Lie group analysis of a Powell–Eyring nanofluid flow over a stretching surface with variable properties

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Received: 29 August 2019 / Accepted: 5 December 2019 / Published online: 19 December 2019
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Abstract

New applications of nanofluids that have enhanced thermo-physical properties have spurred new studies into the flow and heat transfer in nanofluids in the last decade. Most reported studies have considered the case where the fluid viscosity and thermal conductivity depend only on the size of nanoparticles. However, experimental data show that these properties may depend on the size of nanoparticles and the temperature. In this study, we investigate the flow and heat transfer in a Powell–Eyring nanofluid flow past a stretching surface using the nanofluid viscosity and thermal conductivity models derived from experimental data. Using Lie group analysis, the equations describing the flow and energy balance are reduced to a system of coupled differential equations. These equations are then solved using an efficient iterative spectral local linearization method. The computational results show that increasing the nanoparticle volume fraction and thermal radiation parameter enhances the temperature profiles, while an increase in the fluid parameter increased the velocity profiles. In addition, among other results, the Nusselt number increases with an increase in the temperature ratio parameter and thermal radiation. The results from this study may be useful to engineers in designing cooling devices for the enhancement of thermal systems.

Keywords Powell–Eyring model · Nanofluid · Thermal radiation · Variable viscosity and thermal conductivity · Spectral local linearization method

1 Introduction

Efforts to improve the efficiency and performance of industrial and engineering processes have led to the replacement of traditional heat transfer fluids (e.g water, oil or ethylene glycol) with nanofluids. A nanofluid is a colloidal suspension containing metals or oxides, for example, copper oxide, alumina, zinc oxide or iron oxide having a diameter less than 100 nm. The applications of these fluids can be found in solar technology, where nanofluids are used to enhance the productivity and efficiency of a solar thermal system when used as a collector. In biomedical sciences, nanofluids find applications in cancer

therapeutics and cryosurgery. A comprehensive review of other applications of nanofluids is given by Wong and De Leon [1], Robert et al. [2], Devendiran and Amirtham [3] and Munyalo and Zhang [4].

The superiority of nanofluids over traditional heat transfer fluids can be attributed to their stability and higher thermo-physical properties. Due to these remarkable characteristics and new applications of nanofluids, many studies have been carried out on the flow and heat transfer of nanofluids. Khan and Pop [5] studied the thermal boundary layer flow of a nanofluid past a stretching plate. In their study, emphasis was placed on the Brownian motion and thermophoresis effect. The implicit finite

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difference method was used to solve the flow equations. The exact solution to the thermal transport problem of different types of nanofluids was given by Turkiilmazoglu [6]. Sandeep and Gnanaswara [7] scrutinized the effects of nonlinear thermal radiation on the flow of a Cu–water nanofluid. The Runge–Kutta Newton–Raphson algorithm was used to solve the flow equations. Dhlamini et al. [8] discussed the second-grade nanofluid flow over a nonlinearly stretching sheet. Das et al. [9] solved the equations for the Casson nanofluid flow and heat transfer in a porous medium using a spectral quasi-linearization approximation. El-Aziz [10] studied the effect of variable viscosity on the flow and heat transfer of a power-law nanofluid. Recently, Das et al. [11] examined the influence of variable fluid properties on nanofluid flow over a wedge with surface slip.

The studies above considered the viscosity and thermal conductivity of the nanofluid to be a function of nanoparticle size. However, these properties may also change with temperature (Ogunseye et al. [12]). To accurately predict the heat transfer properties of nanofluids, it is important to consider a viscosity and thermal conductivity model that depends on the nanoparticle size and temperature. In recent years, scientists have proposed numerous viscosity and thermal conductivity models that are nanoparticle size and temperature dependent, and among these, the studies by Masoud et al. [13] and Hassani et al. [14] are worth mentioning.

