Electrothermal transport via copper nanoparticles in a microchannel propagated by peristalsis

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Received: 28 March 2020 / Accepted: 12 August 2020 / Published online: 26 August 2020
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Abstract

This communication investigates the simultaneous impact of the magnetic field, heat absorption and mixed convection in electroosmotically induced peristaltic flow of copper particles in the blood. The two-phase flow model is used in the microchannel with flexible walls. The thermal features of nanofluids are studied using copper nanoparticles. The Poisson–Boltzmann equation is employed to accommodate the electrical double layer in the microchannel. With Debye–Hückel, lubrication and long wavelength approximations, the problem becomes non-dimensional. Exact solutions have been calculated for velocity, stream function, pressure rise, pressure gradient and heat transfer coefficient and temperature profile. Impact of different embedded parameters on flow properties is graphically illustrated. It is found that the electroosmotic parameter strongly effects the velocity profile and bolus formation. Also, the decreasing behavior of temperature is presented, which clarifies the nanofluid as a cooling agent. The results show an increment in peristaltic pumping with rise in mixed convection while it decreases with magnetic field. Furthermore, a comparative study of pure blood and copper blood is conducted. The obtained results may be suitable in the peristaltic pump modulation for the efficient operation of a variety of industrial, biomedical and drug delivery systems in the microfluidic device.

Keywords Electroosmosis · Peristalsis · Copper nanoparticles · Mixed convection · Magnetohydrodynamics · Heat transfer · Exact solution

List of symbols

$\tilde{a}$	Channel's half width (L)	k_f	Thermal conductivity of base fluid (ML/T ³ K)
B	Constant heat generation (–)	k_s	Thermal conductivity of nanoparticle (ML/T ³ K)
B_0	Uniform magnetic field (A/L)	k_{nf}	Thermal conductivity of nanofluid (ML/T ³ K)
c	Wave speed (L/T)	m_e	Electroosmotic parameter (–)
e	Electron charge (C)	$n^{\pm}$	Positive, negative ions (AT)
E_x	Electric field (ML/T ³ A)	n_0	Average number of n^+ or n^- ions (AT)
F	Non-dimensional volume flow rate in moving frame (–)	$\tilde{p}$	Dimensional pressure field (ML/T ²)
g	Acceleration due to gravity (L/T ²)	p	Non-dimensional pressure field (–)
G_r	Grashof number (–)	P_r	Prandtl number (–)
$\tilde{h}$	Transverse vibration of wall (L)	P_e	Peclet number (–)
H_r	Hartmann number (–)	$\tilde{Q}$	Heat absorption parameter (ML ² /T ²)
K_B	Boltzmann constant (ML ² /T ² K)	R_e	Reynolds number (–)
		$\tilde{t}$	Dimensional time (T)
		t	Dimensionless time (–)

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$\tilde{T}$	Temperature field (K)
T_0	Constant temperatures of fluid (K)
U_{hs}	Helmholtz–Smoluchowski velocity (L/T)
$\tilde{U}, \tilde{V}$	Dimensional velocity components in stationary frame (L/T)
u, v	Non-dimensional velocity components in wave frame (–)
$\tilde{X}, \tilde{Y}$	Dimensional coordinates in stationary frame (L)
x, y	Non-dimensional coordinates in wave frame (–)
Z	Heat transfer coefficient (–)
z_v	Valence of ions (–)
α	Wave number (–)
α_f	Thermal diffusivity of base fluid (L^2/T)
α_{nf}	Thermal diffusivity of nanofluid (L^2/T)
β_f	Thermal expansion coefficient of base fluid (1/K)
β_s	Thermal expansion coefficient of nanoparticle (1/K)
β_{nf}	Effective thermal expansion coefficient (1/K)
ϵ_0	Permittivity of free space ($T^4 A^2 / ML^3$)
ϵ	Relative permittivity of the medium (–)
θ	Temperature field (K)
Θ	Non-dimensional volume flow rate in fixed frame (–)
λ	Wavelength of peristaltic wave (L)
λ_D	Debye length (L)
χ	Solid volume fraction of nanofluid (–)
μ_f	Dynamic viscosity of base fluid (MLT)
μ_s	Dynamic viscosity of nanoparticle (MLT)
μ_{nf}	Dynamic viscosity of nanofluid (MLT)
ρ_f	Density of base fluid (ML^3)
ρ_s	Density of nanoparticle (ML^3)
ρ_e	Electrical charge density (ML^3)
$(\rho c_p)_f$	Heat capacity of base fluid (ML^2/T^2K)
$(\rho c_p)_s$	Heat capacity of nanoparticle (ML^2/T^2K)
$(\rho c_p)_{nf}$	Effective heat capacity of nanofluid (ML^2/T^2K)
σ_e	Electrical conductivity ($T^3 I^2 / L^3 M$)
ϕ	Dimensional electric potential distribution ($ML^2 / T^2 I$)
Φ	Non-dimensional electric potential distribution (–)
$\tilde{\phi}$	Wave amplitude (L)
ψ	Dimensional stream function (L^2/T)

