Influence of structural parameters on pitching stability of a vertical shiplift

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Abstract

To analyze the pitching stability of the fully balanced hoist vertical shiplift, this paper develops a fluid velocity potential and dynamical model of the vertical shiplift based on an improved energetic exponent method, to study the influence of the primary structural parameters on the pitching stability of an actual shiplift installed in GuoPitan shiplift (GuiZhou province, China). With the linearized potential theory and the undetermined coefficient method, the numerical analytical expression of the velocity potential is obtained by discussing the pitching motion of ship chamber. Applying the Lagrange equation of the second kind to establish a dynamical model of the shiplift having multiple degrees of freedom, the energetic calculated model for ship chamber in respect to the pitching motion is proposed. To accurately analyze the variation of the pitching stability, this paper develops the slope of energetic varied curve, based on the orthogonal polynomial approximation and the subsection averaging method. Also, the influence of the parameters is studied, including the distance of lifting point, the synchronous shaft stiffness, and the hoist radius, with the optimized design of these three structural parameters: $a = 40.0$ m, $C = 2.0 \times 10^9$ N/m, and $R = 1.8$ m. The changing range of the absolute value of energetic exponent for the distance of lifting points is the largest of the three structural parameters (5.00–12.80), indicating that the distance of lifting points has the most remarkable influence on the pitching stability. Moreover, by the Runge–Kutta algorithm, the response of the angle displacement is analyzed before and after optimization, and then the reliability of results is demonstrated. The four angle displacements at the peak values close to 300 s decrease by 11.11%, 13.33%, 15.91%, and 10.86% respectively after optimization. It reveals that the pitching stability of shiplift is enhanced evidently after optimization.

Keywords Energetic exponent method · Fully balanced hoist vertical shiplift · Optimization · Pitching stability · Structural parameter

1 Introduction

Lately in China, there has been a huge demand for water conservancy projects and their related innovations. Under these social circumstances, the paper aims to provide some technical advises for the second step of GuoPitan Shiplift, mainly focusing on the pitching stability. Full balanced hoist vertical shiplift, a kind of navigation structure,

is used for ship to overcome the difference of water level between the upstream and downstream of water conservancy project. As is shown in Fig. 1, the primary structure of full balanced hoist vertical shiplift is composed of ship chamber, hoisting system and tower column. More specifically, the part A in Fig. 1 shows that the hoisting system consists of winding drums lifting ship chamber, and synchronous shaft making the lifting of four winding drums

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Fig. 1 Primary structure of full balanced hoist vertical shiplift

as synchronous as possible. The part B in Fig. 1 shows that the cross-sectional view of ship chamber, a is the distance of two lifting points and one lifting point is formed by each two winding drums in one side of ship chamber. Counterweights include gravity counterweights connected to ship chamber with wire ropes, and torque counterweights connected to winding drums with wire ropes. In this paper, the primary three parameters, including distance of two lifting points a , stiffness of synchronous shaft C , and radius of winding drum R , are the keys of discussion.

In a full balanced hoist vertical shiplift, with the closed rectangular synchronous shafts, long and narrow ship chamber, and the larger elasticity of wire ropes, it is difficult to accurately quantify the various influential factors for pitching stability and their influential degree [1–3]. If the ship chamber capsizes due to instability, it will cause huge economic loss and intolerable social impact. Therefore, with the aim of distinguishing the parameters with great influence on pitching stability, it is very significant to study the accurate quantitative characterization for the influence of structural parameters on pitching stability.

Regarding the pitching stability of ship chamber, a discriminant of instability condition for ship chamber was derived based on the static equilibrium, with respect to water leakage condition [4]. The natural vibration characteristics and stability of full balanced hoist vertical shiplift, containing ship chamber, wire ropes, gravity counterweights, water and ship in ship chamber, was studied to develop the critical influential factors for natural vibration characteristics as well as the critical distance of lifting

