Significances of prescribed heat sources on magneto Casson nanofluid flow due to unsteady bi-directionally stretchable surface in a porous medium

Muhammad Faisal¹ · Iftikhar Ahmad¹ · Tariq Javed²

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Abstract

In this communication, bidirectional flow of Casson nanomaterial driven by an unsteady moveable surface in the region of boundary layer is analyzed. Moreover, the significances of porous space, magnetic field, prescribed surface temperature (PST), and prescribed surface heat flux are also incorporated. Furthermore, the aspects of Brownian motion and thermophoresis are also comprised through Buongiorno nanofluid model. Governing equations are firstly transformed into system of ordinary differential equations by using a suitable combination of variables, and then computational assessment is made through Keller-Box method. Graphical illustrations for temperature distribution, concentration distribution, local Nusselt number and local Sherwood number against escalating amounts of pertinent parameters are presented. It is observed that escalating amounts of unsteady parameter and temperature controlled indices reduce the temperature distribution, as well as the concentration distribution. It is also observed that increasing amounts of Casson parameter enhances the rate of heat transfer, and reduces the rate of mass transfer for PST case. Finally, a comparison benchmark for limited case has been presented to validate the present methodology.

Keywords Casson nanofluid · Eckert number · Keller-Box method · MHD · Porous medium · Prescribed heat sources · Unsteady bi-directional stretching

List of symbols

a, b, c	Stretching rates (s^{-1})	K	Permeability of porous medium (m^2)
B_o	Magnitude of magnetic force ($kg\ s^{-2}\ A^{-1}$)	k_c	Dynamic viscosity ($m^{-1}\ kg\ s^{-1}$)
C	Concentration ($kg\ m^{-3}$)	M	Magnetic parameter ($-$)
c_f	Specific heat of liquid ($m^2\ s^{-2}\ K^{-1}$)	N_b	Brownian motion parameter ($-$)
c_p	Specific heat of nanomaterial ($m^2\ s^{-2}\ K^{-1}$)	N_t	Thermophoresis parameter ($-$)
C_∞	Ambient concentration ($kg\ m^{-3}$)	Nu_x	Local Nusslet number ($-$)
C_w	Surface concentration ($kg\ m^{-3}$)	Sh_x	Local Sherwood number ($-$)
C_{fx}, C_{fy}	Skin-friction coefficients ($-$)	n_p	Number of grid points ($-$)
D_B	Brownian coefficient ($m^2\ s^{-1}$)	Pr	Prandtl number ($-$)
D_T	Thermophoresis coefficient ($m^2\ s^{-1}$)	Re_x, Re_y	Reynolds numbers ($-$)
Ec_x, Ec_y	Eckert numbers ($-$)	r, s	Indices ($-$)
f, g	Similarity functions for velocity ($-$)	S	Unsteady parameter ($-$)
h_j	Step size for grids (m)	T	Temperature (K)
j	Suffix ($-$)	T_∞	Ambient temperature (K)
		T_w	Surface temperature (K)

✉ Muhammad Faisal, muhammad.faisal@ajku.edu.pk | ¹Department of Mathematics, Azad Jammu and Kashmir University, Muzaffarabad 13100, Pakistan. ²Department of Mathematics and Statistics, International Islamic University, Islamabad 44000, Pakistan.

t	Time (s)
u, v, w	Velocity components (m s^{-1})
x, y, z	Space coordinates (m)

Greek letters

α	Stretching ratio (–)
α_m	Thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)
β	Casson fluid parameter (–)
δ	Unknown vector (–)
ε	Porosity parameter (–)
η	Similarity variable (–)
θ	Similarity function for temperature (–)
ϑ	Kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)
λ	Thermal conductivity ($\text{m kg s}^{-3} \text{K}^{-1}$)
ρ	Density of liquid ($\text{m}^{-3} \text{kg}$)
σ	Electrical conductivity ($\text{m}^{-3} \text{kg}^{-1} \text{s}^3 \text{A}^2$)
τ_o	Yield stress ($\text{m}^{-1} \text{kg s}^{-2}$)
τ	Adiabatic index (kg m^{-1})
ϕ	Similarity function for concentration (–)

1 Introduction

Stretching surface phenomenon is a widely used mechanism to analyze the fluid flow in the expanse of boundary layer, and it has intended many researchers owing to its multiplicity of usages in technological industries and engineering fields. Pioneer work of Crane [1] to obtain the series solution of the liquid flow caused by a moveable surface is settled the foundation to use this phenomenon for industrial and engineering purposes. This phenomenon has lot of applications like: fibrous insulations, construction of energy efficient buildings, solidification of casting, winding of electrical machinery, transpiration cooling, energy efficient dryers, ground water hydrology, grain storage installation, separation processes, parasitic losses, polymerase chain reaction, oil refining process and grinding, etc. Later on, hydromagnetic flow driven by a permeable stretchable surface has pulled the focus of various examiners due to its applications in polymerases technologies and metallurgy. Chakrabarti and Gupta [2] firstly examined the hydromagnetic flow over a stretchable surface to obtain the resemblance solution for velocity and heat transport characteristics through porous space. Analytical solution for generalized bidirectional hydromagnetic flow driven by a stretchable sheet saturated porous medium is obtained by Hayat and Javed [3]. The concept of fluid flow driven by a bi-directionally stretchable surface with prescribed heat mechanisms was introduced by Liu and Andersson [4], where heat transport characteristic for boundary layer flow is analyzed through similarity transformations. Elbashbeshy and Basid [5] firstly analyzed the heat transport mechanism for fluid flow over an unsteady

stretchable surface. This type of study has been carried out through references [6–11] under the various aspects of fluid flow characteristics.