Lie group symmetry analysis is a powerful technique for finding similarity solutions to a given set of partial differential equations. Using Lie group analysis, we can find similarity transformations that reduces m independent variables of a partial differential equation into $m - 1$ independent variables. Many authors have applied Lie group symmetry analysis to fluid flow models. Akgül and Pakdemirli [15] studied the transient flow of a power-law fluid using the Lie group symmetry analysis. Jalil and Asghar [16] analysed the boundary layer flow of a Powell–Eyring fluid using the Lie group symmetry analysis. The scaling group of transformations, a special form of the Lie group symmetry, was used by Rehman et al. [17] in studying the heat and mass transfer in a Powell–Eyring fluid flow past a stretching plate. Afify and El-Aziz [10] discussed the scaling group for the flow and heat transfer behaviour in a power-law nanofluid. Other studies using the Lie group analysis are reported in [18–21].

The main focus of this study is the flow and heat transfer analysis in a non-Newtonian nanofluid with variable viscosity and thermal conductivity using the Lie group symmetry analysis. The Powell–Eyring model [22] is adopted due to its diverge advantages over other non-Newtonian fluid models. The model is derived from the molecular theory of fluids and not on empirical relations. Further, under

low and high shear rates the Powell–Eyring fluid is reduced to a Newtonian fluid. A considerable number of studies on the Powell–Eyring fluid flow with constant viscosity have been reported by several authors such as Javed et al. [23], Jalil and Asghar [16], Hayat et al. [24], Mahanthesh et al. [25], Agbaje et al. [26] and Ramzan et al. [27]. However, the Powell–Eyring nanofluid flow with variable properties is yet to be considered. The second-order partial differential equation that models the thermal transportation problem is transformed into an ordinary differential equation using the classical Lie group symmetry approach. The equations are solved using an efficient iterative spectral local linearization method. The viscosity and thermal conductivity adopted here are derived from experimental data. The findings in this study may be useful for engineers in the design of heat exchangers and thermal solar collectors.

2 Formulation of the problem

A steady, two-dimensional, laminar flow of an incompressible Powell–Eyring nanofluid past a stretching surface is considered. The flow is restricted to the region $\bar{y} > 0$, and the stretching velocity is assumed to be $u_w(\bar{x})$. The nanofluid is aluminium oxide Al_2O_3 –water system. Further, the physical properties of the fluid are assumed to vary with the nanoparticle size and temperature.

Following the work of Javed et al. [23] and under the usual boundary layer approximation, the continuity, equations of momentum and energy balance for the Powell–Eyring nanofluid are as follows,

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \quad (1)$$

$$\rho_{nf} \left(\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right) = \frac{\partial}{\partial \bar{y}} \left(\mu_{nf} \frac{\partial \bar{u}}{\partial \bar{y}} \right) + \frac{1}{\beta \gamma} \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{1}{2\beta \gamma^3} \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}, \quad (2)$$

$$\rho_{nf} C_{nf} \left(\bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} \right) = \frac{\partial}{\partial \bar{y}} \left(k_{nf} \frac{\partial \bar{T}}{\partial \bar{y}} \right) + \frac{16\sigma_s}{3k_m} \frac{\partial}{\partial \bar{y}} \left(\bar{T}^3 \frac{\partial \bar{T}}{\partial \bar{y}} \right). \quad (3)$$

The relevant boundary conditions to Eqs. (1)–(3) are

$$\begin{aligned} \bar{y} = 0 : \quad & \bar{u} = U_0 u_w \left(\frac{\bar{x}}{L} \right), \quad \bar{v} = V_0 v_w \left(\frac{\bar{x}}{L} \right), \quad \bar{T} = T_w, \\ \bar{y} \rightarrow \infty : \quad & \bar{u} \rightarrow 0, \quad \bar{T} \rightarrow T_\infty, \end{aligned} \quad (4)$$

where $(\bar{u}, \bar{v})$ are the velocity components in the $(\bar{x}, \bar{y})$ directions, β and γ are Powell–Eyring fluid material constants, $\bar{T}$ is the nanofluid temperature, σ_s is the Stefan–Boltzmann constant, k_m is the mean absorption coefficient, U_0 and V_0 are the reference velocities, L is the characteristic length, T_w is the wall temperature and T_∞ is the nanofluid temperature far away from the wall.