1 Introduction

Electroosmotic flow (EOF) is the well-known method in electrokinetics since it can proficiently propel fluids along the microchannel devices like Lab-on-a-chip (LOC), DNA analyzers, organ-on-a-chip so on. Electrokinetics includes electrophoresis, electroosmosis and several other

phenomena. Recently, electroosmotic flow have penetrated many different nano- and microscale technological applications, including nanoscale electro-fluid thruster designs in-space propulsion, ion-exchange membrane designs, micro-channels, non-adsorbent polymer injection systems, biochip fabrication, microbial fuel cells, corrosion mitigation in civil engineering, dewatering, nano-bot propulsion for medical use and new pump proposals for the microfluidic chromatography. Patankar et al. [1] studied the impact of electroosmotic flow via microchannel. Also, Yang et al. [2] inspected the electroosmotic flow in microfluidics devices. Glasgow et al. [3] analyzed electroosmotic mixing in the microchannel. Xuan et al. [4] considered the EOF for different geometry and different charge distribution. Later, Park et al. [5] examined EOF through a comparison of the Poisson–Boltzmann model and Nernst–Planck model.

A mechanism of flowing the fluid due to natural contraction and expansion of smooth muscles is called peristalsis. Many modern chemical pumps and biomedical devices have been designed by using the peristaltic mechanism, i.e., diabetic pumps, heart–lung machines, waste migration control pumps, pharmacological drug delivery systems, finger and roller pumps. Due to these important practical applications researchers have put their attention toward peristaltic flows [6–12]. Nanofluids are fluids containing nanoparticles. Nanofluids have unique properties that enhance thermal conductivity. Nanofluids are used in a variety of industrial processes, such as disease treatment, heat exchangers, coolants in atomics reactors, geothermal extraction, indicative testing, mechanical cooling and microfluidic applications. With such a wide range of uses, some theoretical and experimental studies have been performed to analyze different aspects of nanofluids [13–18]. Mixed convection, including natural and forced convection, has many engineering importance, including MEMS applications, cooling of the electronic equipment, solar collectors, home ventilation, fire protection, chemical processing equipment, etc. Mixed convection has been widely used due to their wide application in industrial and engineering fields, it has attracted the attention of researchers [19–23].

These studies do throw light on the important characteristics of augmentation of peristaltic flow. The combined effects of electroosmosis and peristalsis are critical in many applications, such as biomimetic pumping and microfluidic devices. Electro-kinetic effects are produced, when the ionized fluid moves relative to the stationary charged surface under an externally applied electric field, called electroosmosis. The term electroosmosis was first time reported in an experimental investigation of peristalsis [24]. The idea of fluid transportation through electroosmotic force has been formed for decades and remarkable

Fig. 1 Geometry of the problem

Table 1 Thermophysical properties of Copper and Blood [37]

Physical Properties	Blood	Cu
ρ (kg m ⁻³)	1063	8933
$\beta \times 10^{-5}$ (K ⁻¹)	0.18	1.67
c_p (J kg ⁻¹ K ⁻¹)	3594	385
k (W m ⁻¹ K ⁻¹)	0.492	400

works in this view include different configurations with various fluids. The fluid dynamics of the electroosmotic flow (EOF) are considerably different from the pressure-driven flow, so the heat transfer dynamics is supposed to be very different. The magnitude of thermal energy source has significant impact on the temperature distribution and heat transfer of nanofluids. The heat transfer in microfluidic devices is of immense importance in the blood flow dilution technology, the destruction of cancer tissue, and the microfabrication technology. Shit et al. [25] inspected the peristaltic transport of biofluid via the microchannel. Tripathi et al. [26] worked on the peristaltic transport of

