Modelling of very high voltage transmission lines inspection's quadrotor

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Abstract

In this study, we present a general mathematical model of a quadrotor unmanned aerial vehicle (UAV), that is dedicated to the transmission lines inspection. It takes into account the mechanical forces and torques, based on the quaternion approach to avoid the singularity problem, and the interference effects of the electromagnetic field, generated by the high voltages transmission lines, on the quadrotor. In order to establish a mathematical model that considers all the constraints of our mission type and to release a specific inspection transmission lines quadrotor, contrary to the mathematical models presented in most of the research works which are mechanical models that describe just a general flight case of a UAV. The choice of the quadcopter was made by comparing it with other methods of transmission lines inspection, as it is the cheapest, easiest, safest, and most efficient to meet our need. To express mechanical and electromagnetic equations and laws, we suppose that the quadrotor's structure is rigid and symmetrical, the propellers are rigid, the center of mass and the origin of the body frame coincide, the lift and drag forces are proportional to the squares of the rotation speed of the rotors, the earth is a perfect conductor and the root-mean-square line voltages are constant.

Keywords UAV · Quadrotor · Transmission lines inspection · Mathematical model · Electromagnetic interference

1 Introduction

Electricity supply operators play a very important role in improving the standard of living of citizens and in the country's economic competitiveness by mobilizing resources to meet electricity demand at the best cost and quality of service conditions, and to keeping up the electrical energy consumption growth driven by countries socioeconomic development. They aim to prepare the electricity grid for the requirements of tomorrow, so they strengthen the transport network that links between generating stations and demand points and which it is constituted of very high and high voltage transmission lines. And they attach great importance to maintenance operations to satisfy the electrical demand with the best

service quality and to guarantee human and environmental security.

Due to the great geographical diversity of Morocco, the electrical transmission lines cross thousands of kilometers through mountains, forests, deserts and inaccessible areas. They are composed of 60 kV, 150 kV, 225 kV, and 400 kV lines, it covers almost all national territory with a total length of 25.288 km in 2017 using more than 60.000 pylons.

Morocco has some regions with difficult climate, such as the temperature can decrease below -20°C and it can rise up to 49°C during the year, and the wind speed can go up to 170 km/h, so these climatic conditions contribute to the transmission lines degradation. It is also characterized by a large desert area and two long coastlines

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which will cause silting and oxidation problems, in addition to vandalism acts.

The inspection of electrical transmission lines is mainly carried out by manned aerial vehicles or patrols while these methods are in some way ineffective and expensive. Recently, UAVs have been developed in several research centers around the world due to their potential applications such as inspection of power transmission lines as they are more efficient, less expensive and safer [1]. They can be sent rapidly anytime to check and have more details on the reported issues. They can provide images and videos of possible problem areas which can then be analyzed with software to find indications for any problem in the electrical transmission lines [2].

UAV's mathematical model has been presented in different works without specifying its field of application, as in [3–5], where it is presented its mathematical system which takes into account just the physical efforts and torques for a general flight mode. For the paper [6], the author specified the field of application of the UAV considered, which is the inspection of transmission lines, but without taking into consideration the electromagnetic forces generated by the lines and which will have to be taken due to their impact on the UAV's operation near power lines.

There is a lot of studies that are interested in the transmission lines inspection with UAVs, and due to the absence of electromagnetic mathematical modeling, they maintain a long-distance between lines and UAVs to avoid any problems that may be generated due to the electromagnetic interferences [7, 8]. Such as in [9], a 10 m was be considered as a better safety distance from a 220 kV lines, and to plan the referenced flight path in [10], a large safety distance has been taken between UAVs and transmission lines according to the inspection company guidelines.

That is why, this paper aims to establish a general mathematical model of a transmission lines inspection's quadrotor, which takes into account, in addition to the mechanical efforts, those electromagnetic. So, we will present, firstly, the transmission lines inspection, interventions types and problems encountered. Secondly, we will list the benefits of using a UAV in transmission lines inspection and the justification of choosing a multicopter as an adequate solution for this type of inspection. We will detail the different forces and torques that is sustained by the quadrotor, and the quaternion approach which we will use to elaborate a quadrotor mathematical model. After that, we will present the electromagnetic effects on UAV in the vicinity of the transmission lines to be able to establish a general mathematical model that takes into account the mechanical and electromagnetic effects.

