Design of multiobserver for nonlinear systems represented by fuzzy T–S models with sensor faults

Hassen Hichri¹ · Mounir Ayadi¹

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Abstract

This paper deals with the estimation of sensor faults for nonlinear systems represented by a Takagi–Sugeno (T–S) fuzzy model. The basic idea is to introduce an augmented state vector which groups together the states of the fuzzy T–S model and the sensor faults. In order to ensure the robustness of the proposed multiobserver and guarantee the convergence conditions, analysis and stability study are based on the candidate Lyapunov function and the formulation of Linear Matrix Inequality (LMI). The impact of the derivation of sensor faults has been attenuated using the H_∞ performance in the proposed multiobserver. To illustrate the validation of this observer, an academic example will be introduced in this work.

Keywords Sensor faults · Augmented fuzzy T–S model · Multiobserver · Linear matrix inequality (LMI) · H_∞ performance

1 Introduction

For the majority of real processes, a nonlinear model is best suited to get a good representation. However, in contexts such as estimation and control, a nonlinear modeling is more recommended than a linear one which is valid on a limited operating area. Therefore, the fuzzy T–S structure [1, 2] is introduced as a modeling approach that enables us to represent a nonlinear system.

The development of modern systems nowadays has resulted in probable system failures. Sensor faults are now considered as the main source of malfunctions in the system. In order to protect them from the malfunctions, sensor faults should be clearly detected. Over the last decade, several researches have been done on the Fault Detection and Isolation (FDI) and Fault Tolerant Control (FTC); see [3, 4].

In this paper, we will present the design of multiobserver in the presence of sensor faults in the case where the effects of the sensor faults derivatives are not zero. The impact of the derivation of sensor faults was attenuated

using the H_∞ performance [5–7] in the multiobserver that we will propose.

The outline of this paper is as follows: in the second section, we will work out the modeling phase T–S with the aim of obtaining the closest presentation ever to the behaviour of the considered nonlinear system (T–S model validation). In the third section, we will formulate the problem of multiobserver design. In the next section, we will present the design of multiobserver in the presence of sensor faults [8, 9]. The analysis and stability study are based on the candidate Lyapunov function and the formulation of Linear Matrix Inequality (LMI) [10, 11]. The simulation results will be introduced in Sect. 5. Then in Sect. 6, we will discuss the results. Finally, we will conclude the paper in Sect. 7.

2 Validation of the T–S model

The example of the nonlinear system considered in this paper is defined by (1), [12, 13].

✉ Hassen Hichri, hassenhichri.ahi@gmail.com; Mounir Ayadi, mounir.ayadi@enit.utm.tn | ¹National Engineering School of Tunis, LR-Automatique-ENIT, University of Tunis EL Manar, Tunis, Tunisia.

$$\begin{cases} \dot{x}_1(t) = -x_1(t) + u(t) \\ \dot{x}_2(t) = x_1(t) - |x_2(t)|x_2(t) - 10 \\ y(t) = x_2(t) \end{cases} \quad (1)$$

with $x(t) = [x_1(t) \ x_2(t)]^T$ the state vector.

The representation of state T-S of the nonlinear system (1) is written as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^2 \mu_i(\theta)(A_i x(t) + B_i u(t)) \\ y(t) = C x(t) \end{cases} \quad (2)$$

The matrices of local models are given by (3).

$$\begin{aligned} A_1 &= \begin{bmatrix} -1 & 0 \\ 1 & -2\theta_{\max} \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 0 \\ 1 & -2\theta_{\min} \end{bmatrix} \\ B_1 &= B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{et} \quad C = [0 \ 1] \end{aligned} \quad (3)$$

The activating functions of each local model are given by (4) and (5).

$$\mu_1(\theta(x(t))) = \frac{\theta(x(t)) - \theta_{\min}}{\theta_{\max} - \theta_{\min}} \quad (4)$$

$$\mu_2(\theta(x(t))) = 1 - \mu_1(\theta(x(t))) = \frac{\theta_{\max} - \theta(x(t))}{\theta_{\max} - \theta_{\min}} \quad (5)$$

with $\theta(x(t)) = |x_2(t)|$ The premise variable varying arbitrarily [14] in the interval $[0 \ 10]$. $\theta_{\min}$ The minimum bound of the premise variable. $\theta_{\max}$ The maximum bound of the premise variable.

The simulation results of the fuzzy T-S model validation are shown in Fig. 1.

