Static output-feedback H_∞ control for T–S fuzzy vehicle lateral dynamics

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Received: 20 October 2019 / Accepted: 11 December 2019 / Published online: 17 December 2019
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Abstract

This paper deals with static output-feedback H_∞ control for a system of the vehicle lateral dynamics, represented by Takagi–Sugeno (T–S) fuzzy models. Sufficient conditions of the existence of H_∞ control based on the static output-feedback are presented. The bilinear matrix inequalities are converted to a set of linear matrix inequalities, with the aid of some special derivations. Simulation results demonstrate the effectiveness of the proposed method.

Keywords Static output control · H_∞ Control · Vehicle lateral dynamics · T–S fuzzy systems · Bicycle model

1 Introduction

In the field of road safety, the most important is to control the vehicle in critical driving situations. With the new control laws and the existence of artificial intelligence in today's vehicles, vehicles are becoming easier to control. Nevertheless, their efficiency, their stability limits and their technical possibilities will be more and more developed. Changing the vehicle's path in difficult driving situations is a major challenge for drivers. For this, several works are devoted to the control of vehicles. To this end, several driver assistance systems have emerged in recent years, such as the anti-lock braking system (ABS), the traction control system (TCS) and the electronic stability control (ESC) [1–3]. They depend on the service provided by the electronic systems for various activities of the vehicle in hazardous driving situations. In the event of a deviation in an undesirable direction, the computer makes immediate decisions to react with corrective actions to ensure the stability of the vehicle. The influence of the vehicle speed and the variation of road adhesion is presented in the works [4, 5]. The control based on the estimated state feedback is

widely studied in recent years as in [7, 8]. The authors have proposed an approach to stabilize the system of vehicle dynamics in the presence of disturbances. In [6, 8, 9], the authors discuss a control method that tolerates sensor and actuator faults for vehicle dynamics. The H_∞ Control For Vehicle Active Suspension Systems In Finite Frequency Domain is studied in [11]. The authors propose a technique to estimate the vehicle states and the crosswind in [10, 12]. In order to improve the stability of the vehicle as well as its safety despite the presence of parametric/modeling disturbances.

Often system states are not always fully accessible, designing the controller using other methods is very necessary. Notice that the most suitable control is based on static output feedback because, at a lower cost, it can be easily implemented. However, the issue of stabilization through the static output-feedback control of vehicle dynamics systems is rarely investigated, although in practice, it is important and useful. This motivates our work. In this study, a new H_∞ control law for the vehicle's lateral dynamics system is developed to ensure vehicle stability and avoid slippage in critical driving situations. The lateral dynamics of the vehicle

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in the presence of disturbances is expressed by the fuzzy model of Takagi–Sugeno, which is much studied to solve the control question for complex non-linear systems [13, 14]. The proposed design makes it possible to design a static output-feedback based controller, avoiding the problem of bi-linearity and without any equality constraint, such as the technique used in [15]. This planning has a superiority compared to some techniques in the bibliography like [19, 21]. Using the Lyapunov's quadratic function, sufficient asymptotic stability conditions, that do not require transformation matrices or equality constraints are given as linear matrix inequalities (LMIs). Thus, it is easy to recover the gains of the controller by using the LMI-Toolbox solver [16].

This paper is organized as follows. The second section deals with the T–S fuzzy modeling of the vehicle lateral dynamics; while the third section presents the main results. The fourth section is devoted to the numerical illustration. Finally, a conclusion is given in the last section.

2 T–S fuzzy vehicle model and problem formulation

Vehicle modeling has been extensively studied in the literature. To simply analyze, we consider a two-degree-of-freedom (2-DOF) model called “bicycle”, whose suspensions and wheel inertia are neglected [17]. Here, we used this model for capturing the lateral dynamics of the vehicle without loss of generality (see Fig. 1) [18]. The following differential equations describe the dynamics presented by this model:

$$\begin{cases} \dot{v}_y = \frac{1}{m_v}(2F_{yf} + 2F_{yr}) - v_x\dot{\psi} \\ \dot{\psi} = \frac{1}{I_z}(2l_f F_{yf} - 2l_r F_{yr} + M_z) \end{cases} \quad (1)$$

where v_x and v_y are longitudinal and lateral velocities, respectively. $\dot{\psi}$ is the yaw rate, m_v is the vehicle mass, M_z is external vehicle yaw moment. l_f and l_r are distances from the center of gravity to front and rear axles, respectively.

