Elastic wave velocities as indicators of lithology-based geomechanical behaviour of sedimentary rocks: an overview

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Abstract

Different geomechanical properties such as elastic modulus/deformation modulus, Poisson's ratio (ν), uniaxial compressive strength, shear strength properties, tensile strength and point load index are widely used for rock mass characterisation in geological and geotechnical engineering. However, there are no such direct methods by which these properties can be obtained in the laboratory or in situ, without following time-consuming and a laborious procedure. Thus, ultrasonic technique, an indirect method, was found reliable to determine these properties in rocks. Many researchers have studied the correlation between the compressional wave velocity (V_p) and the geomechanical properties of sedimentary rocks such as sandstones and carbonates. However, these correlations have not considered the change in rock mineralogy, porosity and saturation conditions when deriving relationships. Hence, in this study, the authors attempt to understand the variability of V_p with the help of a fragmented analysis of major mineral constituents, porosity variation and saturation conditions for rock mass classification based on V_p . A review of the existing studies on the relationship between the rock properties and V_p has been used to perform this analysis. The resulting template can be a basis for interpreting more realistic lithology-based geomechanical behaviours and thus highlights the importance of an integrated study involving geological, petrophysical and engineering data. The results derived from fragmented analysis indicate that with an increase in quartz content and a subsequent decrease in feldspar content in sandstones, Young's modulus (E) and V_p increase. With an increase in porosity from 2 to 40%, there is a decrease in E and V_p values for all types of saturation scenarios (water, gas, brine and oil saturated) for sandstones and carbonates.

Keywords Compressional wave · Velocity · Ultrasonic · Elastic modulus

List of symbols

A	Area of specimen	SDI	Slake durability (%)
C	Clay content (%)	S_r	Degree of saturation (%)
D_e	Effective diameter	t	Transit time
E	Elastic modulus (GPa)	UCS	Unconfined compressive strength (MPa)
G	Shear modulus (GPa)	V	Loss of volume on wear ($\text{cm}^3/50 \text{ cm}^2$)
I_s	Point load strength (MPa)	V_p	P-wave velocity (km/s)
K	Bulk modulus (GPa)	V_s	S-wave velocity (km/s)
L	Length of specimen	τ	Shear strength (MPa)
n	Porosity (%)	ν	Poisson's ratio
P	Breaking load	λ	Lame's parameter
		σ_t	Indirect tensile strength (MPa)

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1 Introduction

Most of the engineering projects related to rocks such as petroleum reservoir geomechanics, foundation and slopes, civil and mining require the knowledge of the subsurface properties during the initial phase of any exploration activity. The behaviour of a rock mass in response to a specific loading system is influenced by its mechanical and the strength properties. However, the present set of established correlations do not consider the change in rock mineralogy, porosity and saturation conditions. The factors such as lithology, density, porosity, permeability, clay volume, bedding planes, weathering, porewater, anisotropy, grain size, mineralogy and saturation play an influential role in affecting the geomechanical properties of rocks [8, 21, 70, 83]. The geomechanical properties such as elastic modulus/deformation modulus, Poisson's ratio (ν), uniaxial compressive strength (UCS), shear strength properties, tensile strength, point load index and rock hardness play a decisive role while assessing stability criteria for studies related to geotechnical problems that involve civil, mining, slope stability, underground excavations, oil recovery, nuclear waste disposal and well bore instabilities during drilling [11, 52]. The rock strength is a crucial geomechanical property, and it is categorised as compressive (UCS) and tensile. The rock's bearing capacity may be indicative by the UCS value of rock, and elastic modulus and Poisson's ratio can be estimated from UCS [80]. Tensile strength is indicative of the resistance of failure of the rock mass, and it is imperative when dealing with stability of underground openings [65]. Modulus of elasticity (E) can be used for intact rock classification and rock failure criteria. Among the elastic properties, Poisson's ratio (ν) plays a fundamental role in the elastic deformation of rocks and rock masses subjected to static/dynamic stresses [26]. The slake durability test estimates the resistance of rock samples to weakening and breakdown resulting from a standard cycle of wetting and drying [70, 72]. Point load strength is also a measure for the determination of rock strength. Point load test is an alternate method to estimate UCS since it is a reliable, economical and easy technique [52]. The conventional tests used for determining the geomechanical properties in laboratory involve rigorous, time-consuming and destructive tests [45]. Moreover, these procedures are expensive which involve the requirement of specimens to be precisely, carefully prepared (following the specifications that are laid as per standards). There are no such direct methods by which these properties can be obtained in the laboratory or in situ, without following time-consuming and a laborious procedure [47].

The difficulty mostly arises due to the sampling issues encountered during the boundary strength conditions of a weak rock [61]. These difficulties motivated researchers to develop non-destructive techniques so as to predict strength and deformability properties which can be measured from relatively easier and reliable techniques such as ultrasonic methods [45].

The application/usage of compressional wave velocity, also known as V_p or p-wave (primary wave), is a common non-destructive method for rock mass characterisation in laboratory and on-site [69, 81]. The elastic wave velocity such as V_p provides a comprehensive index for determining the petrophysical and mechanical properties of rock [12]. The elastic waves can be reproduced in the laboratory with the help of ultrasonic techniques [45, 66]. In other words, in comparison with the mechanical tests, ultrasonic techniques are simple to use, rapid to analyse and more economical than any other technique [27, 44]. As a result, to determine the quality of rock without destructing the rock, ultrasonic techniques are widely used in geotechnical, rock science and mining practices [48]. Researchers have obtained a strong correlation between geomechanical properties (UCS, indirect tensile strength, E , ν , slake durability, point load strength) and V_p , V_s (longitudinal and shear wave) for different rocks (sandstone, shale, limestone, coal, marlstone, siltstone, conglomerate, marl, dolomitic limestone, dolomite, argillaceous limestone, claystone). Table 1 presents the summary of techniques used by various researchers on different rocks reported in the literature. Different empirical relations have also been reported in the literature correlating these geomechanical properties with V_p , V_s for different rock types. These correlations were achieved based on numerous testing from different types of formations with different lithology and different parts of the world (Fig. 1). The use of such empirical correlations is sometimes the only method to evaluate the strength under certain conditions where rock cores cannot be obtained for conducting laboratory tests. However, the relationship between the geomechanical properties and V_p , V_s also depends on some influential factors such as grain size and shape, rock type, rock texture and structure, mineralogical composition, density, porosity, temperature, porewater, anisotropy, weathering, confining pressure, joint properties, alteration zones and bedding planes [83]. Some of these factors such as mineralogy, saturation conditions and porosity effects have been considered in this study for the estimation of lithology-based geomechanical behaviour on the basis of V_p .