points maintaining stability [5, 6]. The influence of synchronous shaft in the main lifting system was neglected in all the above studies. By establishing a series of the multivariable nonlinear differential equations, and using numerical calculation based on the Runge–Kutta method, the stability of ship chamber subjected to the rigid vertical vibration was addressed [7, 8]. A theoretical formulation was developed to analyze the critical distance of lifting points, by establishing a dynamical model without gravity counterweights [9]. Assuming a time-harmonic pitching excitation, a fluid dynamic model is presented to predict the capsizing moments based on the Housner theory [10]. A semi-analytical scheme is proposed to analyze the pitching stability of the high-lift wire ropes vertical shiplift [11]. In this study, a new 9-DOF coupled dynamics model of the complete main hoist system, shiplift chamber motion and shallow water sloshing was established. Combining the Lyapunov motion stability theory, the critical distance of suspension points (the important indicator of pitch stability) would be numerically calculated. When it comes to the theoretical studies about stability, there are many mathematical analytical methods including the Routh–Hurwitz theory, Lyapunov stability theory, Floquet theory, and Mathieu equation etc. More specifically, Lyapunov stability theory is suitable only for linear and nonlinear systems, rather than multi-free linear time-invariant systems, time-varying linear systems and nonlinear systems [12]. The Floquet theory and Mathieu equation are usually applied to study the stability of solutions for linear homogeneous periodic coefficient differential equations [13–15]. As for

constant coefficient linear system, a characteristic equation can be obtained by transforming coefficient matrix of differential equations. Under the Routh-Hurwitz theory, stability condition can be obtained in accordance with the root of the characteristic equation [16, 17]. However, without intuitive characterization of the changing law of stability, the stability criterion derived from the above theoretical analysis is considerably complicated. In terms of the stability of parametrically excited vibration systems, there are several primary research methods including Bolotin method, perturbation method, multiscale method, and numerical continuation method, etc. [18–20]. With drawing method, these works have been devoted to identify the unstable regions and to predict the possibility of instability. Nevertheless, it is exceedingly difficult to identify the unstable region through this method, because the instability under different incentives is highly different for a great number of nonlinear systems. Therefore, an increasing number of researchers have conducted numerous works about innovative methods of evaluating instability. The growth rate of instability characteristic is evaluated by the exponent in Floquet theory [21, 22]. Subsequently, an energetic exponent method was firstly proposed to analyze the stability of two-dimensional fluid sloshing in the container of various shape, and the mathematical model of energetic exponent is derived based on Mathieu equation [23, 24]. These two papers indicate that the energetic exponent method is very convenient and efficient to accurately quantify the stability of two dimensional fluid–solid coupling system by analyzing the mechanical energy of system. However, the traditional energetic exponent method cannot present the change of the pitching stability directly because of its nonlinear characteristic. In order to compensate for this weakness, an improved energetic exponent method is proposed to perform an approximation solution of linear polynomial for each part of the slope of energetic exponent curve, by using the Best square approximation method for Legendre orthogonal polynomials as well as the subsection averaging method.

Section 3 of this paper derives a mathematical model for velocity potential of fluid subjected to pitching excitation. Section 4 proposes an improved energetic exponent method based on the dynamical model of shiplift. In Sect. 5, it investigates the influence of the three structural parameters with the improved energetic exponent method. As for Sect. 6, the three structural parameters of the second step of the shiplift of GouPitan water conservancy is optimized to increase the stability, and a formulation is proposed to analyze the change of the stability. Finally, a set of numerical results is obtained by using the Runge–Kutta algorithm, and the comparison of the numerical and the theoretical results proves the validity of the theoretical analysis.

2 Velocity potential of water in ship chamber

Figure 2 illustrates that the ship chamber of the full balanced hoist vertical shiplift is a two dimensional rectangular container filled with fluid, h means the depth of fluid and L means the length of ship chamber. It is assumed that the fluid is incompressible and inviscid, and the coordinate system origin O is located at the height of $h/2$, with ϕ being the fluid velocity potential. Thus, the fluid continuity equation is as follows:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0. \quad (1)$$

The boundary conditions of the free surface are written as:

$$\left(\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} \right) \bigg|_{z=\frac{h}{2}} = 0 \quad (2)$$

where t means time. Assuming $a(t)$ is the pitching motion of the ship chamber boundaries, it can be given by:

$$\frac{\partial \phi}{\partial z} \bigg|_{z=-\frac{h}{2}} = -x \dot{a}(t) \quad (3)$$

$$\frac{\partial \phi}{\partial x} \bigg|_{x=\pm \frac{L}{2}} = z \dot{a}(t). \quad (4)$$

