Improved averaging techniques for solving multi-objective optimization (MOO) problems

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Abstract

This study evaluates the existing averaging techniques used for solving multi-objective optimization (MOO) problems. The problems are solved using mean, geometric and harmonic mean averaging techniques. The solutions obtained using the existing averaging techniques were not appropriate. Improved averaging techniques using mean, geometric and harmonic mean are proposed in this study. These techniques have been tested with the suitable examples and found superior to existing MOO averaging techniques.

Keywords Averaging technique · Improved averaging technique · Mean · Geometric mean · Harmonic mean

1 Introduction

Making an appropriate decision with conflicting objectives is not easy. Multi-objective optimization (MOO) techniques are helpful in improving the decision-making process in such a situation. Weighted sum techniques are more popular [1] in solving MOO problems. Several MOO averaging techniques [2–6] have been developed during the recent past. Most of these techniques have been tested with non conflicting objectives that were not appropriate. The MOO problem with multiple conflicting objectives is the prerequisite for the application of any MOO technique. Further the existing averaging techniques have been evaluated on the basis of the values of combined multi-objective function which was also not appropriate. The solutions should have been evaluated on the basis of the values of the original objective functions only. There are few more problems with these techniques [1, 7–9] in obtaining an acceptable solution of conflicting objectives. However the Sen's MOO technique [10] is efficient in generating appropriate solutions of the problems with conflicting objectives. This technique has been successfully applied for resource use

planning in agriculture [11–21] with conflicting objectives of maximization of farm income and employment with minimum use of fertilizer, irrigation, plant protection chemicals, etc. However, the Sen's MOO technique have been declared inefficient in several studies using averaging techniques. After analyzing the comments and limitations of the prevailing averaging techniques, the improved average techniques are proposed in this study. Section 2 deals with the structure of MOO averaging techniques. The algorithms of averaging techniques have been presented in the Sect. 3. The examples solved with these techniques are given in Sect. 4. Section 5 deals with the discussion of the results and finally the findings are concluded in the last Sect. 6.

2 MOO averaging techniques

2.1 Existing MOO averaging techniques

For achieving 'n' objective functions with maximization of 'r' objective functions and minimization of remaining

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'n-r' objective functions, the multi-objective optimization model is formulated as follows:

$$\text{Optimize } Z = [\text{Max. } Z_1, Z_2 \dots Z_r, \text{Min. } Z_{r+1}, \dots Z_n]$$

Subject to :

$$AX = b \text{ and } X \geq 0$$

2.2 Individual optimization

Individual optimization is a prerequisite for assessing the need for multi-objective optimization. First, all the 'n' objectives are optimized to obtain 'n' individual optimal solutions. There are 'n' optimal values of 'n' objective functions along with the 'n' set of values of decision variables X_i . Using each set of the values of decision variables, the suboptimal values of remaining n-1 objectives are estimated. The solution of individual optimization is arranged in an 'n x n' matrix as shown in Table 1.

The columns are the optimal solutions of 'n' objective functions. The rows show the optimal and suboptimal values of all the objective functions. If all the columns contain the same values as in all the rows, the solutions are unique and multi-objective optimization is not required. When all 'n' or lesser solutions are different, multi-objective optimization is needed.

2.3 Multi-objective function

The multi-objective function is constructed by scalarizing each objective function by the mean, geometric and harmonic mean of optimal values of objective functions under the existing MOO averaging techniques as follows:

$$\text{Maximize } Z = \frac{\sum_{j=1}^r Z_j}{|\theta_1|} - \frac{\sum_{j=r+1}^s Z_j}{|\theta_2|}$$

Subject to :

$$AX = b \text{ and } X \geq 0$$

$$\theta_j \neq 0 \text{ for } j = 1, 2 \dots s.$$

where $|\theta_1|$, Mean, Geometric Mean and Harmonic Mean of optimal values of maximization objective functions. $|\theta_2|$ = Mean, Geometric Mean and Harmonic Mean of optimal values of Minimization Objective functions.

Note: The estimation of θ_1 and θ_2 is not logical when objective functions are of different dimensions.

Furthermore, the multi-objective function is formulated for the improved averaging techniques as detailed below:

$$\text{Maximize } Z = \frac{\sum_{j=1}^r Z_j}{|\theta_i|} - \frac{\sum_{j=r+1}^s Z_j}{|\theta_j|}$$

Subject to :

$$AX = b \text{ and } X \geq 0$$

$$\theta_j \neq 0 \text{ for } j = 1, 2 \dots s.$$

where $|\theta_i|$, Mean, Geometric Mean and Harmonic Mean of optimal and suboptimal values of ith Maximization Objective function. $|\theta_j|$, mean, Geometric Mean and Harmonic Mean of optimal and suboptimal values of jth Minimization Objective function.