This paper discusses the lithology-based assessment of geomechanical properties of sedimentary rocks on the basis of V_p and hence provides a methodology for future-based similar studies. Initially, the paper collates the research conducted by different researchers on V_p and its relation with

Table 1 Summarised data details reported in the literature

Designation		Rock type	Laboratory technique used	References
A	A	Limestone, mudstones, psammite, greenstones, granites	–	D'Andrea et al. [15] and extended by McCann et al. [54]
B	B	Sandstone	–	Freyburg [23]
C	C	Sandstone	–	Fjaer [22]
D	D	Sandstone	Through transmission, P-wave frequency 1 MHz and S wave 0.6 MHz, V_p measured on water-saturated rocks at different confining pressures (up to 40 MPa)	Han et al. [28]
E	E	Sandstone	–	Moos et al. [58]
F	F	Limestone	Through transmission, transducer frequency 54 kHz	Kahraman [42]
G	G	Carbonates	Through transmission	Yasar and Erdogan [83]
H	H1	Sandstone	Not mentioned	Chang et al. [11]
	H2	Shale		
	H3	Limestone		
	H4	Dolomite		
I	I1	Limestone	Through transmission	Kilic and Teymen [48]
	I2	Sandstone		
J	J1	Limestone	Not mentioned	Çobanoglu and Çelik [14]
	J2	Sandstone		
L	L	Carbonate rocks	Through transmission transducer frequency 54 kHz	Kahraman and Yeken [44]
M	M1	Sandstone	Through transmission	Sharma and Singh [70]
	M2	Coal		
	M2	Shaly rock		
N	N1	Limestone	Through transmission, transducer frequency 1 MHz	Moradian and Behnia [59]
	N2	Sandstone		
	N3	Marlstone		
O	O1	Sandstone	Ultrasonic pulse technique	Soroush et al. [73]
	O2	Siltstone		
	O3	Conglomerate		
	O4	Limestone		
	O5	Marl		
P	P1	Limestone	Through transmission	Sarkar et al. [66]
	P2	Sandstone		
	P3	Dolomitic limestone		
	P4	Dolomite		
	P5	Coal		
	P6	Shale		
Q	Q1	Marl	Through transmission	Karakul and Ulusay [45]
	Q2	Sandstone		
	Q3	Argillaceous limestone		
	Q4	Limestone		
	Q5	Sandstone		
	Q6	Clay stone		

Table 1 (continued)

Designation		Rock type	Laboratory technique used	References
R	R	Limestone	Not mentioned	Parent et al. [64]
S	T	Dolostone	High-emission MAE device with probes of frequency 53 kHz	Pappalardo [63]
T	T	Limestone	Ultrasonic pulse generator operating at 54 kHz	Kurtulus et al. [51]
U	U	Carbonate rocks	Through transmission	Madhubabu et al. [52]

Fig. 1 Location map of different sedimentary rock samples reported in this study. Legends: B-Thuringia, Germany, C-Gulf Coast, USA, D-Gulf of Mexico, E-Cook Inlet, Alaska, USA, F-Turkey, G-Adana, Turkey, I-Southern Anatolia, Turkey, J-Turkey, L-Nigde, Kayseri, Konya and Antalya, Turkey, M-Lower Shiwalik and Lower

Gondwana (Singrauli and Jharia), India, N-Iran, P-Jutogh, Lower Shiwalik and Lower Gondwana, India, Q-Turkey, S-Peloritani Mountains, NE Sicily, T-Cebecikoy, Hereke, Akveren, Soguck and Bakirkoy, Turkey, U-Kutch, Gujrat, India

the geomechanical properties of rocks. These relations may be used when it is difficult to obtain cores from weak, highly fractured and weathered rocks. Such indirect estimation, however, leads to an over- or underestimation of test results when correlated without taking into the consideration the lithology, saturation conditions and mineralogy. Thus, the later part of the paper inculcates the variability of major mineral constituents, saturation conditions and porosity on the geomechanical properties of sedimentary rocks. Such a fragmented analysis would help to demonstrate the benefits of lithology-based geomechanical behaviour/classification on the basis of V_p . The conclusions provide a summary of the state of play that should be understood by the users of this type of data. V_p and V_s are indicators of geomechanical behaviour, and it is important to stress that it is all they are. It is important to understand the mineralogy, density and porosity in order to avoid time and costs of running into unexpected problems during any underground excavations,

oil recovery, nuclear waste disposal and well bore instabilities during drilling.

2 Applied methodology

2.1 Compilation of the dataset

V_p , V_s have been correlated with different mechanical properties from dataset available in the literature to understand the existing association between the two and its importance in reservoir characterisation. The datasets in this review are based on mainly sandstones and carbonate rocks under ambient conditions. Moreover, the datasets considering confining pressure have not been considered in this review. This study was conducted by using literature datasets generated from experimental measurements under ambient conditions in order to have

Table 2 Instruments/tests used/conducted in the literature

Parameter analysis	Instrument/test
Vp and Vs	UPV meter
UCS	UCS test
Indirect tensile strength	Brazilian tensile strength test
Durability	Slake durability test
Mineralogy	XRD, thin-section studies

Table 3 Standards for determination of different parameters of rocks

Parameter	Standard
P-wave velocity (V_p)	ASTM D2845, IS 13311 Part 1:1992
Point load index	ASTM D5731, IS 8764-1998
UCS	IS 9143-1979, ASTM D2938-2002
Brazilian tensile strength	IS 10082-1981, ASTM D3967-08
Durability (slake durability)	IS 10050-1981

a common reference condition. Other datasets reported at confining pressures in the literature [28, 50, 74] were found to be derived under different stress values rather than a common standard value. Hence, it will be challenging to establish a common reference condition to correlate V_p with the different geomechanical parameters. In order to allow experimental results to be extrapolated to natural conditions at a depth z , such a study [38] may be referred when temperature and pore pressure effects also need to be considered. The datasets have been plotted together, and graphs of variation of compressional wave velocity with UCS, σ_v , E , ν , slake durability and point load strength have been shown. Table 2 highlights the instruments/tests used/conducted by researchers for their study to generate the datasets. Due to scarcity of available data, the rocks considered in this review, namely sandstones and carbonate rocks are neither from any particular geological time period nor any preference of a geographical region has been taken. The map (Fig. 1) shows the locations of the rock types considered, and it illustrates all the available datasets from the entire world have been considered for this literature review. It was seen that datasets for the measured shear wave velocity (V_s), available from experiments, were rare, while relating V_s and geomechanical properties of rocks. One of the reasons for this may be calculation of V_s from V_p values, based on methods, suggested by earlier researchers [9]. The foundation of those methods is several empirical relations, such as $V_p/V_s = 1.65-2$ [41], $V_p/V_s = 1.5-1.7$ for quartzose sandstones and $V_p/V_s = 1.9$ for pure limestones [1]. Therefore, by using these correlations, the relationship between V_s and geomechanical properties can be assessed. Moreover,

the datasets reported in the manuscript belong to tests conducted on dry rocks. Furthermore, to measure V_p , V_s in laboratory, researchers have used different ultrasonic techniques: through transmission and pulse-echo for different rock samples [10, 24, 25, 40, 43, 46]. The V_p , V_s values in the rock samples can be measured either by using a pulse generator unit, a pair of transducers (one acting as a transmitter and the other one as a receiver) of frequencies varying generally from 54 kHz to 1 MHz, along with an oscilloscope [60, 76] or by using portable ultrasonic non-destructive digital indicating tester, widely known as PUNDIT [55]. The literature reference regarding laboratory technique used by different researchers is summarised in Table 1. Two different types of transducers: a longitudinal (P-wave) transducer for measuring V_p and a shear (S-wave) transducer for measuring V_s , have been used. The test procedures for determining V_p , point load index, UCS and E have been standardised by ISRM [29, 31, 32] and ASTM [2-4] (Table 3). Table 3 summarises the standards for determination of different parameters of rocks by following a given set of protocols/specifications that are followed worldwide. As per recommendations [30], the minimum lateral dimension must not be less than ten times the wavelength. If the wavelength is considerably greater than the grain size, then the measured velocity is not frequency dependent [30].