Fig. 1 Vehicle body motions and vehicle lateral dynamics model

The slip angles are proportional to lateral forces when they are very small. Then, The linear model works perfectly. However, a nonlinear model must be considered in the case of an increasing slip. To solve this problem, Recent studies such as [12, 20], suggest that a model with two T–S fuzzy rules represent the nonlinear behavior of lateral forces. We introduce in this analysis, an approach to describing the behavior of these forces via a T–S fuzzy four-ruler model. For this, we have modeled the front and rear lateral forces by the following rules:

if $|\alpha_f(t)|$ is M_1 then $F_{yf} = C_{f1}\alpha_f(t)$

if $|\alpha_f(t)|$ is M_2 then $F_{yf} = C_{f2}\alpha_f(t)$

if $|\alpha_r(t)|$ is N_1 then $F_{yr} = C_{r1}\alpha_r(t)$

if $|\alpha_r(t)|$ is N_2 then $F_{yr} = C_{r2}\alpha_r(t)$

Where $M_1(M_2)$ and $N_1(N_2)$ are fuzzy sets, respectively, for small (large) front and rear slip angles, whose fuzzy meaning is given by $\omega_1(\omega_2)$ and $\omega_3(\omega_4)$, respectively. C_{fi} , C_{ri} are the front and rear tire cornering stiffness which depend of road adhesion and vehicle mass m_v , $\alpha_f(t)$ and $\alpha_r(t)$ are slip angles of front and rear tires, respectively. Which are given in [5] as follows:

$$\begin{cases} \alpha_f(t) = \delta_f(t) - \beta(t) - \frac{l_f\dot{\psi}(t)}{v_x} \\ \alpha_r(t) = -\beta(t) + \frac{l_r\dot{\psi}(t)}{v_x} \end{cases} \quad (2)$$

where $\delta_f(t)$ is the steering angle given by the driver and $\beta(t)$ is the sideslip angle which characterizes the ratio of the lateral velocity to the longitudinal velocity.

$$\beta(t) = \frac{v_y(t)}{v_x} \quad (3)$$

To have the fuzzy model T–S, the lateral forces will be modeled by the following fuzzy rules :

$$\begin{cases} F_{yf}(t) = \sum_{i=1}^2 \lambda_{fi}(\theta_f(t))C_{fi}\alpha_f(t) \\ F_{yr}(t) = \sum_{i=1}^2 \lambda_{ri}(\theta_r(t))C_{ri}\alpha_r(t) \end{cases} \quad (4)$$

Membership functions $\lambda_{fi}(\theta_f(t))$ and $\lambda_{ri}(\theta_r(t))$ are given as follows:

$$\lambda_{fi}(\theta_f(t)) = \frac{\omega_{fi}(\theta_f(t))}{\sum_{i=1}^2 \omega_{fi}(\theta_f(t))}; \quad \lambda_{ri}(\theta_r(t)) = \frac{\omega_{ri}(\theta_r(t))}{\sum_{i=1}^2 \omega_{ri}(\theta_r(t))} \quad (5)$$

with:

$$\begin{cases} \omega_{f_i}(\theta_f(t)) = \frac{1}{\left(1 + \left|\frac{\theta_f(t) - c_{1i}}{a_{1i}}\right|\right)^{2b_{1i}}} \\ \omega_{r_i}(\theta_r(t)) = \frac{1}{\left(1 + \left|\frac{\theta_r(t) - c_{2i}}{a_{2i}}\right|\right)^{2b_{2i}}} \end{cases} \quad (6)$$

where $\theta_f(t) = |\alpha_f(t)|$ and $\theta_r(t) = |\alpha_r(t)|$. To get the T-S model membership parameters and the coefficient of tire stiffness, we use the Levenberg–Marquardt algorithm-based classification approach together with the least square technique [7].