The evolution of the modeling error for the example studied in this paper is demonstrated in Fig. 2.

Fig. 1 Validation of the fuzzy T-S model

Fig. 2 Evolution of the modeling error

3 Problem formulation

The fuzzy T-S model with sensor faults of the nonlinear system (1) is written as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^2 \mu_i(\theta)(A_i x(t) + B_i u(t)) \\ y(t) = C x(t) + F_s f_s(t) \end{cases} \quad (6)$$

where $x(t) \in \mathbb{R}^n$ represents the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $f_s(t) \in \mathbb{R}^q$ is the sensor fault vector and $y(t) \in \mathbb{R}^p$ the output vector, $A_i \in \mathbb{R}^{n \times n}$ is the state matrix, $B_i \in \mathbb{R}^{n \times m}$ the input matrix, $F_s \in \mathbb{R}^{p \times q}$ the sensor fault matrix and $C_i \in \mathbb{R}^{n \times m}$ the output matrix.

Assumption 1 The matrices A_i and C are observable.

The basic idea is to introduce an augmented state vector, which groups together the states of the fuzzy T-S model and the sensor faults f_s . Then, we note with $z = [x^T(t) \ f_s^T]^T$ the augmented state vector. The augmented fuzzy T-S model is then written as follows:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^2 \mu_i(\theta)(\bar{A}_i z(t) + \bar{B}_i u(t)) + E_s \dot{f}_s(t) \\ y(t) = \bar{C} z(t) \end{cases} \quad (7)$$

with:

$$\begin{aligned} \bar{A}_i &= \begin{pmatrix} A_i(n \times n) & 0(n \times q) \\ 0(q \times n) & 0(q \times q) \end{pmatrix}, \quad \bar{B}_i = \begin{pmatrix} B_i(n \times m) \\ 0(q \times m) \end{pmatrix}, \\ E_s &= \begin{pmatrix} 0(n \times q) \\ I_q \end{pmatrix}, \quad \bar{C} = [C \ F_s] \end{aligned} \quad (8)$$

The augmented fuzzy T-S model is now built for the system (6) as in (7) with the new state $z(t)$ of the augmented system which is constituted by the sensor faults

$f_s(t)$ and the state vector $x(t)$. Therefore, the exact estimation of $x(t)$ and $f_s(t)$ can be obtained simultaneously from $\hat{x}(t) = [I_n \ 0] \hat{z}(t)$ and $\hat{f}_s(t) = [0 \ I_q] \hat{z}(t)$.

4 Design of multiobserver

The schematic diagram of the proposed multiobserver is given by Fig. 3

The multiobserver of the augmented fuzzy T–S model (7) is written as follows:

$$\begin{cases} \dot{\hat{z}}(t) = \sum_{i=1}^2 \mu_i(\theta) (\bar{A}_i \hat{z}(t) + \bar{B}_i u(t) + L_i (y(t) - \hat{y}(t))) \\ \hat{y}(t) = \bar{C} \hat{z}(t) \end{cases} \quad (9)$$

We define the estimation error of the augmented fuzzy T–S model (7) by (10):

$$e(t) = z(t) - \hat{z}(t) = \begin{pmatrix} x(t) - \hat{x}(t) \\ f_s(t) - \hat{f}_s(t) \end{pmatrix} \quad (10)$$

The dynamics of the estimation error of augmented fuzzy T–S model is written as follows:

$$\dot{e}(t) = \sum_{i=1}^2 \mu_i(\theta(t)) (\bar{A}_i - L_i \bar{C}) e(t) + E_s \dot{f}_s(t) \quad (11)$$

The goal is to determine gains L_i of the multiobserver (9) in order to ensure convergence to zero of the dynamics of the estimation error. To do this, we consider the quadratic Lyapunov function defined by:

$$V(e(t)) = e^T(t) P e(t) \quad (12)$$

The derivative of the quadratic Lyapunov function is written as follows:

Fig. 3 Schematic diagram of the proposed multiobserver

$$\dot{V}(e) = 2e^T \sum_{i=1}^2 \mu_i(\theta) (P \bar{A}_i - P L_i \bar{C}) e + 2e^T P E_s \dot{f}_s \quad (13)$$

The derivative of the sensor faults appears at the term given by (14):

$$2e^T P E_s \dot{f}_s \quad (14)$$

To minimize the effects of this sensor faults derivative, the optimization technique is used by the criterion H_∞ defined by (15).

$$J = \int_0^\infty (e^T e - \delta^2 \dot{f}_s^T \dot{f}_s) dt \quad (15)$$

with δ a positive scalar which characterizes the mitigation level of noise.

