Lattice Boltzmann simulation of magneto-hydrodynamic double-diffusive natural convection in an enclosure with internal heat and solute source using nanofluids

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Abstract

Magneto-hydrodynamic double-diffusive natural convection in a cavity is numerically investigated in the present work. Al_2O_3 , Cu, and Ag based nanofluids in a square cavity with a thermal and solute source in the center, and uniform magnetic field are considered. The Patel model is employed to estimate the thermal conductivity of applied nanofluids, and the numerical calculation is based on the implementation of the lattice Boltzmann method and utilizing the standard D_2Q_9 model to study the heat transfer, and also species concentration. A parametric study is carried out to observe the influence of the type, the volume fraction of nanoparticle ($\phi = 0 - 5\%$), Rayleigh ($Ra = 10^4 - 10^5 - 10^6$), Lewis (0.1 – 2 – 10), and Hartmann number (10 – 20 – 100) on average Nusselt and Sherwood numbers, flow fields, temperature, and concentration distribution. The results show that an augment in nanoparticle volume fraction and Rayleigh number increased the average Nusselt and Sherwood numbers. Lewis number showed a significant impact on mass transfer rate. However, it did not affect the heat transfer rate remarkably. Applying a magnetic field caused a reduction in flow circulation which resulted in a decline in heat and mass transfer. A comparison of the simulation outcome with the available data shows that the generated code perfectly captures the fluid flow behavior and heat transfer process as a function of the affecting parameters.

Keywords MHD double-diffusive natural convection · Square enclosure · Nanofluids · Lattice Boltzmann method

List of symbols

Ar	Aspect ratio	k	Thermal conductivity, Wm^{-1}K
B	The magnitude of the magnetic field, T	L	The side of the square cavity, m
c	Lattice speed, m/s	Le	Lewis number
c_i	Discrete particle speed, m/s	Nu	Average Nusselt number
c_s	Sound velocity, m/s	p	Pressure, Pa
C	Concentration	Pr	Prandtl number
C_p	Specific heat, Jkg^{-1}K	Ra	Rayleigh number
D	Mass diffusivity, m^2/s	Sh	Average Sherwood number
f, g, h	Discrete distribution function	Sc	Schmidt number
F	External force	T	Temperature, K
$f_i^{eq}, g_i^{eq}, h_i^{eq}$	Local equilibrium distribution functions	u	x-component of velocity, m/s
g	Gravity acceleration, $9.81 m/s^2$	v	y-component of velocity, m/s
Ha	Hartmann number		

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Greek letters

α	Thermal diffusivity, m^2/s
B_{T_f}	The volumetric coefficient of thermal expansion, K^{-1}
B_{C_f}	Volumetric coefficient of solutal expansion, m^3/kg
ω	Weighting factor
μ	Dynamic viscosity, $\text{kgm}^{-1}\text{S}^{-1}$
σ	Electrical conductivity, $\Omega^{-1}\text{m}^{-1}$
ν	Kinematic viscosity, m^2S^{-1}
ρ	Density, kgm^{-3}
φ	Electric potential (Volt)
ϕ	Nanoparticle volume fraction

Subscript

f	Fluid
nf	Nanofluid
c	Cold walls
h	Hot walls
np	Nanoparticle

1 Introduction

Studying and modeling heat transfer and fluid flow in cavities is highly favorable due to the numerous applications in both industrial and academic fields, such as electronic cooling, heat exchangers, solar collector-receivers, solidification processes, energy storage systems, and many other devices that are run by thermal buoyancy, in which the major heat transfer mechanism is considered to be natural convection. Those applications were reviewed by Ostrach [1]. The problem with conventional convective systems is the low thermal conductivity of the working fluids [2]. An advanced technique to improve heat transfer efficiency is by using dispersed solid nanoparticles (smaller than 100 nm) in a working fluid which is known as nanofluid [3]. The implementation of this technique has been reported extensively over the past years to enhance heat transfer rate in industrial applications such as double pipe heat exchangers and channels, and the results have shown success in achieving this goal [4, 5].

Applying nanoparticles in the base fluid contributes to the heat and mass transfer enhancement in two ways, the one contribution is due to the Brownian movement of nanoparticles which transports heat (diffusion of nanoparticles), and the other contribution is due to an improvement in fluid thermal features [6]. Wong and Castillo [7] analyzed the impact of nanofluids in heat transfer through various experimental models, and theories regarding the heat transfer mechanism. Kolsi et al. [8] considered a cubical enclosure which contains water- Al_2O_3 nanofluids, and investigated the impact of various parameters such as

Rayleigh number and solid volume portion on its heat transfer. A control volume finite element method has been implemented for this work. The results of this work indicate that the increase in solid particles percentage and Rayleigh number can result in an enlargement of Nusselt number. Jou and Tzeng [9] studied free convection of a nanofluid in a rectangular cavity with the implementation of a stream function-vorticity formulation and by using a finite difference method. Heat transfer characteristics and related fluid flow features inside enclosures as a function of affecting parameters of the system such as Rayleigh number, the aspect ratio of the considered enclosure, and the volume fraction of applied nanoparticles were presented and discussed. Their results showed that by increasing volume fraction of nanoparticles the average heat transfer coefficient increases. Ho, et al. [10] investigated effects of adding Al_2O_3 nanoparticles on viscosity and thermal conductivity of working fluid in a buoyant driven enclosure. The major effects of employing different models for viscosity and thermal conductivity on fluid flow inside the cavity and related thermal performance were extracted and reported.

Kumar et al. [11] numerically investigated fluid flow and thermal field in Cu-water nanofluid considering single-phase thermal dispersion and semi-explicit finite volume solver in a collocated grid. The influence of nanoparticles on the density and viscosity of fluid were investigated, and it was concluded that increasing the volume fraction of the suspended nanoparticle results in an increment in the surface area and the heat transfer capacity of the base fluid, which leads to a higher Nusselt number on the walls.

Chen et al. [12] implemented LBM to study free and mixed convection of Al_2O_3 containing nanofluid. The designed geometry in their study was a square enclosure. Numerical results were extracted to investigate the effects of buoyancy, natural convective influence relative to forced convective effects, and the convective heat transfer effects of nanofluids. As a result of this work, it was found that in natural convection, the temperature and flow patterns are highly influenced by Rayleigh number as in mixed convection Reynolds number finds the dominating role. Furthermore, the convective heat transfer effect is expanded as Richardson number increases, and the structure of the flow field changes.

