Mathematics of uncertainty: an exploration on semi-elliptic fuzzy variable and its properties

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Abstract

Uncertainty is more or less encountered in industrial and medical systems as well. Uncertainty theory is an upgraded theory which comprises of possibility measure, necessity measure and credibility measure plays significant role in modeling uncertainty. In connection with uncertainty modeling, a special and intricate fuzzy variable viz., semi-elliptic fuzzy variable (SEFV) is thrashed out here. Subsequently, an attempt has been made to derive possibility, necessity and credibility measure of the SEFV first. Later, some other properties such as expected value, variance, rational upper bound etc are presented and based on that mean and variance ranking of SEFVs are proposed. Afterwards reliability analysis and medical diagnosis cases are carried out which exhibit the efficiency and novelty of the derived SEFV. In this work done, it is observed that the present work has the capability to resolve problems under uncertain complex situations.

Keywords Uncertainty · Fuzzy variable · Semi-elliptic fuzzy variable · Possibility measure · Necessity measure · Credibility distribution

1 Introduction

Uncertainty occurs due to lack of precision, deficiency in data, diminutive sample sizes, foreseeable man-made/artificial mistakes etc., is an unavoidable component of real world problems. To deal with this type of uncertainty fuzzy set theory (FST) [1] is explored. In power system planning reliability investigation is an extremely significant feature. The electrical energy production and consumption are the essential operating characteristic of the power system those are operated simultaneously and consequently, the investigation of reliability obligation for power system is incredibly elevated. Generally probabilistic approaches of reliability investigation are explored. However, due to association of uncertainty in the system classical probability approaches are seemed to be inappropriate and subsequently, fuzzy reliability investigation models are taken

into consideration [2]. Some recent applications in reliability investigations can be found in [3–15]. On the other hand, in medical diagnosis, usually a disease is characterized by several unswervingly perceptible symptoms which persuade the patient to visit a consultant or practitioner. A set of clinical inspections are commenced to make out the incidence of a disease. In the sphere of medical diagnosis, plenty of variables persuade the decision making process and subsequently, discriminate the judgments of the consultant or practitioner. Furthermore, mainly the medical diagnosis quandary engages dealing with uncertainties and so needed to integrate all the information into investigation. Therefore, fuzzy sets are explored to represent uncertainty and to perform medical diagnosis as well [16]. Some recent development in medical diagnosis can also be encountered in [17–24]. Afterwards Zadeh [25] himself developed possibility theory which was thought to be

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better to treat uncertainty and further studied by acolyte researchers such as Dubois and Prade [26], Klir [27], Yager [28] etc. Furthermore, Dubois and Prade [29] studied mean value of fuzzy numbers, Ban [30] discussed fuzzy valued measure and conditional expectations of fuzzy numbers, Heilpern [31] studied expected value of fuzzy numbers, Carlsson and Fuller [32] developed possibilistic mean and variance of fuzzy numbers, Chen and Tan [33] further developed mean value and variance of multiplication of fuzzy numbers.

Nevertheless in the absence of self duality measure the earlier studies lead to the exaggeration of the reality. keeping this in mind, Liu and Liu [34] initiated a concept termed as credibility theory. Li and Liu [35] presented a sufficient and necessary condition for credibility measures. Further Liu and Liu [36] systematically studied and developed credibility theory. After that some extended studies on credibility theory can be observed in Liu [37], Zhou et al. [38], Yi et al. [39], Garai et al. [40].

Although various types of fuzzy variables are encountered, however, an exceptional and intricate fuzzy variable SEFV in terms of credibility theory is not deliberated yet. This paper presents an approach to derive possibility, necessity and credibility measure of the SEFV. Furthermore, expected value, variance, rational upper bound etc of SEFV are presented. Then ranking of SEFNs through expected value and variance is proposed. Finally, novelty and applicability has been exhibited by performing reliability analysis and medical diagnosis cases.

2 Preliminaries

Uncertainty is an important as well as unavoidable ingredient of decision making process. Depending on the nature and accessibility of data, information, uncertainty is generally modelled using fuzzy set, possibility theory and Credibility theory.