couple-stress fluid via the microchannel. Tripathi et al. [27] analyzed the peristaltic bio-rheological flow in the asymmetric microchannel. Tripathi et al. [28] also examined the buoyancy effect of nanofluids via microchannel. Furthermore, Prakash et al. [29] explored the EOF of Williamson ionic fluid through the tapered microchannel. Jhorar et al. [30] investigated the biomechanical transport in EOF via asymmetric microchannel. Shehzad et al. [31] observed the EOF of MHD power law nanofluid through microchannel. Moreover, Noreen et al. [32] studied the peristaltic transportation of nanofluids with double-diffusive convection via microchannel. Waheed et al. [33] inspected the effect of heat and mass for third-order fluid via peristaltic microchannel. Noreen et al. [34] proposed the transportation of MHD nanofluid via peristaltic asymmetric microchannel. Prakash et al. [35] studied the impact of heat transfer in peristaltic transportation of nanofluids via microchannel. Prakash et al. [36] analyzed the effect of convective heat transfer in ionic nanofluids flow powered by peristalsis and electroosmosis. Prakash et al. [37] explored the EOF of hybrid nanofluid through the asymmetric microchannel. Prakash et al. [38] worked on the EOF of pseudoplastic nanofluid through microchannel. Moreover, Noreen et al. [39] investigated the electrothermal transport of Williamson fluid via peristaltic pumping.

To the author's knowledge, despite the many practical applications in nanofluidic and microfluidic pumps, the combined electroosmotic peristaltic flow of nanofluids has received little attention. Aforementioned, studies have however not addressed EOF of nanofluid through a vertical microchannel with magnetic field, mixed convection, and heat absorption. Therefore, the purpose of this communication is to study a mathematical model of heat transfer in peristalsis modulated EOF of nanofluid through a vertical microchannel with magnetic field, mixed convection, and heat absorption. The two-phase flow model is utilized in the microchannel with flexible walls. The thermal features of nanofluids are studied using copper nanoparticles. The current study addresses this configuration for the very first time. The governing equations are simplified by using reliable approximation, namely lubrication and Debye–Hückel approximation. Exact solutions have been calculated for velocity, stream function, pressure rise, pressure gradient and heat transfer coefficient and temperature profile. The detailed mathematical formulation and exact solutions are reported in Sect. 2 and 3. Comparative analysis of the pure blood and the copper blood is shown in Sect. 4. Analysis of graphical results is done in the Sect. 5. Conclusions are discussed in the Sect. 6.

2 Problem formulation

2.1 Flow regime

Consider the two-dimensional EOF of incompressible electrically conducting nanofluid through the vertical microfluidic channel, shown in Fig. 1.

The channel is filled with incompressible Cu-blood based nanofluid. An incompressible nanofluid consists of blood and nanomaterial. Both materials are in thermal equilibrium and they flow at the same velocity c along the microchannel. Table 1 shows the thermophysical characteristics of blood with Cu. Heat transfer analysis with the nanofluid is also considered. A wave propagating wall of the microchannel is given a constant temperature T_0 . Here, $\tilde{X}$ measured in wave propagation direction and $\tilde{Y}$ direction is transverse to it. Moreover, the wavy microchannel is exposed to the uniform magnetic field of strength B_0 in $\tilde{Y}$ -direction and electric field E_x in the $\tilde{X}$ -direction.

Mathematical representation of microchannel wall is [26]:

$$\tilde{h}(\tilde{X}, \tilde{t}) = \tilde{a} - \tilde{\varphi} \cos^2 \left(\frac{(\tilde{X} - c\tilde{t})\pi}{\lambda} \right), \quad (1)$$

here $\tilde{a}$, $\tilde{X}$, $\tilde{\varphi}$, $\tilde{t}$, c and λ are the channel's half width, axial coordinate, wave amplitude, time, wave speed and wavelength, respectively.

2.2 Modeling

The governing equations of motion and energy for magneto-nanofluid with body forces are [28]:

$$\frac{\partial \tilde{U}}{\partial \tilde{X}} + \frac{\partial \tilde{V}}{\partial \tilde{Y}} = 0, \quad (2)$$

$$\rho_{\text{nf}} \left(\frac{\partial \tilde{U}}{\partial \tilde{t}} + \tilde{U} \frac{\partial \tilde{U}}{\partial \tilde{X}} + \tilde{V} \frac{\partial \tilde{U}}{\partial \tilde{Y}} \right) = -\frac{\partial \tilde{P}}{\partial \tilde{X}} + \mu_{\text{nf}} \left(\frac{\partial^2 \tilde{U}}{\partial \tilde{X}^2} + \frac{\partial^2 \tilde{U}}{\partial \tilde{Y}^2} \right) + \sigma_e B_0^2 \tilde{U} + (\rho\beta)_{\text{nf}} g(\tilde{T} - T_0) + \rho_e E_x, \quad (3)$$