Corcione, et al. [13] studied nanofluid buoyancy-driven convection in a square enclosure by utilizing a two-phase mixture model. The finite volume method was adopted to discretize the governing equations and coupled equations and to solve them numerically by the SIMPLE-C algorithm. As a result, to conclude the influence of the slip motion occurring between solid and fluid phases, the amount of heat transferred across the enclosure by the nanofluid is significantly higher than the transferred amount from the

pure fluid. Moreover, the heat transfer within the enclosure was found as a rising function of nanofluid concentration up to an optimized particle concentration. It was also found that heat transfer within the cavity is increased by increasing the nanofluid average temperature and the cavity width, and is decreasing by an increment in the nanoparticle size. The above literature review deals with fluid motion only caused by a temperature gradient. However, the fluid motion may be induced due to gradients of other scalar quantities. The solute gradients can lead to motion in the absence or presence of temperature field, and can also play a crucial role in this process which this system is known as the double-diffusive convection. Nanoparticles can influence the fluid flow pattern and heat and mass transfer characteristics. The double-diffusive phenomenon can exist everywhere in nature and also in industries. In nature, it often happens in oceans, lakes, solar ponds, shallow coastal waters, and the atmosphere as well as industrial examples including chemical processes, crystal growth, energy storage, freezing, and food processing [14].

Simulation of combined double-diffusive convection and volumetric radiation in a square cavity was conducted by Moufekkik et al. [4]. The proposed numerical method relay on a coupling between the hybrid lattice Boltzmann, the finite difference method, and the discrete ordinates method.

Moufekkik et al. [15] studied double-diffusive convection and volumetric radiation in an ordinated and differentially heated square enclosure employing multiple relaxation time lattice Boltzmann (MRT-LB) and finite difference method (FDM). The reported data include the influence of Rayleigh number, buoyancy number, the optical thickness, and the tilted angle on the flow fields and the heat and mass transfer. Kefayati [16] studied double-diffusive natural convection of Bingham fluid in a cavity with different orientations. The finite-difference lattice Boltzmann method (FDLBM) was adopted, and the effect of Soret and Dufour were analyzed in the cavity in different simulation conditions for various values of Rayleigh number, Bingham number, Lewis number, Dufour parameter, Soret parameter, Eckert number, inclined angle, and the Buoyancy ratio. Sheng Chen et al. [17] studied laminar and turbulence regimes of double-diffusion natural convection in a square cavity containing nanofluid by using LBM. In addition, the influence of the ratio of buoyancy forces on the improvement of double-diffusive convection of nanofluid was presented and discussed. The results imply the difference caused by adding nanoparticles into the base fluid for laminar and turbulence regimes.

Investigation on the fluid flow and thermal performance in the presence of a magnetic field is technically important due to the possibility to induce and control the fluid circulation and heat and mass transfer

phenomenon by utilizing an external magnetic field, which can contribute to a vast range of applications including magnetically controlled thermosiphons for technological purposes, geothermal reservoirs, thermal insulation, oil tanks, improvement in heat transfer for cooling of high-power electric transformers, magnetically controlled heat transfer in energy conversion systems and variety of industrial applications.

Hussein et al. [18] applied a finite difference method to model laminar steady MHD natural convection in a tiled T-shaped enclosure filled with different types of nanofluids. The results showed that the mean Nusselt number rises with an increment of Rayleigh number, inclination angle, aspect ratio, heat source location, and volume fraction of nanoparticles, while, it decreases when the Hartmann number and heat source length increase.

Rudraiah et al. [19] numerically studied the impact of a magnetic field on free convection of an electrically conducting fluid in a rectangular enclosure. The results indicated an enhancement in the value of the magnetic field suppressing fluid flow and as a result that the average Nusselt number was decreased, in a strong magnetic field, the value of Nusselt number approaches to a unit value. Magneto-hydrodynamic natural convection in an open enclosure was analyzed by Hussein et al. [20]. This work considers the nanofluid as water–Cu, and numerous elements such as the effective conductivity, the impact of Hartmann number, nanofluid solid volume fraction, etc. are calculated numerically using LBM, and in the presence of a magnetic field. Kahveci et al. [21] investigated the natural convection flow and heat transfer fields in a laterally heated enclosure with an eccentric partition. It was concluded that the magnetic field that was anticipated in the x-direction is more effective in damping convective flow than the y-direction magnetic field, and it showed an average reduction rate of heat transfer with an increase in the distance of the partition from the hot wall. Ece and Buyuk [22] conducted a parametric study regarding the laminar MHD free convection in a rectangular enclosure with heated and cooled adjacent walls. The damping effect of the magnetic field on fluid flow patterns and heat transfer rate was observed. The different aspect ratios of the enclosure and orientation of geometry and magnitude of exerted magnetic field were also investigated and a significant effect of the fluid flow and temperature fields were observed.

The finite-difference lattice Boltzmann method was adopted to model heat transfer and entropy generation of laminar natural convection of non-Newtonian nanofluids in a square cavity in the presence of horizontal magnetic source by Kefayati [23]. Results indicate that the heat transfer drops by an increase in Hartmann number, and also

affects the power-law index and nanoparticles influence on heat transfer.

Sheikholeslami and Ellahi [24] adopted the lattice Boltzmann method to study MHD hydrothermal treatment in a cubic cavity heated from the bottom of the cube. The results were obtained by considering different values of Hartmann number, nanoparticle volume fraction, and Rayleigh number. Streamlines, isotherms, and isokinetic energy, as well as the Nusselt number, were presented and discussed. It can be inferred that heat transfer caused by fluid flow decreased by applying a magnetic field as a result of an exerted force in the opposite direction of fluid flow. In addition, the Nusselt number was shown to be an increasing function of Rayleigh number and nanofluid volume fraction and a decreasing function of the Hartmann number.

Teamah et al. [25] proposed a numerical simulation to study the impact of a magnetic field on double-diffusive natural convection in a trapezoidal enclosure. A finite volume method was applied to solve the governing equations. The results were shown for different values of Grashof number, the inclination angle of the wall of the cavity, Hartmann number, and buoyancy ratio. The streamlines, isotherms, and iso-concentrations and results for both the local and average Nusselt and Sherwood numbers were presented and justified. It can be concluded that heat and mass transfer rates decrease with inclination angle increment and decrease with Hartmann number increment.

Farooq et al. [26] investigated the effect of magneto-hydrodynamic on squeezing the flow of nanoparticles in the middle of two restricted boundaries. In this work, fundamental explanations have been practiced for convectively heated Maxwell nanofluid by using Buongiorno's model in the mathematical development of the assumed flow problem. The result of this work shows that the velocity profile illustrates cross flow features for increasing the values of the material parameter. Also, for governing values of Biot number, the convective impact is dominant on the plate surface and helps the temperature and nanoparticle concentration increase drastically close to the surface. The effect of using nanofluids in heat and mass transfer besides the controlling capability of the external magnetic field on nanofluids introduced a new field in fluid Mechanics and heat and mass transfer called magnetic nano-fluids (MNF) [27].