Definition Let Θ be a non-empty set, and P the power set of Θ , and Pos is possibility measure. Then, the triplet (Θ, P, Pos) is known as a possibility space. A fuzzy variable is a mapping from possibility space (Θ, P, Pos) to the set of real numbers [37, 39].

Definition Let ζ be a fuzzy variable defined on the credibility space (Θ, P, Pos) . Then its membership function (MF) defined from the credibility measure is given by [37, 39]

$$\mu_{\zeta}(x) = \min(2Cr\zeta = 1); x \in \mathbb{R}.$$

Definition The α -cut of a fuzzy variable A is defined as

$${}^{\alpha}A = \{x \in X : \mu_A(x) \geq \alpha\}.$$

Definition Let A be a fuzzy variable, μ be the MF of A , and r be any real number. Then, the possibility measure of A is defined as [25]

$$Pos\{A \leq r\} = \sup_{x \leq r} \mu_A(x)$$

Definition Let A be a fuzzy variable, μ be the MF of A , and r be any real number. Then, the possibility measure of A is defined as [25]

$$Nec\{A \leq r\} = 1 - \sup_{x > r} \mu_A(x)$$

Definition A credibility measure (Cr) is a non-negative set function holds the following [35]

$$\begin{aligned} Cr(\Theta) &= 1 \\ Cr(A) &\leq Cr(B) \text{ for whenever } A \subset B \\ Cr(A) + Cr(A^c) &= 1 \text{ for any } A \\ Cr\{\cup A_i\} &= \sup_i Cr\{A_i\} \text{ for any events } A_i \text{ with} \\ &\sup_i Cr\{A_i\} < 0.5 \end{aligned}$$

If the fuzzy variable A is given by its MF μ , then

$$Cr(A \leq r) = \frac{1}{2} \{ \sup_{x \leq r} \mu_A(x) + 1 - \sup_{x > r} \mu_A(x) \}, x, y \in \mathbb{R}$$

Definition The credibility distribution $\Phi_A : \mathbb{R} \rightarrow [0, 1]$ of a fuzzy variable A is defined as [36]

$$\Phi_A(x) = Cr\{\theta \in \Theta : \zeta(\theta) \leq x\}.$$

That is, the credibility that the fuzzy variable ζ takes a value less than or equal to x .

Definition The credibility density function of credibility distribution defined as $\phi_A : \mathbb{R} \rightarrow [0, \infty)$ of any fuzzy variable A , is a function such that [37]

$$\phi_A(x) = \int_{-\infty}^x \phi(y) dy, \forall x \in \mathbb{R}$$

Definition [38] A credibility distribution Φ_A of a fuzzy variable A is called regular if it is a continuous and strictly increasing function w.r.t x such that $0 < \Phi_A < 1$ and if $\lim_{x \rightarrow -\infty} \Phi_A = 0$, and $\lim_{x \rightarrow \infty} \Phi_A = 1$.

Definition [38] Let A be a fuzzy variable with a regular credibility distribution Φ_A , then the inverse function Φ_A^{-1} is called the inverse credibility distribution of A .

3 Construction of semi-elliptic fuzzy variable

Let's consider the general equation of the horizontal ellipse centred at (a, b) is

$$\frac{(x-a)^2}{h^2} + \frac{(y-b)^2}{k^2} = 1$$

To construct a normal semi-circular fuzzy variable (SEFV), it is needed to consider $b = 0$ and $k = 1$ i.e.,

$$\frac{(x-a)^2}{h^2} + y^2 = 1$$

Thus, the required membership function of the SEFV $A = S_E(a, h)$ is

$$\mu_A(x) = \sqrt{1 - \frac{(x-a)^2}{h^2}}, a-h \leq x \leq a+h$$

where a indicates the mean/core of the fuzzy variable while h determines width of the fuzzy variable A .

The α -cut of the SEFV $A = S_E(a, h)$ is

$${}^{\alpha}A = [a - h\sqrt{1 - \alpha^2}, a + h\sqrt{1 - \alpha^2}]$$

Suppose $A = S_E(10, 7)$ is SEFV representing a uncertain variable. The MF of the SEFV A is

$$\mu_A(x) = \sqrt{1 - \frac{(x-10)^2}{49}}, 3 \leq x \leq 17$$

The graphical representation of A is depicted in Fig. 1.