Hydromagnetic viscoelastic liquid flow driven by a bi-directionally moveable surface with heat transport characteristic in the expanse of boundary layer is numerically treated by Ahmad and Nazar [12] using Keller-Box method, and they concluded through this study that temperature rises for escalating amount of magnetic constraint. Now, in the classifications of non-Newtonian fluids, Casson liquid has distinctive structure because of its rheological identities corresponds to the shear stress–strain relations. It looks like an elastic solid at small shear strain, and beyond a critical stress amount, it behaves alike a Newtonian fluid. A Casson fluid can be best explained as a shear thinning liquid with zero viscosity at an infinite rate of shear, and infinite viscosity at a zero rate of shear. Most common examples having these type of characteristics are jelly, honey, tomato sauce, ketchup, jell, concentrated fruit juices, soup, chocolate, ice cream, and human blood, whereas industrial applications include molten plastics, slurries, and polymeric products. Nadeem et al. [13] has considered the hydromagnetic Casson fluid flow by reason of bi-directionally moveable surface in the expanse of boundary layer embedded porous medium, computationally by using shooting technique, and has deduced that flow velocity decreases for large amount of Casson material constraint. Spectral relaxation method is used by Mahanta and Shaw [14] for the investigation of Casson fluid flow because of bi-directional stretching porous surface with the influence of convective heat transport.

Recently, the investigation of nanofluid flow has achieved an incredible peak because of its lot of usages in chemical and pharmaceutical industries, heat transfer devices associated with condensers and boilers, cooling system of nuclear reactors and automobile engines, solar collectors and nanolubricants, macro plus micro-electronics, geothermal extraction and nano emulsification, improvement of power plants and thermal reservoirs, microelectromechanical systems, cooling of microchips, nanoliquid detergent, space technology, and many more. Buongiorno [15] illustrated a novel mathematical model to investigate the framework of nanofluid that describes the features of thermophoresis and Brownian movement. Khan and Pop [16] described the solution of nanomaterial flow over a moveable surface in the expanse of boundary layer, numerically, which was probably the first attempt on nanofluid flow over a moveable surface by using two-phase nanoliquid model. Convective heat and mass transportations on magnetized Casson nanoliquid flow caused by a moveable surface was illustrated by Haq et al. [17], numerically. Nadeem et al. [18] deliberated the convective Casson nanofluid flow due to moving boundaries,

and optimal solution of the modeled problem has been presented. Nadeem et al. [19] also discussed the hydromagnetic Casson nanofluid flow owing to bi-directionally moveable surface in the expanse of boundary layer with the stimulus of convective heat transport, numerically. Hayat et al. [20] addressed the mixed convective nanofluid flow of Casson liquid as a result of linearly stretchable surface in the boundary layer region under the influences of heat generation and chemical reaction, analytically. Hussain et al. [21] explored the series solution of Casson nanofluid flow owing to linearly stretchable surface under viscous dissipation effect. More studies are carried out for Casson nanofluid flow due to stretchable surface by many researchers [22–27], under the influences of various mechanisms like: Joule heating, porous medium, convective heat transport, hydromagnetic flow, slip conditions, thermal radiation, inclined surface, etc. Chamkha et al. [28] discussed the similarity solution for unsteady nanofluid flow because of moveable surface in the expanse of boundary layer, under the inspiration of chemical reaction through porous media with suction/injection parameters. Pushpalatha et al. [29] firstly described the unsteady boundary layer flow of Casson nanofluid caused by a moveable surface under the impact of magnetic force with convectively heated wall, where series solution has been presented by using perturbation method.

In the sight of above literature, we tried to explore the unsteady boundary layer flow of Casson nanofluid driven by a bi-directionally moveable surface through porous space with the effects of magnetic field and prescribed heat source, namely, PST and PHF. The novelty of this work is that a comprehensive study related to the Casson nanomaterial has been presented to check the influences of prescribed heat sources on rates of heat and mass transferes. The whole manuscript consists on the four major sections. In the first section, mathematical modeling of the physical model has been presented. In the second section, numerical solution has been obtained through Keller-Box method [30–37]. In the third section, graphical and tabular illustrations have been made to analyze the temperature distribution, concentration distribution, rate of heat transfer, and the rate of mass transfer, against wide ranges of involved parameters. In the last section, conclusion of the whole parametric study has been made and implication of the present work has been discussed.

2 Modeling

Consider the hydromagnetic Casson nanofluid flow owing to an unsteady bi-directional stretching surface in the region of boundary layer through porous media with the controls of prescribed heat mechanisms, namely, PST and

PHF. Frame of Cartesian coordinate is chosen for flow field analysis, where x - and y -axes are supposed to be in the directions of stretching velocities, while z -axis is adjusted at right angle to the moveable surface, and fluid occupies the region $z > 0$. Initially, at $t \leq 0$ both the fluid and the surface are kept at rest, and suddenly motion activates for $t > 0$ with expanding velocities $U_w(x, t) = \frac{ax}{1-ct}$ along x -direction and $V_w(y, t) = \frac{by}{1-ct}$ along y -direction. The special influences of magnetic field $\vec{B}$ and porous medium are incorporated through Lorentz force and Darcy's law, respectively, while the effects of induced and electric magnetic fields are ignored. Moreover, the influence of magnetic field is also incorporated into the heat equation, and the analysis of Casson nanofluid is conceded out using two-phase nanofluid model. Furthermore, the physical explanation of the current investigation is described through Fig. 1.