A study on free convection of magnetic nanofluid considering MFD viscosity was conducted by Sheikholeslami et al. [28] in enclosure geometry. The employed numerical method reported CVFEM, and the fluid is treated as a single phase. A parametric study was conducted to investigate the effect of non-dimensional numbers including viscosity parameter, Hartmann number, and Rayleigh number on the hydrothermal behavior of the fluid.

The selection of an optimized nanoparticle in the based fluid has a key role in heat and mass transfer efficiencies. The affecting parameters include allowable concentration and size, shape, aspect ratio, temperature, base fluid, pH, stability, etc. The present research tries to address and investigate the heat and mass transfer of Al_2O_3 , Cu, and Ag water-based nanofluid from the thermos-physical perspective. Accordingly, the rightful based fluid for Ag nanoparticles is toluene and water, for Cu nanoparticles are water, oil, ethylene glycol, acetone, and for Al_2O_3 is water, ethylene glycol, pump oil, water + glycerine [29]. Some of the important features of nanofluid industrialization including stability are missing, and are currently under research. There are not solid available data on the number of industrial-scale successful adaptation of nanofluids.

The detailed analysis of the literature presented above demonstrates a lack of deep understanding of a general study about the effect of internally positioned heat and solute source in an enclosure with permeable walls. The present study can be a good representative for modeling contaminant and heat removal systems, drug and food treatment processes and absorption and adsorption in chemical and petrochemical processing in the absence of magnetic field but by combining via magnetic field effect this problem is classified as a new field in fluid Mechanics and heat and mass transfer called magnetic nano-fluids (MNF).

The main application of MNF is in heat exchanger industries, geothermal energy, microelectronics, MEMS, and anywhere which magnetic fields influence fluid flow.

The main goal of the present work is to numerically investigate the double-diffusive natural heat and mass transfer in an enclosure with an internally located heat and solute source filled with nanofluid in the presence of an imposed horizontal magnetic field. This study investigates the combined effect of temperature and solute gradient in the presence of magnetic field in the particular geometry and the relevant fluid flow pattern due to the temperature and solute concentration gradients. The adopted geometry is a simple applicable geometry in many industrial uses. The lattice Boltzmann method for numerical calculation and standard D_2Q_9 model are utilized to model the heat transfer, fluid flow, and species concentration as a more novel method than the conventional method. The flow is considered as steady-state and laminar, and the Patel model is used to predict the thermal conductivity of nanofluid, restricted to spherical nanoparticles.

The parametric study is performed for multifarious values of the dimensionless governing parameters, such as Rayleigh, Lewis and Hartmann number, nanoparticles volume fraction, average Nusselt and Sherwood number, and different types of nanoparticles.

A detailed review has been presented in this section to give a better understanding of the recent works, and the importance of the present study. The following section (Sect. 2), includes problem description, the classical governing equations, lattice Boltzmann numerical method formulation, classical and lattice Boltzmann boundary conditions definitions, and finally the required correlation to approximate and calculate nanofluid thermos-physical properties. The next section (Sect. 3) is devoted to illustrate the iterative numerical method algorithm, grid independency study, and validating the generated code with regards to previous studies. The results are presented and discussed in Sect. 4, which is divided into four subsections for investigation each parameter individually. Finally, Sect. 5 presents a conclusion of the study, in which the research findings are concisely discussed.

2 Problem description and modeling

2.1 Classical mathematical modeling

The two-dimensional square cavity is considered and is presented schematically in Fig. 1. Both the inner and the outer walls of the enclosure are uniformly permeable enough to maintain solute absorption from the inner contaminated secondary fluid and adsorption of the outer clean secondary fluid. It is also assumed that the secondary fluid is bulky enough to maintain constant temperature and concentration on internal and external enclosure walls. This condition is achievable by a steady axial flow of the secondary fluid. Under the preceding circumstances, enclosure's external walls are exposed to a constant temperature and concentration of magnitudes T_c, C_c , and the enclosure's internal

walls are kept at constant temperature and concentration of magnitudes T_h, C_h . It is worth mentioning that in the present research the radiation heat transfer from walls to nanofluid did not consider and the nanofluids treated as a single phase liquid.

A uniform magnetic field of magnitude B is available in the horizontal direction. The fluid in the enclosure is a water-based nanofluid containing different types of solid particles such as Al_2O_3 , Cu, and Ag. The nanofluid is considered Newtonian, single-phase fluid, incompressible, and laminar.

The nanoparticles volume fraction is assumed uniform in the convective flow domain in the cavity. Although nanoparticles distribution can be affected by electromagnetic forces as the fluid is being circulated continuously due to temperature and solute concentration gradients, the nanoparticles are redistributed repeatedly and therefore uniformity in the nanoparticles volume fraction is a realistic assumption in this problem. Additionally, the base fluid and the nanoparticles are assumed to be at thermal equilibrium. The properties of adopted nanoparticles and the pure fluid are presented in Table 1 and are considered to be constant. The density in the buoyancy force term is calculated by the Boussinesq's approximation. Besides, a magnetic field operates in the horizontal direction at all points of the flow field. According to Ohm's law, law of conservation of charge and Lorentz force law, the electric current density J and the electromagnetic force F are given by the relations as bellow [28, 30, 31].

$$J = \sigma_{nf}(-\nabla\varphi + \mathbf{V} \times \mathbf{B}) \quad (1)$$

$$\nabla \cdot \mathbf{J} = 0 \quad (2)$$

$$\mathbf{F} = \mathbf{J} \times \mathbf{B} \quad (3)$$

where V is the velocity field, φ is the electric potential, σ is electrical conductivity and B is the magnetic field vector with a magnitude of B , respectively. According to Ohm's law and the law of conservation of electric charge (Eqs. 1 and 2), the electric potential equation is given as follow:

Fig. 1 Schematic of the problem geometry

Table 1 The properties of adopted nanoparticles and pure fluid [28]

Physical properties	Water	Ag	Cu	Al_2O_3
$C_p (J/Kg.k)$	4179	235	385	765
$\rho (kg/m^3)$	997.1	10,500	8933	3970
$k (W/mK)$	0.613	429	400	40
$\beta_{Ti} \cdot E^{-5} (1/K)$	21	1.89	1.67	0.85
$\sigma (\Omega \cdot m)^{-1}$	0.05	$3.6 \times 10^{+7}$	$5.96 \times 10^{+7}$	1×10^{-12}

$$\nabla^2 \varphi = \nabla \cdot (\mathbf{V} \times \mathbf{B}) \quad (4)$$

The boundary condition for the electric potential is given by:

$$\frac{\partial \varphi}{\partial n} = (\mathbf{V} \times \mathbf{B})_{\text{boundary}} \cdot \mathbf{n} \quad (5)$$

where $\mathbf{n}$ is the normal unit vector of the boundaries. For an insulating boundary (electrically non-conductive boundaries) $\varphi = \varphi_0$, where φ_0 is the specified potential at the boundary. Therefore, for electrically non-conductive boundaries, the electric potential is constant. Under these Conditions Eqs. 1 and 3 lead to the following equations:

$$\mathbf{J} = \sigma_{nf}(\mathbf{V} \times \mathbf{B}) \quad (6)$$

$$\mathbf{F} = \sigma_{nf}(\mathbf{V} \times \mathbf{B}) \times \mathbf{B} \quad (7)$$

The imposed magnetic field in the horizontal direction causes a magnetic force influencing the flow domain in the vertical direction.