The density (ρ_{nb}) and specific heat capacity (C_{nb}) of the nanofluid are given by the expressions (see Khanafer and Vafai [28]),

$$\begin{aligned}\rho_{nf} &= (1 - \phi)\rho_{bf} + \phi\rho_p, \\ C_{nf} &= (1 - \phi)\rho_{bf}C_{bf} + \phi\rho_pC_p,\end{aligned}\quad (5)$$

where the subscripts *bf* and *p* represent the base fluid and nanoparticle, respectively, and ϕ is the volume fraction of the nanoparticle.

Following Masoud et al. [13], the nanofluid viscosity is defined by

$$\begin{aligned}\frac{\mu_{nf}}{\mu_{bf}} &= \exp \left[m + \alpha \left(\frac{\bar{T}}{T_\infty} \right) \right. \\ &\quad \left. + \beta_1 \phi \left(1 + \frac{2r}{d_p} \right)^3 + \gamma_1 \left(\frac{d_p}{1+r} \right) \right].\end{aligned}\quad (6)$$

The thermal conductivity is determined by the new empirical correlations proposed by Hassani et al. [14], which is expressed as,

$$\begin{aligned}\frac{k_{nf}}{k_{bf}} &= \frac{k_p + 2k_{bf} - 2(k_{bf} - k_p)\phi}{k_p + 2k_{bf} + (k_{bf} - k_p)\phi} + 5 \\ &\quad \times 10^4 \frac{\beta_2 \phi \rho_{bf} C_{bf}}{k_{bf}} \sqrt{\frac{\kappa \bar{T}}{\rho_p d_p}} f(\bar{T}, \phi),\end{aligned}\quad (7)$$

where

$$\begin{aligned}f(\bar{T}, \phi) &= (2.8217 \times 10^{-2} \phi + 3.917 \times 10^{-3}) \left(\frac{\bar{T}}{T_\infty} \right) \\ &\quad - (3.0669 \times 10^{-2} \phi + 3.91123 \times 10^{-3}),\end{aligned}$$

where μ is the viscosity, k is the thermal conductivity, m is a factor that depends on the nanoparticles, the base fluid and their interaction, α , β_1 , β_2 and γ_1 are empirical parameters determined from experimental data, d_p is the diameter of the nanoparticle, r is the capping layer thickness and κ is the Boltzmann constant. The values of these empirical parameters are given in Table 1, and the thermo-physical properties of water and aluminium oxide Al_2O_3 are presented in Table 2.

Table 1 Empirical parameters for aluminium oxide Al_2O_3 –water nanofluid [13, 14]

m	α	β_1	γ_1	β_2	Volume fraction
0.72	−0.485	14.94	0.0105	$8.4407(100\phi)^{-1.07304}$	$1\% \leq \phi \leq 10\%$

Table 2 Thermo-physical properties of water and aluminium oxide Al_2O_3

	ρ	c_p (J kg ^{−1} K)	k (Wm ^{−1} K)
Water fluid	997.1	4179	0.613
Al_2O_3	3970	765	40

Equations (1)–(4) are nondimensionalized by introducing the following dimensionless variables:

$$\begin{aligned}x &= \frac{\bar{x}}{L}, \quad y = \frac{\bar{y}}{L} \left(\frac{U_0 \rho_{bf} L}{\mu_{bf}} \right)^{\frac{1}{2}}, \quad u = \frac{\bar{u}}{U_0}, \\ v &= \frac{\bar{v}}{U_0} \left(\frac{U_0 \rho_{bf} L}{\mu_{bf}} \right)^{\frac{1}{2}}, \quad T = \frac{\bar{T} - T_\infty}{T_w - T_\infty}.\end{aligned}\quad (8)$$