The rheological tensor τ_{ij} for an incompressible and isotropic flow of Casson liquid can be composed as: (see Refs. [7–9])

$$\tau_{ij} = \begin{cases} 2\left(k_c + \frac{\tau_0}{\sqrt{2\pi}}\right)e_{ij} & \text{if } \pi > \pi_c, \\ 2\left(k_c + \frac{\tau_0}{\sqrt{2\pi_c}}\right)e_{ij} & \text{if } \pi < \pi_c, \end{cases} \quad (1)$$

Here e_{ij} symbolized for the (i, j) th element of the deformation rate, π symbolized for the multiplication of e_{ij} with itself, k_c symbolized for the dynamic viscosity of Casson liquid, τ_0 symbolized for the yield-stress of the liquid, π_c symbolized for the critical value of π and e_{ij} can be expressed as: (see Refs. [13, 14])

$$e_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \quad (2)$$

The principal equations for mass, momentum, and energy conservations with nanoparticle concentration using above considerations can be composed as: (see Refs. [22, 29, 30])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (3)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \vartheta \left(1 + \frac{\tau_0}{k_c \sqrt{2\pi_c}} \right) \frac{\partial^2 u}{\partial z^2} - \left(\frac{\vartheta}{K} + \frac{\sigma B_0^2}{\rho} \right) u, \quad (4)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \vartheta \left(1 + \frac{\tau_0}{k_c \sqrt{2\pi_c}} \right) \frac{\partial^2 v}{\partial z^2} - \left(\frac{\vartheta}{K} + \frac{\sigma B_0^2}{\rho} \right) v, \quad (5)$$

Fig. 1 Physical description of the present investigation

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_m \frac{\partial^2 T}{\partial z^2} + \tau \left[D_B \left(\frac{\partial C}{\partial z} \frac{\partial T}{\partial z} \right) + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial z} \right)^2 \right] + \frac{\sigma B_0^2}{(\rho c)_f} (u^2 + v^2), \quad (6)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = D_B \frac{\partial^2 C}{\partial z^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial z^2}. \quad (7)$$

Here t signifies the time, (u, v, w) symbolize the velocity components along (x, y, z) directions, correspondingly, T indicates the temperature, C indicates the concentration, $\theta = \frac{k_c}{\rho}$ indicates the kinematic viscosity consists on the ratio form of dynamic viscosity k_c with the density ρ of fluid, K represents the permeability of the porous space, σ signifies the electric conductivity, B_0 represents the magnitude of magnetic field vector, $\alpha_m = \frac{\lambda}{(\rho c)_f}$ indicates the thermal diffusivity consists on the ratio form of thermal conductivity λ with the volumetric heat capacity $(\rho c)_f$, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ denotes the adiabatic index consists on the ratio of thermal capacity of nanomaterial $(\rho c)_p$ with the thermal capacity of base liquid $(\rho c)_f$, D_B describes the Brownian diffusion, while D_T describes the thermophoresis diffusion, and T_∞ represents the surrounding temperature. The typical boundary restrictions in the view of present continuation are (see Refs. [4, 31])

$$u = \frac{ax}{1-ct}, \quad v = \frac{by}{1-ct}, \quad w = 0 \text{ at } z = 0, \quad u \rightarrow 0, \quad v \rightarrow 0 \text{ as } z \rightarrow \infty, \quad (8)$$

Here, (a, b, c) are the stretching rates, (r, s) represent the regulatory indices, C_∞ represents the surrounding concentration, and (T_0, C_0, T_1, C_1) are positive constants having intrinsic dimensions. Moreover, it is also deduced that Eqs. (8) and (9) are valid only for $t \leq \frac{1}{c}$ with $c \neq 0$. Now, we are introducing the following combination of variables to change the principal equations into dimensionless forms (see Refs. [32])

$$u = \frac{ax}{1-ct} f'(\eta), \quad v = \frac{ay}{1-ct} g'(\eta), \quad w = -\sqrt{\frac{\theta a}{1-ct}} (f(\eta) + g(\eta)), \quad \eta = \sqrt{\frac{a}{\theta(1-ct)}} z, \\ \text{PSTcase: } \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \quad \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}, \\ \text{PHFcase: } T - T_\infty = \frac{T_1}{\lambda} \sqrt{\frac{\theta}{a(1-ct)}} x' y' \theta(\eta), \quad C - C_\infty = \frac{C_1}{D_B} \sqrt{\frac{\theta}{a(1-ct)}} x' y' \phi(\eta). \quad (10)$$

Here f, g, θ and ϕ are the similarity functions for stretching velocity along x -direction, stretching velocity along y -direction, temperature distribution and concentration distribution, respectively, while η is the independent similarity variable. It is noted that the Eq. (3) is straightforwardly fulfilled and remaining Eqs. (4)–(7) are transformed under the influence of Eq. (10) into the following forms:

$$\left(1 + \frac{1}{\beta}\right) f''' + (f+g)f'' - f'^2 - S \left(f' + \left(\frac{\eta}{2}\right) f'' \right) - (\varepsilon + M^2) f' = 0, \quad (11)$$

$$\text{PST case: } T = T_\infty + T_0 \left(\frac{x' y'}{1-ct} \right), \quad C = C_\infty + C_0 \left(\frac{x' y'}{1-ct} \right) \text{ at } z = 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ as } z \rightarrow \infty, \\ \text{PHF case: } -\lambda \frac{\partial T}{\partial z} = T_1 \left(\frac{x' y'}{1-ct} \right), \quad -D_B \frac{\partial C}{\partial z} = C_1 \left(\frac{x' y'}{1-ct} \right) \text{ at } z = 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ as } z \rightarrow \infty. \quad (9)$$

$$\left(1 + \frac{1}{\beta}\right)g''' + (f + g)g'' - g'^2 - S\left(g' + \left(\frac{\eta}{2}\right)g''\right) - (\epsilon + M^2)g' = 0, \quad (12)$$