The assumption of velocity potential [18] is written as:

$$\phi = \sum_{n=1}^{\infty} A_n \sinh \left[\frac{(2n-1)\pi z}{h} \right] \sin \left[\frac{(2n-1)\pi x}{h} \right] \quad (5)$$

where A_n is the amplitude of velocity potential. Next, the Fourier series expansion is carried out for x in Eq. (3), and set $\lambda_n = (2n-1)\pi$, the equation can be written as:

$$x = \sum_{n=1}^{\infty} \left(\frac{2h^2}{\lambda_n^2} \right) (-1)^{n-1} \sin \left(\frac{\lambda_n x}{h} \right). \quad (6)$$

Fig. 2 2-D water area for ship chamber

Based on Eqs. (3), (5) and (6), the equation is as follows as:

$$\sum_{n=1}^{\infty} A_n \frac{\lambda_n}{h} \cosh\left(\frac{\lambda_n}{2}\right) \sin\left(\frac{\lambda_n x}{h}\right) = - \sum_{n=1}^{\infty} \left(\frac{2h^2}{\lambda_n^2}\right) (-1)^{n-1} \sin\left(\frac{\lambda_n x}{h}\right) \dot{\alpha}(t). \quad (7)$$

Thus, time function A_n is written as:

$$A_n = 2(-1)^n \left(\frac{h}{\lambda_n}\right)^3 \frac{\dot{\alpha}(t)}{\cosh\left(\frac{\lambda_n}{2}\right)}. \quad (8)$$

The A_n be obtained by substituting Eq. (8) into Eq. (5), in the form:

$$\phi = \sum_{n=1}^{\infty} \frac{\dot{\alpha}(t)(2h^3)(-1)^n}{\cosh\left(\frac{\lambda_n}{2}\right)\lambda_n^3} \sinh\left(\frac{\lambda_n z}{h}\right) \sin\left(\frac{\lambda_n x}{h}\right). \quad (9)$$

3 Improved energetic exponent method

3.1 Formulation of energetic exponent

The energetic expression for pitching motion of ship chamber is written as [24]:

$$T(t) = \iiint \frac{\rho}{2} \left\{ \left[\frac{\partial \phi(x, z)}{\partial x} \right]^2 + \left[\frac{\partial \phi(x, z)}{\partial z} \right]^2 \right\} dx dy dz \quad (10)$$

$$V(t) = B \int_{\partial S_f} \frac{\rho g}{2} z^2(x, t) ds \quad (11)$$

where $T(t)$ and $V(t)$ are the kinetic energetic and potential energy of the fluid system, respectively. B is the width of ship chamber, ρ is the density of liquid, g is acceleration of gravity.

Based on Eqs. (9), (10) and (11), the function of energetic varies with time $E(t)$ can be obtained, in the form:

$$E(t) = T(t) + V(t) = I_1 [\dot{q}(t)]^2 + I_2 [q(t)]^2 \quad (12)$$

where I_1 ($i = 1, 2$), $q(t)$ and $\dot{q}(t)$ can be calculated as follows:

$$I_1 = \frac{B\rho\pi(-1+2n)\left(h\sin\left[\frac{L(1-2n)\pi}{h}\right] - L\sinh[(1-2n)\pi]\right)}{4h} \quad (12a)$$

$$I_2 = -\frac{(-1+2n)\pi\cosh\left[\frac{\pi-2n\pi}{h}\right]^2\left(L(-1+2n)\pi + h\sinh\left[\frac{L(1-2n)\pi}{h}\right]\right)\rho g B}{4h^2} \quad (12b)$$

$$\dot{q}(t) = A_n \quad (12c)$$

$$q(t) = \frac{\alpha(t)(2h^3)(-1)^n}{\cosh\left(\frac{\lambda_n}{2}\right)\lambda_n^3}. \quad (12d)$$

The formulation of energetic exponent $F(t)$ is arranged in the form:

$$F(t) = \ln \left[\frac{E(t, \xi_1)}{E(0, \xi_1)} \right]. \quad (13)$$

3.2 Dynamic model of vertical shiplift

Vertical shiplift, most commonly used form of shiplift, has been widely used in China where seven vertical shiplifts are functioning for primary water conservancy projects. GouPitan shiplift, discussed in this paper, is most representative vertical shiplift. Figure 3 illustrates the dynamic model of fully balanced hoist vertical shiplift, with nine degrees of freedom together: $\alpha(t)$ being pitching angle displacement of ship chamber, $\theta(t)$ being pitching angle displacement of free surface, $\varphi_1(t)$ and $\varphi_2(t)$ being rotating angle displacement of hoist, $z_1(t)$ and $z_4(t)$ being vertical displacement of the torque counterweights, $z_2(t)$ and $z_3(t)$ being vertical displacement of the gravity counterweights, $z_5(t)$ being vertical displacement of the ship chamber. J_1 denotes the equivalent moment of inertia for the hoist and torque counterweights, with J_2 being the equivalent moment of inertia for the pulleys, J_3 being the equivalent moment of inertia for the pitching motion of the ship chamber and liquid, γ_i ($i = 1, 2, 3$) respectively are the damping coefficient of the torsional vibration of the winding