4 Possibility, necessity and credibility measures of SEFV

In this section, the possibility measure, necessity measure and credibility measure of the SEFV are derived.

4.1 Possibility measures of SEFV

Suppose $A = S_E(a, h)$ is a SEFV.

Then, the possibility measure of $(A \leq x)$ and $(A \geq x)$ are respectively

$$\begin{aligned} \text{Pos}(A \leq x) &= \begin{cases} 1, & \text{if } x \geq a, \\ \sqrt{1 - \frac{(x-a)^2}{h^2}}, & \text{if } x < a, \end{cases} \\ \text{Pos}(A \geq x) &= \begin{cases} 1, & \text{if } x \leq a, \\ \sqrt{1 - \frac{(x-a)^2}{h^2}}, & \text{if } x > a, \end{cases} \end{aligned}$$

The possibility measure of the SEFV $A = S_E(10, 7)$ for $(A \leq x)$ and $(A \geq x)$ are depicted in Figs. 2 and 3 respectively.

4.2 Necessity measures of SEFV

Suppose $A = S_E(a, h)$ is a SEFV.

Then, the necessity measure of $A \leq x$ and $A \geq x$ are respectively

$$\begin{aligned} \text{Nec}(A \leq x) &= \begin{cases} 0, & \text{if } x \leq a, \\ \sqrt{1 - \frac{(x-a)^2}{h^2}}, & \text{if } x > a, \end{cases} \\ \text{Nec}(A \geq x) &= \begin{cases} 0, & \text{if } x \geq a, \\ \sqrt{1 - \frac{(x-a)^2}{h^2}}, & \text{if } x < a, \end{cases} \end{aligned}$$

The necessity measure of the SEFV $A = S_E(10, 7)$ for $(A \leq x)$ and $(A \geq x)$ are depicted in Figs. 4 and 5 respectively.

4.3 Credibility measures of SEFV

Suppose $A = S_E(a, h)$ is a SEFV.

Fig. 1 The SEFV $A = S_E(10, 7)$

Fig. 2 The possibility measure of SEFV $A = S_E(10, 7)$ for $(A \leq x)$

Fig. 3 The possibility measure of SEFV $A = S_E(10, 7)$ for $(A \geq x)$

Fig. 4 The necessity measure of SEFV $A = S_E(10, 7)$ for $(A \leq x)$

Fig. 5 The necessity measure of SEFV $A = S_E(10, 7)$ for $(A \geq x)$

Then, the credibility measure of $A \leq x$ and $A \geq x$ are respectively

$$Cr(A \leq x) = \begin{cases} \frac{1}{2} \sqrt{1 - \frac{(x-a)^2}{h^2}}, & \text{if } a-h \leq x \leq a, \\ 1 - \frac{1}{2} \sqrt{1 - \frac{(x-a)^2}{h^2}}, & \text{if } a \leq x \leq a+h, \end{cases}$$

$$Cr(A \geq x) = \begin{cases} 1 - \frac{1}{2} \sqrt{1 - \frac{(x-a)^2}{h^2}}, & \text{if } a-h \leq x \leq a, \\ \frac{1}{2} \sqrt{1 - \frac{(x-a)^2}{h^2}}, & \text{if } a \leq x \leq a+h, \end{cases}$$

The necessity measure of the SEFV $A = S_E(10, 7)$ for $(A \leq x)$ and $(A \geq x)$ are depicted in Figs. 6 and 7 respectively.

4.4 Credibility distribution of SEFV

As the credibility distribution $\Phi_A : \mathbb{R} \rightarrow [0, 1]$ of a fuzzy variable A is defined as $\Phi_A(x) = Cr\{\theta \in \Theta : \zeta(\theta) \leq x\}$.