Above physical quantities in the non-dimensional forms can be expressed as:

$$\frac{1}{Pr}\theta'' + (f + g)\theta' - (rf' + sg')\theta - S\left(\theta + \left(\frac{\eta}{2}\right)\theta'\right) + N_b\theta'\phi' + N_t\theta'^2 + M^2(Ec_x f'^2 + Ec_y g'^2) = 0, \quad (13)$$

$$\phi'' + PrLe\left((f + g)\phi' - (rf' + sg')\phi - S\left(\phi + \left(\frac{\eta}{2}\right)\phi'\right)\right) + \frac{N_t}{N_b}\theta'' = 0, \quad \sqrt{Re_x}C_{fx} = \left(1 + \frac{1}{\beta}\right)f''(0), \quad \left(\frac{x}{y}\right)\sqrt{Re_x}C_{fy} = \alpha\left(1 + \frac{1}{\beta}\right)g''(0), \quad (14)$$

$$f(0) = g(0) = 0, \quad f'(\eta = 0) = 1, \quad g'(\eta = 0) = \alpha, \quad f'(\eta \rightarrow \infty) \rightarrow 0, \quad g'(\eta \rightarrow \infty) \rightarrow 0, \quad (15)$$

PST case : $\theta(\eta = 0) = 1, \quad \phi(\eta = 0) = 1, \quad \theta(\eta \rightarrow \infty) \rightarrow 0, \quad \phi(\eta \rightarrow \infty) \rightarrow 0,$

PHF case : $\theta'(\eta = 0) = -1, \quad \phi'(\eta = 0) = -1, \quad \theta(\eta \rightarrow \infty) \rightarrow 0, \quad \phi(\eta \rightarrow \infty) \rightarrow 0.$

Here, $\beta = \frac{k_c \sqrt{2\pi_c}}{\tau_0}$ symbolizes the Casson fluid parameter, $S = \frac{\epsilon}{a}$ indicates the unsteady constraint consists on the ratio of stretching rates, $M^2 = \frac{\sigma B_0^2}{\rho \mu}$ symbolizes the magnetic constraint, $\epsilon = \frac{\theta}{aK}$ represents the porosity constraint, $Pr = \frac{\theta}{\alpha_m}$ specifies the Prandtl number, $N_b = \frac{\tau D_B (C_w - C_\infty)}{\theta}$ specifies the Brownian movement constraint, $N_t = \frac{\tau D_T (T_w - T_\infty)}{\theta T_\infty}$ specifies the thermophoretic constraint, $Ec_x = \frac{U_w^2}{c_f (T_s - T_\infty)}$ symbolized for the Eckert number along x-direction, $Ec_y = \frac{V_w^2}{c_f (T_s - T_\infty)}$ symbolized for the Eckert number along y-direction, $Le = \frac{\alpha_m}{D_B}$ symbolizes the Lewis number, and $\alpha = \frac{b}{a}$ symbolizes the ratio parameter. The quantities of physical interest for present investigation are wall skin-friction C_{fx} along x-direction stretching side, wall skin-friction C_{fy} along y-direction stretching side, local Nusselt number Nu_x for the prediction of heat transport, and local Sherwood number Sh_x for the prediction of mass transport. Mathematically, these quantities can be expressed as: (see Refs. [18, 19, 32]):

$$PST \text{ case : } Nu_x = -\theta'(0)\sqrt{Re_x}, \quad Sh_x = -\phi'(0)\sqrt{Re_x}, \quad Nu_y = -\theta'(0)\sqrt{Re_y}, \quad Sh_y = -\phi'(0)\sqrt{Re_y}, \quad (20)$$

$$PHF \text{ case : } Nu_x = \frac{1}{\theta(0)}\sqrt{Re_x}, \quad Sh_x = \frac{1}{\phi(0)}\sqrt{Re_x}, \quad Nu_y = \frac{1}{\theta(0)}\sqrt{Re_y}, \quad Sh_y = \frac{1}{\phi(0)}\sqrt{Re_y}. \quad (21)$$

Here, $(Re_x = \frac{xU_w}{\nu}, Re_y = \frac{yV_w}{\alpha \theta})$ represent the local Reynolds numbers.

3 Keller-Box scheme

The next step after modeling of the mathematical model is the choice of the suitable method to solve the modeled system. In this continuation, we preferred the Keller-Box method (KBM) introduced by Keller [37], which is implicit in nature, in order to solve the set of nonlinear ODEs. Equations (11)–(14) subject to the classical boundary conditions

$$C_{fx} = \frac{\tau_{xz}}{\rho U_w^2}, \quad C_{fy} = \frac{\tau_{yz}}{\rho V_w^2}, \quad Nu_x = \frac{xq_s}{\lambda(T_w - T_\infty)}, \quad Sh_x = \frac{xj_s}{D_B(C_w - C_\infty)}, \quad Nu_y = \frac{yq_s}{\lambda(T_w - T_\infty)}, \quad Sh_y = \frac{yj_s}{D_B(C_w - C_\infty)}. \quad (16)$$

Here, τ_{xz} labels the local shear stress along x-direction, while τ_{yz} labels the local shear stress along y-direction, q_s labels the local surface heats flux, while j_s symbolizes the local surface mass flux and these are defined as:

$$\tau_{xz} = \left(k_c + \frac{\tau_0}{\sqrt{2\pi_c}}\right)\left(\frac{\partial u}{\partial z}\right)_{z=0}, \quad \tau_{yz} = \left(k_c + \frac{\tau_0}{\sqrt{2\pi_c}}\right)\left(\frac{\partial v}{\partial z}\right)_{z=0}, \quad (17)$$

$$q_s = -\lambda\left(\frac{\partial T}{\partial z}\right)_{z=0}, \quad j_s = -D_B\left(\frac{\partial C}{\partial z}\right)_{z=0}. \quad (18)$$

Eq. (15), numerically. This method has second-order accuracy, and has a lot of advantages over other numerical approaches (SLM, shooting, BVP4c, RK, etc.). These advantages are listed below:

- It has faster convergence than other numerical approaches.
- Mixed boundary conditions can be easily tackled thorough Keller-Box approach.
- Accuracy of the solution can be easily achieved by reducing the step-size.