Fig. 3 Model diagram of the full balanced hoist vertical shiplift

drums, the damping coefficient of the pitching motion for ship chamber and the damping coefficient of the vertical motion for ship chamber. K being the stiffness of the wire ropes, C being the stiffness of the synchronous shaft, R being the radius of the winding drums, t_i ($i = 1, 2, 3$) being the proportional coefficient of wire ropes and synchronous shaft, m_1 being the mass of liquid and ship chamber, m_2 being the mass of gravity counterweights, m_3 being the mass of gravity counterweights, F being the loading of ship chamber in the vertical direction.

For the convenience of solution, each free degree can be written as:

$$[\alpha(t), \theta(t), \varphi_1(t), \varphi_2(t), z_1(t), z_2(t), z_3(t), z_4(t), z_5(t)] = e^{\xi_i t} \sin t, (i = 1, 2, \dots, 9). \quad (14)$$

The kinetic energy and potential energy of shiplift is defined as:

$$T = \frac{1}{2}J_1(\dot{\varphi}_1^2 + \dot{\varphi}_2^2) + \frac{1}{2}J_2(\dot{\varphi}_1^2 + \dot{\varphi}_2^2) + \frac{1}{2}J_3\dot{\alpha}^2 + \frac{1}{2}m_1\dot{z}_5^2 + \frac{1}{2}m_2\dot{z}_2^2 + \frac{1}{2}m_2\dot{z}_3^2 + \frac{1}{2}m_3\dot{z}_1^2 + \frac{1}{2}m_3\dot{z}_4^2 \quad (15a)$$

$$V = \frac{1}{2}K(R\varphi_1 - z_5 - \frac{1}{2}a\alpha)^2 + \frac{1}{2}K(R\varphi_2 - z_5 + \frac{1}{2}a\alpha)^2 + \frac{1}{2}t_3K(R\varphi_1 + z_1)^2 + \frac{1}{2}t_2K(R\varphi_1 + z_2)^2 + \frac{1}{2}t_2K(R\varphi_2 + z_3)^2 + \frac{1}{2}t_3K(R\varphi_2 + z_4)^2 + \frac{1}{2}C(\varphi_1 - \varphi_2)^2 + \frac{1}{2}t_1C(\varphi_1^2 + \varphi_2^2). \quad (15b)$$

Using the Lagrange equation of the second kind, the dynamical equations are as follows:

$$(J_3 + 2J_s)\ddot{\alpha} + \gamma_2\dot{\alpha} - \frac{1}{2}aK(R\varphi_1 - z_5 - \frac{1}{2}a\alpha) + \frac{1}{2}aK(R\varphi_2 - z_5 + \frac{1}{2}a\alpha) - C_s(\alpha - \theta) - \frac{10gh}{L^2}J_s\theta = 0 \quad (16a)$$

$$(J_3 + 2J_s)\ddot{\theta} + \gamma_2\dot{\alpha} - \frac{1}{2}aK(R\varphi_1 - z_5 - \frac{1}{2}a\alpha) + \frac{1}{2}aK(R\varphi_2 - z_5 + \frac{1}{2}a\alpha) - C_s(\alpha - \theta) + \frac{10gh}{L^2}(J_s + J_3)\theta = 0 \quad (16b)$$

$$(J_1 + J_2)\ddot{\varphi}_1 + \gamma_1\dot{\varphi}_1 + KR(R\varphi_1 - z_5 - \frac{1}{2}a\alpha) + t_3KR(R\varphi_1 + z_1) + t_2KR(R\varphi_1 + z_2) + C(\varphi_1 - \varphi_2) + t_1C\varphi_1 = 0 \quad (16c)$$

$$(J_1 + J_2)\ddot{\varphi}_2 + \gamma_1\dot{\varphi}_2 + KR(R\varphi_2 - z_1 + \frac{1}{2}a\alpha) + t_2KR(R\varphi_2 + z_3) + t_3KR(R\varphi_2 + z_4) - C(\varphi_1 - \varphi_2) + t_1C\varphi_2 = 0 \quad (16d)$$