With zero initial conditions, the criterion (15) is written again as follows:

$$J = \int_0^\infty (e^T e - \delta^2 \dot{f}_s^T \dot{f}_s + \dot{V}) dt - \int_0^\infty \dot{V} dt \quad (16)$$

with:

$$\int_0^\infty \dot{V} dt = V(\infty) - V(0) = 0 \quad (17)$$

The Eq. (16) is then written as follows:

$$J = \int_0^\infty (e^T e - \delta^2 \dot{f}_s^T \dot{f}_s + \dot{V}) dt \quad (18)$$

Taking into account the Eq. (13), the criterion J is written again as follows:

$$J = \int_0^\infty \left(e^T e - \delta^2 \dot{f}_s^T \dot{f}_s + 2e^T \sum_{i=1}^2 \mu_i (P \bar{A}_i - P L_i \bar{C}) e + 2e^T P E_s \dot{f}_s \right) dt \quad (19)$$

By noting $Z_i = P L_i$ and $\bar{e} = [e^T \ \dot{f}_s^T]^T$, the criterion J is finally written as follows:

$$J = \int_0^\infty \bar{e}^T \begin{pmatrix} H_i & P E_s \\ E_s^T P & -\delta^2 I_q \end{pmatrix} \bar{e} dt \quad (20)$$

with:

$$H_i = \sum_{i=1}^2 \mu_i(\theta) (P \bar{A}_i + \bar{A}_i^T P - Z_i \bar{C} - \bar{C}^T Z_i) + I_{n+q} \quad (21)$$

To determine gains L_i ($i = 1, 2$) of multiobserver (9), we need to resolve the condition of LMI (22).

$$\begin{bmatrix} P\bar{A}_i + \bar{A}_i^T P - Z_i \bar{C} - \bar{C}^T Z_i + I_{n+q} & P E_s \\ E_s^T P & -\delta^2 I_q \end{bmatrix} < 0 \quad (22)$$

The resolution of LMIs enables us to find gains L_i of multiobserver (9). The Lyapunov matrix P and matrices Z_i for $i = \{1, 2\}$ are given by:

$$L_1 = 10^3 [0.0000 \quad -0.0004 \quad 2.0862]^T$$

$$L_2 = 10^3 [-0.0000 \quad -0.0000 \quad 2.1711]^T$$

$$P = \begin{bmatrix} 245.3146 & 15.3131 & -0.0036 \\ 15.3131 & 32.4237 & -0.0014 \\ -0.0036 & -0.0014 & 0.0004 \end{bmatrix}$$

$$Z_1 = 10^3 [-9.1380 \quad -14.5612 \quad 0.7995]^T$$

$$Z_2 = 10^3 [-8.1918 \quad -3.2501 \quad 0.8316]^T$$

5 Simulation results

This section is devoted to the presentation of the simulation results for the following two cases:

- **Case 1** The effects of derivation of sensor faults are not zero ($\dot{f}_s(t) \neq 0$).
- **Case 2** The effects of derivation of sensor faults are zero ($\dot{f}_s(t) = 0$).

The sensor fault considered in this work is given by (23).

$$f_s(t) = \begin{cases} 0 & \text{if } t \leq 6 \\ 1 & \text{if } 6 < t \leq 8 \\ -1 & \text{if } 8 < t \leq 12 \\ \sin(t) & \text{if } 12 < t \leq 18 \\ 0 & \text{if } t \geq 18 \end{cases} \quad (23)$$

5.1 Simulation results (case 1)

This subsection is devoted to the presentation of the simulation results in the case where the effects of derivation of sensor faults are assumed to be non-zero. So, we noticed that the synthesized multiobserver is able to reproduce the states of the fuzzy T-S model and the sensor faults in an exact and precise way as shown in Figs. 4, 6 and 7.

The simulation results of the estimation of the fuzzy T-S model states, the activating functions, the sensor fault as well as the estimation error are given by Figs. 4, 5, 6, 7, 8 and 9.