Hence, the credibility distribution of the SEFV $A = S_E(a, h)$ is

$$\Phi_A(x) = \begin{cases} \frac{1}{2} \sqrt{1 - \frac{(x-a)^2}{h^2}}, & \text{if } a-h \leq x \leq a, \\ 1 - \frac{1}{2} \sqrt{1 - \frac{(x-a)^2}{h^2}}, & \text{if } a \leq x \leq a+h, \end{cases}$$

The credibility distribution function of the SEFV $A = S_E(10, 7)$ is

$$\Phi_A(x) = \begin{cases} \frac{1}{2} \sqrt{1 - \frac{(x-10)^2}{49}}, & \text{if } 3 \leq x \leq 10, \\ 1 - \frac{1}{2} \sqrt{1 - \frac{(x-10)^2}{49}}, & \text{if } 10 \leq x \leq 17, \end{cases}$$

4.5 Inverse credibility distribution (ICD) of SEFV

The credibility distribution of the SEFV $A = S_E(a, h)$ is

$$\Phi^{-1}(\alpha) = \begin{cases} a - h\sqrt{1 - 4\alpha^2}, & \text{if } 0 \leq \alpha \leq 0.5, \\ a + h\sqrt{1 - 4(1-\alpha)^2}, & \text{if } 0.5 \leq \alpha \leq 1, \end{cases}$$

Fig. 6 The credibility measure of SEFV $A = S_E(10, 7)$ for $(A \leq x)$

Fig. 7 The credibility measure of SEFV $A = S_E(10, 7)$ for $(A \geq x)$

5 Expected value

Using the idea of credibility distribution, Liu and Liu [34] provided the expected value of fuzzy variables. Zhou et al. [38] presented expected value of fuzzy variables via ICD.

5.1 Expected value via credibility distribution

If A is a fuzzy variable then the expected value of A in terms of credibility distribution is defined as [34]

$$E(A) = \int_0^{\infty} Cr\{A \geq r\} dr - \int_{-\infty}^0 Cr\{A \leq r\} dr$$

Then, the expected value of the SEFV $A = S_E(a, h)$ is

$$\begin{aligned} E(A) &= \int_0^{a-h} dr + \int_{a-h}^a \frac{1}{2} \sqrt{1 - \frac{(r-a)^2}{h^2}} dr \\ &\quad + \int_a^{a+h} \left\{ 1 - \frac{1}{2} \sqrt{1 - \frac{(r-a)^2}{h^2}} \right\} dr \\ &= (a-h) + h + \int_{a-h}^a \frac{1}{2} \sqrt{1 - \frac{(r-a)^2}{h^2}} dr \\ &\quad - \int_a^{a+h} \frac{1}{2} \sqrt{1 - \frac{(r-a)^2}{h^2}} dr \\ &= a + \frac{\pi h}{8} - \frac{\pi h}{8} \\ &= a \end{aligned}$$

5.2 Expected value via ICD

If A is a SEFV then the expected value of A in terms of ICD is defined as [38]

$$E[A] = \int_0^1 \Phi^{-1}(\alpha) d\alpha$$

Then, the expected value of the SEFV $A = S_E(a, h)$ is

$$\begin{aligned} E(A) &= \int_0^{0.5} \left\{ a - h\sqrt{1 - 4\alpha^2} \right\} d\alpha \\ &\quad + \int_{0.5}^1 \left\{ a + h\sqrt{1 - 4(1-\alpha)^2} \right\} d\alpha \\ &= a - \frac{h}{2} \int_0^1 \sqrt{1 - t^2} dt - \frac{h}{2} \int_1^0 \sqrt{1 - t^2} dt \\ &= a \end{aligned}$$

Thus, in both the approaches, it is obtained that the expected value of the SEFV A is $e = a$.

Remark If $A = S_E(h_1, h_2)$ is a asymmetric SEFV where h_1 and h_2 are left spread and right spread of A . Then, the expected value of A is

$$E[A] = a + \frac{\pi}{8}(h_2 - h_1).$$

6 Variance

In this section, variance of SEFV is calculated in terms of regular credibility distribution.

If A is a fuzzy variable with expected value e , then the variance of A is defined as [36]

$$V[A] = E[(A - e)^2]$$

It should be noted that if the expected value e of the fuzzy variable is finite, then the variance satisfies

$$V[A] = E[(A - e)^2] = \int_0^{+\infty} Cr\{(A - e)^2 \geq r\} dr$$

6.1 Variance of a SEFV

To evaluate variance $V[A]$ of a SEFV A , it is needed to calculate the MF of $(A - e)^2$ first and to find MF of SEFV $A = S_E(a, h)$, α -cut technique is explored here. It is already obtained that the expected value of SEFV is $e = a$

The α -cut of the SEFV $A = S_E(a, h)$ is ${}^\alpha A = [a - h\sqrt{1 - \alpha^2}, a + h\sqrt{1 - \alpha^2}]$.