2 Transmission lines inspection

To ensure the transport of electrical energy, electricity supply operators give great importance to maintenance operations whether corrective or preventive. After recognizing the presence of a problem in a line, there is no way of locating it without the intervention of the patrols. The linemen will have to go out to inspect the line to determine the defect position, the route of the line is done on foot, it can be during the day as it can be also during the night and it takes, in cases, days. As part of preventive maintenance, many diagnostic operations are scheduled to control the state of lines and the safety of their surroundings. It relies on patrols, mounted tours or even helicopters to monitor cables, accessories and towers that require a detailed diagnosis to see the condition of the metal, solid masses, and bolts and nuts, and for the surveillance of nearby human settlements, the presence of new plants and anything that may cause problems in the future, such as bird nests.

As we have seen above, the traditional diagnostic methods adopted need a lot of time such that the control of only one pylon can take a whole day which causes a delay to have information, decreases the frequency of diagnostics and increases the risk of having a big failure: a break in a cable can lead to the destruction of pylons. Sometimes these operations disrupt the network because we need to cut power to climb to the line. So, the time of these operations, a large number of linemen and the use of helicopters make the cost of these operations very high. For linemen, these operations are insecure because they will be exposed to electromagnetic fields, which has influences on the human health [11], they are often asked to intervene in difficult access areas due to most transmission lines are in isolated regions, and even sometimes they use primitive means to arrive there, such as mules. Owing to the human error and the difficulty of helicopter maneuverability, the diagnostic results are not precise and unreliable to ensure excellent preventive transmission lines maintenance and these methods do not allow intervening quickly when the problem has already occurred. So, our proposition aims to develop the maintenance lines means and to go beyond these traditional methods in order to decrease the operations cost, time and difficulty, increase their quality and frequency, and mainly to protect linemen.

3 The use of UAVs in power lines inspection

In the past few years, UAV has attracted the attention of several researchers and its use in various applications has become the desired choice due to its different benefits and its ability to replace man in hostile and dangerous environments [12]. It is recommended to replace different technologies of transmission lines inspection [13], such as: manned aerial vehicles [14] that can flight near transmission lines to let take pictures and videos, but this method is very expensive and it put the human life and the crew in danger. The Rolling on Wire Robots, as the Expliner robot [15, 16]. They are called also the climbing robots and they are developed to be connected to transmission lines and move on them. They can inspect lines without interrupting the power supply and they allow to place sensors directly over or around the lines. But they are difficult to realize due to their complexity, they are a slower inspection method and they are expensive. This inspection's form inspects just cables and it does not inspect pylons, accessories, and line's surroundings which have a big importance in these operations to delete anything that can produce problems in the future.

The use of UAVs in transmission lines inspection makes linemen safe because they can stay away from the lines without the need to get close. It makes operations faster and more accurate, which increases the frequency and the reliability of inspections and the quality of results. It also reduces operations cost by minimizing diagnostic time, logistics means, manpower and maintenance cost. By processing the photos and videos captured by the UAV cameras, visible or thermal, we will be able to detect and locate the various line defects. And with a programmed platform, we can have a real time reading of captured data with all the necessary detail. And in [17], it was presented the different advantages of using UAVs in transmission lines inspection.

But they also have major inconvenient that need to be taken into consideration, such as the modest autonomy, the determined range, the limited payload, and the recommended climatic conditions, all of which impact the mission's nature and progress.

By comparing with the other existing methods, UAV showed that it is a perfect choice, because it is the cheapest, safest, easiest and fastest method. And the most important thing is that it can inspect all the element that we want: transmission cables, pylons, accessories, and lines surroundings, in order to guarantee the transmission network's good state.

The choice of a Vertical Take Off and Landing (VTOL) UAV (multi-rotors) is made according to its ability to

do a thorough diagnosis of cables and pylons by taking pictures and videos from different viewing points thanks to its great maneuverability and its possibility to do vertical and horizontal inspections which is exactly our need. And due to safety, power, stability, payload, simplicity and cost, we have chosen a quadrotor, which is a multirotor with four rotors, for the transmission line inspection operation.

4 Quadrotor's mathematical model

The objective of this part is to model the quadrotor by exploiting its dynamics equations. And to that, we must take into consideration the different hypotheses below [18, 19]: the multi-rotors structure is supposed rigid and symmetrical, the propellers are supposed to be rigid, the center of mass and the origin of the body frame coincide, and the lift and drag forces are proportional to the squares of the rotation speed of the rotors.

In what follows, we will present a general mathematical model and for that we will use the notation presented in equations system (1):

$$\xi = (x, y, z)^T \quad (1a) \quad \eta = (\varphi, \theta, \Psi)^T \quad (1b)$$

4.1 Physical effects

4.1.1 Forces

The forces acting on the system, expressed in the inertial frame, are:

- Weight: $F_w = (0 \ 0 \ -mg)^T$ (2)

With m is the quadrotor mass and g is the gravitational constant.