3 Solving MOO problems

3.1 Algorithm of existing averaging techniques

An algorithm for solving MOO problems using averaging techniques is given as:

Table 1 Individual optimization matrix

Obj. function	Individual optimization						
	Max. Z_1	Max. Z_2	–	Max. Z_r	Min. Z_{r+1}	–	Min. Z_n
Z_1	θ_{11}	θ_{12}	–	θ_{1r}	θ_{1r+1}	–	θ_{1n}
Z_2	θ_{21}	θ_{22}	–	θ_{2r}	θ_{2r+1}	–	θ_{2n}
–	–	–	–	–	–	–	–
–	–	–	–	–	–	–	–
Z_r	θ_{r1}	θ_{r2}	–	θ_{rr}	θ_{rr+1}	–	θ_{rn}
Z_{r+1}	θ_{r+11}	θ_{r+12}	–	θ_{r+1r}	θ_{r+1r+1}	–	θ_{r+1n}
–	–	–	–	–	–	–	–
Z_n	θ_{n1}	θ_{n2}	–	θ_{nr}	θ_{nr+1}	–	θ_{nn}

θ_{ij} = Value of ith objective function on the optimal solution of jth objective function

θ_{ii} = Optimal value of the ith objective function

Step I: Obtain optimal solution of all the objectives individually.

Step II: Calculate the averages of the optimal values of Maximization objective functions and Minimization objective function separately.

Step III: Divide all the maximization objective functions by the averages of individual optima of maximization objectives and minimization objective functions by the averages of the individual optima of minimization objectives.

Step IV: Sum the weighted maximization and minimization objective functions separately.

Step V: Construct the combined objective function by subtracting weighted minimization objective function from weighted maximization objective function.

Step VI: Maximize the combined objective function.

3.2 Algorithm of improved averaging techniques

The algorithm of improved averaging techniques is described below:

Step I: Same as 3.1

Step II: Calculate the averages of optimal and sub optimal values of each objective function separately.

Step III: Divide each objective function by the averages of its optimal and sub optimal values estimated in step II.

Step IV: Same as 3.1

Step V: Same as 3.1

Step VI: Same as 3.1

4 Numerical examples

Example 1 The example of the study [6] has been reproduced here for present analysis.

$$\text{Max. } Z_1 = 12500X_1 + 25100X_2 + 16700X_3 + 23300X_4 + 20200X_5$$

$$\text{Max. } Z_2 = 21X_1 + 15X_2 + 13X_3 + 17X_4 + 11X_5$$

$$\text{Min. } Z_3 = 370X_1 + 280X_2 + 350X_3 + 270X_4 + 240X_5$$

$$\text{Min. } Z_4 = 1930X_1 + 1790X_2 + 1520X_3 + 1690X_4 + 1720X_5$$

Subject to :

$$X_1 + X_2 + X_3 + X_4 + X_5 = 4.5$$

$$2X_1 \geq 1.0$$

$$3X_4 \geq 1.5$$

4.1 Solution

The example was solved for achieving each objective and the results are presented in Table 2. Each column gives the optimal value of the mentioned objective function and suboptimal values of the remaining objective functions. The values of decision variables are given in the X_i row.

Table 2 Individual optimization matrix

Item	Individual optimization			
	Max. Z_1	Max. Z_2	Min. Z_3	Min. Z_4
X_i	0.5, 3.5, 0, 0.5, 0	4, 0, 0, 0.5, 0	0.5, 0, 0, 0.5, 3.5	0.5, 0, 3.5, 0.5, 0
Z_1	105750	61650	88600	76350
Z_2	71.5	92.5	57.5	64.5
Z_3	1300	1615	1160	1545
Z_4	8075	8565	7830	7130

The maximum value of Z_1 is 105750 obtained on $X_1 = 0.5$, $X_2 = 3.5$ and $X_4 = 0.5$. On the same solution the remaining objective functions Z_2 , Z_3 and Z_4 got suboptimal values of $Z_2 = 71.5$, $Z_3 = 1300$ and $Z_4 = 8075$. Similar results have been observed in optimization of objective function Z_2 , Z_3 and Z_4 . All the solutions of individual optimizations are different. None of the solutions optimize all the objective functions simultaneously. The conflicts among objective functions clearly necessitate the use of multi-objective optimization.

The existing MOO averaging techniques using mean, geometric mean and harmonic mean have been applied for solving the Example 1. The example was also solved using the improved averaging techniques with mean, geometric mean and harmonic mean for comparison. The estimation of mean, geometric mean and harmonic mean for both existing and improved techniques are shown in Table 3. Under the improved averaging techniques, all the optimal and suboptimal values of each objective function in a row have been considered for estimation of mean, geometric and harmonic mean. For the existing averaging techniques, only optimal values of the objective functions of maximization and minimization have been considered separately for the estimation of mean, geometric and harmonic mean.

The example was solved with the existing as well as improved averaging techniques using mean, geometric and harmonic mean for formulation of multi-objective functions. The solution of multi-objective optimization is presented in Table 4.

The results of multi-objective optimization with existing averaging techniques using mean, geometric and harmonic mean are all the same and achieve the first objective only. The remaining three objectives are not met. However, the improved averaging techniques have generated a compromising solution in achieving all the four objectives simultaneously. The improved averaging techniques have generated a comprehensive more acceptable solution than the existing averaging techniques. All these averaging techniques have been tested again on another example as described below.