$$m_3\ddot{z}_1 + \gamma_3\dot{z}_1 + t_3K(R\varphi_1 + z_1) = 0 \quad (16e)$$

$$m_2\ddot{z}_2 + \gamma_3\dot{z}_2 + t_2K(R\varphi_1 + z_2) = 0 \quad (16f)$$

$$m_2\ddot{z}_3 + \gamma_3\dot{z}_3 + t_2K(R\varphi_2 + z_3) = 0 \quad (16g)$$

$$m_3\ddot{z}_4 + \gamma_3\dot{z}_4 + t_3K(R\varphi_2 + z_4) = 0 \quad (16h)$$

$$m_1\ddot{z}_5 + \gamma_3\dot{z}_5 - K(R\varphi_1 - z_5 - \frac{1}{2}a\alpha) - K(R\varphi_2 - z_5 + \frac{1}{2}a\alpha) = F \quad (16i)$$

$$\text{where } J_s = (1/24)\rho BhL^3, C_s = (1/12)\rho BhL^3.$$

3.3 Slope of energetic exponent curve exponent

A series of nonlinear equations with respect to ξ_i ($i = 1, 2, \dots, 9$) can be elicited by combining Eqs. (14) and (16). Subsequently, with the application of Broyden quasi-Newton method [8], ξ_1 and $\alpha(t)$ are obtained. Finally, $F(t, \xi_1)$ is spelled out based on the Eqs. (12d) and (13).

The slope of energetic exponent curve cannot present the change of the pitching stability directly because of its nonlinear characteristic. In order to compensate for this weakness, an improved energetic exponent method is proposed to perform an approximation solution of linear polynomial for the slope of energetic exponent curve, by

using the Best square approximation method for Legendre orthogonal polynomials as well as the subsection averaging method. As is shown in Fig. 4, beginning from [0, 1], the interval is narrowed continuously by changing its right endpoint until the approximation error of linear polynomial is lower than the controlling range, and the slope of this linear polynomial at this time is named as K_1 . Next, it is essential that the right endpoint is transformed into the left endpoint of the next interval, and a new interval is obtained by repeating the previous steps until the left endpoint of the N interval becomes 100, with attaining a series of the slopes $K_2, K_3, \dots, K_n$.

Subsequently, the slope of energetic exponent for vertical shiplift K is written as:

$$K = \frac{1}{n} \sum_{i=1}^n K_i. \quad (17)$$

Finally, the criteria for stability is arranged in the form:

$$K = \begin{cases} > 0 \text{ stability} \\ = 0 \text{ stability boundary} \\ < 0 \text{ instability} \end{cases}. \quad (18)$$

4 Influence of structural parameters

To analyze the influence of the primary structural parameters including the distance of critical lifting point a , the synchronous shaft stiffness C , as well as the hoist radius R , taking the second step of shiplift of GouPitan water conservancy, with the greatest lifting height and lifting mass ($H=113.0$ m, $m_1=m_2=1075$ t) throughout China, as a sample calculation. Founded on the design experience

Fig. 4 Solution of slope for energetic exponent curve

of the vertical shiplift, the variable range of its parameters implies: $a \in [30.0 \text{ m}, 40.0 \text{ m}]$, $C \in [1.0 \times 10^9 \text{ N/m}, 2.0 \times 10^9 \text{ N/m}]$, $R \in [1.8 \text{ m}, 2.6 \text{ m}]$, respectively. In the corresponding range of values, a series of nodes are selected at the same distance, and the corresponding value K of each node is solved by the improved energetic exponent method.

The influence of distance for lifting point a on energetic exponent K is shown in Fig. 5. The increasing lifting point leads to the increasing reverse overturning moment caused by tension difference of wire ropes at the two lifting points and the increasing pitching acceleration of ship chamber, contributing to the increasing anti overturning capability of ship chamber. The changing range for the absolute value of energetic exponent is 5.00–12.80. Therefore, the energetic exponent decreases with the increasing distance of lifting point a , contributing to the increasing stability.

The influence for stiffness of synchronous shaft C on energetic exponent K is shown in Fig. 6. The synchronous shaft stiffness increases with increasing reverse torsional moment, contributing to the increasing reverse rotation angular acceleration at the two lifting points, and the rotation speed of the hoist rapidly decays to zero. The changing range for the absolute maximum absolute value of energetic exponent is 4.50~9.50. Thus, the energetic exponent decreases with increasing synchronous shaft stiffness C , leading to the increasing stability.