We define the stream function ψ as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}.\quad (9)$$

Substituting Eqs. (8) and (9) into Eqs. (2)–(4) yields:

$$\begin{aligned}\frac{\partial}{\partial y} \left(\exp(A_1 + \alpha[1 + \Delta T]) \frac{\partial^2 \psi}{\partial y^2} \right) + \lambda \frac{\partial^3 \psi}{\partial y^3} - \lambda \delta \left(\frac{\partial^2 \psi}{\partial y^2} \right)^2 \frac{\partial^3 \psi}{\partial y^3} \\ - A_2 \left(\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} \right) = 0,\end{aligned}\quad (10)$$

$$\begin{aligned}\frac{\partial}{\partial y} \left([A_3 + A_4[1 + \Delta T]^{\frac{3}{2}} - A_5[1 + \Delta T]^{\frac{1}{2}}] \frac{\partial T}{\partial y} \right) \\ + \text{Nr} \frac{\partial}{\partial y} \left([1 + \Delta T]^3 \frac{\partial T}{\partial y} \right) - \text{Pr} A_6 \left(\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right) = 0\end{aligned}\quad (11)$$

$$\begin{aligned}y = 0 : \quad \frac{\partial \psi}{\partial y} = u_w(x), \quad \frac{\partial \psi}{\partial x} = F_w v_w(x), \quad T = 1, \\ y \rightarrow \infty : \quad \frac{\partial \psi}{\partial y} \rightarrow 0, \quad T \rightarrow 0.\end{aligned}\quad (12)$$

where λ and δ are fluid parameters, Pr is the Prandtl number, Nr is the thermal radiation parameter, Δ is the temperature difference, F_w is the suction or injection parameter and A_i , ($i = 1, \dots, 5$) are constants. These parameters and constants are defined as

$$\begin{aligned}
A_1 &= m + \beta_1 \phi \left(1 + \frac{2r}{d_p} \right)^3 + \gamma_1 \left(\frac{d_p}{1+r} \right), \\
A_2 &= \left((1-\phi) + \frac{\rho_p}{\rho_{bf}} \phi \right), \quad A_3 = \frac{k_p + 2k_{bf} - 2(k_{bf} - k_p)\phi}{k_p + 2k_{bf} + (k_{bf} - k_p)\phi}, \\
F_w &= -V_0 \left(\frac{L\rho_f}{U_0\mu_\infty} \right) \\
A_4 &= (1.4109 \times 10^3 \phi^2 + 1.9585 \times 10^2 \phi) \frac{\beta_2 \rho_{bf} C_{bf}}{k_{bf}} \sqrt{\frac{\kappa T_\infty}{\rho_p d_p}}, \\
A_5 &= (1.5334 \times 10^3 \phi^2 + 1.9556 \times 10^2 \phi) \frac{\beta_2 \rho_{bf} C_{bf}}{k_{bf}} \sqrt{\frac{\kappa T_\infty}{\rho_p d_p}}, \\
A_6 &= (1-\phi) + \frac{\rho_p C_p}{\rho_{bf} C_{bf}} \phi, \quad \lambda = \frac{1}{\mu_{bf} \beta \gamma}, \quad \delta = \frac{U_0^3 \rho_{bf}}{2LC^2 \mu_{bf}}, \\
Pr &= \frac{C_{bf} \mu_{bf}}{k_{bf}}, \quad Nr = \frac{16\sigma_s T_\infty^3}{3k_m k_{bf}}, \quad \Delta = \theta_w - 1, \quad \theta_w = \frac{T_w}{T_\infty}.
\end{aligned} \quad (13)$$