2.2 Fragmented analysis of major mineral constituents and saturation conditions for rock mass classification based on V_p

Since there are multiple parameters that alter the elastic and mechanical properties of sedimentary rocks, an attempt is made to introduce and then analyse its variability with some of the mechanical properties. Since rock is composed of multiple minerals, varying the amount of these major minerals would reflect the change in V_p , and this change in V_p could thereby be related to get an inference on a certain property under investigation. For instance, in sandstones, the major minerals present include quartz and feldspar, while for carbonates, the major minerals present include calcite and dolomite. These minerals have certain parameter values such as $V_{p_{\text{quartz}}} = 6.057$ km/s, $V_{p_{\text{feldspar}}} = 4.68$ km/s, $V_{p_{\text{calcite}}} = 6.259$ km/s and $V_{p_{\text{dolomite}}} = 6.93$ km/s [16, 53, 62, 71]. These values are the maximum, i.e. for zero porosity. V_p of feldspar is taken as 4.68 km/s ("average" feldspar [53]). However, based on composition of feldspar, values may vary. Similarly, the other parameter values such as V_s , density, ν , K , G and E for these minerals are given in Table 4.

For changing V_p , the density of material was varied by changing the mineralogy of the rock as well as the

Table 4 Different parameter values for different minerals (these values are the maximum (zero porosity)) [16, 53, 62, 71]

Minerals	Vp (km/s)	Vs (km/s)	Density (g/cc)	Poisson's ratio	Bulk modulus <i>K</i> (GPa)	Shear modulus <i>G</i> (GPa)	Young's modulus <i>E</i> (GPa)
Quartz	6.057	4.153	2.65	0.08	37	44	95
Calcite	6.259	3.243	2.71	0.316	76.8	32	84.2
Dolomite	6.93	4.23	2.87	0.202	76.4	49.7	119.4
Feldspar	4.68	2.39	2.56	0.32	37.5	15	39.6
<i>Vp variation</i>							
Vp order	Mineralogy	Dolomite > calcite > quartz > feldspar					
Vp order	Fluids	Brine > water > oil > gas					

Table 5 Tabulated datasets of Poisson's ratio and Vp obtained by varying the volume percentage of major minerals for sandstones and carbonates

Sandstones				Carbonates			
Fraction of quartz	Fraction of feldspar	Vp (=Vp _{matrix})	Poisson's ratio (<i>u</i>)	Fraction of calcite	Fraction of dolomite	Vp (=Vp _{matrix})	Poisson's ratio (<i>u</i>)
0.75	0.25	5.71275	0.14	0.75	0.25	6.42675	0.2875
0.8	0.2	5.7816	0.128	0.8	0.2	6.3932	0.2932
0.85	0.15	5.85045	0.116	0.85	0.15	6.35965	0.2989
0.9	0.1	5.9193	0.104	0.9	0.1	6.3261	0.3046
0.95	0.05	5.98815	0.092	0.95	0.05	6.29255	0.3103
1	0	6.057	0.08	1	0	6.259	0.316

saturation of pores. Thus, we assume several volume percentages of these minerals so as to change the density of the material. For sandstones, the volume percentage was varied from 75% quartz and 25% feldspar to 100% quartz and 0% feldspar content. Similarly, for carbonates, volume percentage was varied from 75% calcite and 25% dolomite to 100% calcite and 0% dolomite content. Based on these varying volume mineral percentages, Vp was calculated as:

$$(V_{\text{Quartz}}) \times (V_{\text{pQuartz}}) + (V_{\text{Feldspar}}) \times (V_{\text{pFeldspar}}) = V_{\text{p}} \quad (1)$$

$$(V_{\text{Calcite}}) \times (V_{\text{pCalcite}}) + (V_{\text{Dolomite}}) \times (V_{\text{pDolomite}}) = V_{\text{p}} \quad (2)$$

where V_{Quartz} , V_{Feldspar} , V_{Calcite} and V_{Dolomite} are the volume percentages (in fraction) of these minerals.

These equations are based on the Voigt averaging (isostrain conditions) [5] which may overestimate the velocities of the mixtures. At any given volume fraction of constituents, Vp will fall between the bounds but its precise value depends on the geometric details. Since the geometric details of the datasets in the literature are not known, an allowable theoretical range of seismic velocity values or the upper bound values were computed by using Voigt averaging method. Other methods like Voigt–Reuss–Hill average can also be used. Moreover,

Table 6 Compressional wave velocity and bulk moduli values for some fluids [68]

Fluids	Vp (km/s)	Bulk modulus <i>K</i> (GPa)
Brine	1.635	2.6
Water	1.5	2.2
Oil	1.2	1.1
Gas	0.6	0.1

various studies [33, 37, 39] may be referred to estimate the different parameters based on phenomenological approach.

The tabulated datasets of Vp obtained by varying the volume percentage of major minerals for sandstones and carbonates are presented in Table 5. Doing such an exercise would provide the range of Vp values for different proportions of quartz and feldspar content for sandstones, calcite and dolomite content for carbonates, which would help in understanding the variability of major minerals with Vp. The common aim is to construct a template that honours the data (known data) and extends the interpretation to unknown situations, in different exploration cases. Table 6 shows Vp and bulk moduli values for some fluids [68].

In order to forecast the effective elastic moduli of a mixture of grains and pores, Hashin–Shtrikman–Walpole bounds [5, 77] were used for evaluating bulk and shear moduli (K and G , refer Eqs. 3, 4, 5 and 6). Thereafter, E was calculated by using Eq. 10. The range of datasets of E and V_p obtained by varying the volume percentage of major minerals for sandstones and carbonates, with a porosity variation from 2 to 40% for different geological scenarios (fluid saturations—water, gas, brine and oil), is provided in supplementary information (Table S1). The formulation of Hashin–Shtrikman–Walpole bounds [5] is presented in the form of equations:

$$K^{HS+} = \wedge(G_{\max}), \quad (3)$$

$$K^{HS-} = \wedge(G_{\max}), \quad (4)$$

$$G^{HS+} = \Gamma(\mathcal{E}(K_{\max}, G_{\max})), \quad (5)$$

$$G^{HS-} = \Gamma(\mathcal{E}(K_{\min}, G_{\min})) \quad (6)$$

where

$$\wedge(z) = \left\{ \frac{1}{K(r) + \frac{4}{3}z} \right\}^{-1} - \frac{4}{3}z \quad (7)$$

$$\Gamma(z) = \left\{ \frac{1}{G(r) + z} \right\}^{-1} - z \quad (8)$$

$$\mathcal{E}(K, G) = \frac{G}{6} \left\{ \frac{9K + 8G}{K + 2G} \right\} \quad (9)$$

$$E = \frac{9KG}{G + 3K}. \quad (10)$$

The brackets $\{\cdot\}$ in Eqs. 7, 8 and 9 specify an average over the medium (which is similar as average over the constituents weighted by their volume fractions).