The global membership functions $\mu_i(\theta_f(t), \theta_r(t))$ are set as follows

$$\mu_i(\theta_f(t), \theta_r(t)) = \lambda_{fk}(|\alpha_f(t)|) \cdot \lambda_{rl}(|\alpha_r(t)|), \quad k = 1, 2; l = 1, 2. \quad (7)$$

which satisfy the following conditions

$$\begin{cases} \sum_{i=1}^4 \mu_i(\theta_f(t), \theta_r(t)) = 1 \\ 0 \leq \mu_i(\theta_f(t), \theta_r(t)) \leq 1, \quad i = 1, \dots, 4 \end{cases} \quad (8)$$

and the fuzzy rules of the system are defined as follows,

if $|\alpha_f(t)|$ is M_1 and $|\alpha_r(t)|$ is N_1 then

$$\begin{cases} \dot{x}(t) = A_1 x(t) + B_{21} u(t) + B_{11} \omega(t) \\ y(t) = C_{21} x(t) + D_{21} \omega(t) \\ z(t) = C_{11} x(t) + D_{11} u(t) \end{cases}$$

if $|\alpha_f(t)|$ is M_1 and $|\alpha_r(t)|$ is N_2 then

$$\begin{cases} \dot{x}(t) = A_2 x(t) + B_{22} u(t) + B_{12} \omega(t) \\ y(t) = C_{22} x(t) + D_{22} \omega(t) \\ z(t) = C_{12} x(t) + D_{12} u(t) \end{cases}$$

if $|\alpha_f(t)|$ is M_2 and $|\alpha_r(t)|$ is N_1 then

$$\begin{cases} \dot{x}(t) = A_3 x(t) + B_{23} u(t) + B_{13} \omega(t) \\ y(t) = C_{23} x(t) + D_{23} \omega(t) \\ z(t) = C_{13} x(t) + D_{13} u(t) \end{cases}$$

if $|\alpha_f(t)|$ is M_2 and $|\alpha_r(t)|$ is N_2 then

$$\begin{cases} \dot{x}(t) = A_4 x(t) + B_{24} u(t) + B_{14} \omega(t) \\ y(t) = C_{24} x(t) + D_{24} \omega(t) \\ z(t) = C_{14} x(t) + D_{14} u(t) \end{cases}$$

That can be translated as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^4 \mu_i(\theta_f(t), \theta_r(t)) [A_i x(t) + B_{2i} u(t) + B_{1i} \omega(t)] \\ y(t) = \sum_{i=1}^4 \mu_i(\theta_f(t), \theta_r(t)) [C_{2i} x(t) + D_{2i} \omega(t)] \\ z(t) = \sum_{i=1}^4 \mu_i(\theta_f(t), \theta_r(t)) [C_{1i} x(t) + D_{1i} u(t)] \end{cases} \quad (9)$$

where $x(t) = [\beta(t) \ \dot{\psi}(t)]^T$ is the system state, $z(t) = \dot{\psi}(t)$ is the controlled output, $y(t) = [a_y(t) \ \dot{\psi}(t)]^T$ is the measured output, where $a_y(t)$ is lateral acceleration. $\omega(t)$ is the disturbances. We consider the signal input $u(t)$ is an assistant steering angle $\delta_c(t)$ added to the driver's steering angle $\delta_f(t)$. $A_i, B_{1i}, B_{2i}, D_{1i}, D_{2i}, C_{1i}$ and C_{2i} are constant matrices with compatible dimensions.

To get the matrices A_i and B_{2i} , we substitute the front and rear sliding expressions (2) into (4), we will have

$$\begin{cases} F_{yf}(t) = \sum_{i=1}^2 \lambda_{\bar{n}}(\theta_f(t)) C_{\bar{n}} \left(\delta_f(t) - \beta(t) - \frac{l_f \dot{\psi}(t)}{v_x} \right) \\ F_{yr}(t) = \sum_{i=1}^2 \lambda_{\bar{n}}(\theta_r(t)) C_{\bar{n}} \left(-\beta(t) + \frac{l_r \dot{\psi}(t)}{v_x} \right) \end{cases} \quad (10)$$