3 Lie symmetry analysis

In this section, we seek the similarity solution to Eqs. (10)–(12) using the Lie symmetry group approach. Finding symmetry group is equivalent to finding the infinitesimal generator that renders Eqs. (10)–(12) invariant. We consider a one-parameter group Lie group of infinitesimal transformations with Lie group parameter ϖ defined as:

$$\begin{aligned}
x^* &= x + \varpi \xi_1(s, y, \psi, T) + \mathcal{O}(\varpi^2), \\
y^* &= y + \varpi \xi_2(s, y, \psi, T) + \mathcal{O}(\varpi^2), \\
\psi^* &= \psi + \varpi \varphi_1(s, y, \psi, T) + \mathcal{O}(\varpi^2), \\
T^* &= T + \varpi \varphi_2(s, y, \psi, T) + \mathcal{O}(\varpi^2),
\end{aligned} \quad (14)$$

the infinitesimal generator is prolonged to first, second and third derivatives, and it is defined by:

$$\begin{aligned}
X &= \xi_1 \frac{\partial}{\partial x} + \xi_2 \frac{\partial}{\partial y} + \varphi_1 \frac{\partial}{\partial \psi} + \varphi_2 \frac{\partial}{\partial T} + \varphi_{1x} \frac{\partial}{\partial \psi_x} + \varphi_{1y} \frac{\partial}{\partial \psi_y} \\
&\quad + \varphi_{1xy} \frac{\partial}{\partial \psi_{xy}} + \varphi_{1yy} \frac{\partial}{\partial \psi_{yy}} + \varphi_{1yyy} \frac{\partial}{\partial \psi_{yyy}} \\
&\quad + \varphi_{2x} \frac{\partial}{\partial T_x} + \varphi_{2y} \frac{\partial}{\partial T_y} + \varphi_{2yy} \frac{\partial}{\partial T_{yy}},
\end{aligned} \quad (15)$$

where

$$\begin{aligned}
\varphi_{1R} &= D_R \varphi_1 - \psi_x D_R \xi_1 - \psi_y D_R \xi_2, \\
\varphi_{2R} &= D_R \varphi_2 - \varphi_{2x} D_R \xi_1 - \varphi_{2y} D_R \xi_2, \\
\varphi_{1SR} &= D_R \varphi_{1S} - \psi_{Sx} D_R \xi_1 - \psi_{Sy} D_R \xi_2, \\
\varphi_{2SR} &= D_R \varphi_{2S} - \psi_{Sx} D_R \xi_1 - \psi_{Sy} D_R \xi_2, \\
D_x &= \frac{\partial}{\partial x} + \psi_x \frac{\partial}{\partial \psi} + T_x \frac{\partial}{\partial x} + \psi_{xx} \frac{\partial}{\partial \psi_x} \\
&\quad + T_{xx} \frac{\partial}{\partial T_x} + \psi_{xy} \frac{\partial}{\partial \psi_y} + \dots, \\
D_y &= \frac{\partial}{\partial y} + \psi_y \frac{\partial}{\partial \psi} + T_y \frac{\partial}{\partial y} + \psi_{yy} \frac{\partial}{\partial \psi_y} \\
&\quad + T_{yy} \frac{\partial}{\partial T_y} + \psi_{xy} \frac{\partial}{\partial \psi_x} + \dots,
\end{aligned} \quad (16)$$

R, S represent either x or y depending on the component.

Using X on Eqs. (10) and (11), we obtain the following infinitesimals after some algebraic simplification:

$$\xi_1 = c_1 x + c_2, \quad \xi_2 = \frac{1}{3} c_1 y + g(x), \quad \varphi_1 = \frac{2}{3} c_1 \psi + c_3, \quad \varphi_2 = 0. \quad (17)$$

where $c_j (j = 1, \dots, 3)$ are arbitrary constants. Thus, we obtain a three-dimensional space of operator. By setting $g(x) = 0$ and any of two the constants to zero, the following infinitesimal generators can be obtained:

$$X_1 = x \frac{\partial}{\partial x} + \frac{y}{3} \frac{\partial}{\partial y} + \frac{2}{3} \frac{\partial}{\partial \psi}, \quad X_2 = \frac{\partial}{\partial x}, \quad X_3 = \frac{\partial}{\partial \psi}. \quad (18)$$

The implication of Eq. (18) is that Eqs. (10) and (11) admit three one-parameter transformation groups. X_1 corresponds to scaling group of transformation, while X_2 and X_3 are translation groups of transformation.