$$\rho_{\text{nf}} \left(\frac{\partial \tilde{V}}{\partial \tilde{t}} + \tilde{U} \frac{\partial \tilde{V}}{\partial \tilde{X}} + \tilde{V} \frac{\partial \tilde{V}}{\partial \tilde{Y}} \right) = -\frac{\partial \tilde{P}}{\partial \tilde{Y}} + \mu_{\text{nf}} \left(\frac{\partial^2 \tilde{V}}{\partial \tilde{X}^2} + \frac{\partial^2 \tilde{V}}{\partial \tilde{Y}^2} \right), \quad (4)$$

$$(\rho c_p)_{\text{nf}} \left(\frac{\partial \tilde{T}}{\partial \tilde{t}} + \tilde{U} \frac{\partial \tilde{T}}{\partial \tilde{X}} + \tilde{V} \frac{\partial \tilde{T}}{\partial \tilde{Y}} \right) = k_{\text{nf}} \left(\frac{\partial^2 \tilde{T}}{\partial \tilde{X}^2} + \frac{\partial^2 \tilde{T}}{\partial \tilde{Y}^2} \right) + \tilde{Q}. \quad (5)$$

Here $\tilde{U}$ and $\tilde{V}$ are the velocities along $\tilde{X}$ and $\tilde{Y}$ directions and $\tilde{T}$ represents temperature. ρ_{nf} represents nanofluid effective density, μ_{nf} is the nanofluid dynamic viscosity, σ_e is electrical conductivity, β_{nf} is the effective thermal expansion coefficient, g is the acceleration due to gravity, ρ_e is the electrical charge density, $\tilde{Q}$ is heat absorption parameter, $(\rho c_p)_{\text{nf}}$ is the nanofluid effective heat capacity, E_x represents applied electric field and k_{nf} represents the thermal conductivity of the nanofluid, which are defined as [37]

$$\begin{aligned} (\rho c_p)_{\text{nf}} &= (1 - \chi)(\rho c_p)_f + \chi(\rho c_p)_s, \quad \rho_{\text{nf}} = (1 - \chi)\rho_f + \chi\rho_s, \\ (\rho\beta)_{\text{nf}} &= (1 - \chi)(\rho\beta)_f + \chi(\rho\beta)_s, \quad \mu_{\text{nf}} = \mu_f / (1 - \chi)^{2.5}, \\ \alpha_{\text{nf}} &= \frac{k_{\text{nf}}}{(\rho c_p)_{\text{nf}}}, \quad \frac{k_{\text{nf}}}{k_f} = \frac{(k_s + 2k_f) - 2\chi(k_f - k_s)}{(k_s + 2k_f) + \chi(k_f - k_s)}. \end{aligned} \quad (6)$$

Here α_{nf} is nanofluid's effective thermal diffusivity, χ is solid volume fraction of nanofluid, μ_f is base fluid dynamic viscosity. Also, ρ_f , ρ_s , β_f , β_s , k_f , k_s , $(c_p)_f$, $(c_p)_s$ denote densities, thermal expansion coefficient, thermal conductivities and specific heat of base fluid and nanoparticles, respectively.

For microchannel, the Poisson–Boltzmann equation [38] is stated as:

$$\nabla^2 \tilde{\phi} = -\frac{\rho_e}{\epsilon_0 \epsilon}. \quad (7)$$

where ϵ is the relative permittivity, $\rho_e (= -z_v e(\tilde{n}^- - \tilde{n}^+))$ is total charge density, $\tilde{\phi}$ is the electric potential, ϵ_0 is free space permittivity, $\tilde{n}^+$ is the cation, $\tilde{n}^-$ is the anion and defined as $\tilde{n}^\pm = \tilde{n}_0 e^{\left(\pm \frac{ze_v \tilde{\phi}}{T_{\text{av}} K_B} \right)}$. Additionally, e , z_v , n_0 , K_B and T_{av} represents electronic charge, charge balance, bulk concentration, Boltzmann constant and average temperature.