- Lift: $F_l = T \left(0 \ 0 \ \sum_{i=1}^n b \omega_i^2 \right)^T$ (3)

With b is the lift coefficient and ω_i are rotors angular velocities.

- Drag: $F_d = (-K_{fx}\dot{x} \ -K_{fy}\dot{y} \ -K_{fz}\dot{z})^T$ (4)

With K_{ft} are the drag coefficients.

4.1.2 Torques

There are several acting moments on the quadrotor, they are due to the forces and the gyroscopic effects:

- Drag moment: $\tau_d = (M_x \ M_y \ M_z)^T$ (5)

These moments are calculated, for the adopted model in the development of the mathematical model presented in section IV.C, by the following equations:

$$\begin{cases} M_x = bl(\omega_1^2 - \omega_3^2) \\ M_y = bl(\omega_4^2 - \omega_2^2) \\ M_z = d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{cases} \quad (6)$$

- Moment resulting from aerodynamic friction:

$$\tau_a = (K_{fax}\dot{\phi}^2 \ K_{fay}\dot{\theta}^2 \ K_{faz}\dot{\psi}^2)^T \quad (7)$$

With K_{fa} are the aerodynamic friction coefficients.

- Gyroscopic moment of the propellers:

$$\tau_{gh} = (\Omega_r J_r \dot{\theta} \ -\Omega_r J_r \dot{\phi} \ 0)^T \quad (8)$$

With $\Omega_r = \sum_{i=1}^n (-1)^{i+1} \omega_i$ and J_r is the inertia of the rotor.

- Gyroscopic moment due to the quadrotor's movements:

$$\tau_{gm} = (\dot{\theta}\dot{\psi}(I_z - I_y) \ \dot{\psi}\dot{\phi}(I_x - I_z) \ \dot{\phi}\dot{\theta}(I_y - I_x))^T \quad (9)$$

With: I_x , I_y and I_z are respectively the moments of inertia around the x , y and z axis.

4.2 Frames

The expression of solid movement in referential depends on six parameters which are the three coordinates describing the position of its center of mass (or any point of the solid) and three angles named Euler angles: ϕ , θ , and ψ .

So, the matrix T is the transformation matrix which related coordinates of the body frame with that of the inertial frame, and it is defined by the matrix (10):

$$T = \begin{pmatrix} c_\psi c_\theta & c_\psi s_\phi s_\theta - c_\phi s_\psi & c_\phi c_\psi s_\theta + s_\phi s_\psi \\ c_\phi s_\psi & s_\phi s_\psi s_\theta + c_\phi c_\psi & c_\phi s_\psi s_\theta - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\phi c_\theta \end{pmatrix} \quad (10)$$

The matrix that related speeds is defined by the matrix W :

$$W = \begin{pmatrix} 1 & 0 & -s_\theta \\ 0 & c_\phi & c_\theta s_\phi \\ 0 & -s_\phi & c_\theta c_\phi \end{pmatrix} \quad (11)$$

With: $c_\phi = \cos(\phi)$ and $s_\phi = \sin(\phi)$

$c_\theta = \cos(\theta)$ and $s_\theta = \sin(\theta)$

$c_\psi = \cos(\psi)$ and $s_\psi = \sin(\psi)$

If we calculate the inverse of the matrix W , we find that it is written as follows:

$$W^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ t_\theta s_\phi & c_\phi & sc_\theta s_\phi \\ t_\theta c_\phi & -s_\phi & sc_\theta c_\phi \end{pmatrix} \quad (12)$$

With: $t_\theta = \tan(\theta)$ and $sc_\theta = \sec(\theta) = \frac{1}{\cos(\theta)}$.

It is defined if and only if: $\theta \neq \frac{\pi}{2} + k\pi, (k \in \mathbb{Z})$, because of the term $\tan\theta$. Hence the generation of a singular point (Gimbal Lock) this means the loss of a freedom degree.

So, to overcome this problem, we can consider different presentations such as the Quaternions where the rotation in one frame regarding another frame will be presented by 4 parameters, and the advantages of using an approach based on it consist not only in the absence of singularities but also in the simplicity of computation.