The procedure is presented below.

$$\begin{aligned} ({}^\alpha A - a)^2 &= \left[(a - h\sqrt{1 - \alpha^2} - a)^2, (a + h\sqrt{1 - \alpha^2} - a)^2 \right] \\ &= \left[(-h\sqrt{1 - \alpha^2})^2, (h\sqrt{1 - \alpha^2})^2 \right] \\ &= [h^2(1 - \alpha^2), h^2(1 - \alpha^2)] \end{aligned}$$

Now, taking $x = h^2(1 - \alpha^2)$ gives $\alpha = \sqrt{1 - \frac{x}{h^2}}, 0 \leq x \leq h^2$.

Thus, the MF of $(A - e)^2$ is

$$\mu_{(A-e)^2}(x) = \sqrt{1 - \frac{x}{h^2}}, 0 \leq x \leq h^2.$$

Since $Cr\{(A - e)^2 < r\} = \frac{1}{2} \{ \sup_{x < r} \mu_{(A-e)^2}(x) + 1 - \sup_{x \geq r} \mu_{(A-e)^2}(x) \}$

$$\therefore Cr\{(A - e)^2 < r\} = \frac{1}{2} \left\{ 1 + 1 - \sqrt{1 - \frac{x}{h^2}} \right\}, 0 \leq x \leq h^2.$$

Again, $Cr\{(A - e)^2 \geq r\} = 1 - Cr\{(A - e)^2 < r\}$. Hence,

$$Cr\{(A - e)^2 \geq r\} = \begin{cases} 1 - Cr\{(A - e)^2 < r\}, & 0 \leq r \leq h^2 \\ 0, & r > h^2 \end{cases}.$$

Then, the variance of SEFV $A = S_E(a, h)$ is

$$\begin{aligned} V[A] &= \int_0^\infty Cr\{(A - e)^2 \geq r\} dr \\ &= \int_0^{h^2} \frac{1}{2} \sqrt{1 - \frac{r}{h^2}} dr \\ &= \frac{h^2}{3} \end{aligned}.$$

For example, the variance of the SEFV $A = S_E(10, 7)$ is $\frac{7^2}{3} = 16.33$.

Remark If the width h of a SEFV is unit then SEFV will represent a semi-circular fuzzy variable (SCFV). Then, it can be derived that for all SCFV the expected value is 0.33.

7 Rational upper bound of the variance

Yi et al. [39] derived the concept of rational upper bounded of the variance (RUBV) along with some definitions and results in terms of credibility distribution and ICD.

Definition Let A be a fuzzy variable with credibility distribution Φ and finite expected value e . The RUBV is defined as [39]

$$\bar{V}[A] = \int_0^\infty (1 - \Phi(e + \sqrt{x})\Phi(e - \sqrt{x}))$$

Definition Let A be a fuzzy variable with credibility distribution Φ . If the expected value is e , then RUBV is evaluated as [39]

$$\bar{V}[A] = \int_0^1 (\Phi^{-1}(\alpha) - e)^2 d\alpha$$

Corollary The RUBV of SEFV depends on the width h and it is $\frac{2h^2}{3}$.

Consider the SEFV $A = S_E(a, h)$.

$$\begin{aligned} &\int_0^{0.5} h^2(1 - 4\alpha^2) d\alpha + \int_{0.5}^1 h^2(1 - 4(1 - \alpha)^2) d\alpha \\ &= h^2 + h^2 \int_0^{0.5} 4\alpha^2 d\alpha - h^2 \int_{0.5}^1 4(1 - \alpha)^2 d\alpha \\ &= h^2 - \frac{h^2}{2} \int_0^1 t^2 dt + \frac{h^2}{2} \int_1^0 t^2 dt \\ &= h^2 - h^2 \int_0^1 t^2 dt \\ &= h^2 - \frac{h^2}{3} \\ &= \frac{2h^2}{3} \end{aligned}$$

Corollary Let $A = S_E(a, h)$ be SEFV. Then, $\bar{V}[A] = 2V[A]$.