Based on the obtained values of K and G , $\mathcal{E}$ parameter was calculated $\{\mathcal{E}(K_{\max}, G_{\max})$ and $\mathcal{E}(K_{\min}, G_{\min})\}$. Since $G_{\min} = 0$, theoretically lower bound $E = 0$. If the low Hashin–Shtrikman bound is zero for the Young's modulus, it does not mean that this bound does not exist. This means that the bounds are wide, and the modulus may vary within 200%. Hence, the upper bound is plotted in Fig. 4d. Different clusters or trends of data can be interpreted, or classified, based on this plot. The plot may also be useful for understanding or extracting values for similar geological conditions as well as to understand the variability for conditions that are yet to be explored for interpreting potential moduli values or V_p values.

To analyse the effect of mineralogy on Poisson's ratio (ν), the volume percentage of major minerals was varied and V_p was calculated based on Eq. 2. Thereafter, ν was calculated as

$$(V_{\text{Calcite}}) \times (\nu_{\text{Calcite}}) + (V_{\text{Dolomite}}) \times (\nu_{\text{Dolomite}}) = \nu. \quad (11)$$

The tabulated datasets of ν and V_p obtained by varying the volume percentage of major minerals for sandstones and carbonates are presented in Table 5. A theoretical model represented in the form of Eq. 11 was used to analyse the effect of mineralogy on Poisson's ratio. The limitation of this model is that it does not take into account the effect of porosity and pore geometry.

3 Results and discussion

The different mechanical properties of rocks, V_p and V_s reported in this literature review were based on laboratory measurements under ambient conditions. Among the different literature data analysed, datasets labelled as B, C,

Table 7 Instrument used for evaluating UCS in the literature

Instrument/technique used	Associated literature	Sample	Location
UCS instrument	1. Madhubabu et al. [52]	Carbonate rocks	Gujarat, India
	2. Kilic and Teymen [48]	Sandstone	Turkey
	3. Pappalardo [63]	Sandstone	Sicily
	4. Sarkar et al. [66]	Sandstone	Jutogh, India
	5. Kurtulus et al. [51]	Limestone	Turkey
	6. Moradian and Behnia [59]	Sandstone and limestone	Iran

Table 8 Instrument used for evaluating indirect tensile strength in the literature

Instrument/technique used	Associated literature	Sample	Location
Brazilian tensile strength instrument	1. Kilic and Teymen [48]	Sandstones	Turkey
	2. Kurtulus et al. [51]	Limestones	Turkey

Table 9 Summary of correlation laws between ultrasonic velocity (V_p) and UCS available in the literature

S No.	Parameters	Empirical relation	ss^2	Reference
1	UCS (MPa)– V_p (km/s)	$UCS = 35.4V_p - 55$	0.80	A
2	UCS (MPa)– V_p (m/s)	$UCS = 0.035V_p - 31.5$	–	B
3	UCS (MPa)– V_p (m/s)	$UCS = 3.3 \times 10^{-20} \rho^2 V_p^4 [(1 + \nu)/(1 - \nu)]^2 (1 - 2\nu)[1 + 0.78V_{clay}]$	–	C
4	UCS (MPa)– V_p (m/s)	$UCS = 1.745 \times 10^{-9} \rho V_p^2 - 21$	–	E
5	UCS (MPa)– V_p (km/s)	$UCS = 9.95V_p^{1.21}$	0.83	F
6	UCS (MPa)– V_p (m/s)	$V_p = 0.0317UCS + 2.0195$	–	G
7	UCS (MPa)– V_p (km/s)	$UCS = 56.71V_p - 192.93$	0.67	J
8	UCS (MPa)– V_p (km/s)	$UCS = 2.304V_p^{2.4315}$	0.94	I
9	UCS (MPa)– V_p (m/s)	$UCS = 165.05e^{(-4451.07/V_p)}$	0.70	N
10	UCS (MPa)– V_p (m/s)	$UCS = 43V_p + 1000$	–	O
11	UCS (MPa)– V_p (m/s)	$UCS = 0.038V_p - 50$	0.934	P
12	UCS (MPa)– V_p (km/s)	$UCS = 7.812V_p^{1.6}$ for $S_r = 0$	–	Q
13	UCS (MPa)– V_p (km/s)	$UCS = 6.070V_p^{1.633}$ for $S_r = 0.2$	–	Q
14	UCS (MPa)– V_p (km/s)	$UCS = 4.937V_p^{1.673}$ for $S_r = 0.5$	–	Q
15	UCS (MPa)– V_p (km/s)	$UCS = 4.401V_p^{1.684}$ for $S_r = 0.7$	–	Q
16	UCS (MPa)– V_p (m/s)	$UCS = 5.61 \times 10^{-9} V_p^{2.75}$	0.86	R
17	UCS (MPa)– V_p (km/s)	$V_p = 3.110 + 0.0289UCS$	0.88	S
18	UCS (MPa)– V_p (m/s)	$UCS = 0.018V_p - 18.405$	0.93	T

Table 10 Summary of correlation laws between velocities (V_p , V_s) and indirect tensile strength available in the literature

S. no.	Parameters	Empirical relation	R^2	Reference
1	σ_t (MPa)– V_p (km/s)	$\sigma_t = 0.4935V_p^{1.8723}$	0.9216	I
2	σ_t (MPa)– V_p (m/s)	$\sigma_t = 0.348e^{0.0004V_p}$	0.53	O
3	σ_t (MPa)– V_s (m/s)	$\sigma_t = 0.277e^{0.0008V_s}$	0.52	O
4	σ_t (MPa)– V_p (km/s)	$\sigma_t = 1.050V_p^{1.389}$ for $S_r = 0$	–	Q
5	σ_t (MPa)– V_p (km/s)	$\sigma_t = 0.919V_p^{1.416}$ for $S_r = 0.2$	–	Q
6	σ_t (MPa)– V_p (km/s)	$\sigma_t = 0.767V_p^{1.444}$ for $S_r = 0.5$	–	Q
7	σ_t (MPa)– V_p (km/s)	$\sigma_t = 0.694V_p^{1.447}$ for $S_r = 0.7$	–	Q
8	σ_t (MPa)– V_p (km/s)	$\sigma_t = 0.583V_p^{1.473}$ for $S_r = 1$	–	Q

D, E represent sandstones, datasets labelled as A, F, G, L, S, T, U represent carbonate rocks, while datasets labelled as H, I, J, N, O, P, Q consist of an amalgamation of a variety of rocks as summarised in Table 1. The correlations/empirical formulations among mechanical properties and V_p , V_s for reservoir rocks were examined/plotted/tabulated.

3.1 Rock strength

Suitable core for testing is not always available, and in the absence of cores, correlations can be used to estimate

strength. [14, 15, 20, 22, 23, 42, 45, 48, 51, 54, 57–59, 63, 64, 66, 70, 73, 75, 83] established empirical correlations between UCS, tensile strength and V_p . These relationships are shown in Tables 7 and 8, respectively. Tables 9 and 10 present the instrument/technique used for evaluating UCS and indirect tensile strengths in the literature.