The influence of distance for lifting point a on energetic exponent K is shown in Fig. 7. The lower reverse rotation angular acceleration is attributed to the increasing moment of inertia of hoist caused by the increasing hoist radius when the reverse torsional moment of synchronous shaft is constant. The changing range for the maximum absolute value of energetic exponent is 1.87–3.20. Consequently, the instability enhances with the extending

Fig. 5 Influence of distance for lifting point a on energetic exponent K

Fig. 6 Influence of synchronous shaft stiffness C on energetic exponent K

decay time of the rotation angular speed for hoist at the two lifting points.

5 Optimization of shiplift

With the aim of the avoiding complicated for the optimization of the shiplift, an approximated mathematical model of the energetic exponent slope K is obtained to bring a great convenience for the engineering design. The five data points are selected for each structural parameter according to the equal distance, and a $5 \times 5 \times 5$ three dimensional data set is formed by crossing each other. In this set, based on each group of a , C , R , the corresponding ξ_1 is calculated in accordance of Eq. (16). A function is obtained by using the three dimensional fitting, in the form:

$$\xi_1 = 0.07524 + 2.052 \times 10^{-4}a - 3.867 \times 10^{-5}R + 2.587 \times 10^{-14}C. \quad (19)$$

In the change of time t range $[0, 100]$, the energetic exponent slope K is assumed to the following two equations:

$$K_1 = -\frac{1}{3} \left[\frac{F(10n, \xi_1) - F(0, \xi_1)}{10n} + \frac{F(100 - 10n, \xi_1) - F(10n, \xi_1)}{100 - 20n} + \frac{F(100, \xi_1) - F(100 - 10n, \xi_1)}{10n} \right] \quad (20a)$$

$$K_2 = -\frac{1}{4} \left[\frac{F(10n, \xi_1) - F(0, \xi_1)}{10n} + \frac{F(100 - 10n, \xi_1) - F(10n, \xi_1)}{50 - 10n} + \frac{F(100, \xi_1) - F(100 - 10n, \xi_1)}{10n} \right]. \quad (20b)$$

Fig. 7 Influence of radius of winding drums R on energetic exponent K

After taking $n = 1, 2, 3, 4$, respectively, Table 1 illustrates the difference of the results from Eq. (20) and the improved energetic exponent method, and displays that the error is the lowest when $n = 4$, 3.8%. Therefore, founded on Eqs. (12), (13) and (20b), the approximated mathematical model of the energetic exponent slope is arranged in the form:

$$K_2 = \frac{-1}{160} \left[\ln \left(\frac{l_1 A_1 + l_2 A_2}{4l_1 h^6} \right) + 4 \ln \left(\frac{l_1 A_3 + l_2 A_4}{l_1 A_1 + l_2 A_2} \right) + \ln \left(\frac{l_1 A_5 + l_2 A_6}{l_1 A_3 + l_2 A_4} \right) \right] \quad (21)$$

Table 1 Results of for critical distance of lifting points of seven most representative full balanced hoist vertical shiplift in China

Parameters	$n=1$	$n=2$	$n=3$	$n=4$
K_1	-3.402	-1.587	-0.975	-0.645
Error	580.4%	217.4%	95%	29%
K_2	-3.320	-1.501	-0.882	-0.519
Error	564.0%	200.2%	76.4%	3.8%

Fig. 8 Responses of pitching angular displacements for ship chamber α

$$A_1 = [2e^{40\xi_1} (0.643\xi_1 + 0.766)h^3]^2 \quad (21a)$$

$$A_2 = [1.286e^{40\xi_1} \xi_1 h^3]^2 \quad (21b)$$

$$A_3 = [2e^{60\xi_1} (0.866\xi_1 + 0.5)h^3]^2 \quad (21c)$$

$$A_4 = [1.732e^{60\xi_1} \xi_1 h^3]^2 \quad (21d)$$

$$A_5 = [2e^{100\xi_1} (0.985\xi_1 - 0.174)h^3]^2 \quad (21e)$$

$$A_6 = [1.970e^{100\xi_1} \xi_1 h^3]^2 \quad (21f)$$

By using Eq. (19), for the enhancement of pitching stability, taking $K_2(a, C, R)$ as the objective function, the minimum K_2 can be obtained. The optimal boundary conditions are written as:

$$a \in [30.0, 40.0] \quad (22a)$$

$$C \in [1.0 \times 10^9, 2.0 \times 10^9] \quad (22b)$$

$$R \in [1.8, 2.6]. \quad (22c)$$

Finally, the minimization of the objective function $K_2(a, C, R)$ is -15.18 , with $a=40.0$ m, $C=2.0 \times 10^9$ N/m, and $R=1.8$ m.