we replace again the lateral forces expression (10) in (1), taking into consideration (3), we get

$$\begin{cases} \dot{x}(t) = \sum_{k=1}^2 \sum_{l=1}^2 \lambda_{f_k}(\theta_f(t)) \lambda_{r_l}(\theta_r(t)) A_{kl} x(t) + B_{2kl} u(t) \\ y(t) = \sum_{k=1}^2 \sum_{l=1}^2 \lambda_{f_k}(\theta_f(t)) \lambda_{r_l}(\theta_r(t)) C_{2kl} x(t) \end{cases} \quad (11)$$

with:

$$\begin{aligned} A_{kl} &= \begin{bmatrix} -2 \frac{C_{f_k} + C_{r_l}}{m_v v_x} & -2 \frac{l_f C_{f_k} - l_r C_{r_l}}{m_v v_x^2} - 1 \\ -2 \frac{l_f C_{f_k} + l_r C_{r_l}}{I_z} & -2 \frac{l_f^2 C_{f_k} - l_r^2 C_{r_l}}{I_z v_x} \end{bmatrix}; \\ B_{2kl} &= \begin{bmatrix} 2 \frac{C_{f_k}}{m_v v_x} \\ 2 \frac{l_f C_{f_k}}{I_z} \end{bmatrix} \\ C_{2kl} &= \begin{bmatrix} -2 \frac{C_{f_k} + C_{r_l}}{m_v} & -2 \frac{l_f C_{f_k} - l_r C_{r_l}}{m_v v_x} \\ 0 & 1 \end{bmatrix} \end{aligned}$$

By using (7), we obtain matrices in accordance with the letters of the indices.

The expression of the control law based on the static output-feedback for the system of the vehicle lateral dynamics represented by the Takagi–Sugeno fuzzy models can be described as follows

$$\delta_c(t) = \sum_{i=1}^4 \mu_i(\theta_f(t), \theta_r(t)) F_i y(t) \quad (12)$$

By simply replacing (12) into (9), the vehicle dynamics fuzzy system can be defined as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^4 \sum_{j=1}^4 \sum_{l=1}^4 \mu_i \mu_j \mu_l [\bar{A}_{ijl} x(t) + \bar{B}_{ijl} \omega(t)] \\ z(t) = \sum_{i=1}^4 \sum_{j=1}^4 \sum_{l=1}^4 \mu_i \mu_j \mu_l [\bar{C}_{ijl} x(t) + \bar{D}_{ijl} \omega(t)] \end{cases} \quad (13)$$

where

$$\begin{aligned} \bar{A}_{ijl} &= A_i + B_{2i} F_j C_{2l}, & \bar{B}_{ijl} &= B_{1i} + B_{2i} F_j D_{2l}, \\ \bar{C}_{ijl} &= C_{1i} + D_{1i} F_j C_{2l}, & \bar{D}_{ijl} &= D_{1i} F_j D_{2l} \\ \mu_i &= \mu_i(\theta_f(t), \theta_r(t)), & \mu_j &= \mu_j(\theta_f(t), \theta_r(t)), \\ \mu_k &= \mu_k(\theta_f(t), \theta_r(t)) \end{aligned} \quad (14)$$

To simplify the calculation, we define the following matrices:

$$\begin{aligned} \bar{A} &= \sum_{i=1}^4 \sum_{j=1}^4 \sum_{l=1}^4 \mu_i \mu_j \mu_l \bar{A}_{ijl}, \\ \bar{B} &= \sum_{i=1}^4 \sum_{j=1}^4 \sum_{l=1}^4 \mu_i \mu_j \mu_l \bar{B}_{ijl}, \\ \bar{C} &= \sum_{i=1}^4 \sum_{j=1}^4 \sum_{l=1}^4 \mu_i \mu_j \mu_l \bar{C}_{ijl}, \\ \bar{D} &= \sum_{i=1}^4 \sum_{j=1}^4 \sum_{l=1}^4 \mu_i \mu_j \mu_l \bar{D}_{ijl}. \end{aligned} \quad (15)$$

To demonstrate our results, the following lemmas are required.