Correlation between UCS and ultrasonic tests on 13 samples of numerous carbonate rock types around Antara, Turkey, reported a R^2 of 0.80 [83], by using simple linear relations, while analysis [59] in three different sedimentary rock types (sandstone, limestone and marlstone) around various dam sites in Iran reported R^2 0.70. However, velocity values were multiplied by density values to obtain a better regression coefficient between UCS and V_p . Correlation coefficient improved to 0.75 as against 0.70. R^2 of 0.9022 was reported [70] on tests carried out on seven types of rock (one igneous, three sedimentary and three metamorphic). Similarly, an empirical correlation was reported [42] between V_p and UCS with R^2 0.83. A correlation with R^2 0.67 [14] was described for two rock types only, while [66] reported R^2 0.934. A strong linear dependence on UPV with a R^2 0.93 was reported [51]. Analysis [48] carried out on 19 rock types comprising of ten igneous rocks (seven of which were volcanic), seven sedimentary and two metamorphic of southern Anatolia location, correlated the V_p and the indirect tensile strength and reported a strong correlation with a R^2 0.9216. Empirical equations were presented [45] between V_p and the indirect tensile strength for different degrees of saturation values.

Fig. 2 Variation of uniaxial compressive strength with the compressional wave velocity **a** as per studies of several researchers, **b** including general trend derived from this study using simple regression analysis and confidence interval, **c** data plotted on a log–log scale indicating nonlinear nature of the relationship, **d** graphical representation of UCS versus Vp, as reported by different researchers along with their trend line [51, 59, 63, 64, 66, 70, 73,

83]. The marked rectangle shows the range of Vp values for sandstones (5.7–6 km/s) when considering the effect of varying mineral contents (* major minerals Q—quartz and F—feldspar with varying proportions (75% Q, 25% F, 80% Q, 20% F, till 100% Q, 0% F), while for carbonates, Vp varies from 6.25 to 6.43 km/s for varying major minerals (# C—calcite and D—dolomite) proportions (100% C, 0% D till 75% C, 25% D)

Fig. 3 Variation of indirect tensile strength with the compressional wave velocity **a** as per studies of several researchers, **b** including general trend derived from this study using simple regression analysis and confidence interval, **c** data plotted on a log–log scale indicating nonlinear nature of the relationship, **d** graphical representation of indirect tensile strength versus V_p , as reported by different researchers along with their trend line [48, 51, 73]. The marked rectangles show the range of V_p values for sandstones (5.7–6 km/s) when considering the effect of varying mineral contents (* major minerals Q—quartz and F—feldspar with varying proportions (75% Q, 25% F till 65% Q, 35% F), while for carbonates, V_p varies from 6.25 to 6.43 km/s for varying major minerals (# C—calcite and D—dolomite) proportions (100% C, 0% D till 75% C, 25% D)

The variation of UCS and σ_t with V_p observed by various researchers is presented in Figs. 2 and 3, respectively. It is observed from Fig. 2 that the UCS value of limestone from Turkey location (label T in Table 1) varies from 19 to 46 MPa with a change in V_p from 3.4 to 5.8 km/s. Comparatively, the UCS value of limestone from Paris location (label R in Table 1) varies from 20 to 135 MPa with a change in V_p from 3 to 6 km/s. The percentage increase thus observed in UCS from Turkey location (label T) is 142%, while the percentage increase in UCS from Paris location (label R) is 575%. The indirect tensile strength values, as observed from Fig. 3, vary from 0.7 to 19.7 MPa and V_p range is 1.47–6.75 km/s. Significant results [54] showed wide variability in the velocity values for individual rock types as against a limited range of UCS values. They obtained a correlation coefficient of 0.94, R^2 value of 0.88. For a multiplicative model of the form, $y = ax^b$. In spite of obtaining a good correlation between UCS and V_p , still researcher [54] suggested caution in deriving strength values and vice versa owing to the scattering of data points (large variation in V_p values for a limited range of UCS values) indicating nonlinear nature of the relationship. However, to improve the accuracy of deriving strength values from velocity measurements, the mineralogy of the rock needs to be investigated. For instance, the dependency of velocity measurements on an increase or decrease in quartz or feldspar content for sandstones and calcite or dolomite content for carbonates may be examined. Nevertheless, velocity measurements may be utilised to give a varied indication of disparity in rock mass strength.

To understand the link between the strength properties and V_p , the combined graphical representation of variation of UCS and tensile strength (indirect tensile strength) with V_p is plotted in Figs. 3 and 4, respectively, where Figs. 3 and 4a illustrate data as reported by different researchers; Figs. 3 and 4b illustrate data including general trend derived from this study using simple regression analysis and confidence interval; and Figs. 3 and 4c illustrate data plotted on a log–log scale indicating nonlinear relationship. The confidence intervals were set at 95%, as most of the raw data fall within this range. Hence, from the

plotted graphs, it is observed that both UCS and tensile strength increase with an increase in V_p . However, there is a wide variation in the strength values, and for a particular value of V_p , there exist two values of rock strength (UCS and tensile strength). This scattering of data results may be due to some other influential factors like mineralogical composition or higher clay content leading to an alteration in mechanical and elastic properties of rocks, thereby showing the dispersion of data points in the graph. Moreover, it is observed that carbonate rocks have a higher value of UCS as compared to sandstones which can be attributed to a greater value of density, thereby resulting in a higher value of V_p for carbonate rocks.

An attempt is made to present the nonlinear relationship between UCS, indirect tensile strength with V_p , as shown in Figs. 2c and 3c. A log–log graph is plotted since a reasonable straight line relationship is obtained on doing so; however, obtained R^2 is 0.28 (presented in the form of $y = ax^b$) for UCS– V_p and R^2 is 0.49 (of the form $y = ax^b$) for indirect tensile strength– V_p plot. The scatter is observed due to the presence of outliers. For UCS– V_p plot, the variation in V_p is observed to be as low as 2–6.5 km/s. Similarly, for tensile strength– V_p plot, the variation in V_p is observed to be as low as 2.2–6.75 km/s. Ideally, major mineral present in sandstone has a V_p of 6.057 km/s (quartz's grain density is 2.65 g/cc) [16], while, for the case of carbonates, V_p is 6.259 km/s corresponding to major mineral calcite with grain density of 2.71 g/cc [16]. However, these V_p values are the maximum (for zero porosity). Table 4 presents a compilation illustrating different parameter values for different minerals. Thus, any variation in the V_p value corresponds to an alteration in porosity value. For instance, minimum value of V_p obtained from the study is 2 km/s, maybe indicating high porosity of the same sample. This is because an increase in porosity is always associated by an equivalent decrease in V_p , since seismic velocity in the pores is less than the grains of which the rocks are composed of. The V_p values, so obtained by fragmented analysis, are plotted in Fig. 2d for UCS– V_p plot and Fig. 3d for indirect tensile strength– V_p plot. The marked rectangles in Figs. 2d and 3d show the range of V_p values for sandstones (5.7–6 km/s) and carbonates (6.25–6.43 km/s) when considering the effect of varying mineral contents. However, apart from mineralogy, porosity and saturation also have an influence on the alteration of the geomechanical properties. The presence of clay minerals could also lead to a deviation in V_p values. Also, V_p increases with increasing saturation and also depends on the type of fluid (Tables 4 and 6). Hence, to be more precise when assessing rock mass conditions, this inclusive study (considering all influential parameters) may be carried out so as to be more reliable when dealing in different stages of an investigation. However, it is important here to mention