Quaternion is a vector in a four-dimensional plane defined as:

$$q = q_0 e + q_1 i + q_2 j + q_3 k \quad (13a)$$

$$\bar{q} = q_0 e - q_1 i - q_2 j - q_3 k \quad (13b)$$

With the following condition:

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \quad (14)$$

If θ is the angle of rotation around the unit vector $u = (u_1, u_2, u_3)$, it is possible to define the quaternion as following:

$$q = \left(\cos \frac{\theta}{2}, u \sin \frac{\theta}{2} \right) \quad (15)$$

$$q = e \cos \frac{\theta}{2} + u_1 \sin \frac{\theta}{2} + u_2 \sin \frac{\theta}{2} + u_3 \sin \frac{\theta}{2} \quad (16)$$

In order to deduct the transformation matrix from the initial frame to the relative frame of coordinates and velocities, we need to exploit the expressions below by using a set of equations that connect the four elements of the canonical basis (e, i, j, k) , presented in [20]:

$$x_R = qX_B\bar{q} \quad (17a) \quad y_R = qY_B\bar{q} \quad (17b) \quad z_R = qZ_B\bar{q} \quad (17c)$$

So, with the exploitation of the three Eqs. (17), we will have the quaternion transformation matrix (18) of coordinates T_q :

$$T_q = \begin{pmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_0q_3 + q_1q_2) & 2(q_1q_3 - q_0q_2) \\ 2(q_1q_2 - q_0q_3) & q_0^2 + q_2^2 - q_1^2 - q_3^2 & 2(q_0q_1 + q_2q_3) \\ 2(q_0q_2 + q_1q_3) & 2(q_2q_3 - q_0q_1) & q_0^2 + q_3^2 - q_1^2 - q_2^2 \end{pmatrix} \quad (18)$$

And, the quaternion transformation matrix that related speeds is presented as follows:

$$W_q = \begin{pmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{pmatrix} \quad (19)$$

4.3 Quadrotor quaternion dynamic model

To present the state equations, identify the control inputs, and simulate the system, we chose a quadrotor whose scheme is shown below (see Fig. 1).

For the dynamic study of the system, we will be based on the Newton–Euler formulation which describes the combined translational and rotational dynamics of a rigid body:

$$\begin{pmatrix} F \\ \tau \end{pmatrix} = \begin{pmatrix} m & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} \ddot{\xi} \\ \ddot{\eta} \end{pmatrix} + \begin{pmatrix} 0 \\ \dot{\eta} \times (I\dot{\eta}) \end{pmatrix} \quad (20)$$

From forces and torques Eqs. (2), ..., (9), and from the Newton–Euler formulations (20), we can deduce the equations system (21) that describe the system's state representation in the inertial frame:

Fig. 1 The adopted quadrotor's diagram

$$\begin{cases} \ddot{x} = -\frac{K_{fx}}{m}\dot{x} + \frac{2}{m}(q_1q_3 - q_0q_2) \sum_{i=1}^n b\omega_i^2 \\ \ddot{y} = -\frac{K_{fy}}{m}\dot{y} + \frac{2}{m}(q_0q_1 - q_2q_3) \sum_{i=1}^n b\omega_i^2 \\ \ddot{z} = -\frac{K_{fz}}{m}\dot{z} + \frac{1}{m}(q_0^2 + q_3^2 - q_1^2 - q_2^2) \sum_{i=1}^n b\omega_i^2 - g \\ \ddot{\phi} = \frac{1}{I_x}(bl(\omega_1^2 - \omega_3^2) - K_{fax}\dot{\phi}^2 - \Omega_r J_r \dot{\theta} - \dot{\theta}\dot{\psi}(I_z - I_y)) \\ \ddot{\theta} = \frac{1}{I_y}(bl(\omega_4^2 - \omega_2^2) - K_{fay}\dot{\theta}^2 + \Omega_r J_r \dot{\phi} - \dot{\psi}\dot{\phi}(I_x - I_z)) \\ \ddot{\psi} = \frac{1}{I_z}(d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) - K_{faz}\dot{\psi}^2 - \dot{\phi}\dot{\theta}(I_y - I_x)) \end{cases} \quad (21)$$

U_1, U_2, U_3 , and U_4 are the control inputs of our system which command the Altitude z , Roll ϕ , Pitch θ , and Yaw ψ . They are defined by the Eqs. (22):

$$\begin{cases} U_1 = b \sum_{i=1}^n \omega_i^2 \\ U_2 = bl(\omega_1^2 - \omega_3^2) \\ U_3 = bl(\omega_4^2 - \omega_2^2) \\ U_4 = d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{cases} \quad (22)$$

With: l is the distance between the rotor and the center of gravity, and d is the drag coefficient and it depends on the propeller manufacturing.