The relations express the transformation between fixed

frame $(\tilde{X}, \tilde{Y})$ and moving frame $(\tilde{x}, \tilde{y})$ for coordinate velocities, pressure and temperature:

$$\begin{aligned} \tilde{u}(\tilde{x}, \tilde{y}) &= \tilde{U}(\tilde{X}, \tilde{Y}, \tilde{t}) - c, \\ \tilde{v}(\tilde{x}, \tilde{y}) &= \tilde{V}(\tilde{X}, \tilde{Y}, \tilde{t}), \quad \tilde{x} = \tilde{X} - c\tilde{t}, \quad \tilde{y} = \tilde{Y}, \\ T(\tilde{x}, \tilde{y}) &= \tilde{T}(\tilde{X}, \tilde{Y}, \tilde{t}), \quad \tilde{p}(\tilde{x}, \tilde{y}) = \tilde{P}(\tilde{X}, \tilde{Y}, \tilde{t}). \end{aligned} \quad (8)$$

Using Eq. (8) in Eqs. (2)–(5), afterward introducing non-dimensional variables:

$$\begin{aligned} h &= \frac{\tilde{h}}{\tilde{a}}, n = \frac{\tilde{n}}{n_0}, m_e = \frac{\tilde{a}}{\lambda_D}, x = \frac{\tilde{x}}{\lambda}, y = \frac{\tilde{y}}{\tilde{a}}, t = \frac{c\tilde{t}}{\lambda}, u = \frac{\tilde{u}}{c}, v = \frac{\tilde{v}}{\alpha c}, \alpha = \frac{\tilde{a}}{\lambda}, \varphi = \frac{\tilde{\varphi}}{a}, B = \frac{\tilde{a}^2 \tilde{Q}}{T_0 k_f}, \\ \theta &= \frac{\tilde{T} - T_0}{T_0}, U_{hs} = -\frac{E_x \epsilon_0 \in T_{av} K_B}{e z_v c \mu_f}, \phi = \tilde{\phi} \frac{e z_v}{T_{av} K_B}, P_e = \frac{c \tilde{a}}{D}, F = \frac{q}{c \tilde{a}}, \Theta = \frac{\tilde{K}}{c \tilde{a}}, p = \frac{\tilde{a}^2}{c \lambda \mu_f} \tilde{p}, \\ P_r &= \frac{\mu_f (c_p)_f}{k_f}, H_r^2 = \frac{\tilde{a}^2 \sigma_e B_0^2}{\mu_f}, R_e = \frac{\rho_f c \tilde{a}}{\mu_f}, G_r = \frac{g \tilde{a}^2 (\rho \beta)_f T_0}{c \mu_f}, u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}. \end{aligned} \quad (9)$$

In the above, $m_e, x, y, \lambda_D, \alpha, U_{hs}, p, \theta, \varphi, R_e, P_e, P_r, G_r, H_r, B, \Theta$ and F denotes electroosmotic parameter, dimensionless axial coordinate and transverse coordinate, Debye length, peristaltic wave number, Helmholtz–Smoluchowski velocity, pressure, dimensionless temperature, amplitude of wave, Reynolds number, ionic Peclet number, Prandtl number, Grashof number, Hartmann number, constant heat generation, dimensionless flow rate in fixed frame and in moving frame. Now, applying Debye–Hückel linearization approximation [38] and Eqs. (9), (7) becomes

$$\frac{\partial^2 \phi}{\partial y^2} = \phi m_e^2. \quad (10)$$

Here, $m_e^2 = \frac{2 \tilde{a}^2 n_0 (e z_v)^2}{\epsilon_0 \in K_B T_0} = \frac{\tilde{a}^2}{\lambda_D^2}$ (electroosmotic parameter).

The appropriate boundary conditions for Eq. (10) are

$$\begin{aligned} \frac{\partial \phi}{\partial y} &= 0, \text{ at } y = 0, \\ \phi &= 1, \text{ at } y = h. \end{aligned} \quad (10a)$$

Therefore, the analytical solution of Eq. (10) is

$$\phi = \frac{\cosh(m_e y)}{\cosh(m_e h)}. \quad (11)$$

Fig. 2 Velocity profile for different values of Electroosmotic parameter for Pure blood and Cu-blood