Lemma 1 [23] Given a symmetric matrix $\Sigma \in \mathbb{R}^{p \times p}$ and two matrices X, Z of column dimension p , there exists a matrix Y such that the LMI

$$\Sigma + \text{sym}\{X^T Y Z\} < 0 \quad (16)$$

holds if and only if the following two projection inequalities with respect to Y are satisfied:

$$X^\perp{}^T \Sigma X^\perp < 0, \quad Z^\perp{}^T \Sigma Z^\perp < 0. \quad (17)$$

where $X^\perp$ and $Z^\perp$ are arbitrary matrices whose columns form a basis of the null spaces of X and Z , respectively.

Lemma 2 [22] For a positive definite matrix $R \in \mathbb{R}^{n \times n}$, matrices X and Y with appropriate dimensions, the following inequality holds:

$$X^T Y + Y^T X \leq X^T R X + Y^T R^{-1} Y$$

Lemma 3 [22] For a positive definite matrix $R \in \mathbb{R}^{n \times n}$, a square matrix $X \in \mathbb{R}^{n \times n}$, and a scalar α , the following inequality holds:

$$-X^T R^{-1} X \leq \alpha^2 R \pm \alpha X \pm \alpha X^T$$

3 Main results

In this section, we will produce an H_∞ control based on static output-feedback for the T-S systems of the vehicle lateral dynamics.

Theorem 1 Consider a scalar $\gamma > 0$, the closed-loop fuzzy system (13) is asymptotically stable and satisfies the H_∞ performances, if there exist symmetric matrices $P > 0$, and matrices S and G , such that the following LMI is satisfied.

$$\Lambda = \begin{bmatrix} -S - S^T & \Lambda_{12} & S\bar{B} & 0 \\ * & \Lambda_{22} & G\bar{B} & \bar{C}^T \\ * & * & -\gamma^2 I & \bar{D}^T \\ * & * & * & -I \end{bmatrix} < 0 \quad (18)$$

where,

$$\Lambda_{12} = P + S\bar{A} - G^T, \quad \Lambda_{22} = G\bar{A} + \bar{A}^T G^T,$$

Proof Let

$$\Sigma = \begin{bmatrix} 0 & P & 0 \\ P & \bar{C}^T \bar{C} & \bar{C}^T \bar{D} \\ 0 & \bar{D}^T \bar{C} & -\gamma^2 I + \bar{D}^T \bar{D} \end{bmatrix},$$

By using the Schur complement, (18) is equivalent to

$$\Sigma + \text{sym}(X^T Y Z) < 0 \quad (19)$$

with

$$X = I, \quad Y = \begin{bmatrix} S^T & G^T & 0 \end{bmatrix}^T, \quad Z = \begin{bmatrix} -I & \bar{A} & \bar{B} \end{bmatrix}$$

$$\text{Selecting } Z^\perp = \begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \\ 0 & I \end{bmatrix} \text{ with application of the Lemma 1,}$$

and through some manipulations, we obtain:

$$\begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \end{bmatrix}^T \begin{bmatrix} 0 & P \\ P & 0 \end{bmatrix} \begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \end{bmatrix} + \begin{bmatrix} \bar{C}^T \bar{C} & \bar{C}^T \bar{D} \\ \bar{D}^T \bar{C} & \bar{D}^T \bar{D} - \gamma^2 I \end{bmatrix} < 0 \quad (20)$$

Inequality (20), can be denoted similarly by

$$\begin{bmatrix} P\bar{A} + \bar{A}^T P & P\bar{B} \\ \bar{B}^T P & 0 \end{bmatrix} + \begin{bmatrix} \bar{C}^T \bar{C} & \bar{C}^T \bar{D} \\ \bar{D}^T \bar{C} & \bar{D}^T \bar{D} - \gamma^2 I \end{bmatrix} < 0 \quad (21)$$

Consider the following Lyapunov function

$$V(t) = x^T(t)Px(t) \quad (22)$$

Calculating the derivative of $V(t)$ results in

$$\dot{V}(t) = \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \quad (23)$$

if (21) holds, it can be easily checked that

$$\dot{V}(t) + z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t) < 0 \quad (24)$$