Fig. 4 The sensor fault f_s and its estimate (case 1)

Fig. 5 Activation functions in the presence of sensor fault (case 1)

Fig. 6 Variable state x_1 and its estimate in the presence of the sensor fault (case 1)

Fig. 7 Variable state x_2 and its estimate in the presence of the sensor fault (case 1)

Fig. 8 The estimation error e_1 in the presence of the sensor fault (case 1)

Fig. 9 The estimation error e_2 in the presence of the sensor fault (case 1)

5.2 Simulation results (case 2)

This subsection is devoted to the presentation of the simulation results in the case where the effects of derivation of sensor faults are assumed to be zero. So, we noticed that

the multiobserver is unable to reproduce the system states and sensor faults in an exact and precise way, as shown in Figs. 10, 11 and 12.

The simulation results of the estimation of the fuzzy T–S model states, the sensor fault as well as the estimation error are given by Figs. 10, 11, 12, 13 and 14.

Fig. 10 The sensor fault f_s and its estimate (case 2)

Fig. 11 Variable state x_1 and its estimate in the presence of the sensor fault (case 2)

Fig. 12 Variable state x_2 and its estimate in the presence of the sensor fault (case 2)

Fig. 13 The estimation error e_1 in the presence of the sensor fault (case 2)

Fig. 14 The estimation error e_2 in the presence of the sensor fault (case 2)

6 Results discussion

A nonlinear modeling is more recommended than a linear one that is valid on a limited operating area. Therefore, the fuzzy T–S model [1] introduced as a modeling approach allows the good representation of a nonlinear system as shown in this work by Figs. 1 and 2. Indeed, at the level of Fig. 1 we notice that the output of the nonlinear system y_{NL} practically coincides with the output of the fuzzy T–S model y_{TS} . This explains the effectiveness of the fuzzy T–S representation. We also note in the same figure that the output of the fuzzy T–S model, y_{TS} , presents a slight overshoot compared to the response of the nonlinear system, y_{NL} . This is due to the precision of the modeling (as shown in Fig. 2), the complexity of the nonlinear system, and the choice of the form of the activation functions.

In the work [13], the authors hypothesize that the effects of the derivation of the sensor faults are zero (case 2), which obliges the authors to treat only the

sensor faults of very low amplitude as indicated in the works [15, 16]. This hypothesis can pose problems at the level of the estimation of the fuzzy T–S model states and the sensor faults which are not reconstructed in a precise way, as shown in Figs. 10, 11 and 12, as well as problems at the level of the development of Fault Tolerant Controls. To avoid the issues raised previously, we have taken into account, in this work, that the effects of derivation of sensor faults are not zero (case 1). Therefore, in this work, the problem of estimating sensor faults was studied in a category of nonlinear systems represented by fuzzy T–S models. However, we have built an augmented fuzzy T–S model. The state vector and sensor faults are put together in an augmented state vector in order to estimate both the states and sensor faults. The results are presented in the form of Linear Matrix Inequality (LMI) with the H_∞ performance. The formulation of the conditions of the LMI as well as the H_∞ performances [17] make it possible to ensure the convergence of the estimation error and to attenuate the impact of the derivation of the sensor faults as illustrated in Figs. 4, 8 and 9. Thus, the multiobserver synthesized in this work is able to reproduce in a precise way the sensor faults which have considerable amplitudes compared to what we find in the works [15, 16].

In Figs. 6 and 7, we note that the states' curves of the fuzzy T–S model and their estimates are perfectly the same, even in the presence of the sensor fault. This shows the efficiency of the synthesized multiobserver in this work.

7 Conclusion

In this article, the estimation of sensor faults has been studied in a category of nonlinear systems represented by fuzzy T–S models. In order to ensure this estimation in a precise way, we have constructed an augmented fuzzy T–S model. The state vector and sensor faults have become components of a new state vector in order to estimate both the system states and sensor faults. The results have been given in the form of an LMI with H_∞ performance which allows us to attenuate the effects of the derivation of the sensor faults. Therefore, in this work, we were able to handle the problem of estimating sensor faults despite their large capacity, without also assuming that their derivation effects are zero, contrary to what has been considered in the research works [15, 16]. This allows authors to estimate only sensor faults with low amplitude, which poses problems in terms of developing a Fault Tolerant Control law. Finally, an academic example of a nonlinear system model has been

used to show the efficiency and interest of the proposed multiobserver. As a result, the future directions of our research work will be about the development of a Fault Tolerant Control for nonlinear systems represented by the fuzzy T–S models.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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