Remark The SCFV also satisfies the above corollary and it is also observed that for all SCFV, the RUBV is 0.66.

Corollary Suppose $A = S_E(a_1, h_1)$ and $B = S_E(a_2, h_2)$ are two SRFV.

Then, $\bar{V}[A + B] \leq 2(\bar{V}[A] + \bar{V}[B])$.

For the two SEFV A and B , the ICV of $A + B$ is

$$\begin{aligned} &\Phi^{-1}(\alpha) \\ &= \Phi_A^{-1}(\alpha) + \Phi_B^{-1}(\alpha) \\ &= \begin{cases} (a_1 + b_1) - (h_1 + h_2)\sqrt{1 - 4\alpha^2}, & \alpha \leq 0.5 \\ (a_1 + b_1) + (h_1 + h_2)\sqrt{1 - 4(1 - \alpha)^2}, & \alpha > 0.5 \end{cases} \end{aligned}$$

Now,

$$\begin{aligned} &\bar{V}[A + B] \\ &= \int_0^{0.5} [(a_1 + b_1) - (h_1 + h_2)\sqrt{1 - 4\alpha^2}]^2 d\alpha \\ &\quad + \int_{0.5}^1 [(a_1 + b_1) + (h_1 + h_2)\sqrt{1 - 4(1 - \alpha)^2}]^2 d\alpha \\ &= \frac{2(h_1 + h_2)^2}{3}. \end{aligned}$$

Again, $\bar{V}[A] = \frac{2(h_1)^2}{3}$ and $\bar{V}[B] = \frac{2(h_2)^2}{3}$.

Since, $\frac{2(h_1 + h_2)^2}{3} \leq \frac{2(h_1)^2}{3} + \frac{2(h_2)^2}{3}$.

Consequently, $\bar{V}[A + B] \leq 2(\bar{V}[A] + \bar{V}[B])$.

Corollary Suppose $A = S_E(a_1, h_1)$ and $B = S_E(a_2, h_2)$ are two SRFVs.

Then, $\sqrt{\bar{V}[A + B]} = \sqrt{\bar{V}[A] + \bar{V}[B]}$.

Since $\sqrt{\bar{V}[A + B]} = \sqrt{\frac{2(h_1 + h_2)^2}{3}}$ and $\sqrt{\bar{V}[A]} = \sqrt{\frac{2h_1^2}{3}}$,

$$\sqrt{\bar{V}[B]} = \sqrt{\frac{2h_2^2}{3}}.$$

$$\text{Thus, } \sqrt{\bar{V}[A+B]} = \sqrt{\bar{V}[A]} + \sqrt{\bar{V}[B]}.$$

8 Arithmetic on SEFVs

In this section, basic operations on SEFVs are reviewed and adopted from [41].

Suppose $A = S_E(a, h)$ and $B = S_E(b, k)$ are two SEFVs defined on a universe of discourse X .

8.1 Addition

The membership function of $A + B$ is

$$\mu_{(A+B)} = \sqrt{1 - \left\{ \frac{x - (a+b)}{h+k} \right\}^2},$$

$$x \in [(a-h) + (b-k), (a+h) + (b+k)]$$

8.2 Subtraction

The membership function of $A - B$ is

$$\mu_{(A-B)} = \sqrt{1 - \left\{ \frac{x - (a-b)}{h+k} \right\}^2},$$

$$x \in [(a-h) - (b+k), (a+h) - (b-k)]$$

8.3 Multiplication

The membership function of AB is

$$\mu_{AB}(x) = \sqrt{1 - \left\{ \frac{(ak+bh) - \sqrt{(ak+bh)^2 - 4hk(ab-x)}}{2hk} \right\}^2},$$

$$x \in (a-h)(b-k) \leq x \leq (a+h)(b+k)$$

8.4 Division

The membership function of A/B is

$$\mu_{\frac{A}{B}}(x) = \sqrt{1 - \left\{ \frac{a-xb}{h+xk} \right\}^2}, \quad \frac{a-h}{b+k} \leq x \leq \frac{a+h}{b-k}$$

9 Rank of SEFVs

In this section, ranking of two SEFVs are defined based on the expected value and variance of the SEFVs. Suppose $A = S_E(a_1, h_1)$ and $B = S_E(a_2, h_2)$ are two SEFVs.