After using Eq. (9), followed by long wavelength and low Reynolds approximation, the Eqs. (3)–(5) reduces to following form:

$$\frac{\partial p}{\partial x} = \frac{\mu_{nf}}{\mu_f} \frac{\partial^3 \psi}{\partial y^3} - H_r^2 \frac{\partial \psi}{\partial y} + \frac{(\rho \beta)_{nf}}{(\rho \beta)_f} G_r \theta + m_e^2 U_{hs} \phi, \quad (12)$$

$$\frac{\partial p}{\partial y} = 0, \quad (13)$$

$$\frac{\alpha_{nf}}{\alpha_f} \frac{\partial^2 \theta}{\partial y^2} + \frac{(\rho c_p)_{nf}}{(\rho c_p)_f} B = 0. \quad (14)$$

$$\frac{\mu_{nf}}{\mu_f} \frac{\partial^4 \psi}{\partial y^4} - H_r^2 \frac{\partial^2 \psi}{\partial y^2} + \frac{(\rho \beta)_{nf}}{(\rho \beta)_f} G_r \frac{\partial \theta}{\partial y} + m_e^2 U_{hs} \frac{\partial \phi}{\partial y} = 0. \quad (15)$$

The non-dimensional boundary conditions are:

$$\frac{\partial^2 \psi}{\partial y^2} = 0, \psi = 0, \theta = 0, \text{ at } y = 0, \quad (16a)$$

$$\frac{\partial \psi}{\partial y} = -1, \psi = F, \theta = 1, \text{ at } y = h(x) = 1 - \varphi \cos^2 x. \quad (16b)$$

Here, dimensionless time mean flow rate Θ in fixed frame is connected to the dimensionless time mean flow rate F in moving frame as:

$$\Theta = 1 + F - \frac{\varphi}{2}, F = \int_0^h \frac{\partial \psi}{\partial y} dy. \quad (17)$$

3 Exact solutions

The exact solution of Eq. (14) is

$$\theta = y \left(\frac{1}{h} + \frac{b_5}{2} (h - y) \right), \quad (18)$$

where

Fig. 3 Axial velocity profile u for **a** m_e , **b** U_{hs} , **c** χ , **d** H_r , with $\varphi = 0.4$, $x = 5.5$, $\Theta = 0.3$, $m_e = 5$, $U_{hs} = -1$, $\chi = 0.01$, $H_r = 1$, $G_r = 0.01$, $B = 2$

$$b_1 = \frac{\alpha_{nf}}{\alpha_f}, b_2 = \frac{(\rho C_p)_{nf}}{(\rho C_p)_f}, b_3 = \frac{\mu_{nf}}{\mu_f}, b_4 = \frac{(\rho\beta)_{nf}}{(\rho\beta)_f}, b_5 = \frac{b_2}{b_1} B. \quad (19)$$

Invoking Eqs. (11) and (14) into Eq. (15), then solving the resultant equation using boundary condition (16a), (16b), again the exact solution of stream function is obtained as:

$$\psi = C_0 + C_1 \sinh(m_e y) + C_2 y + C_3 y^2 + C_4 y^3. \quad (20)$$

The solution of axial velocity ($u = \frac{\partial \psi}{\partial y}$) is as follows:

$$u = C_1 m_e \cosh(m_e y) + C_2 + 2C_3 y + 3C_4 y^2. \quad (21)$$

Now, summoning Eqs. (11), (14) and Eq. (18) into Eq. (12) for the exact solution of pressure gradient as:

$$\frac{dp}{dx} = C_5 + C_6 y + C_7 y^2. \quad (22)$$

Here, $C_0 - C_7$, are constants and omitted due quite bulky expressions.

The pressure rise per wavelength ΔP_λ is as follows:

$$\Delta P_\lambda = \int_0^1 \frac{\partial p}{\partial x} dx. \quad (23)$$

The heat transfer coefficient Z is defined as:

$$Z = h_x \frac{\partial \theta}{\partial y} \Big|_{y=h(x)}. \quad (24)$$

4 Comparison of pure blood and nano-blood

Figure 2 depicts that with the addition of copper to the base fluid, the velocity profile gradually decreases. It is observed that velocity profile certainly declines in central region with an increase in electroosmotic parameter. However, it enhances near the walls. The arrows in the figure show that the velocity profile for base fluid (without any addition of nanoparticles) is more in the central region as compared to walls.

Fig. 4 Pressure gradient dp/dx for **a** m_e , **b** U_{hs} , **c** χ , **d** H_r , **e** G_r , with $\varphi = 0.4$, $\theta = 0.3$, $m_e = 5$, $U_{hs} = -1$, $\chi = 0.01$, $H_r = 1$, $G_r = 0.01$, $B = 2$

5 Discussion

The exact solutions found in the previous section (i.e., velocity profile u , pressure rise ΔP , pressure gradient dp/dx , stream function ψ , temperature profile θ and heat transfer coefficient Z) have been discussed graphically for different physical variables (i.e., electroosmotic parameter m_e , Helmholtz–Smoluchowski velocity U_{hs} , Hartmann number H_r , volume fraction of copper nanoparticles χ ,

Grashof number G_r and heat absorption parameter B) here in this section.

