Deborah numbers (β_1, β_2) , mixed convection constraint λ , and ratio of stretching rates α when $S = r = s = 0.0, N = 0.2 = N_t = \gamma, N_b = 0.5, Pr = 1.0 = Le$ and $\varepsilon_1 = 0.3 = \varepsilon_2$ are fixed

Parameters				$-\theta'(0)$		$-\phi'(0)$	
β_1	β_2	α	λ	Published [29]	Present	Published [29]	Present
0.0	0.2	0.1	0.3	0.62857	0.628571	0.69069	0.690689
0.5	–	–	–	0.60326	0.603260	0.66579	0.665791
1.0	–	–	–	0.58125	0.581252	0.64598	0.645980
0.2	0.0	0.1	0.3	0.60658	0.606580	0.66907	0.669069
–	0.2	–	–	0.61807	0.618070	0.68006	0.680062
–	0.5	–	–	0.63117	0.631171	0.69312	0.693120
0.2	0.2	0.0	0.3	0.60851	0.608511	0.67129	0.671290
–	–	0.2	–	0.62735	0.627352	0.68855	0.688554
–	–	0.5	–	0.65396	0.653961	0.71284	0.712840
0.2	–	0.1	0.0	0.60400	0.604003	0.66672	0.666721
–	–	–	0.4	0.62211	0.622112	0.68394	0.683943
–	–	–	0.6	0.62953	0.629533	0.69108	0.691082

higher amounts of temperature controlled indices diminish the temperature and concentration distributions, as a results heat and mass transport rates are enhanced. It is also perceived via Table 2 that larger amounts of thermal stratification ε_1 enhance the amount of mass transport, whereas amount of heat transport reduces. Additionally, temperature of the surface is reduced by strengthening the amount of thermal stratification constraint ε_1 . Moreover, nanoparticles concentration is also reduced near to the surface for larger amounts of solutal stratification constraint ε_2 , as a result amount of heat transportation

increases, whereas amount of mass transportation is condensed.

A link is developed with the previous published article through Table 3 for heat transport $-\theta'(0)$ as well as for mass transport $-\phi'(0)$ to warrant the accuracy and correctness of the applied methodology. A decent agreement via Table 3 is found between the set of data through present study and through previous published study against various amounts of Deborah numbers (β_1, β_2) , ratio of expansion rate α , mixed convection constraint λ , and buoyancy ratio constraint N .

5 Conclusion

In this article, we discussed the belongings of thermal and solutal stratification for mixed convection Oldroyd-B nanomaterial flow generated by an unsteady bi-directionally expandable surface. The effect of chemical reaction is also included. Moreover, temperature and concentration at the surface and far away from the surface are adjusted through predefined surface temperature, which is not reported earlier. The major outcomes are bulleted below:

- Temperature and concentration curves are showing negative tendencies for larger amounts of heat distribution indices (r, s).
- Larger amount of unsteadiness S is generating low temperature as well as low nanoparticles concentration.
- The behavior of both temperature and concentration distributions are qualitative similar for large amounts of Deborah numbers (β_1, β_2).
- Thermal thickness of boundary layer has a negative tendency for nanomaterial constraints β_1 and β_2 .
- Solutal stratification constraint ε_2 provides an enhancement in concentration profile and its related wideness of boundary layer.
- Thermal stratification constraint ε_1 provides a reduction in temperature profile and its related thermal wideness of boundary layer.
- Rate of heat transference is improved with the escalation in the amounts of unsteady factor, solutal stratification constraint and heat distribution indices.
- Rate of mass transference is developed with the intensification in the amounts of thermal stratification constraint, unsteady constraint and heat distribution indices.

Finally, it is anticipated that this technical exploration will not only deliver useful information to its reader but also will be helpful for industrial and engineering applications, like, coating a sheet with nanomaterial, improvement of the photo-conversion efficiency, production processes of rubber hydrochloride and polyvinyl films, etc. It is stated that this parametric study will be beneficial for upcoming researchers working in this field because of its variety of applications in engineering sectors.

Compliance with ethical standards

Conflict of interest The corresponding author (M. Faisal) states that there is no conflict of interest.

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