Applying X to the boundary conditions, Eq. (12) yields:

$$\begin{aligned}
\frac{du_w}{dx} &= \frac{c_1}{3(c_1 x + c_2)} u_w, \\
\frac{dv_w}{dx} &= -\frac{c_2}{3(c_1 x + c_2)} v_w,
\end{aligned} \quad (19)$$

which implies that:

$$\begin{aligned}
u_w(x) &= k_1 (c_1 x + c_2)^{\frac{1}{3}}, \\
v_w(x) &= k_2 (c_1 x + c_2)^{-\frac{1}{3}},
\end{aligned} \quad (20)$$

where k_1 and k_2 are constants of integration.

Obviously, X_2 and X_3 do not have an invariant solution; hence, we consider only X_3 . Under X_3 , Eq. (20) is transformed into:

$$u_x(x) = x^{\frac{1}{3}} \text{ and } v_w(x) = x^{-\frac{1}{3}}. \quad (21)$$

The characteristic equation to X_1 is:

$$\frac{dx}{x} = \frac{3dy}{y} = \frac{3d\psi}{2\psi} = \frac{dT}{0}, \quad (22)$$

on solving Eq. (22), we obtain the following similarity variables and function:

$$\eta = yx^{-\frac{1}{3}}, \quad \psi = x^{\frac{2}{3}}f(\eta), \quad T = \theta(\eta). \quad (23)$$

Finally, substituting Eqs. (21) and (22) into Eqs. (10) and (12) yields:

$$\begin{aligned} &(\exp(A_1 + \alpha(1 + \Delta\theta)) + \lambda)f'''' + \alpha\Delta \exp(A_1 + \alpha(1 + \Delta\theta))\theta'f'' \\ &- \lambda\delta(f'')^2f''' + A_2\left(\frac{2}{3}ff'' - \frac{1}{3}f'^2\right) = 0, \end{aligned} \quad (24)$$

$$\begin{aligned} &\left(A_3 + A_4(1 + \Delta\theta)^{\frac{3}{2}} - A_5(1 + \Delta\theta)^{\frac{1}{2}} + \text{Nr}(1 + \Delta\theta)^3\right)\theta'' \\ &+ \Delta\left(\frac{3A_4}{2}(1 + \Delta\theta)^{\frac{1}{2}} - \frac{A_5}{2}(1 + \Delta\theta)^{-\frac{1}{2}} + 3\text{Nr}(1 + \Delta\theta)^2\right)\theta'^2 \\ &+ \frac{2}{3}\text{Pr}A_6f\theta' = 0 \end{aligned} \quad (25)$$

$$\begin{aligned} \eta = 0 : \quad &f' = 1, \quad f = F_w, \quad \theta = 1, \\ \eta \rightarrow \infty : \quad &f' \rightarrow 0, \quad \theta \rightarrow 0. \end{aligned} \quad (26)$$

Pantokratoras [29] pointed out that assuming a uniform Prandtl number when thermo-physical properties are temperature dependent may lead to unrealistic results. Hence, to take care of this paradox, the Prandtl number of the nanofluid is defined as

$$\begin{aligned} \text{Pr}_v &= \frac{\mu_{nf}C_{nf}}{k_{nf}} = \frac{A_6 \exp(A_1 + \alpha(1 + \Delta\theta))}{A_3 + A_4(1 + \Delta\theta)^{\frac{3}{2}} - A_5(1 + \Delta\theta)^{\frac{1}{2}}} \\ &\frac{C_{bf}\mu_{bf}}{k_{bf}} \\ &= \frac{\mu_{nf}C_{nf}}{k_{nf}} = \frac{A_6 \exp(A_1 + \alpha(1 + \Delta\theta))}{A_3 + A_4(1 + \Delta\theta)^{\frac{3}{2}} - A_5(1 + \Delta\theta)^{\frac{1}{2}}} \text{Pr}. \end{aligned} \quad (27)$$