6 Numerical validation

In order to identify the difference of the dynamic characteristics before and after optimization of structural parameters, the further validation is performed for the influence of primary structural parameters on the pitching stability, and the displacement responses are analyzed by the application of the Runge–Kutta algorithm. Figures 8 and 9 illustrate the change of the pitching angular displacement α of ship chamber and the pitching angular displacement θ of free surface for the second step of the shiplift of GouPitan water conservancy before and after optimization, respectively. The black solid lines in the figures represent the responses of the pitching angular displacement and the pitching angular displacement of free surface before optimization, and the blue solid lines represent that after optimization. As shown in Fig. 8, the last peak of angle displacement close to 300 s is 0.0126° before optimization while the value at the same location is 0.0112° after optimization. The peak of angle displacement is reduced by 11.11%. On the other hand, the last trough of angle displacement close to 300 s is -0.0127° before optimization while the value at the same location is -0.0114° after optimization. The absolute values for trough of angle displacement is reduced by 10.24%. For the other three angular displacements, the peak values close to 300 s decrease by 13.33%, 15.91%, 10.86% after optimization while absolute values of trough close to 300 s decrease by 13.16%, 15.04%, 10.56% after optimization.

It is revealed that the absolute values of peak and trough of two angular displacements after optimization are smaller than those before optimization. Thus, in the range of time 0 s to 300 s, the time of dynamic response

Fig. 9 Responses of pitching angular displacements for free surface θ

Fig. 10 Responses of angular displacements for winding drum φ_1

Fig. 11 Responses of angular displacements for winding drum φ_2

Table 2 Influence of primary structural parameters on pitching stability

Parameters		t (s)
a (m)	30.0	1.3922
	35.0	0.5849
	40.0	0.5283
C (N/m)	1.0×10^9	0.6122
	1.5×10^9	0.6060
	2.0×10^9	0.6035
R (m)	1.8	0.6062
	2.3	0.6096
	2.8	0.6118

to stability is obviously shortened after optimization, with the evidently-reduced dynamical responses (Figs. 10, 11).

Table 2 presents the influence of the primary structural parameters on the time (the pitching angular displacements decay to 10^{-3} degree). The distance of lifting points and the synchronous shaft stiffness increases gradually with the reducing decaying time. It suggests

that the increasing distance of lifting points and synchronous shaft stiffness arise the more shorten time and increasing stability. Moreover, the hoist radius increases gradually with the increasing decaying time. It indicates that the increasing hoist radius arises the more extended time and decreasing stability.

7 Conclusion

In accordance with the 2D fluid solid coupling of ship chamber and mechanical dynamic of shiplift, an improved energetic exponent method founded on the linear water theory, best square approximation method, and the subsection averaging method is presented. In order to pursue the minimum of energetic exponent, the primary structural parameters of shiplift is optimized. Firstly, using the orthogonal polynomial approximation, and the subsection averaging method, an improved energetic exponent method is proposed to analyze the change of stability for shiplift. Based on the improved energetic exponent method, a formulation is presented to calculate the slope of the energetic exponent curve. For the second step of shiplift of GouPitan water conservancy, the error of the formulation in comparison to the improved energetic exponent method, only 3.80%. Moreover, for the enhancement of pitching stability of shiplift, the primary structural parameters of the second step of shiplift of GouPitan water conservancy is optimized to be: $a = 40.0$ m, $R = 1.8$ m, $C = 2.0 \times 10^9$ N/m. Finally, the stability of shiplift increases with an increasing distance of lifting points and synchronous shaft stiffness, while decreases with an increasing hoist radius. Particularly, distance of two lifting points is the most significant factor to affect the stability.

However, the influence of tower column and hydraulic leveling system are not considered in establishing dynamical system. Future work can focus on discussing the stability of shiplift by a mechanical-hydraulic-structural-fluid coupling system.

Compliance with ethical standards

Conflict of interest The authors declare they have no conflict of interest.

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