- It is a more stable, more effectual, and more flexible to use than other numerical approaches.

The mathematical procedure to implement this numerical approach can be started by considering the new variables H, G, P, Q, L and W to alter the higher-order ODEs into the system of first-order ODEs as follows:

$$\begin{aligned} f' &= H, H' = G, g' = P, P' = Q, \theta' = L, \phi' = W, \\ \left(1 + \frac{1}{\beta}\right)G' + (f + g)G - H^2 - S\left(H + \left(\frac{\eta}{2}\right)G\right) - (\varepsilon + M^2)H &= 0, \\ \left(1 + \frac{1}{\beta}\right)Q' + (f + g)Q - P^2 - S\left(P + \left(\frac{\eta}{2}\right)Q\right) - (\varepsilon + M^2)P &= 0, \\ \frac{1}{Pr}L' + (f + g)L - (rH + sP)\theta - S\left(\theta + \left(\frac{\eta}{2}\right)L\right) + N_b LW + N_t L^2 \\ &+ M^2(Ec_x H^2 + Ec_y P^2) = 0, \\ W' + PrLe\left((f + g)W - (rH + sP)\phi - S\left(\phi + \left(\frac{\eta}{2}\right)W\right)\right) + \frac{N_t}{N_b}L &= 0, \end{aligned} \quad (22)$$

with the boundary restrictions

$$\begin{aligned} f(0) &= g(0) = 0, H(\eta = 0) = 1, P(\eta = 0) = \alpha, H(\eta \rightarrow \infty) \rightarrow 0, P(\eta \rightarrow \infty) \rightarrow 0, \\ PST \text{ case : } \theta(\eta = 0) &= 1, \phi(\eta = 0) = 1, \theta(\eta \rightarrow \infty) \rightarrow 0, \phi(\eta \rightarrow \infty) \rightarrow 0, \\ PHF \text{ case : } L(\eta = 0) &= -1, W(\eta = 0) = -1, \theta(\eta \rightarrow \infty) \rightarrow 0, \phi(\eta \rightarrow \infty) \rightarrow 0. \end{aligned} \quad (23)$$

The discretization scheme for Eq. (22) is expressed as:

$$\begin{aligned} \eta_0 &= 0, \eta_j = \eta_{j-1} + h_j, j = 1, 2, 3, \dots, J; \eta_J = \eta_\infty, H_{j-\frac{1}{2}} = \frac{f_j - f_{j-1}}{h_j}, \\ G_{j-\frac{1}{2}} &= \frac{H_j - H_{j-1}}{h_j}, P_{j-\frac{1}{2}} = \frac{g_j - g_{j-1}}{h_j}, Q_{j-\frac{1}{2}} = \frac{P_j - P_{j-1}}{h_j}, L_{j-\frac{1}{2}} = \frac{\theta_j - \theta_{j-1}}{h_j}, W_{j-\frac{1}{2}} = \frac{\phi_j - \phi_{j-1}}{h_j}, \\ \left(1 + \frac{1}{\beta}\right)\frac{G_j - G_{j-1}}{h_j} + \left(f_{j-\frac{1}{2}} + g_{j-\frac{1}{2}}\right)G_{j-\frac{1}{2}} - H_{j-\frac{1}{2}}^2 - S\left(H_{j-\frac{1}{2}} + \frac{\eta}{2}G_{j-\frac{1}{2}}\right) - (\varepsilon + M^2)H_{j-\frac{1}{2}} &= 0, \\ \left(1 + \frac{1}{\beta}\right)\frac{Q_j - Q_{j-1}}{h_j} + \left(f_{j-\frac{1}{2}} + g_{j-\frac{1}{2}}\right)Q_{j-\frac{1}{2}} - P_{j-\frac{1}{2}}^2 - S\left(P_{j-\frac{1}{2}} + \frac{\eta}{2}Q_{j-\frac{1}{2}}\right) - (\varepsilon + M^2)P_{j-\frac{1}{2}} &= 0, \\ \frac{1}{Pr}\left(\frac{L_j - L_{j-1}}{h_j}\right) + (f + g)L_{j-\frac{1}{2}} - \left(rH_{j-\frac{1}{2}} + sP_{j-\frac{1}{2}}\right)\theta_{j-\frac{1}{2}} - S\left(\theta_{j-\frac{1}{2}} + \frac{\eta}{2}L_{j-\frac{1}{2}}\right) + N_b L_{j-\frac{1}{2}} W_{j-\frac{1}{2}} \\ &+ N_t L_{j-\frac{1}{2}}^2 + M^2\left(Ec_x H_{j-\frac{1}{2}}^2 + Ec_y P_{j-\frac{1}{2}}^2\right) = 0, \\ \frac{W_j - W_{j-1}}{h_j} + PrLe\left(\left(f_{j-\frac{1}{2}} + g_{j-\frac{1}{2}}\right)W_{j-\frac{1}{2}} - \left(rH_{j-\frac{1}{2}} + sP_{j-\frac{1}{2}}\right)\phi_{j-\frac{1}{2}} - S\left(\phi_{j-\frac{1}{2}} + \frac{\eta}{2}W_{j-\frac{1}{2}}\right)\right) \\ &+ \left(\frac{N_t}{N_b}\right)\frac{L_j - L_{j-1}}{h_j} = 0. \end{aligned} \quad (24)$$