Let $\vec{\mathbf{w}}$ be velocity vector:

$$\vec{\mathbf{w}} = u\hat{i} + v\hat{j} \quad (8)$$

where u, v are the x and y -direction velocity components respectively. The governing differential equations for fluid flow are as follows [32–34]:

Continuity equation:

$$\nabla \cdot \mathbf{w} = 0 \quad (9)$$

Momentum equation:

$$(\mathbf{w} \cdot \nabla)(\mathbf{w}) = -\frac{1}{\rho_{nf}} \nabla p + \frac{\mu_{nf}}{\rho_{nf}} \nabla^2 \mathbf{w} + \frac{g}{\rho_{nf}} [(T - T_c)(\phi \rho_{np,0} \beta_{np} + (1 - \phi) \rho_{f,0} \beta_{T_f}) - (c - c_c)(\phi \rho_{np,0} \beta_{np} + (1 - \phi) \rho_{f,0} \beta_{c_f})] \hat{j} - \left(\frac{1}{\rho_{nf}} \sigma_{nf} B^2 v \right) \hat{j} \quad (10)$$

$\rho_{f,0}$ and $\rho_{np,0}$ are reference values for the density of the fluid and nanoparticles respectively. The third and fourth terms in the right-hand side of Eq. (10) are the thermal and solutal gravitational forces respectively which are estimated by Boussinesq's approximation and are the forces that drive the natural convection in the cavity. The last term is the Lorentz force resulting from the external magnetic field effect on electrically conducting nanofluid. Energy equation:

$$(\mathbf{w} \cdot \nabla)(T) = \alpha_{nf} \nabla^2 T \quad (11)$$

Concentration equation is given as below:

$$(\mathbf{w} \cdot \nabla)(c) = D_{nf} \nabla^2 c \quad (12)$$

To transform the above equations to dimensionless form the following dimensionless parameters are used [35]:

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha_f}, V = \frac{vL}{\alpha_f}, \theta = \frac{T - T_c}{T_h - T_c}, \\ P = \frac{\rho L^2}{\rho \alpha_f^2}, Sc = \frac{\mu}{\rho D_f}, Le = \frac{\alpha_f}{D_{nf}} = \frac{Sc}{Pr}, \\ C = \frac{c - c_c}{c_h - c_c}, Pr = \frac{\mu}{\rho \alpha_f}, Ra = \frac{g \beta (T_h - T_c) L^3}{\nu \alpha_f}, \\ Ha = LB \sqrt{\frac{\sigma_{nf}}{\mu_{nf}}} \quad (13)$$

where σ is electrical conductivity, B is the magnitude of the imposed magnetic field, L is the length of the cavity, T denotes temperature, c is solute concentration, μ and α are dynamic viscosity and thermal diffusivity respectively, p is pressure, D is mass diffusivity and ρ is density. In the present study, the Lewis number is defined based on the mass diffusivity in the nanofluid. With the above transformation parameters, Eqs. (9–12) can be written in the following form:

$\vec{\mathbf{W}}$ is velocity vector:

$$\vec{\mathbf{W}} = U\hat{i} + V\hat{j} \quad (14)$$

Continuity equation:

$$\nabla \cdot \mathbf{W} = 0 \quad (15)$$

Momentum equation:

$$(\mathbf{W} \cdot \nabla)(\mathbf{W}) = -\nabla P + \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \nabla^2 \mathbf{W} + \left[RaPr \frac{\rho_{f,0}}{\rho_{nf,0}} \left[1 - \phi + \phi \frac{\rho_{np} \beta_{np}}{\rho_f \beta_{T_f}} \right] (\theta - NC) - Ha^2 \frac{\mu_{nf}}{\rho_{nf} \alpha_f} V \right] \hat{j} \quad (16)$$

where N is the buoyancy ratio defined as:

$$N = \frac{\beta_{c_f} (c_h - c_c)}{\beta_{T_f} (T_h - T_c)} \quad (17)$$

Energy equation:

$$(\mathbf{W} \cdot \nabla)(\theta) = \frac{\alpha_{nf}}{\alpha_f} \nabla^2 \theta \quad (18)$$

Concentration equation:

$$(\mathbf{W} \cdot \nabla)(C) = \frac{1}{Le} \frac{\rho_{nf} C_{p_{nf}}}{\rho_f C_{p_f}} \frac{K_f}{K_{nf}} \nabla^2 C \quad (19)$$

2.2 Lattice Boltzmann method implementation

Lattice Boltzmann method is utilized to simulate different hydrodynamics systems [36, 37]. The reason for the popularity of LBM over conventional CFD methods is the local dynamic and special nature of this method that allows users to easily parallel the computation and simple adoption of the method. The method uses the probability of detecting fluid particles in a certain range of velocity at a certain range of locations instead of tagging each particle as in the computationally-intensive molecular dynamics simulation approach. LBM simulates fluid particles colliding and moving on a uniform square grid to catch macroscopic fluid quantities including velocity, pressure, temperature, and concentration.

The implemented LBM in the present study uses three distribution functions f , g , and h for flow part, temperature and concentration fields, respectively. For the implementation of the LBM method, the fluid domain discretizes to form a uniform Cartesian cell and Each cell holds a fixed number of distribution functions, which represents the number of fluid particles moving in the discrete directions. The general form of the lattice Boltzmann equation is presented below [38, 39]:

For flow field:

$$f_i(x + c_i \Delta t, t + \Delta t) - f_i(x, t) = -\frac{1}{\tau_v} [f_i(x, t) - f_i^{eq}(x, t)] + \Delta t F_i \quad (20)$$

For temperature field:

$$g_i(x + c_i \Delta t, t + \Delta t) - g_i(x, t) = -\frac{1}{\tau_T} [g_i(x, t) - g_i^{eq}(x, t)] \quad (21)$$