We will choose the state vector below for our system's state representation:

$$X = (\xi, \dot{\xi}, q, \dot{q})^T = (x, y, z, \dot{x}, \dot{y}, \dot{z}, q_0, q_1, q_2, q_3, \dot{\phi}, \dot{\theta}, \dot{\psi})^T \quad (23)$$

$$X = (x_1, x_2, x_3, \dots, x_{13})^T \quad (24)$$

Then, we will have the following state representation based on quaternions:

$$\begin{cases} \dot{x}_1 = x_4 \\ \dot{x}_2 = x_5 \\ \dot{x}_3 = x_6 \\ \dot{x}_4 = a_4x_4 + \frac{2}{m}U_1(x_8x_{10} - x_7x_9) \\ \dot{x}_5 = a_5x_5 + \frac{2}{m}U_1(x_7x_8 - x_9x_{10}) \\ \dot{x}_6 = -g + a_6x_6 + \frac{1}{m}U_1(x_7^2 + x_{10}^2 - x_8^2 - x_9^2) \\ \dot{x}_7 = -x_8x_{11} - x_9x_{12} - x_{10}x_{13} \\ \dot{x}_8 = x_7x_{11} - x_{10}x_{12} + x_9x_{13} \\ \dot{x}_9 = x_{10}x_{11} + x_7x_{12} - x_8x_{13} \\ \dot{x}_{10} = -x_9x_{11} + x_8x_{12} + x_7x_{13} \\ \dot{x}_{11} = b_{11}U_2 + a_{11}x_{11}^2 + c_{11}x_{12} + d_{11}x_{12}x_{13} \\ \dot{x}_{12} = b_{12}U_3 + a_{12}x_{12}^2 + c_{12}x_{11} + d_{12}x_{11}x_{13} \\ \dot{x}_{13} = b_{13}U_4 + a_{13}x_{13}^2 + d_{13}x_{11}x_{12} \end{cases} \quad (25)$$

With:

$$\begin{cases} a_4 = \frac{-K_{fxx}}{m}, a_5 = \frac{-K_{fyy}}{m}, a_6 = \frac{-K_{fzz}}{m}, a_{11} = \frac{-K_{fax}}{l_x}, \\ a_{12} = \frac{-K_{fay}}{l_y}, a_{13} = \frac{-K_{faz}}{l_z}, b_{11} = \frac{1}{l_x}, b_{12} = \frac{1}{l_y}, \\ b_{13} = \frac{1}{l_z}, c_{11} = \frac{-\Omega_r J_r}{l_x}, c_{12} = \frac{\Omega_r J_r}{l_y}, \\ d_{11} = \frac{l_z - l_y}{l_x}, d_{12} = \frac{l_x - l_z}{l_y}, d_{13} = \frac{l_y - l_x}{l_z} \end{cases} \quad (26)$$

5 The electromagnetic effect on UAV in the vicinity of high voltages transmission lines

The inspection of high voltages transmission lines requires the presence of the UAV close to high voltages transmission lines, with a line approach distance which differs according to the level of the on-board camera's performance. So, it will undergo an electromagnetic field, generated by the voltages and intensities that run through the transmission lines. That is why we must establish a general mathematical model, because the model presented above takes into account only mechanical forces and torques undergone by the UAV in general use cases. So, in this part, we will study the electromagnetic interference with UAV in vicinity of high voltages transmission lines.

The modelling of electromagnetic fields and interference demands the development of adequate mathematical models, which have to consider a large number of input parameters including those unknown in advance that describe these designs. Therefore, development of such mathematical models is not always feasible [21].

Fig. 2 Cross section of overhead double circuit power line

Below, we will present the results of laboratory tests and electromagnetic simulation presented in research works which are one of the approaches that allow the analysis of the electromagnetic interference resistance of the UAV's electronic systems. And then we will define the UAV's minimum approach distance from transmission lines concluded from empirical models, standards, and physical tests to avoid interference problems with no need to develop the electromagnetic mathematical model.

5.1 The calculation of the electromagnetic field in the vicinity of high voltages transmission lines

To visualize the electromagnetic waves propagation around lines we will consider the most extreme case that it is a 400 kV line, which is the highest Moroccan voltage transmitted, supported by an S-pylon that holds two parallel lines (see Fig. 2).

To calculate the electric and magnetic field we will consider the root-mean-square (rms) line voltages are constant and the earth as a perfect conductor, and we will use the image theorem [22], where we place virtual charges of opposite sign behind the conductive plane to reproduce an electric field distribution that satisfies the boundary conditions on the plane, as it is shown in (Fig. 2). So, the receiver receives the echo from the image load as if the signal emitted by the load was reflected by the conductive plane. The field is two dimensional because it equals along the line and there is no variation at the Z axis.