Table 1 Solution of present model for different refinement levels when $Pr = Le = 1.2$, $S = \varepsilon = M = N_t = N_b = \alpha = 0.5$, $\beta = r = s = 1.0$, $Ec_x = Ec_y = 0.3$

R_i	n_p	$-f''(0)$	$-g''(0)$	PST case		PHF case	
				$-\theta'(0)$	$-\phi'(0)$	$\theta(0)$	$\phi(0)$
1st	500	1.070235	0.494328	1.083489	1.266369	0.880629	0.834445
2nd	1000	1.070235	0.494325	1.083451	1.266414	0.880657	0.834429
3rd	1500	1.070236	0.494325	1.083444	1.266423	0.880662	0.834426
4th	2000	1.070236	0.494324	1.083441	1.266426	0.880664	0.834425
5th	2500	1.070236	0.494324	1.08344	1.266427	0.880665	0.834425
6th	3000	1.070236	0.494324	1.083439	1.266428	0.880665	0.834425
7th	3500	1.070236	0.494324	1.083439	1.266428	0.880665	0.834425

level, the number of elements n_p for the computed domain is chosen five-hundred, whereas for the second refinement level the number of elements is increased up-to one-thousand, and so on. The tolerance level of the convergence is adjusted to 10^{-6} . It is noted that by increasing the number of elements, error in the numerical solution is reducing. The error in the computation is approaches to 0% for fourth refinement level. Hence, through-out this analysis, third refinement level is adopted to predict the amounts of heat and mass transports with $h_j = \frac{\eta_j - \eta_0}{n_p} : \eta_0 = 0, \eta_j = 20$.

5 Validation

In order to ensure the accuracy of the applied methodology, the outcomes obtained through Keller-Box method are compared with the previously published results of "Nadeem et al. [13]" through Table 2. It is concluded through Table 2 that the two set of results for skin-friction coefficients are best matched with each other for limited steady state ($S = 0$) case.

6 Results and discussion

In this portion, the impact of embryonic involved constraints on temperature distribution, concentration distribution, local Nusselt number (for heat flux), and local Sherwood number (for mass flux) are briefly analyzed

through various graphical illustrations. Special focus has been given to PST and PHF mechanisms.

6.1 Temperature and concentration distributions

The impacts of emerging involved constraints, namely, unsteady constraint S and temperature controlled indices (r and s) on temperature $\theta(\eta)$ and concentration $\phi(\eta)$ distributions are analyzed in this section through Figs. 2, 3 and 4.

Figure 2a–d illustrate the impact of unsteady constraint S on temperature distribution $\theta(\eta)$ for PST case, temperature distribution $\theta(\eta)$ for PHF case, concentration distribution $\phi(\eta)$ for PST case, and concentration distribution $\phi(\eta)$ for PHF case, respectively. It is observed through Fig. 2a–d that the escalating amount of unsteady constraint S reduces the temperature $\theta(\eta)$ as well as concentration $\phi(\eta)$ distributions, for both PST and PHF mechanism. Moreover, temperature of the surface through Fig. 2a–b is detected higher for PST source as compared to PHF source against augmenting amounts of S , whereas the thickness of thermal layer is noticed higher for PHF case as compared to PST case. Physically, escalating amount of S increases the electric conductivity of the liquid and this increment in the electric conductivity is higher for PHF case as compared to PST case. As a result, temperature distribution is reduced against higher amounts of S . Furthermore, concentration distribution nearer to the surface through Fig. 2c–d is observed higher for PST case than PHF case against higher amounts of S , whereas the thickness of concentration

Table 2 Comparison of skin-friction coefficient's values for a suitable ranges of M , β , α and ϵ with previous data of "Nadeem et al. [13]" when $S = 0$

M	β	ϵ	$\alpha = 0$		$\alpha = 1.0$	
			$-\left(1 + \frac{1}{\beta}\right)f''(0)$		$-\left(1 + \frac{1}{\beta}\right)g''(0)$	
			Present	"Nadeem et al. [13]"	Present	"Nadeem et al. [13]"
0.0	∞	0.0	1.00000	1.0042	1.17372	1.1748
			1.09545	1.0954	1.28575	1.2857
			1.41421	1.4142	1.65989	1.6599
	1	0.5	1.73205	1.7320	1.93156	1.9315
		1.0	2.00000	2.0000	2.17183	2.1718
10	∞	0.0	3.31662	3.3165	3.36726	3.3667
			3.63318	3.6331	3.68864	3.6886
			4.69042	4.6904	4.76201	4.7620
	1	0.5	4.79583	4.7958	4.86583	4.8658
		1.0	4.89898	4.8990	4.96749	4.9674
100	∞	0.0	10.0499	10.049	10.0665	10.066
			11.00909	11.0091	11.0273	11.0272
			14.21267	14.2127	14.2362	14.2361
	1	0.5	14.24781	14.2478	14.2713	14.2712
		1.0	14.28286	14.2829	14.3062	14.3062

Fig. 2 Impact of unsteady constraint S on temperature distribution $\theta(\eta)$ for [PST case (a) and PHF case (b)] and on concentration distribution $\phi(\eta)$ for [PST case (c) and PHF case (d)]

layer is achieved higher for PHF case as compared to PST case. Mathematically, increasing amount of S reduces the amount of thermal diffusivity and increases the amount of Brownian coefficient, as a result, concentration distribution is diminished against higher amounts of S .