Concentration field:

$$h_i(x + c_i \Delta t, t + \Delta t) - h_i(x, t) = -\frac{1}{\tau_c} [h_i(x, t) - h_i^{eq}(x, t)] \quad (22)$$

where Δt considers the lattice time step, c_i is the discrete lattice velocity in the direction i , F is the external force exerted on the fluid in direction i , τ_v , τ_T and τ_c are lattice relaxation time for flow, temperature fields, and concentration fields, respectively, f_i^{eq} , g_i^{eq} and h_i^{eq} are local equilibrium distribution functions for flow, temperature fields, and concentration fields, respectively. The local distribution functions are defined to model the equilibrium distribution of functions and clarify the type of problem for

solving, the local equilibrium distribution functions are presented as follows [40]:

$$f_i^{eq}(x, t) = \omega_i \rho \left[1 + \frac{c_i \cdot u}{c_s^2} + \frac{1}{2} \frac{(c_i \cdot u)^2}{c_s^4} - \frac{1}{2} \frac{u \cdot u}{c_s^2} \right] \quad (23)$$

$$g_i^{eq} = \omega_i T \left[1 + \frac{c_i \cdot u}{c_s^2} \right] \quad (24)$$

$$h_i^{eq} = \omega_i C \left[1 + \frac{c_i \cdot u}{c_s^2} \right] \quad (25)$$

where ω_i is a weighting factor, ρ , T , and C are lattice density, temperature, and concentration of fluid respectively, and c_s represents the speed of sound. To consider the thermal and solutal gravitational forces and the Lorentz force resulting from the external magnetic field, the external force defines as follows:

$$F = F_x + F_y \quad (26)$$

$$F_x = 0 \quad (27)$$

$$F_y = 3\omega_k \rho \left[\frac{g}{\rho_{nf}} [(T - T_c)(\phi \rho_{np,0} \beta_{np} + (1 - \phi) \rho_{f,0} \beta_{Tf}) - (c - c_c)(\phi \rho_{np,0} \beta_{np} + (1 - \phi) \rho_{f,0} \beta_{c_f})] - \left(\frac{1}{\rho_{nf}} \sigma_{nf} B^2 v \right) \right] \quad (28)$$

In the present study the standard D_2Q_9 model [41] is applied, the reason for this choice is that without the boundary effect, both the D_2Q_5 and D_2Q_9 models have second-order convergence for scalar variable and its derivatives, and the D_2Q_5 model has smaller numerical errors at low and moderate Péclet numbers while the D_2Q_9 model has better accuracy at high Péclet numbers. Clearly, D_2Q_5 is more appealing than D_2Q_9 for the convection-diffusion equation when the convection is not very strong and the boundary effect is significant but in the present study Ra number consider as a key parameter and varies to investigate the heat transfer mechanism regime change from conduction to convection [42]. Therefore, based on the above explanation D_2Q_9 suggested to model low temperature, and concentration fields in strong convective flow. The weighting factors and discrete particles velocity of D_2Q_9 model are presented below [43, 44]:

$$\omega_i = \begin{cases} \frac{4}{9}, & i = 0 \\ \frac{1}{9}, & i = 1 - 4 \\ \frac{1}{36}, & i = 5 - 8 \end{cases} \quad (29)$$

$$c_i = \begin{cases} 0 & i = 0 \\ c \left(\cos \left[\frac{\pi(i-1)}{2} \right], \sin \left[\frac{\pi(i-1)}{2} \right] \right) & i = 1 - 4 \\ c\sqrt{2} \left(\cos \left[\frac{\pi(i-5)}{2} + \frac{\pi}{4} \right], \sin \left[\frac{\pi(i-5)}{2} + \frac{\pi}{4} \right] \right) & i = 5 - 8 \end{cases} \quad (30)$$

where the relation between particle velocity c and sound velocity c_s is written as follows:

$$c_s = \frac{c}{\sqrt{3}} \quad (31)$$

The kinematic viscosity, thermal diffusivity, and mass diffusivity transforms are implemented as follows [38]:

$$\nu = \left[\tau_v - \frac{1}{2} \right] c_s^2 \Delta t \quad (32)$$

$$\alpha = \left[\tau_T - \frac{1}{2} \right] c_s^2 \Delta t \quad (33)$$

$$D = \left[\tau_c - \frac{1}{2} \right] c_s^2 \Delta t \quad (34)$$

where c_s is sound speed, Δt is lattice time step and τ_v , τ_T and τ_c are lattice relaxation time for flow, temperature fields, and concentration fields, respectively. The external force in the direction of lattice velocity can be found from the following direction [38]:

$$F_i = \omega_i F \frac{c_i}{c_s^2} \quad (35)$$

The macroscopic quantities are given by [38]:

$$\rho(x, t) = \sum_i f_i(x, t) \quad (36)$$

$$\rho u(x, t) = \sum_i f_i(x, t) c_i + F \quad (37)$$

$$T = \sum_i g_i(x, t) \quad (38)$$

$$C = \frac{D_{nf}}{\alpha_{nf}} \sum_i h_i(x, t) \quad (39)$$

2.3 The boundary conditions

2.3.1 Classical boundary conditions

The classical boundary conditions of the problem are as follows:

$$U = V = 0, \text{ on the enclosure and the source walls.}$$

$$\theta = C = 0, \text{ on the enclosure walls.}$$

$$\theta = C = 1, \text{ on the source walls.}$$

2.3.2 Lattice Boltzmann boundary conditions

The bounce-back boundary condition applied to walls and base on the standard D_2Q_9 model boundaries are presented as follows:

$$\text{For flow: } f_{6,n} = f_{8,n}, f_{7,n} = f_{5,n}, f_{3,n} = f_{1,n} \quad (40)$$

For Temperature and concentration:

By knowing that external walls are exposed to constant temperature and concentration of magnitude T_c, C_c and the source walls are exposed to constant temperature and concentration of magnitude T_h, C_h , unknown distribution functions on the walls of the cavity and heaters walls are calculated. For instance, the boundary condition for temperature and concentration of the right side of the cavity and source is presented below [45, 46]:

A. cavity side

$$\begin{aligned} g_3 &= T_c(\omega_1 + \omega_3) - g_1, g_7 = T_c(\omega_5 + \omega_7) - g_5, \\ g_6 &= T_c(\omega_6 + \omega_8) - g_8 \\ h_3 &= C_c(\omega_1 + \omega_3) - h_1, h_7 = C_c(\omega_5 + \omega_7) - h_5, \\ h_6 &= C_c(\omega_6 + \omega_8) - h_8 \end{aligned} \quad (41)$$

B. source side

$$\begin{aligned} g_1 &= T_h(\omega_1 + \omega_3) - g_3, g_5 = T_h(\omega_5 + \omega_7) - g_7, \\ g_8 &= T_h(\omega_6 + \omega_8) - g_6 \\ h_1 &= C_h(\omega_1 + \omega_3) - h_3, h_5 = C_h(\omega_5 + \omega_7) - h_7, \\ h_8 &= C_h(\omega_6 + \omega_8) - h_6 \end{aligned} \quad (42)$$