Integration of inequality (24) on both sides from 0 to ∞ gives

$$\begin{aligned} \int_0^\infty \dot{V}(t)dt + \int_0^\infty (z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t))dt \\ = V(x(\infty)) - V(x(0)) \\ + \int_0^\infty (z^T(t)z(t) - \gamma^2 \omega^T(t)\omega(t))dt < 0 \end{aligned} \quad (25)$$

We received zero initial condition

$$\int_0^\infty z^T(t)z(t)dt < \gamma^2 \int_0^\infty \omega^T(t)\omega(t)dt \quad (26)$$

The proof is completed. $\square$

In the following theorem, we will propose new LMI conditions to guarantee the asymptotic stability of the closed-loop T-S fuzzy system (13) with the design of the H_∞ control based on the static output-feedback.

Theorem 2 Consider a scalar $\gamma > 0$, the closed-loop fuzzy system (13) is asymptotically stable and satisfies the H_∞ performances, if for known scalar parameter α there exist symmetric positif matrices P and R and other matrices S, G, H, M , and N_j , such that the following LMIs hold.

$$\begin{bmatrix} \Xi_{iii} & \theta_{iii} \\ * & \Gamma \end{bmatrix} < 0, \quad i = 1, \dots, 4 \quad (27)$$

$$\begin{bmatrix} \Xi_{ijj} + \Xi_{jji} + \Xi_{jji} & \theta_{ijj} & \theta_{jji} & \theta_{jji} \\ * & \Gamma & 0 & 0 \\ * & * & \Gamma & 0 \\ * & * & * & \Gamma \end{bmatrix} < 0, \quad i, j = 1, 2, \dots, 4, \quad i \neq j \quad (28)$$

$$\begin{bmatrix} \Xi_{ijl} + \Xi_{ilj} \\ + \Xi_{jil} + \Xi_{jli} \\ + \Xi_{ijl} + \Xi_{jji} & \theta_{ijl} & \theta_{ilj} & \theta_{jil} & \theta_{jli} & \theta_{lji} \\ * & \Gamma & 0 & 0 & 0 & 0 \\ * & * & \Gamma & 0 & 0 & 0 \\ * & * & * & \Gamma & 0 & 0 \\ * & * & * & * & \Gamma & 0 \\ * & * & * & * & * & \Gamma \end{bmatrix} < 0, \quad (29)$$

$i = 1, \dots, 2, \quad j = i + 1, \dots, 3, \quad l = j + 1, \dots, 4.$

where

$$\begin{aligned} \Xi_{ijl} &= \begin{bmatrix} \Xi_{11} & \Xi_{12ijl} & SB_{1i} + B_{2i}N_jD_{2l} & 0 \\ * & \Xi_{22ijl} & GB_{1i} + B_{2i}N_jD_{2l} & C_{1i}^T H^T + C_{2i}^T N_j^T D_{1i}^T \\ * & * & -\gamma^2 I & D_{2i}^T N_j^T D_{1i}^T \\ * & * & * & I - H - H^T \end{bmatrix} \\ \Xi_{11} &= -S - S^T \\ \Xi_{12ijl} &= P + SA_i - G^T + B_{2i}N_jC_{2l} \\ \Xi_{22ijl} &= \text{sym}\{GA_i + B_{2i}N_jC_{2l}\} \\ \theta_{ijl} &= \begin{bmatrix} 0 & SB_{2i} - B_{2i}M \\ C_{2i}^T N_j^T & GB_{2i} - B_{2i}M \\ D_{2i}^T N_j^T & 0 \\ 0 & HD_{1i} - D_{1i}M \end{bmatrix}, \\ \Gamma &= \begin{bmatrix} -R & 0 \\ 0 & \alpha^2 R - \alpha M - \alpha M^T \end{bmatrix}, \end{aligned} \quad (30)$$

Furthermore, the static output-feedback controller gain matrices are given by $F_j = M^{-1}N_j, j = 1, 2, \dots, 4$.

Proof Assume that the inequality (27) holds, that means that $I - H - H^T < 0$ and $\alpha^2 R - \alpha M - \alpha M^T < 0$ (with $\alpha^2 R > 0$), which guarantees the non singularity of the matrices M and H .