Figure 3a–d describe the impact of index r on temperature distribution (PST case), temperature distribution (PHF case), concentration distribution (PST case), and concentration distribution (PHF case), respectively. It is noted through Fig. 3a–d that higher amount of r diminishes the concentration $\phi(\eta)$ distribution as well as temperature $\theta(\eta)$ distribution for both the prescribed heat sources (PST and PHF). It is perceived through Fig. 3a–b that PHF source produces higher temperature $\theta(\eta)$ distribution than PST source for negative amounts of r , whereas opposite trend is achieved for positive amounts of r . Physically, positive amounts of r enhance the wall temperature T_s whereas negative amounts of r diminish the wall temperature T_s , as a result, temperature distribution $\theta(\eta)$ is higher for negative amounts of r than positive amounts of r . Similar trends are observed through Fig. 3c–d for concentration

$\phi(\eta)$ distribution against augmenting amounts of r under PST and PHF mechanisms.

Figure 4a–d elucidate the influence of index s on temperature distribution (PST case), temperature distribution (PHF case), concentration distribution (PST case), and concentration distribution (PHF case), respectively. It is deduced through Fig. 4a–d that developed amount of r lessens the concentration $\phi(\eta)$ distribution as well as temperature $\theta(\eta)$ distribution for both the prescribed heat sources (PST and PHF). It is perceived through Fig. 4a–b that PHF source produces higher temperature $\theta(\eta)$ distribution than PST source for negative amounts of s , however reverse trend is achieved for positive amounts of s . Actually, positive amounts of s improve the wall temperature T_s while negative amounts of s lessen the wall temperature T_s , as a result, temperature distribution $\theta(\eta)$ is enhanced for negative amounts of r than positive amounts of r . Similar trends are observed through Fig. 4c–d for concentration $\phi(\eta)$ distribution besides extending amounts of s under PST and PHF mechanisms.

Fig. 3 Impact of temperature controlled index r on temperature distribution $\theta(\eta)$ for [PST case (a) and PHF case (b)] and on concentration distribution $\phi(\eta)$ for [PST case (c) and PHF case (d)]

6.2 Heat and mass transports

The present section describes the influences of temperature controlled indices (r and s), Brownian constraint N_b , and thermophoresis constraint N_t on the rate of heat transfer for PST case $\theta'(0)$, the rate of heat transfer for PHF case $\frac{1}{\theta(0)}$, the rate of mass transfer for PST case $\phi'(0)$, and the rate of mass transfer for PHF case $\frac{1}{\phi(0)}$ through Figs. 5a and 6d. Moreover, tabular comparisons are also presented between the rates of heat transferences (for PST and PHF cases) through Table 3 and between the rates of mass transferences (for PST and PHF cases) through Table 4 against escalating amounts of magnetic constraint M , Eckert numbers (Ec_x and Ec_y), Prandtl number Pr , Casson parameter β , Lewis number Le , and unsteady constraint S .

Figure 5a illustrates the impacts of N_t and r on the local Nusslet number for PST case and it is deduced through this figure that increasing amounts of N_t decline the rate of heat transfer, whereas escalating amounts of r enhance the rate of heat transfer. Basically, higher amount of N_t

augments the thermal diffusivity of the liquid, as a result, rate of heat transfer is reduced. Similar trend is achieved for the local Nusslet number for PHF case through Fig. 5b, but the rate of heat transfer is higher for PHF case as compared to PST case. Figure 5c describes the influences of r and N_t on the local Sherwood number (PST case) and it is extracted through this figure that growing amounts of N_t also decline the amount of mass transport whereas growing amounts of r progress the rate of mass transport. Mathematically, higher amount of r reduces the concentration distribution $\phi(\eta)$, as a result, rate of mass flux is augmented for higher amounts of r . Identical tendency is attained for the local Sherwood number for PHF case through Fig. 5d against escalating amounts of r and N_t . It is observed through Fig. 5c–d that the rate of mass flux is higher for PHF case as compared to PST case against growing amounts of r and N_t .

Figure 6a demonstrates the effects of N_b and s on the local Nusslet number for PST situation and it is inferred through this figure that growing amounts of N_b decay

Fig. 4 Impact of temperature controlled index s on temperature distribution $\theta(\eta)$ for [PST case (a) and PHF case (b)] and on concentration distribution $\phi(\eta)$ for [PST case (c) and PHF case (d)]

the level of heat transport, whereas rising amounts of s enrich the level of heat transport. Ultimately, developed amount of N_b reduces the coefficient of Brownian diffusion, as a result, level of heat transport is decayed. Similar trend is seen for the level of heat transport for PHF case through Fig. 6b, but the level of heat transport is higher for PHF case as compared to PST case. Figure 6c pronounces the effects of s and N_b on the local Sherwood number (PST case) and it is noted through this figure that developing amounts of N_b and s enhance the level of mass transport. Mathematically, higher amount of s diminishes the concentration distribution $\phi(\eta)$, as a result, level of mass flux is increased for higher amounts of s . Identical tendencies are attained for the local Sherwood number for PHF case through Fig. 6d against escalating amounts of s and N_b . It is detected through Fig. 6c, d that the level of mass flux is greater for PHF case as compared to PST case against mounting amounts of s and N_b .

Table 3 elaborates the numerical comparison between the level of heat transfer for PST case and the level of heat transfer for PHF case. It is illustrated through Table 4 that growing amounts of magnetic constraint M , Lewis number Le , and Eckert number (Ec_x and Ec_y) reduce the level of heat transport whereas enlarging amounts of Prandtl number Pr , Casson parameter β , and unsteady constraint S enhance the level of heat transport. It is also deduced through Table 3 that PHF source produces the better rate of heat transfer as compared to PST source due to the negative influence of thermal conductivity in the PHF mechanism.