G_1 and G_2 are the guard cables that do not carry current, and they play the role of a lightning rod. C_i are conductors such as C_1, C_3 and C_5 are the three phases of the circuit 1 and C_2, C_4 and C_6 are those of the circuit 2. C'_i are the images. C'_i , with $i = \{1, 2, \dots, 6\}$ have respectively the coordinates $(-10.1; 59.7)$, $(10.1; 59.7)$, $(-10.1; 49.85)$, $(10.1; 49.85)$, $(-10.85; 40)$ and $(10.85; 40)$. C'_i have the same coordinates with negative y_i .

5.1.1 Electric field

It is produced in the region of space between the conductors and between the conductors and ground. It is the responsible for the appearance of corona effect which is an ionization phenomenon of the air surrounding the conductor. Its strength depends on voltage, current, geometric configuration of conductors. The electric field can be calculated as the vector sum below:

$$\vec{E}_M(x, y) = \sum_{i=1}^N \frac{Q_i}{2\pi\epsilon_0} \left(\frac{\vec{r}_i}{R_i} - \frac{\vec{r}'_i}{R'_i} \right) \quad (27)$$

$$\vec{E}_M(x, y) = \begin{cases} E_{Mx}(x, y) = \sum_{i=1}^N \frac{Q_i}{2\pi\epsilon_0} (x - x_i) \left(\frac{1}{R_i^2} - \frac{1}{R_i'^2} \right) \\ E_{My}(x, y) = \sum_{i=1}^N \frac{Q_i}{2\pi\epsilon_0} \left(\frac{y - y_i}{R_i^2} - \frac{y + y_i}{R_i'^2} \right) \end{cases} \quad (28)$$

where N is the number of the conductors, ϵ_0 is the permittivity of vacuum and it is equal to 8.85×10^{-12} F/m, R_i and R_i' are respectively the distances between the point M and the conductor i and its image, (x_i, y_i) are the coordinates of the conductor i , and Q_i is the line charge, that it is calculated by the following expression:

$$[Q]_{N \times 1} = [A]_{N \times N}^{-1} \times [V]_{N \times 1} \quad (29)$$

$$[A]_{N \times N} = \begin{cases} a_{ii} = \frac{1}{2\pi\epsilon_0} \ln \left(\frac{D_i}{r} \right) \\ a_{ij} = \frac{1}{2\pi\epsilon_0} \ln \left(\frac{D'_{ij}}{D_{ij}} \right) \end{cases} \quad (30)$$

where V is a vector of conductor's voltage, r is the conductor radius, D_i , D_{ij} , and D'_{ij} are respectively the distances between the conductor i and its image, the two conductors i and j , and the conductor i and the image j . In the Fig. 2, all these distances are presented in the case of $i = 6$ and $j = 5$.

In the figure below, we will present curves of the electric field propagation in the vicinity of the 400 kV transmission line supported by an S-type pylon for the altitudes: 1 m, 20 m, and 70 m (Fig. 3).

5.1.2 Magnetic field

It is generated by the moving charges in a conductor and it is related to the current and the line configuration. The magnetic flux density produced by transmission lines can be calculated as:

$$\vec{B}_M(x, y) = \sum_{i=1}^N \frac{\mu_0 I_i}{2\pi} \left(\frac{\vec{i}_i \wedge \vec{R}_i}{R_i^2} - \frac{\vec{i}_i' \wedge \vec{R}_i'}{R_i'^2} \right) \quad (31)$$

$$\vec{B}_M(x, y) = \begin{cases} B_{Mx}(x, y) = \sum_{i=1}^N \frac{\mu_0 I_i}{2\pi} \left(\frac{y - y_i}{R_i^2} - \frac{y + y_i + \delta(1-j)}{R_i'^2} \right) \\ B_{My}(x, y) = \sum_{i=1}^N \frac{\mu_0 I_i (x - x_i)}{2\pi} \left(\frac{1}{R_i^2} - \frac{1}{R_i'^2} \right) \end{cases} \quad (32)$$

where: N is the number of the conductors, μ_0 is the magnetic permeability of the whole space, it is taken to be the free space value and it is equal to $4\pi \times 10^{-7}$ H/m, I_i is the intensity of the conductor's current, δ is the earth skin depth and it is equal to $\sqrt{1/(\mu_0 \pi f \sigma)}$, σ is the conductivity

Fig. 3 Propagation of the electric field in the vicinity of the 400 kV transmission line supported by an S-type pylon at altitudes: $y = 1$ m, 20 m, and 70 m

Fig. 4 Propagation of the magnetic field in the vicinity of the 400 kV transmission line supported by an S-type pylon at altitudes: $y = 1$ m, 20 m, and 70 m

of earth, and f is the frequency of the source current, and (x_i, y_i) are the coordinates of the conductor i .