The variation of velocity profile u is shown in Fig. 3a–d for various physical flow parameters. Here, in all the graphs, velocity at the wall and channel center is dissimilar due to boundary resistance. Figure 3a expresses that m_e boosts axial velocity u , i.e., electroosmotic parameter m_e is ratio of channel height to Debye length λ_d , which signifies that λ_d is inversely proportional to EDL thickness. Therefore, more fluid moves in central region. Figure 3b envisages the effect of Helmholtz–Smoluchowski velocity U_{hs}

Fig. 5 Pressure rise Δp_λ for **a** m_e , **b** U_{hs} , **c** χ , **d** H_r , **e** G_r , with $\varphi = 0.4$, $\Theta = 0.3$, $m_e = 5$, $U_{hs} = -1$, $\chi = 0.01$, $H_r = 1$, $G_r = 0.01$, $B = 2$

for u . It is concluded that change in U_{hs} significantly lowers the velocity at the central area of the vertical wavy microchannel. Here negative U_{hs} is taken which corresponds to a positively orientated electric field. Figure 3c illustrates the effect of Cu-nanoparticles for u . It is observed that the magnitude of axial velocity u declines in central region of microchannel with a rise in χ . This tells that the variation in nanoparticles concentration, reduces the velocity at the middle of microchannel, i.e., concentration of copper particles provides a resistance to flow and hence obstructs fluid from attaining high velocities at center part of the microchannel. Figure 3d shows that by increasing

the Hartmann number H_r , the axial velocity u decreases in the center part of the vertical microchannel. Since H_r is the ratio of electromagnetic force to viscous force so viscous forces are highly demonstrated as compared to the magnetic forces. Therefore, velocity profile decreases at the center of the microchannel but at the same time, it increases at the edges of the wavy microchannel.

Figure 4a–e is portrayed to examine the evolution of physical parameters (i.e., electroosmotic parameter m_e ,

Fig. 6 Streamline distribution for **a** $m_e = 2$, **b** $m_e = 5$, **c** $m_e = 10$ with $\varphi = 0.4$, $\theta = 0.3$, $U_{hs} = -1$, $\chi = 0.01$, $H_r = 1$, $G_r = 0.01$, $B = 2$

Helmholtz–Smoluchowski velocity U_{hs} , Hartmann number H_r , volume fraction of copper nanoparticles χ and Grashof number G_r) for pressure gradient dp/dx across the vertical wavy microchannel. Figure 4a emphasizes that by growing the m_e , magnitude of dp/dx decreases. Therefore, more

pressure gradient is required to move the nanoparticles. It indicates that the existence of EDL in the charged surface hinders the flow of nanoparticles. Likewise, it is important that the pumping characteristics can be modified by EDL and it can be systematized by thickening and thinning of

Fig. 7 Streamline distribution for **a** $U_{hs} = -1.5$, **b** $U_{hs} = -1.0$, **c** $U_{hs} = 0.0$ with $\varphi = 0.4$, $\theta = 0.3$, $m_e = 5$, $\chi = 0.01$, $H_t = 1$, $G_t = 0.01$, $B = 2$

Fig. 8 Streamline distribution for **a** $\chi = 0.0$, **b** $\chi = 0.01$, **c** $\chi = 0.02$ with $\varphi = 0.4$, $\Theta = 0.3$, $m_e = 5$, $U_{hs} = -1$, $H_r = 1$, $G_r = 0.01$, $B = 2$

Fig. 9 Streamline distribution for **a** $H_r = 1.0$, **b** $H_r = 2.0$, **c** $H_r = 3.0$ with $\varphi = 0.4$, $\theta = 0.3$, $m_e = 5$, $U_{hs} = -1$, $\chi = 0.01$, $G_r = 0.01$, $B = 2$

Fig. 10 Temperature profile θ for **a** χ , **b** B , with $\varphi = 0.3, x = 5.5, \theta = 0.2, m_e = 5, U_{hs} = -1, \chi = 0.01, G_r = 0.01, B = 2$