Table 4 illustrates the computational comparison between the level of mass transfer for PST case and the level of mass transfer for PHF case. It is detected through Table 4 that growing amounts of magnetic constraint M and Casson parameter β reduce the level of mass transport whereas expanding amounts of Eckert numbers (Ec_x and Ec_y), unsteady constraint S , Prandtl number Pr ,

Fig. 5 Trends of local Nusselt number for [PST case (a) and for PHF case (b)] and local Sherwood number for [PST case (c) and PHF case (d)] against wide ranges of N_t and r

and Lewis number Le enhance the level of mass transport. It is also deduced through Table 4 that PST source produces the better rate of mass transfer as compared to PHF

source due to the negative influence of mass diffusivity in the PHF mechanism.

Fig. 6 Trends of local Nusselt number for [PST case (a) and for PHF case (b)] and local Sherwood number for [PST case (c) and PHF case (d)] against wide ranges of N_b and s

Table 3 Trend of local Nusselt number for PST and PHF mechanisms against escalating amounts of magnetic constraint M , Prandtl number Pr , Lewis number Le , Eckert numbers (Ec_x, Ec_y), Casson parameter β and unsteady constraint S when $\varepsilon = \alpha = 0.3$, $N_t = N_b = 0.7$, $r = s = 1.0$

M	Pr	Le	Ec_x	Ec_y	β	S	$\frac{Nu_x}{\sqrt{Re_x}} = \frac{Nu_y}{\sqrt{Re_y}}$	
							PST case $[-\theta'(0)]$	PHF case $\left[\frac{1}{\theta(0)}\right]$
0.5	1.2	1.2	0.1	0.1	1.0	0.5	0.917115	0.948989
1.0	–	–	–	–	–	–	0.884277	0.904043
1.5	–	–	–	–	–	–	0.837420	0.846168
–	1.0	–	–	–	–	–	0.813953	0.794250
–	1.3	–	–	–	–	–	0.844088	0.868534
–	1.6	–	–	–	–	–	0.849653	0.924621
–	–	0.6	–	–	–	–	0.964351	0.897547
–	–	1.0	–	–	–	–	0.878902	0.917204
–	–	1.4	–	–	–	–	0.825589	0.931045
–	–	–	0.2	–	–	–	0.766833	0.873279
–	–	–	0.3	–	–	–	0.707801	0.821959
–	–	–	0.4	–	–	–	0.648491	0.776068
–	–	–	–	0.2	–	–	0.643005	0.772072
–	–	–	–	0.3	–	–	0.637516	0.768114
–	–	–	–	0.4	–	–	0.632025	0.764194
–	–	–	–	–	5.0	–	0.640043	0.750470
–	–	–	–	–	10	–	0.641015	0.747467
–	–	–	–	–	∞	–	0.641929	0.743954
–	–	–	–	–	–	0.8	0.732775	0.853349
–	–	–	–	–	–	1.1	0.814385	0.953775
–	–	–	–	–	–	1.4	0.889111	1.047541

Table 4 Trend of local Sherwood number for PST and PHF mechanisms against escalating amounts of magnetic constraint M , Prandtl number Pr , Lewis number Le , Eckert numbers (Ec_x, Ec_y), Casson parameter β and unsteady constraint S when $\varepsilon = \alpha = 0.3$, $N_t = N_b = 0.7$, $r = s = 1.0$

M	Pr	Le	Ec_x	Ec_y	β	S	$\frac{Sh_x}{\sqrt{Re_x}} = \frac{Sh_y}{\sqrt{Re_y}}$	
							PST case $[-\phi'(0)]$	PHF case $\left[\frac{1}{\phi(0)}\right]$
0.5	1.2	1.2	0.1	0.1	1.0	0.5	1.279449	1.165181
1.0	–	–	–	–	–	–	1.245482	1.132613
1.5	–	–	–	–	–	–	1.207667	1.092575
–	1.0	–	–	–	–	–	1.032777	0.967284
–	1.3	–	–	–	–	–	1.293007	1.152761
–	1.6	–	–	–	–	–	1.540295	1.325481
–	–	0.6	–	–	–	–	0.728899	0.820085
–	–	1.0	–	–	–	–	1.306616	1.169738
–	–	1.4	–	–	–	–	1.759549	1.471945
–	–	–	0.2	–	–	–	1.793373	1.496479
–	–	–	0.3	–	–	–	1.836422	1.521693
–	–	–	0.4	–	–	–	1.879697	1.547613
–	–	–	–	0.2	–	–	1.883667	1.550003
–	–	–	–	0.3	–	–	1.887638	1.552399
–	–	–	–	0.4	–	–	1.891611	1.554802
–	–	–	–	–	5.0	–	1.779278	1.456543
–	–	–	–	–	10	–	1.758903	1.439395
–	–	–	–	–	∞	–	1.73627	1.420569
–	–	–	–	–	–	0.8	1.865901	1.549862
–	–	–	–	–	–	1.1	1.981162	1.658172
–	–	–	–	–	–	1.4	2.086825	1.753786

7 Conclusions

In this article, MHD Casson nanofluid flow driven by an unsteady bidirectional moving surface subject to prescribed heat mechanism, namely, PST and PHF has been studied. The aspects of Brownian motion plus thermophoresis have been analyzed by using the mathematical relation of Buongiorno model. Similarity variables have been used to transform the governing PDEs into ODEs and then computational treatment has been made through Keller-Box method. It is concluded through present study that PHF mechanism produces the better rate of heat transfer as compared to PST mechanism, whereas PST mechanism produces the better rate of mass transfer as compared to PHF mechanism. Moreover, it is also observed that the escalating amount of Eckert number enhances the rate of mass transfer, and reduces the rate of heat transfer.

Finally, it is hoped that the outcomes through present investigation will not be only helpful for industrial and engineering applications but also will be beneficiary for future endeavors.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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