Example 2 The example has been taken from the study [5] of multi-objective programming.

$$\text{Max. } Z_1 = 7000X_1 + 10000X_2 + 4000X_3 + 5000X_4 + 9000X_5$$

$$\text{Max. } Z_2 = 89X_1 + 50X_2 + 100X_3 + 79X_4 + 95X_5$$

$$\text{Min. } Z_3 = 10X_1 + 15X_2 + 14X_3 + 13X_4 + 12X_5$$

$$\text{Min. } Z_4 = 150X_1 + 120X_2 + 140X_3 + 100X_4 + 130X_5$$

Subject to :

$$X_1 + X_2 + X_3 + X_4 + X_5 = 7$$

$$X_1 \geq 0.7$$

$$X_2 \geq 0.3$$

$$X_3 \geq 0.4$$

$$X_5 \geq 0.6$$

4.2 Solution

All the objectives have been optimized individually to find the optimal values of each objective. The results are presented in Table 5. Each optimal solution achieved the single objective only ignoring the remaining three objectives. The solution indicates the presence of conflicts among all

the four objectives. The mean, geometric mean and harmonic mean of optimal values are estimated and given in Table 6.

The multi-objective functions for both existing and improved averaging techniques have been formulated and optimized with given constraints. The results of both the techniques are presented in the Table 7. The existing averaging techniques are achieving the first objective only. Whereas, all the four objectives have been achieved simultaneously by the improved averaging techniques.

5 Discussion

It is clear from the results of both the examples that the improved averaging techniques are efficient in solving MOO problems. The solutions of multi objective optimization of both the examples 1 and 2 indicate that all the existing averaging techniques are unable to provide the compromising solutions. These solutions have achieved the first objective only ignoring all the remaining three objectives. However, the improved averaging techniques have achieved all the four objectives simultaneously.

Table 3 Mean, geometric mean and harmonic mean

Objective function	Existing averaging techniques			Improved averaging techniques		
	Mean	G. Mean	H. Mean	Mean	G. Mean	H. Mean
Z_1	52921.25	3127.59	184.83	83087.5	81491.82	79902.32
Z_2				71.5	70.37	69.33
Z_3	4145	2875.89	1995.36	1405	1392.75	1380.38
Z_4				7900	7882.79	7865.35

Table 4 Multi-objective optimization

Item	Existing averaging techniques			Improved averaging techniques		
	Mean	Harmonic mean	Geometric mean	Mean	Harmonic mean	Geometric mean
X_i	0.5, 3.5, 0, 0.5, 0	0.5, 3.5, 0, 0.5, 0	0.5, 3.5, 0, 0.5, 0	0.5, 0, 0, 4, 0	0.5, 0, 0, 4, 0	0.5, 0, 0, 4, 0
Z_1	105750	105750	105750	99450	99450	99450
Z_2	71.5	71.5	71.5	78.5	78.5	78.5
Z_3	1300	1300	1300	1265	1265	1265
Z_4	8075	8075	8075	7725	7725	7725

Table 5 Individual optimization

Item	Individual optimization			
	Max. Z_1	Max. Z_2	Min. Z_3	Min. Z_4
X_i	0.7, 5.3, 0.4, 0, 0.6	0.7, 0.3, 5.4, 0, 0.6	5.7, 0.3, 0.4, 0, 0.6	0.7, 0.3, 0.4, 5.0, 0.6
Z_1	64900	34900	49900	39900
Z_2	424.3	674.3	619.3	569.3
Z_3	99.3	94.3	74.3	89.3
Z_4	875	975	1025	775

Table 6 Mean, geometric mean and harmonic mean

Objective function	Existing averaging techniques			Improved averaging techniques		
	Mean	G. Mean	H. Mean	Mean	G. Mean	H. Mean
Z_1	32787.15	6615.29	1334.73	47400	46082.48	44860.98
Z_2				571.8	563.56	554.70
Z_3	424.65	239.96	135.59	89.3	88.78	88.24
Z_4				912.5	907.32	902.03

Table 7 Multi-objective optimization

Item	Existing averaging techniques			Improved averaging techniques		
	Mean	Harmonic mean	Geometric mean	Mean	Harmonic mean	Geometric mean
Xi	0.7, 5.3, 0.4, 0, 0.6	0.7, 5.3, 0.4, 0, 0.6	0.7, 5.3, 0.4, 0, 0.6	0.7, 0.3, 0.4, 0, 5.6	0.7, 0.3, 0.4, 0.5, 6	0.7, 0.3, 0.4, 0.5, 6
Z_1	64900	64900	64900	59900	59900	59900
Z_2	424.3	424.3	424.3	649	649	649
Z_3	99.3	99.3	99.3	84.3	84.3	84.3
Z_4	875	875	875	925	925	925

6 Conclusion

Existing and improved averaging MOO techniques using mean, geometric and harmonic mean have been applied for solving two examples of multi-objective optimization problem. The results show the superiority of the improved averaging techniques over existing averaging techniques for solving multi-objective optimization problems.

Compliance with ethical standards

Conflict of interest The author declares that there is no conflict of interest.

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