Then rank of A and B are defined as

- I. $A \leq B$ if $E[A] \leq E[B]$.
- II. If $E[A] = E[B]$ then

$$A \leq B \text{ if } V[A] \geq V[B]$$

Example 9.1 Suppose $A = [-4, 0, 4]$ and $B = [-2, 0, 2]$ are two fuzzy variables adopted from [42]. It is observed that the approaches [42–47] fail to compare the fuzzy variables. Reforming these fuzzy variables to SEFVs as $A = S_E(0, 4)$ and $B = S_E(0, 2)$ and applying the present approach it is obtained that $A \leq B$ which is consistent with human intuitions. Here, $E[A] = E[B] = 0$, but $V[A] = 5.33 \geq V[B] = 1.33$ and consequently, it can be adopted that $A \leq B$. A details comparison has been presented in Table 1.

Example 9.2 Consider the fuzzy variables $A = [0.2, 0.5, 0.8]$ and $B = [0.4, 0.5, 0.6]$. The approaches [44, 47–49, 51] are not applicable to distinguish A and B while [52–54] and [50] produces illogical output. Here also the general or human intuition is that $A \leq B$. Applying the present approach we obtain $E[A] = E[B] = 0.5$, but $V[A] = 0.1 \geq V[B] = 0.033$ which gives $A \leq B$. A detail discussion is presented in Table 2.

From the above analysis it can be opined that the present ranking approach has the capability to overcome the drawbacks of the existing approaches.

10 Application of SEFVs

In this segment, application of SEFVs are performed in structural reliability analysis and medical diagnosis. For structural reliability arithmetic of SEFVs are taken into consideration while in medical diagnosis arithmetic as well as ranking of SEFVs are adopted.

Table 1 Ranking of fuzzy variables for example 8.1

Approaches	A	B	Result
Abbasbandy et al. [42]	0	0	$A \sim B$
Wang [43]	0	0	$A \sim B$
Asady [44]	0.9	0.9	$A \sim B$
Asady [45]	0	0	$A \sim B$
Asady [46]	0.444	0.444	$A \sim B$
Abbasbandy and Hajjari [47]	0.117	0.074	$A > B$
Present approach	$E[A] = 0.5$ $V[A] = 5.33$	$E[B] = 0.5$ $V[B] = 1.33$	$A \leq B$

Table 2 Ranking of fuzzy variables for example 8.2

Approaches	Decision-level	A	B	Result
Yager [48]		0.5	0.5	$A \sim B$
Abbasbandy and Hajjari [47]		0.5	0.5	$A \sim B$
Asady [44]		0.374	0.374	$A \sim B$
Chen and Sanguansat [51]		0.5	0.5	$A \sim B$
Chen et al. [49]		0.444	0.444	$A \sim B$
Vincent et al. [52, 53]	$\alpha = 1$	0.117	0.074	$A > B$
Vincent and Dat [54]	$\alpha = 1$	0.45	0.35	$A > B$
Rezvani [50]		0.078	0.062	$A > B$
Present Approach		$E[A] = 0.5$ $V[A] = 0.1$	$E[B] = 0.5$ $V[B] = 0.033$	$A \leq B$

Fig. 8 Beam associated with its bending moment

10.1 Application in structural reliability

Depictions of the elements of structural analysis are more often SEFVs in some circumstances. In such circumstances structural failure can be evaluated using arithmetic of SEFVs. Consider the following problem of structural failure adopted from Dutta [3].

Example 10.1 Suppose a beam of height $h = 9\text{mm}$, length $L = 1250\text{mm}$ and the force density $f = 78.5 \times 10^{-5} \text{kN/mm}^3$. The load w , breadth of the beam b and ultimate bending moment M_o are uncertain input variables represented by SEFVs where $w = S_E(400, 15) \text{kN}$, $b = S_E(40, 5)$ and $M_o = S_E(2.05 \times 10^5, 0.05 \times 10^5) \text{kN} - \text{mm}$. which is depicted in Fig. 8.

The limit state function is $g(b, f, h, w, M_o, L) = M_o - \left(\frac{wL}{4} + \frac{fbhL^2}{8} \right)$.

It is needed to evaluate the structural failure of the beam.