Fig. 4 Variation of Young's modulus with the compressional wave velocity **a** as per studies of several researchers, **b** including general trend derived from this study using simple regression analysis and confidence interval, **c** data plotted on a log–log scale indicating nonlinear nature of the relationship, **d** graphical representation of E versus V_p for different geological scenarios along with incorporating the effect of varying porosity and mineral composition, including literature dataset

that it is improbable that V_p values may ever substitute the mechanically derived parameters, although a suggestion of the variation can be noted or a comparative study of different geomechanical properties of various lithological units can be investigated. These seismic measurements can be viewed as complementary and equally important for studies related to the evaluation of rock mass performance (geomechanical classification system).

3.2 Modulus of Elasticity

The prediction of elastic modulus (E) can be made from the measurement of V_p and V_s since E is an important parameter for intact rock classification and criteria for rock failure. The laboratory determination of E involves rigorous, time-consuming and destructive testing. Hence, ultrasonic techniques can be utilised to provide a direct evaluation of the rock mass. Thus, studies by [51, 59, 63, 64, 83] were conducted to establish a relationship between the two as shown in Fig. 4. Many correlations have been reported as shown in Table 11. The range of Young's modulus values found from graph is 2–69.14 GPa and V_p range is 1.47–6.75 km/s. Yasar and Erdogan [83] found regression coefficient 0.86 for studies conducted on carbonate rocks, Parent et al. [64] and Kurtulus et al. [51] 0.77 and 0.76, respectively, for limestones, Pappalardo [63] 0.80 for dolostone, while [59] reported the highest regression coefficient value of 0.92.

To understand the correlation between E and V_p , different datasets were compiled from the literature and a combined graphical representation of its variation is plotted in Fig. 4, where Fig. 4a illustrates datasets as reported by different researchers; Fig. 4b illustrates datasets including general trend derived from this study using simple regression analysis and confidence interval; and Fig. 4c illustrates data plotted on a log–log scale indicating nonlinear relationship (of the form $y = ax^b$, with $R^2 = 0.55$). The confidence intervals were set at 95%. From the plotted graph, modulus of elasticity increases with an increase in V_p . However, for a particular value of V_p there exist two values of E . Such a wide variation may be due to the change in mineralogical composition (rock composition and texture, clay content, anisotropy, etc.) leading to an alteration in mechanical and elastic properties of rocks. Moreover, there are fundamental

differences between sandstone and carbonate rocks. Carbonates are autochthonous, while sandstones are allochthonous; carbonate minerals have a greater chemical reactivity [13, 19, 56]. These differences in sandstones and carbonates lead to the variation in parameters. The density of minerals that constitute sandstones and carbonates varies as summarised in Table 4. The density of major minerals in carbonates (calcite and dolomite) has a relatively higher value than the density of major minerals in sandstones (quartz and feldspar). This variation in density also leads to the alteration in the geomechanical properties of sandstones and carbonates. As discussed in methodology, by using the formulation of Hashin–Shtrikman–Walpole bounds [5], it was found that lower bound does not exist and only upper bound is prevailing. Based on this, the variation of E with V_p is analysed and the template is presented in Fig. 4d. This template can be used to predict the elastic properties for different geological scenarios that include water-saturated, gas-saturated, brine-saturated and oil-saturated conditions for sandstones and carbonates along with the literature dataset ([51]—label T, [48]—label I and [52]—label U). The plotted datasets highlight several salient features.

- With an increase in porosity from 2 to 40%, there is a decrease in E and V_p values for all types of saturation scenarios for sandstones and carbonates;
- With an increase in quartz content and subsequent decrease in feldspar content in sandstones, E and V_p increase (owing to higher density of quartz mineral as compared to feldspar mineral), while for carbonates, with an increase in calcite content and subsequent decrease in dolomite content, E and V_p decrease (owing to higher density of dolomite mineral as compared to calcite mineral);
- Carbonates have a higher value of E and V_p than sandstones (higher density of minerals found in carbonates than sandstones) for all particular sets of saturation conditions.
- More or less, there is an approximate 50% reduction in V_p and E values for a porosity increase from 2 to 40%.

Thus, the plotted figure will help to understand the variability of V_p values associated with the change in saturation conditions for a reservoir rock under investigation. Such a template would aid in the interpretation of geomechanical behaviour of rocks on the basis of V_p values, even in the absence of well log data. However, the present template considers the effect of only major minerals and porosity. Other factors such as the presence of other minor minerals and clay content may also alter the properties, which may be one of the reasons for literature

Table 11 Summary of correlation laws between ultrasonic velocities (V_p , V_s) and Young's modulus available in the literature

S. no.	Parameters	Empirical relation	R^2	Reference
1	E (GPa)– V_p (km/s)	$V_p = 0.0937E + 1.7528$	0.86	G
2	E (GPa)– V_p (m/s)	$E = 0.965V_p^{0.810}$	0.77	R
3	E (GPa)– V_p (m/s)	$E = 2.06V_p^{2.78}$	0.92	N
4	E (MPa)– V_p (km/s)	$E = -15718.21 + 5075.79V_p$	0.80	S
5	E (GPa)– V_p (m/s)	$E = 0.0114V_p + 3.7059$	0.76	T

Table 12 Summary of correlation laws between ultrasonic velocity (V_p) and Poisson's ratio available in the literature

S. no.	Parameters	Empirical relation	R^2	Reference
1	ν – V_p (m/s)	$\nu = 0.152V_p^{0.126}$	0.67	R
2	ν – V_p (m/s)	$\nu = -5E - 0.5V_p + 0.6$	0.85	T

dataset falling below the modelled template dataset. Thus, it can be inferred that a material with higher E will have a higher V_p , due to higher density and presence of lower pores (lower porosity). A comprehensive study on the correlations between porosity, elastic moduli, Poisson's ratio and seismic velocity for solid materials and rocks may be referred [34, 85].

However, it is to be noted that the modulus of elasticity estimated from the measurement of V_p , V_s is dynamic elastic modulus and not static modulus [45, 54]. For most design purposes, static modulus will be required to obtain deformation behaviour. Dynamic modulus can be used where dynamic loading is involved. Nevertheless, the static modulus is generally lesser than the dynamic modulus and the literature establishing the correlations between the two for various types of rock is available [7]. This variation in modulus values may possibly be due to the broadly used strain levels at which two sets of measurements are made [54]. It was also suggested [82] that majority of the variation between static and dynamic elastic moduli is because of viscoelastic effects or inelastic deformation occurring inside the rock, since the frequency variation among dynamic and static measurements is sufficiently large to permit viscoelastic deformation in static tests that would not happen on deformation timescale of elastic wave used to estimate dynamic moduli [67]. However, if testing is done at high stress levels and under pressure, there is an agreement between the two measurements [49]. Nonetheless, E values obtained from V_p , V_s measurements may be used for qualitative comparative measurements as an indicator of geomechanical behaviour.