2.4 Modeling suspended nanoparticles in the main fluid

The thermal conductivity of the nanofluid is presented by Patel [47] according to the following equation:

$$\frac{k_{nf}}{k_f} = 1 + \frac{k_{np} A_{np}}{k_f A_f} + \tau k_{np} Pe \frac{A_{np}}{k_f A_f} \quad (43)$$

where τ is constant and is determined experimentally. $\frac{A_{np}}{A_f}$ and Pe (Peclet number) is defined as the following equations respectively:

$$\frac{A_{np}}{A_f} = \frac{\phi d_{np}}{(1 - \phi) d_f}, Pe = \frac{\mu_{np} d_{np}}{\alpha_f} \quad (44)$$

where d_{np} is the diameter of nanoparticles and is considered to be equal and constant to 100 nm, d_f is the molecular size of liquid, taken as $2A^\circ$ for water, ϕ is the volume fraction of nanoparticles and μ_{np} is the Brownian motion velocity of the nanoparticle, defined as follows:

$$\mu_{np} = \frac{2K_b T}{\pi \mu_f d_{np}^2} \quad (45)$$

where K_b is the Boltzmann constant. The viscosity of the nanofluid is approximated by Brinkman's equation as follows [48]:

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \quad (46)$$

The effective properties of the nanofluid at the reference temperature were presented in [49]. The effective density of the nanofluid is:

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_{np} \quad (47)$$

The specific heat capacity of the nanofluid is:

$$(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_{np} \quad (48)$$

The volumetric coefficient of thermal expansion is:

$$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_{np} \quad (49)$$

The thermal diffusivity of the nanofluid is defined as:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \quad (50)$$

The electrical conductivity of the nanofluid is given by Maxwell's model [50] for Cu and Ag:

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_{np}}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_{np}}{\sigma_f} + 2\right) - \left(\frac{\sigma_{np}}{\sigma_f} - 1\right)\phi} \quad (51)$$

Results that are obtained from the Eq. (51) for alumina do not fit available experimental data represented by [51] and therefore experimental results are used directly in the simulation in the case of alumina. The average Nusselt and Sherwood numbers [52] are obtained as bellow:

$$\begin{aligned} \overline{Nu}_{vertical} &= -\frac{1}{L} \int_0^L \left(\frac{\partial \theta}{\partial X} \right)_{on vertical walls} dY \\ \overline{Nu}_{horizontal} &= -\frac{1}{L} \int_0^L \left(\frac{\partial \theta}{\partial Y} \right)_{on horizontal walls} dX \end{aligned} \quad (52-a)$$

$$\overline{Nu} = \frac{\overline{Nu}_{vertical} + \overline{Nu}_{horizontal}}{2} \quad (52-b)$$

$$\overline{Sh}_{vertical} = -\frac{1}{L} \int_0^L \left(\frac{\partial C}{\partial X} \right)_{on vertical walls} dY \quad (53-a)$$

$$\overline{Sh}_{horizontal} = -\frac{1}{L} \int_0^L \left(\frac{\partial C}{\partial Y} \right)_{on horizontal walls} dX$$

$$\overline{Sh} = \frac{\overline{Sh}_{vertical} + \overline{Sh}_{horizontal}}{2} \quad (53-b)$$

3 Algorithm, validation and grid independency

The algorithm can be summarized as follows:

1. Initialize ρ , u , $f_i(x, t)$, $f_i^{eq}(x, t)$, $g_i(x, t)$, $g_i^{eq}(x, t)$, $h_i(x, t)$, $h_i^{eq}(x, t)$
2. $f_i(x, t) \rightarrow f_i(x + c_i \Delta t, t + \Delta t)$ in the direction of ω_i
3. Calculate ρ and u then T and C by applying Eqs. (36–39)
4. Calculate $f_i^{eq}(x, t)$, $g_i^{eq}(x, t)$ and $h_i^{eq}(x, t)$ by applying Eqs. (23–25)
5. Update distribution function by applying Eqs. (20–22)
6. Repeat steps 2–5.

The LBM algorithm is adopted on MHD double-diffusive free convection of nanofluid, and a computer code has been created in Fortran. An extensive grid testing procedure was conducted to guarantee a grid-independent solution scheme, the average Nusselt and Sherwood numbers for three different grid resolutions and the lattice sizes is carried out as shown in Table 2., for a specific case in which these parameters are considered $Ra = 10^6$, $Le = 2$, $Ha = 20$ and $\phi = 5\%$.

Based on the results that are illustrated in the table and considering the accuracy of the results required and computational time involved, a 100×100 grid size is used for all of the subsequent numerical calculations.

To check the accuracy of the present results, the results were compared with those of Teamah et al.

Table 2 Result of grid independence examination ($Ra = 10^6$, $Le = 2$, $Ha = 20$ and $\phi = 5\%$)

Grid size	$\overline{Nu}$	$\overline{Sh}$
70 × 70	41.9321	42.7321
90 × 90	42.8521	43.4412
100 × 100	42.8691	43.4514
150 × 150	42.8913	43.4801

Table 3 Average Nusselt and Sherwood numbers compared with results by Teamah et al. [53] and Oueslati [54] at $Ra = 10^5$, $Le = 2$ and $Pr = 1$.

	$Ar = 1, N = -1$			$Ar = 2, N = -0.8$		
	Oueslati [54]	Present work	Error (%)	Teamah [53]	Present work	Error (%)
$\overline{Nu}$	3.0100	3.0098	0.0066	3.4613	3.4582	0.089
$\overline{Sh}$	3.8200	3.8121	0.2	4.3767	4.3712	0.12

[53] and Oueslati et al. [54]. The simulation was performed for double-diffusive natural convection flow in a vertical square enclosure with boundary conditions (T_h, C_h) on the enclosure left wall and (T_c, C_c) on the enclosure right wall. A comparison was performed for $Ra = 10^5$, $Le = 2$ and $Pr = 1$.

Table 3. Shows that the obtained Nusselt and Sherwood numbers of the present paper are in excellent agreement with Nusselt and Sherwood numbers of

Teamah et al. [53] and Oueslati et al. [54]. In addition a comparison of the isotherms, streamlines, and isoconcentrations between present simulation and the study of Teamah et al. [53] has been conducted and presented in Fig. 2. Based on this successful validation, the computer code was used to simulate the present problem.