Applying the arithmetic of SEFVs on the problem the value of structural reliability or g is obtained as $S_E(0.248 \times 10^5, 0.1658 \times 10^5)$.

10.2 Application in medical diagnosis

It is observed that patient's explanations, medical information even medical assessment process tainted with imprecision/vagueness/uncertainty. On the other hand, knowledge base correlating the symptom-disease relationship

Table 3 Patient-symptom relation

R_1	S_1	S_2	S_3
P_1	$S_E(3, 1)$	$S_E(6, 1)$	$S_E(2, 1)$
P_2	$S_E(7, 1)$	$S_E(2, 1)$	$S_E(4, 2)$
P_3	$S_E(2, 1)$	$S_E(3, 1)$	$S_E(7, 2)$

Table 4 Symptom-disease relation

R_2	D_1	D_2	D_3
S_1	$S_E(6, 2)$	$S_E(4, 2)$	$S_E(3, 1)$
S_2	$S_E(2, 1)$	$S_E(6, 1)$	$S_E(2, 1)$
S_3	$S_E(3, 1)$	$S_E(2, 1)$	$S_E(7, 2)$

encompasses of ambiguity and uncertainty in medical assessment process. Accordingly to deal with such uncertainties FST is being adopted and became most demanding area in medical assessment process. Here, SEFVs are considered to represent uncertain information.

Consider the Patient-symptom and Symptom-disease relations presented in Tables 3 and 4, respectively.

Using the multiplication and addition of SEFVs the resultant Patient-disease relation is evaluated and presented in Table 5. Then, ranking of SEFVs is adopted to obtain crisp values of resultant SEFVs and presented in Table 6. It should be noted that maximum value in each row indicates that the patient is likely to have the disease. Here, $\{P_1, P_2, P_3\}$, $\{S_1, S_2, S_3\}$ and $\{D_1, D_2, D_3\}$ are the set of patients, symptoms and diseases, respectively. From Table 6, it is clear that the maximum value (the bold value) in 1st row is 59.0686 which associates patient P_1 and

Table 5 Patient-Disease relation

R	D_1	D_2	D_3
P_1	$S_E(36, 17, 40)$	$S_E(52, 20, 38)$	$S_E(35, 19, 33)$
P_2	$S_E(58, 29, 35)$	$S_E(48, 29, 28)$	$S_E(53, 30, 28)$
P_3	$S_E(39, 23, 29)$	$S_E(40, 23, 27)$	$S_E(61, 32, 27)$

Table 6 Crisp values of the patient-disease relation

R	D_1	D_2	D_3
P_1	45.0321	59.0686	40.4978
P_2	60.3562	47.6073	52.2146
P_3	41.3562	41.5708	59.0365

disease D_2 . That is, patient P_1 is likely to have the disease D_2 . Similarly, from 2nd and 3rd row (bold values in the Table 6) it can be opined that patient P_2 is suffering from disease D_1 and patient P_3 is suffering from disease D_3 .

11 Conclusions

Uncertainty is an integral part of real world problems such as reliability assessment as well as medical diagnosis problems. To cope with uncertainty a handful number of fuzzy variables are demonstrated yet in literature. However, a special complicated fuzzy variable SEFV is not deliberated well and in this regard here SEFV has been introduced in terms of credibility theory first. Then, some important properties such as possibility measure, necessity measure, credibility measure, credibility distribution and ICD were presented. Afterwards, investigations on expected value of SEFV using credibility distribution and ICD along with variance and RUVB of SEFV have been performed and established relationship between them. Another important concept ranking of SEFVs is introduced based on expected value of SEFV and if it fails then variance of SEFVs concept has been utilized to evaluate order of SEFVs. Comparative numerical illustrations have been presented where results of existing methods and present method were compared and exhibited that present method smoothly over come the limitations of earlier methods. Finally, reliability analysis has been performed using arithmetic of SEFV while a medical diagnosis is performed using arithmetic and rank of SEFVs as well. The present model successfully solves both the problems which exhibits the novelty and applicability of the present model. However, the limitation of this present model is that it can't work properly when asymmetric SEFVs come into picture. Therefore, as an extension of this work, asymmetric SEFV will be investigated.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human or animal subjects.

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