3.3 Poisson's ratio

Among the elastic properties, ν is the least studied, but at the same time the most interesting [26]. ν is defined as the ratio of a decrease in transverse dimension to that of an increase in axial dimension when subjected to uniaxial stress for an isotropic material. At a given temperature and a given pressure, ν is constant for an isotropic material and its values lie between -1 and 0.5 . A comprehensive analysis [35] on ν and auxetic properties of natural rocks may be referred for further study. Empirical equations were reported [51, 64] as shown in Table 12. Parent et al. [64] and Kurtulus et al. [51] gave empirical equations with regression coefficient values of 0.67 and 0.85 , respectively, for limestone. To understand the association between ν and V_p , combined graphical representation of its variation is presented in Fig. 5, where Fig. 5a illustrates data as reported by different researchers; Fig. 5b illustrates data including general trend derived from this study using simple regression analysis and confidence interval; and Fig. 5c illustrates data plotted with calcite mineral values showing alteration. The confidence intervals were set at 95% . The range of Poisson's ratio values found from graph is 0.23 – 0.4 and V_p range is 2.2 – 5.8 km/s. From the plotted graph, it is observed that as V_p increases, ν decreases or there is a negative linear correlation between them. Hence, a decrease in ν is linked with an increase in V_p , which further leads to an increase in E . Ideally for limestone samples, the major mineral present would be calcite with a value of Poisson's ratio 0.316 and V_p 6.259 km/s. However, as explained earlier, due to the influence of several factors (mineralogy, porosity, saturation), there occurs a deviation from these values. As discussed in methodology, to account for the mineralogy effect on V_p , the generated dataset of V_p – ν is plotted with the existing dataset from the literature, as shown in Fig. 5d. The circled part depicts the decrease in Poisson's ratio values with a decrease in calcite and an increase in dolomite content, thereby highlighting the effect of mineral content on V_p values and thus on Poisson's ratio. Moreover, the effect of porosity and clay content (if present) was not considered which may also add to the dispersion in the values. The influence of porosity on ν of natural rocks is limited due to lack of experimental data over different porosity and pore geometry ranges [85]. However, studies [18, 78, 85] provide a comprehensive review on ν and may be referred for further study.

Different correlations [6] among V_p , V_s , density, ν for a wide variety of common lithology have been analysed. These measurements were compiled from wireline borehole logs, vertical seismic profiles, laboratory or field measurements and in situ estimates from seismic tomography studies. Correlations between V_p , V_s and ν were analysed

Fig. 5 Variation of Poisson's ratio with the compressional wave velocity **a** as per studies of several researchers, **b** including general trend derived from this study using simple regression analysis and confidence interval, **c** data plotted with calcite mineral values showing alteration, **d** graphical representation of Poisson's ratio versus V_p , as reported by different researchers along with their

trend line [51, 52]. Label # represents limestone major minerals (C—calcite and D—dolomite) different proportions (75% C, 25% D, 80% C, 20% D, till 100% C, 0% D). The circled part depicts the decrease in Poisson's ratio values with a decrease in calcite and an increase in dolomite content, thereby highlighting the effect of mineral content on V_p values and thus on Poisson's ratio

Fig. 6 Variation of slake durability with the compressional wave velocity **a** as per studies of several researchers, **b** including general trend derived from this study using simple regression analysis and confidence interval, **c** data plotted showing linear nature of the relationship

Table 13 Summary of correlation laws between ultrasonic velocity (V_p) and slake durability index available in the literature

S. no.	Parameters	Empirical relation	R^2	Reference
1	SDI (%)– V_p (m/s)	$SDI = 0.0069V_p + 78.577$	0.7831	M
2	SDI (%)– V_p (m/s)	$SDI = 0.0014V_p + 92.98$	0.904	P

[78, 79] for different rocks (common rocks and sulphide ores) based on high-pressure laboratory measurements of V_p and V_s . Moreover, this study [36] analysed in detail the characteristic Lamé's (λ) parameter values of common rocks in the Earth's crust and upper mantle and presented λ as a function of pressure, temperature and mineralogy.

3.4 Other parameters

Some of the other parameters that can be predicted using ultrasonic velocity include slake durability index (SDI) and point load strength [70]. In this paper, the data corresponding to slake durability are limited (low number of datasets available). A probable reason may be since slake durability is partially controlled by strength, indicating a coherent relation among them, and studies related to V_p –strength (UCS, indirect tensile strength) correlation were already studied by different researchers, as also discussed in the earlier sections. The presence of clay minerals would also play a major role for slake durability studies.

To understand the association between SDI and V_p , combined graphical representation of its variation is presented in Fig. 6, where Fig. 6a illustrates data as reported by different researchers; Fig. 6b illustrates data including general trend derived from this study using

Fig. 7 Variation of point load strength with the compressional wave velocity **a** as per studies of several researchers, **b** including general trend derived from this study using simple regression anal-

ysis and confidence interval, **c** data plotted on a log–log scale indicating nonlinear nature of the relationship

Table 14 Summary of correlation laws between ultrasonic velocities (V_p , V_s) and other parameters available in the literature

S. no.	Parameters	Empirical relation	R^2	Reference
1	V_p (km/s)– V_s (km/s)	$V_p/V_s = 1.55 + 0.56n + 0.43C$	–	D
2	V_p (km/s)– V_s (km/s)	$V_p = 1.26V_s + 1.07$	–	D
3	$V(\text{cm}^3/50 \text{ cm}^2)$ – V_p (km/s)	$V = 57997V_p^{-2.4279}$	0.8452	I
4	V_s (m/s)– V_p (m/s)	$V_s = 0.456V_p + 264.3$	0.90	O
5	τ (MPa)– V_p (m/s)	$\tau = 0.0019V_p - 2.6545$	0.90	T
6	I_s (MPa)– V_p (m/s)	$I_s = 0.0005V_p + 0.659$	0.83	T
7	I_s (MPa)– V_p (m/s)	$I_s = 0.022V_p + 30.631$	0.70	T

simple regression analysis and confidence interval; and Fig. 6c illustrates data plotted showing linear nature of the relationship (in the form $y = ax + b$ with R^2 0.64). The confidence intervals were set at 95%. The range of slake durability values found from graph is 93.1–99.6% and V_p range is 2–5 km/s. From the plotted graph, it is observed

that slake durability increases with an increase in V_p . [66] found a linear correlation between V_p and SDI, with a regression coefficient of 0.903. [70] also reported a similar linear correlation, but they found low correlation coefficient 0.7831. Table 13 presents the summary of correlation

laws between ultrasonic velocity (V_p) and SDI as proposed by researchers.