In the figure below, we will present curves of the magnetic field propagation in the vicinity of the 400 kV transmission line supported by an S-type pylon for the altitudes: 1 m, 20 m, and 70 m (Fig. 4).

5.2 Electromagnetic interference with the UAV

The UAV is an independent aerial vehicle, it fully relies on complex electronics systems and it is supposed to be connected with the ground station to control it, to survey its sensor payload and for the real-time transmission data. So, due to the impacts of the electromagnetic waves propagation on electronic circuits, signals, and materials, the UAV's exposition to the electromagnetic field will jeopardize its functionality.

Unfortunately, most new UAVs manufactured are based on non-metallic materials, like carbon-fiber composites that is pervasively used to meet strong, light-weight structural requirements. But UAVs will be especially susceptible to electromagnetic interference due to the poorer screening and conductive capabilities of carbon-fiber composites compared to conventional metal materials [21].

In the other hand, various parts of the UAV control system are connected by cables that are good receivers of

electromagnetic interference, which can lead to a partial or total violation of the information signal, malfunction of on-board equipment, loss of control and even damage to the vehicle because, according to Maxwell's equations, when a time-varying electromagnetic field incident on a conductor will produce time-varying currents on the conductor, which can present a "noise" on electronic circuits, which can compromise the operation of the circuit [23].

5.3 The results of electromagnetic interference's physical models

In the absence of a transmission lines inspection UAV's mathematical model while taking care of all the constraints which it undergoes, there are many research studies that focus on electromagnetic simulations or laboratory tests to analyze the electromagnetic interference resistance of its electronic circuits and communication signals.

For example, in [24], a small quadrotor has been tested by performing electromagnetic compatibility tests, by reproducing the flight conditions near power lines. The experiment was carried out inside a laboratory where a voltage up to 200 kV could be generated. To describe the physical behavior, a list of parameters was examined, concerning the telemetry data signal, the Global Positioning System (GPS) signal, and the measurements of power

supply, Inertial Measurement Unit (IMU), and attitude. The results were proved that the multirotor can operate, correctly, close to these voltages power lines, except the number of satellites detected by the GPS antenna that was sometimes limited.

In the paper [25], the reliability of a carbon fiber UAV platform through radio frequency immunity tests carried out under a high electromagnetic field was evaluated in an anechoic chamber. The results show that the UAV can be controlled even in the presence of strong electromagnetic fields and that its telemetry subsystem reliably operates under such conditions. Even though there are potential problems related to GPS signal's loss during testing in the 1.4–2.7 GHz frequency range. So, the autonomous GPS based functions are not secure when operating near radio frequency sources of this band. The article [22] it was proposed a physical model of electromagnetic interference in communication links of a UAV to have a basis of a practical technique to study the interference resistance to transmission lines electromagnetic field impact of the onboard electronic systems of a UAV.

In [26, 27] it was presented the methodological foundations of a practical technique as well as the physical model of electromagnetic interference in the UAV communication lines under the influence of electric transport contact network switching magnetic fields to study the noise immunity of electronic systems of UAVs. And in [28] is a summary of the analysis of the influence of different types of electromagnetic pulses on the quality of an image transmitted via the UAV downlink, which will provide a theoretical basis for classification of electromagnetic pulses, methods of anti-interference suppression, and design optimization of UAV in the future.

And as future works, the paper [29] present the design of a low frequency magneto-static simulation based on 3D simulations using finite elements in order to develop a magnetic field acquisition system, which could provide information and thus detect the magnetic field strength and automatically locate the UAV at a safe distance from transmission lines.

So, the physical models result differ according to the chosen test conditions, such as the type of the UAV, its characteristics, the voltage of the line, etc. And now, even companies rely mainly on tests to determine the recommended use requirements to the customer of their transmission line inspection UAVs.

5.4 UAV's minimum approach distance

The UAV's minimum approach distance (MAD) in vicinity of transmission lines is the shortest distance in air that must be between an energized conductor and the closest point of UAV. So, it defines the UAV's Safe-Fly Zones where it can

fly without causing breakdown with the transmission lines or vice versa.

Several experiments are made to compare empirical models and standards as in [30] where it is judged that the Minimum Helicopter Approach Distance of IEEE Standard for the MAD are very similar to the empirical model of Rizk's predictions for the UAV model. So according to this Standard [31], the UAV's Minimum Approach Distance is expressed as follows:

$$D = (((C + a)VTA) + M)H \quad (33)$$

With: C is a rod gap withstand, a is the adjustment ratio to compensate for air saturation expressed as a ratio of distance to kV, V is the nominal voltage across the air gap, T is the maximum anticipated per-unit Transient Over-Voltage, which can occur across the air gap, A is the altitude correction factor, M is the inadvertent movement factor, and H is the UAV factor.