Fig. 2 Comparison of the isotherms, streamlines, and isoconcentrations between preset simulation colored contours and the study of Teamah et al. [53]

4 Results and discussion

As it was mentioned at the beginning of the article in the introduction section, there are a large number of heat transfer systems based on the concept of free movement and fluid motion based on temperature gradient [1]. Which by applying nanofluids, the problems of low heat transfer coefficient of conventional fluids were largely solved [3]. However, it should be noted that temperature gradient is not the only parameter that induces motion, or alters fluid motion, and changes in heat and mass transfer patterns, but also other influential parameters, including concentration gradient, and applying a magnetic field can be mentioned [14, 17, 19, 27]. In this article, it has been tried to deal with the interaction of temperature gradient, the concentration, size of the magnetic field, and the type of nanofluid. Fluid geometry is a heat and concentration square shaped source that is located in the middle of a cavity filled with nanofluids. This geometry is a useful geometry for many industries. The specific geometry of this study along with the implementation of lattice Boltzmann method as a newer and more modern numerical modeling method compared to conventional methods such as volume control, finite difference or the combination of such method [34, 55, 56], are the main distinguishing features of this article from previous studies. In the following, the effect of each parameter on a different subsection is examined, and each subsection can be a model for the process of various industries such as steelmaking, geothermal, heat exchangers, solidification processes, and so on. Therefore:

Numerical computations were performed for a wide range of $Ra(10^4, 10^5, 10^6)$, $\phi(0 - 5\%)$, $Le(0.1, 2, 10)$, $Ha(10, 20, 100)$ to study the effect of preceding parameters and type and volume fraction of various nanoparticles on heat transfer rate and fluid flow circulation. Ranges of all dimensionless parameters are chosen based on their calculations from nearest to real values involving physical parameters. The Prandtl number (Pr) and the buoyancy ratio (N) are fixed at 6.2 and -0.8 , respectively.

4.1 Effect of different type of nanoparticles

The presence of nanoparticles in water improves both the effective thermal conductivity and diffusion effects which leads to an increase in the intensity of fluid flow. Most articles examine the effect of different parameters on one type of nanofluid. However, only a few works have studied several quantities of various nanofluids. In the present research, three different types of nanoparticles are worked on. The innovation of this part of the study is

Fig. 3 Variation of the average Nusselt number as a function of nanoparticle volume fraction, for different types of nanoparticle ($Ra = 10^6$, $Le = 2$, and $Ha = 0$)

Fig. 4 Variation of the average Sherwood number as a function of nanoparticle volume fraction, for different types of nanoparticle ($Ra = 10^6$, $Le = 2$, and $Ha = 0$)

the use of the Patel conductivity calculation model [47] for nanoparticles, which compared to the conventional models [57–59], takes the Brownian motion effect of particles, and the increase in heat transfer due to the increase in surface area (Increase in the specific surface area as well as Brownian motion) into account using a micro-convection

model, which has a wider range of performance and higher accuracy than older models.

Figures 3 and 4, display the variation of average Nusselt and Sherwood numbers with volume fraction using different nanoparticles for constant values of Rayleigh and Lewis numbers ($Ra = 10^6$, $Le = 2$, and $Ha = 0$).

The figures show that the Nusselt and Sherwood numbers increase with increasing volume fraction of all different types of nanoparticles. The lowest amount of heat transfer rate has been obtained for Al_2O_3 . This is expected since it has the lowest value of thermal conductivity compared to Cu and Ag nanoparticles. The difference in values of Ag and Cu is negligible, but it can be seen that the Ag nanoparticles have better heat and mass transfer properties as compared to other particles.

4.2 Effect of Rayleigh number

Rayleigh number is a dimensionless number in fluid mechanics that is closely related to the heat transfer regime of the floating current or free movement. This parameter has been extensively studied in previous studies. Based on the problem conditions and the type of geometry, the change in heat transfer mechanism from conduction to convection occurs in a specific Rayleigh number for this work and cannot be extracted from previous studies [60, 61].

Figures 5 and 6 illustrate average Nusselt and Sherwood numbers as a function of both the Rayleigh number and Ag nanoparticle concentration at $Le = 2$, and $Ha = 0$. Results show that an increase in Rayleigh number results in larger

Fig. 5 Effect of the Rayleigh number on the average Nusselt number for various nanoparticle volume fraction for Ag nanoparticles ($Le = 2$ and $Ha = 0$)

Fig. 6 Effect of the Rayleigh number on average Sherwood number for various nanoparticle volume fraction for Ag nanoparticles ($Le = 2$ and $Ha = 0$)

values of heat and mass transfer characteristics in terms of Nusselt and Sherwood. Similarly, an increase in heat and mass transfer rate is observable by increasing nanoparticle volume fraction due to its incremental effects on fluid conductivity. More illustrations of these effects are discussed by flow patterns in Fig. 7. As it can be seen in Fig. 7, at $Ra = 10^4$, the weak buoyancy force leads to a weak flow circulation and heat transfer rate, so isotherms and isoconcentration contours are almost uniform in most of the cavity with no plumes at the top of the cavity, referring to the point that the dominant heat transfer mechanism is conduction. By increasing the Rayleigh number to $Ra = 10^5$ and 10^6 , the flow strengthens and the center of the eddy moves up forward the top of the cavity and spreads further. This increases the flow circulation and intensity; Thus, the heat transfer showed by Nusselt number changed from 7.49 at $Ra = 10^4$ to 10.17 at $Ra = 10^5$ and 25.97 at $Ra = 10^6$ for the nanoparticle volume fraction of 0.05 on the top of the cavity. This is due to the forming of eddies in near top of the cavity.

Furthermore, an increment of Rayleigh number causes enhanced convective heat transfer, so isotherms and isoconcentration lines change completely and thermal and solutal plumes tend to rise, as it can be seen clearly in Fig. 7.

4.3 Effect of Lewis number

Lewis number is defined as the ratio of thermal diffusivity to mass diffusivity. Accordingly, an increase in the mass transfer rate is expected by an increase in the Lewis number.

Fig. 7 Streamlines, isotherms, and isoconcentrations for $\phi = 0.0 - 0.05$ of Ag nanoparticles at $Ra = 10^4 - 10^5 - 10^6$, $Le = 2$ and $Ha = 0$.

Alterations in Lewis number result in changes in conductivity and the way fluid moves in the cavity. The effect of Lewis number is investigated in most double diffusive heat transfer studies. However, it should be noted that in each geometry and specific conditions, the parameters governing it lead to different results.

An increase in Lewis number results in a decrease in thickness of concentration boundary layer. Accordingly

via confining the solutal driving force in thinner regime of velocity boundary layer, the intensity of induced solutal flow decreases, which is reported as the stream function as well as flow, temperature, and concentration lines.

In the present study, the Lewis number has been changed from 0.1 to 10 to determine its effect on fluid flow. The increase in Lewis number is accompanied by a decrease in the thickness of the mass boundary layer, and

Fig. 8 Average Nusselt number as a function of Lewis number for various Ag nanoparticles volume fraction ($Ra = 10^6$ and, $Ha = 0$)

Fig. 9 Average Sherwood number as a function of Lewis number for various Ag nanoparticles volume fraction ($Ra = 10^6$ and, $Ha = 0$)

this decrease in thickness causes the forces produced by mass gradients to be located in thinner areas of the thermal boundary layer, which reduces the current induced by the mass gradients. It is reported based on the values of stream functions and contours of current, temperature, and concentration lines. These values are specified based on the unique geometry and specific conditions of the present study.