The other parameter, i.e. point load strength, is also a measure for the determination of rock strength. The combined graphical representation of change of point load strength with V_p is presented in Fig. 7, where Fig. 7a illustrates data as reported by different researchers; Fig. 7b illustrates data including general trend derived from this study using simple regression analysis and confidence interval; and Fig. 7c illustrates data plotted on a log–log scale (since a reasonable straight line is obtained on doing so), indicating nonlinear nature relationship (of the form $y = ax^b$, with $R^2 = 0.59$). The confidence intervals were set at 95%. Table 14 presents the summary of different correlations between ultrasonic velocity (V_p) and point load strength, along with V_p – V_s , V_p –shear strength and V_p – V (loss of volume on wear, $\text{cm}^3/50 \text{ cm}^2$) as proposed by researchers. The range of point load strength values found from graph is 0.35–8.6 MPa and V_p range is 1.47–6.75 km/s. From the plotted graph, it is noted that point load strength increases proportionately with V_p .

Several theoretical velocity models were reported in the literature [1, 45] to predict V_p . [45] in their study reported that for rocks with high effective clay content, the utilisation of theoretical velocities is not favourable since the comparison between theoretical Gassmann–Biot velocities and experimental results showed inconsistency when calculated at varying degrees of saturation. Similarly, [1] reported that, for low-porosity rocks, the theoretical velocities (predicted by both Gassmann and Biot theories) overestimated the measured velocities, due to the negligence of rock–fluid interaction. Also, when compared

to Biot–Gassmann velocity model, time average and Raymer's relationships overestimated V_p [1]. Thus, it can be concluded that the use of theoretical models may yield over or under predictions.

In general, the properties of the dataset obtained from the literature cover a wide range of exploration interests. The statistical summary of different geotechnical properties (UCS, indirect tensile strength, E , ν , slake durability, point load strength) and their corresponding V_p obtained from the literature is presented in Table 15. Studies related to improving the fracturing index-based brittleness prediction [84] and V_s estimation by combining classification and regression methods [17] may help to build a more realistic model.

4 Critical Summary and Conclusion

The study presents a comprehensive review of all the correlations and relationships that exist between several geomechanical properties of sedimentary rocks with the ultrasonic velocity. Based on the comprehensive scrutiny from the available literature, it can be concluded that it is extremely important to focus the research towards an inclusive study incorporating the petrophysical, geological and mechanical factors so as to identify and address all the scenarios that lead to an alteration in the geomechanical and elastic properties of rocks. Past reviews provide a good understanding and development related to the relationships that exist between several geomechanical properties and V_p . Seismic measurements can be utilised to provide a direct evaluation of the nature of rock mass (indicator

Table 15 Statistical summary of different geotechnical properties and their corresponding V_p obtained from the literature

Point load strength (MPa)	Mean $\pm$ SD	2.69 $\pm$ 1.46	V_p (km/s)	4.34 $\pm$ 1.11
	Minimum	0.35		1.47
	Maximum	8.6		6.75
Slake durability (%)	Mean $\pm$ SD	97.17 $\pm$ 2.02	V_p (km/s)	3.16 $\pm$ 0.99
	Minimum	93.1		2
	Maximum	99.6		5
Poisson's ratio	Mean $\pm$ SD	0.34 $\pm$ 0.038	V_p (km/s)	4.36 $\pm$ 0.86
	Minimum	0.23		2.204
	Maximum	0.4		5.8
Young's modulus (GPa)	Mean $\pm$ SD	33.94 $\pm$ 24.48	V_p (km/s)	4.35 $\pm$ 1.09
	Minimum	2		1.47
	Maximum	69.14		6.75
Tensile strength (MPa)	Mean $\pm$ SD	6.23 $\pm$ 3.75	V_p (km/s)	4.53 $\pm$ 1.09
	Minimum	0.7		1.47
	Maximum	19.7		6.75
UCS (MPa)	Mean $\pm$ SD	54.45 $\pm$ 31.67	V_p (km/s)	4.36 $\pm$ 1.16
	Minimum	10		2
	Maximum	145		6.5

of geomechanical behaviour). With the use of this geophysical technique, information related to exploration studies can be increased since rock types can be assessed qualitatively. However, it is important to mention that it is unlikely that these measurements would ever replace the mechanically derived geomechanical tests, although a variation can be estimated. Nevertheless, following salient points can be summarised after the critical scrutiny of the literature:

- It is evident from the literature that there exists a definite link between V_p , V_s with mechanical properties of rocks, and hence, it can be considered as a dependable parameter to provide a link with these properties. It is observed that UCS, Brazilian tensile strength, point load index and E increase with an increase in V_p , but ν decreases. Moreover, a number of rock strength correlations have been summarised which may be useful in geotechnical applications (design and planning for use in general practice with greater safety and stability) by providing lower bound strength estimate when no other information is available, thus saving time and cost in performing tedious and rigorous test methods. These relations will also be helpful where no prior knowledge on rock strength is available or in situations where there is an absence of rock cores for performing laboratory tests in situ rocks. However, measurement of V_p can be viewed as complementary and cannot simply replace the mechanical testing of rock specimens under certain testing procedures. Nevertheless, the correlation listed in this review study may be generalised for almost all types of rock and help to identify the different alterations in rock mass strength.
- Development of empirical and/or semi-analytical equations/models needs attention. The empirical relations established in the literature are mere site specific (lack generalisation) and may lead to mis-classification. The datasets exhibit wide ranges for velocities and geomechanical parameters. This is because there are several other parameters such as mineralogy, pore geometry, rock framework, saturation and porosity that influence these geomechanical properties. Furthermore, several trends of geomechanical parameters conclude that there is an uncovered trend that affects these parameters and a combined analysis involving the integration of geological, petrophysical and engineering properties (geomechanical properties) is required to understand this uncovered trend. In order to widen the acceptability of these empirical equations and to improve the existing ones, the influential parameters such as degree of saturation, effective clay content, the rock's composition (matrix), mineral constituents and void space (pore geometry/pore structure/pore archi-

ture/pore size distribution) of the rock also need to be taken into account. All these petrophysical parameters including the difference in depositional environments, composition, degree of weathering, texture and complex diagenesis processes influence the internal structure of the rock, and neglecting these parameters would result in an under or overestimated predictions. Such studies should be undertaken in future research so as to understand the effects which would be favourable in interpreting the overall picture. An attempt is made in this study to understand the variability of V_p with the help of a fragmented analysis of major mineral constituents, porosity variation and saturation conditions for rock mass classification. The resulting template so generated may provide a basis for interpreting lithology, thereby favourable in facilitating more realistic geomechanical behaviours of rocks (rock mass performance).

- The modulus obtained from ultrasonic testing only provides the modulus in the very low strain elastic range (dynamic modulus), whereas for most design purposes, modulus values are from much higher strains (static modulus) often with strain of a per cent or more. The method of testing can also have a significant effect depending on whether strain is measured over the length and diameter of the sample or from local strain gauges such as direct contact strain gauges. The static modulus is generally lesser than the dynamic modulus, and the literature establishing the correlations between the static and dynamic moduli of various varieties of rock is available.

Extensive combined analysis involving the integration of geological, petrophysical and engineering properties (mechanical properties) is presently scarce. Much attention is required to further understand and develop standard guidelines/predictions for field experimentation so as to reinforce the technique as an equally important, complementary technique.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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