Fig. 11 Heat transfer rate Z for **a** χ , **b** B , with $x = 0.2, \varphi = 0.4, \theta = 0.3, m_e = 5, U_{hs} = -1, \chi = 0.01, G_r = 0.01, B = 2$

EDL width. Figure 4b shows that for increasing values of U_{hs} , dp/dx increases. Here, the negative values of U_{hs} are taken. The negative and positive values of U_{hs} refers to aligned axial electric field and reverse axial electric field. Figure 4c depicts that for gradually rising χ , the values of dp/dx decreases. Figure 4d portrays that by increasing Hartmann number H_r , the magnitude of dp/dx increases. Since H_r is the ratio of electromagnetic force to viscous force, higher values of H_r signify stronger Lorentz force, thus more pressure is required to overcome Lorentz force. Figure 4e reveals that an increase in the magnitude of G_r creates an increase in dp/dx . As, G_r is ratio of buoyancy forces to the viscous forces, i.e., when buoyancy forces are high than viscous forces, dp/dx increases.

Variation in pressure rise ΔP_λ against flow rate θ are shown in Fig. 5a–e. Figure 5a portrays that ΔP_λ declines as m_e rises which physically interprets that EDL formation on charged surface resists the Cu-blood based nanofluid flow. Also, Fig. 5b illustrates that ΔP_λ increases as U_{hs} increases. This exemplifies that pumping is controlled in electric field presence. Figure 5c–e shows that ΔP_λ is growing function

of Hartmann number H_r , volume fraction of copper nanoparticles χ and Grashof number G_r .

Effects of electroosmotic parameter m_e , Helmholtz–Smoluchowski velocity U_{hs} , nanoparticles fraction χ and H_r on streamlines are examined through Fig. 6, 7, 8 and 9. Figure 6a–c shows that dimensions of trapped bolus increases with increasing the electroosmotic parameter m_e . It shows that as the electroosmotic parameter m_e increases, the trapping bolus strongly appears in the EDL. Figure 7a–c reveals that number and size of enclosed bolus decreases as Helmholtz–Smoluchowski velocity U_{hs} increases. Since U_{hs} is directly proportional to electric field. Therefore, stronger Helmholtz–Smoluchowski velocity refers to stronger electric field, which strongly effects the bolus formation. Figure 8a–c displays streamline formation for various values of nanoparticles fraction χ . It reveals that number of enclosed bolus decreases as χ increases. It is due to resistance proposed by nanoparticles to fluid motion. Moreover, Fig. 9a–c discloses that magnitude and volume of enclosed bolus grows for different H_r values.

Since the prime use of the nanofluids is to act as a facilitator of heat transfer, heat transfer analysis for current flow configuration is performed through Fig. 10a, b. Figure 10a infers the magnitude of θ starts decreasing for more values of nanoparticles fraction χ . This means that nanoparticles are used as a cooling agent in present work. Figure 10b shows that θ boosts for higher values of B . Since, temperature is directly proportional to B . It is concluded that temperature is more rapid in case of Cu-blood based nanofluid flow

Figure 11a, b tells the variations in heat transfer coefficient Z for various values of nanoparticles fraction χ and heat absorption parameter B . Figure 11a illustrates that Z increases for more values of nanoparticles fraction χ . This is due to greater thermal conductivity of the nanofluids obtained by adding Cu-nanoparticles. This boosted thermal conductivity accelerates the heat transfer between solid boundary and fluid. Figure 11b illustrates that Z decreases as the heat absorption parameter B increases.

6 Concluding remarks

Here heat absorption, mixed convection and magnetic field effects in two-dimensional electroosmotic flow of viscous Cu-blood nanoparticles are discussed exactly. The two-phase flow model is used. The main outcomes are:

- Velocity of nanofluid is inversely proportional to U_{hs} , χ and H_r .
- Electroosmotic parameter m_e strongly affects the velocity profile.
- The magnitude of pressure rise increases as U_{hs} , χ , H_r and G_r increases.
- It is seen that the pressure gradient and the pressure rise increase as G_r increases.
- It is also noted that bolus becomes larger for greater values to m_e and becomes smaller for U_{hs} , χ and H_r .
- B have opposite effect on the heat transfer rate and temperature of nanofluid.
- Increase in χ reduces the temperature and it increases Z when the thermal conductivity of fluid is enhanced.
- With addition of copper to base fluid (pure blood), velocity profile gradually decreases as compare to pure blood.
- Nanoparticles behaved as a cooling agent in the vertical microchannel.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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