In Figs. 8 and 9, the Rayleigh number is kept constant at $Ra = 10^6$, to investigate the effect of variation in Lewis number. These figures represent that Lewis number has a considerable influence on the mass transfer rate in terms of Sherwood number but has less effect on the heat transfer rate in terms of Nusselt number. This can be explained by the fact that the mass diffusivity coefficient and as a result of Lewis number only appears in the diffusion term of concentration equation and will not perturb other equations significantly. For the same reason, as shown in Fig. 10, the Lewis number has not a significant effect on the streamlines and isotherms. Therefore, the average Nusselt number is almost independent of the Lewis number. This is the result of negligible changes in thermal contours concerning Lewis number for the preceding reasons. The results illustrate significant variations in the concentration contours due to the increment in the mass diffusivity value of the solute species through nanofluid as the Lewis number increases.

4.4 Effect of Hartmann number

Hartmann number is defined as the ratio of the electromagnetic force to the viscous force. As it was mentioned in the literature review section, in recent year the impact of magnetic field on fluid flow has gained attention. Applications include MEMS, microelectronics, geothermal, and more. The high importance has created a branch in fluid mechanics as magnetic nano fluid [27]. The innovative section of the present study in this part is the investigation of the impact of the magnetic field on various nanofluids to see which particle is influenced more from this field. Moreover, the effects of the magnetic field for the specific geometry of the problem is reported as the stream function as well as flow, temperature, and concentration lines.

In Figs. 11 and 12, the effect of Hartmann number on the average Nusselt and Sherwood number for a particular case ($Ra = 10^6$, $Le = 2$, $Ha = 0 - 20 - 100$) is shown for different nanoparticles volume fraction. For a specific value of nanoparticle volume fraction, the Hartmann number represents the intensity of the imposed magnetic field to the fluid flow. Results demonstrate that at constant nanoparticle volume fraction effect of an increase in the intensity of the magnetic field in terms of Hartmann number is to decrease heat and mass transfer rate in the cavity which represented in terms of reduction in Nusselt and Sherwood numbers. This reduction is more observable in nanofluids with higher values of nanoparticle volume fraction. The effect can be explained in Fig. 13 for both cases of pure fluid and nanofluid convection in the cavity. As can be seen from Fig. 13 for both values of $\phi = 0.0$ and $\phi = 0.05$, the external horizontal magnetic field dampens flow circulation in the cavity due to induced bulk forces exerted

Fig. 10 Streamlines, isotherms, and isoconcentrations for $\phi = 0.0, 0.05$ of Ag nanoparticles ($Ra = 10^6$, $Le = 0.1 - 2 - 10$ and $Ha = 0$)

in the opposite direction of fluid flow which prevents the flow to circulate in the cavity and consequently reduces heat and mass transfer in terms of Nusselt and Sherwood number respectively. In other words, bulk Lorentz forces

attributed to the magnetic field oppose bulk gravitational forces in the flow domain.

Additionally, as Hartmann number is an increasing function of both magnetic field intensity and nanoparticle

Fig. 11 Hartmann number effects on the average Nusselt number as a function of Ag nanoparticles volume fraction at $Ra = 10^6$, and $Le = 2$

Fig. 12 Hartmann number effects on the average Sherwood number as a function of Ag nanoparticles volume fraction at $Ra = 10^6$, and $Le = 2$

volume fraction, for a constant value of Hartmann number increase in volume fraction implies a reduction in the magnitude of the magnetic field. Figures 11 and 12

disclose that at constant Hartmann number, an increase in nanoparticle volume fraction dominates a decrease in magnetic intensity effect, and eventually heat and mass transfer rate increase.

As shown in Table 4, the applied magnetic field has a greater effect on the nanofluids containing Ag and Cu and Al_2O_3 particles, respectively.

5 Conclusion

The present study analyzes double-diffusive natural convection in a cavity filled with water-based nanofluids subjected to an external magnetic field. The square enclosure's outer walls are exposed to a constant temperature and concentration with the magnitude of T_c , C_c , where the internal heater walls are exposed to a constant temperature and concentration with the magnitude of T_h , C_h .

Lattice Boltzmann method, known as (LMB) under laminar and steady-state conditions is employed. Patel model is applied to estimate the effective thermal conductivity of the nanofluid. The results of the numerical calculations are analyzed for different values of the volume fraction of nanoparticles, Rayleigh number, Lewis number, and Hartmann number to make it possible for studying the impact of mentioned parameters on heat and mass transfer on aforementioned nanofluids. The outcome of which are concisely stated as follows.

The effectiveness order of applied water based nanofluids were Ag, Cu, Al_2O_3 in terms of heat and mass transfer rate. Moreover, the average Nusselt and Sherwood numbers increased by increasing Rayleigh number, as it was expected. An increase in the volume fraction of the nanoparticles led to a greater average Nusselt number, particularly at high Rayleigh Numbers (where convection was the main heat transfer mechanism). An increase in Lewis number resulted in an increase in the value of the average Sherwood number. However, it did not affect the average Nusselt number significantly. By increasing Hartmann number for a constant value of nanoparticle volume fraction, the values of average Nusselt and Sherwood numbers decreased. The flow circulation in the cavity was damped by increasing magnetic field magnitude. The applied magnetic field has the greatest impact on Ag nanofluid in terms of reducing heat and mass transfer. After Ag, there are Cu and Al_2O_3 .

Fig. 13 Streamlines, isotherms, and isoconcentrations for $\phi = 0.0 - 0.05$ of Ag nanoparticles ($Ra = 10^6$, $Le = 2$, and $Ha = 0, 20, 100$)

Table 4 The influence of exerted magnetic field on applied nanoparticles in terms of average Nusselt and Sherwood numbers ($Ra = 10^6$, $Le = 2$)

Ha	0	20	100
Ag, $\phi(0.05)$	$\overline{Nu} = 57.16$ $Sh = 48.01$	$\overline{Nu} = 42.86$ $Sh = 43.45$	$\overline{Nu} = 30.00$ $Sh = 19.54$
Cu, $\phi(0.05)$	$\overline{Nu} = 55.22$ $Sh = 47.58$	$\overline{Nu} = 41.53$ $Sh = 43.13$	$\overline{Nu} = 28.56$ $Sh = 19.45$
Al ₂ O ₃ , $\phi(0.05)$	$\overline{Nu} = 25.60$ $Sh = 30.49$	$\overline{Nu} = 22.62$ $Sh = 2.87$	$\overline{Nu} = 11.06$ $Sh = 12.18$

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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