From (27)-(29) we have

$$\sum_{i=1}^4 \sum_{j=1}^4 \sum_{l=1}^4 \mu_i \mu_j \mu_l \Psi_{ijl} < 0 \quad (31)$$

where

$$\Psi_{ijl} = \begin{bmatrix} \Xi_{ijl} & \theta_{ijl} \\ * & \Gamma \end{bmatrix} \quad (32)$$

From the lemma 3 and the exploitation of the Schur complement, the inequality (31) ensures that

$$\begin{aligned} \bar{\Psi}_{ijl} = & \Xi_{ijl} + \begin{bmatrix} 0 \\ C_{2l}^T N_j^T \\ D_{2l}^T N_j^T \\ 0 \end{bmatrix} R^{-1} \begin{bmatrix} 0 \\ C_{2l}^T N_j^T \\ D_{2l}^T N_j^T \\ 0 \end{bmatrix}^T \\ & + \begin{bmatrix} SB_{2i} - B_{2i}M \\ GB_{2i} - B_{2i}M \\ 0 \\ HD_{1i} - D_{1i}M \end{bmatrix} \\ & (M^T R^{-1} M)^{-1} \begin{bmatrix} SB_{2i} - B_{2i}M \\ GB_{2i} - B_{2i}M \\ 0 \\ HD_{1i} - D_{1i}M \end{bmatrix}^T < 0 \end{aligned} \quad (33)$$

By using lemma 2 and defining $N_j = MF_j$, we can check that (33) is equivalent to

$$\begin{aligned} \bar{\Psi}_{ijl} = & \Xi_{ijl} + \begin{bmatrix} SB_{2i} - B_{2i}M \\ GB_{2i} - B_{2i}M \\ 0 \\ HD_{1i} - D_{1i}M \end{bmatrix} \begin{bmatrix} 0 \\ C_{2l}^T F_j^T \\ D_{2l}^T F_j^T \\ 0 \end{bmatrix}^T \\ & + \begin{bmatrix} 0 \\ C_{2l}^T F_j^T \\ D_{2l}^T F_j^T \\ 0 \end{bmatrix} \begin{bmatrix} SB_{2i} - B_{2i}M \\ GB_{2i} - B_{2i}M \\ 0 \\ HD_{1i} - D_{1i}M \end{bmatrix}^T < 0 \end{aligned} \quad (34)$$

Replacing (30) with its expression in (34) gives the following inequality

$$\begin{aligned} \bar{\Psi}_{ijl} = & \begin{bmatrix} \bar{\Xi}_{11} & \bar{\Xi}_{12ijl} & \bar{\Xi}_{13ijl} & 0 \\ * & \bar{\Xi}_{22ijl} & \bar{\Xi}_{23ijl} & \bar{\Xi}_{24ijl} \\ * & * & -\gamma^2 I & \bar{\Xi}_{34ijl} \\ * & * & * & I - H - H^T \end{bmatrix} < 0, \\ \bar{\Xi}_{11} = & -S - S^T \\ \bar{\Xi}_{12ijl} = & P + SA_i - G^T + SB_{2i}F_jC_{2l} \\ \bar{\Xi}_{22ijl} = & \text{sym}\{GA_i + GB_{2i}F_jC_{2l}\} \\ \bar{\Xi}_{13ijl} = & SB_{1i} + SB_{2i}F_jD_{2l} \\ \bar{\Xi}_{23ijl} = & GB_{1i} + GB_{2i}F_jD_{2l} \\ \bar{\Xi}_{24ijl} = & C_{1i}^T H^T + C_{2l}^T F_j^T D_{1i}^T H^T \\ \bar{\Xi}_{34ijl} = & D_{2l}^T F_j^T D_{1i}^T H^T \end{aligned} \quad (35)$$

The simulations are carried out with the front steering angle profile given in Fig. 2. We considered car speed and

Multiplying (35) by $\text{diag}\{I, I, I, H^{-1}\}$ on the left and its transpose on the right, and replacing $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\bar{D}$, by their expressions in (15), we get the inequality in (18).