All these factors are detailed in [31], And it is the user's responsibility to incorporate safety factors based on application, weather conditions, and the ability of the UAV to adhere to flight paths.

The realized bibliographic study allowed us to define the electric and magnetic fields in the vicinity of the transmission lines, to understand the electromagnetic interference effects on the UAV's functioning, to note the results of the physical tests already elaborated, and to discover the UAV's minimum approach distance that is concluded from standards, empirical models and physical tests.

6 Results and discussion

From the state of the art established of electromagnetic interference with the UAV in the vicinity of high voltage transmission lines, we can deduce a general mathematical model of a quadrotor, that takes into account the specific conditions of our type of operation that requires the permanent presence near the electromagnetic field's sources, contrary to the mathematical models presented in most of the research works which describe only the general flight case of a UAV.

So, the mathematical model presented below takes into consideration all the external effects: forces, torques, and the electromagnetic field's interference by the addition of the minimum approach distance's equation of a quadrotor to the mechanical model, expressed in the inertial frame.

$$\begin{cases}
 \ddot{x} = -\frac{K_{fmx}}{m}\dot{x} + \frac{2}{m}(q_1q_3 - q_0q_2) \sum_{i=1}^n b\omega_i^2 \\
 \ddot{y} = -\frac{K_{fmy}}{m}\dot{y} + \frac{2}{m}(q_0q_1 - q_2q_3) \sum_{i=1}^n b\omega_i^2 \\
 \ddot{z} = -\frac{K_{fz}}{m}\dot{z} + \frac{1}{m}(q_0^2 + q_3^2 - q_1^2 - q_2^2) \sum_{i=1}^n b\omega_i^2 - g \\
 \ddot{\phi} = \frac{1}{I_x}(bl(\omega_1^2 - \omega_3^2) - K_{fmx}\dot{\phi}^2 - \Omega_r J_r \dot{\theta} - \dot{\theta}\psi(l_z - l_y)) \\
 \ddot{\theta} = \frac{1}{I_y}(bl(\omega_4^2 - \omega_2^2) - K_{fay}\dot{\theta}^2 + \Omega_r J_r \dot{\phi} - \psi\dot{\phi}(l_x - l_z)) \\
 \ddot{\psi} = \frac{1}{I_z}(d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) - K_{faz}\dot{\psi}^2 - \dot{\phi}\dot{\theta}(l_y - l_x)) \\
 D = (((C + a)VTA) + M)H
 \end{cases} \quad (34)$$

Most of the research works, which present the VTOL UAV mathematical model, are not interested in its mission, they establish a mathematical model which represents a normal case where the drone is subjected only to the mechanical effects, as in [32–34], but our mission requires the quadrotor presence close to the transmission lines so it will undergo the electromagnetic field generated.

Several works have mentioned these electromagnetic effects or even they have established physical tests, [22, 24, 25, 28, 35], but they have not proposed a general mathematical model of a transmission lines inspection UAV and thus for a quadrotor.

So, our work's results are the proposition of a mechanical and electromagnetic mathematical model that takes into consideration all external efforts related to this specific mission of a quadrotor, in addition to the mechanical effects those electromagnetic that are caused by the electromagnetic field generated by the high voltage transmission lines.

7 Conclusion

Our work's goal is to achieve a quadrotor intended primarily for transmission lines inspection. This solution will be proposed to the National Office of Electricity and Drinking Water (ONEE), Electricity Branch, because it will allow it to overcome the various problems generated by traditional methods and to win on several levels: cost, time, reliability and security. The inspection of the transmission lines will put the quadrotor under, in addition to the mechanical forces and torques, the interference of the electromagnetic field generated by the lines. So, to establish a general mathematical model that takes into account mechanical and electromagnetic effects, we firstly developed a mathematical mechanical model and then we presented a summary of the established bibliographic study concerning UAV's electromagnetic interference where we calculated the electric and magnetic

fields in vicinity of the transmission lines, detailed their effects on UAV, presented the results of the physical tests that are already performed, and defined the minimum approach distance of the UAV in the vicinity of the transmission lines. So, in this paper, we got the general mathematical model of an inspection transmission lines quadrotor that takes into consideration the external effects related to this type of operation, to be simulated and tested in future works, in order to release a specific inspection transmission lines quadrotor.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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