This completes the proof. $\square$

4 Numerical illustration

A test illustrating a critical driving situation is performed under MATLAB software, to show the efficiency of the static output-feedback based H_∞ control for vehicle lateral dynamics model (1) represented by T-S fuzzy system (9). The parameters of the vehicle are shown in Table 1.

By using Theorem 2, when $\alpha = 5$, we can obtain the lower bounded of H_∞ level $\gamma_{\min} = 0.0618$ and the following matrices:

$$\begin{aligned} M = & [0.0792], P = \begin{bmatrix} 1.9267 & -5.2949 \\ -5.2949 & 73.3009 \end{bmatrix}, \\ S = & \begin{bmatrix} 0.0482 & -0.6132 \\ -0.6126 & 7.8777 \end{bmatrix} \\ G = & \begin{bmatrix} 1.5016 & 0.1704 \\ 0.3965 & 0.1209 \end{bmatrix}, R = [8.9426 \times 10^{-04}], \\ H = & [1.0014] \\ N_1 = & \begin{bmatrix} 0.0530 \\ -0.0005 \end{bmatrix}^T, N_2 = \begin{bmatrix} 0.0601 \\ -0.0007 \end{bmatrix}^T, \\ N_3 = & \begin{bmatrix} 0.0507 \\ -0.0012 \end{bmatrix}^T, N_4 = \begin{bmatrix} 0.0589 \\ -0.0015 \end{bmatrix}^T \\ F_1 = & \begin{bmatrix} 0.6692 \\ -0.0068 \end{bmatrix}^T, F_2 = \begin{bmatrix} 0.7590 \\ -0.0084 \end{bmatrix}^T, \\ F_3 = & \begin{bmatrix} 0.6406 \\ -0.0155 \end{bmatrix}^T, F_4 = \begin{bmatrix} 0.7440 \\ -0.0185 \end{bmatrix}^T \end{aligned}$$

Table 1 Parameters for the vehicle simulation

Constants	Value	Unit
m_v	1832	kg
v_x	50	m s ⁻¹
I_z	2988	kg m ²
C_{f1}	55234	N rad ⁻¹
C_{f2}	15544	N rad ⁻¹
C_{r1}	49200	N rad ⁻¹
C_{r2}	13543	N rad ⁻¹
l_r	1.77	m
l_f	1.18	m

Fig. 2 Profile of the front wheel steering angle provided by the driver **Fig. 3** Time evolution of sideslip angle **Fig. 4** Time evolution of yaw rate

steering angle variations, to demonstrate the effectiveness of the proposed control law.

We applied fuzzy H_∞ control based on the static output-feedback (12) to the system (9), the Figs. 3 and 4 show the response of the system (9) for an initial condition $x(0) = [0.01 \ -0.01]^T$, as well as the Fig. 5 shows the performance of our algorithm to eliminate disturbances. We simply select $\omega(t) = 0.5 \sin(12t)$ in these simulations.

The understeer performance of the vehicle is presented for the steering maneuvers considered in Fig. 2. We can see in Figures 3 and 4, that the H_∞ controller based on the proposed static output-feedback, with disturbances rejection, helps to keep and improve stability and safety under difficult driving conditions. We also notice that the additional disturbances have been dismissed.

Fig. 5 The performance of disturbances attenuations with H_∞ control

From Fig. 5, we can note that the performance of the attenuation perturbation is satisfactory. However, the H_∞ requirement on

$$\int_0^\infty z^T(t)z(t)dt < \gamma^2 \int_0^\infty \omega^T(t)\omega(t)dt$$

is achieved.

5 Conclusion

In this manuscript, we have developed a method of H_∞ control based on the static output-feedback, for the vehicle lateral dynamics system represented by the Takagi–Sugeno fuzzy model. The stability of the whole system is studied with the Lyapunov's quadratic function; appropriate conditions for the existence of this control are analyzed and their conceptions are expressed in the form of linear matrix inequalities (LMI). This study answers to some problems related to vehicle dynamics control. The simulation results tested and validated the efficacy in a crucial driving situation of the proposed solution for controlling the vehicle dynamics. As perspectives, we aimed to work on the problems of uncertainties and delay, and make an application on the CarSim software.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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