Using Eq. (27) in Eq. (25) gives a modified thermal boundary layer equation with a variable Prandtl number, that is,

$$\begin{aligned} &\left(A_3 + A_4(1 + \Delta\theta)^{\frac{3}{2}} - A_5(1 + \Delta\theta)^{\frac{1}{2}} + \text{Nr}(1 + \Delta\theta)^3\right)\theta'' \\ &+ \Delta\left(\frac{3A_4}{2}(1 + \Delta\theta)^{\frac{1}{2}} - \frac{A_5}{2}(1 + \Delta\theta)^{-\frac{1}{2}} + 3\text{Nr}(1 + \Delta\theta)^2\right)\theta'^2 \\ &+ \frac{2}{3}\text{Pr}_v f\theta' \left(\frac{A_3 + A_4(1 + \Delta\theta)^{\frac{3}{2}} - A_5(1 + \Delta\theta)^{\frac{1}{2}}}{\exp(A_1 + \alpha(1 + \Delta\theta))}\right) = 0 \end{aligned} \quad (28)$$

The skin friction coefficient C_f and the Nusselt number Nu_x which are of interest in thermal engineering design are defined as follows:

$$C_f = \frac{\tau_w}{\rho_f U_0^2} \text{ and } \text{Nu}_x = \frac{\left(\frac{x}{L}\right)q_w}{k_{nf}(\bar{T})(T_w - T_\infty)} \quad (29)$$

and the wall shear stress τ_w and the wall flux q_w are expressed as:

$$\begin{aligned} \tau_w &= \left(\mu_{nf} + \frac{1}{\beta\gamma}\right)\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right) - \frac{1}{6\beta\gamma^3}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^3 \Big|_{\bar{y}=0} \\ \text{and } q_w &= -\left(k_{nf} + \frac{16\sigma_s \bar{T}^3}{3k_m}\right)\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right) \Big|_{\bar{y}=0}. \end{aligned} \quad (30)$$

Equation (29) can be written as:

$$C_f(Re)^{\frac{1}{2}} = (\exp(A_1 + 2\alpha) + \lambda)f''(0) - \frac{\lambda\delta}{3}(f''(0))^3, \quad (31)$$

$$\text{Nu}_x\left(\frac{x}{L}\right)(Re)^{-\frac{1}{2}} = -\left(1 + \frac{\text{Nr}\theta_w^3}{A_7}\right)\theta'(0). \quad (32)$$

where $Re = \frac{U_0 L}{\nu_\infty}$ represents the local Reynolds number and $A_7 = A_3 + A_4\theta_w^{\frac{3}{2}} - A_5\theta_w^{\frac{1}{2}}$.

4 Method of solution

Spectral methods are a class of numerical methods used to solve differential equations arising in applied mathematics, science and engineering. The name spectral methods is derived from the fact that the solution is expressed as a

series of orthogonal eigenfunctions of some linear operator. Although the SQLM has a very high convergence, it is limited to cases where there is only one independent variable. In this section, an efficient iterative spectral local linearization method (SLLM) which was proposed by Motsa [30] is used to numerically integrate the coupled nonlinear differential Eqs. (24) and (28) with the boundary condition Eqs. (26). To apply this technique, we consider the following nonlinear differential operators

$$\Omega_f = (\exp(A_1 + \alpha(1 + \Delta\theta_n)) + \lambda)f_n''' + \alpha\Delta \exp(A_1 + \alpha(1 + \Delta\theta_n))\theta_n' f_n'' - \lambda\delta(f_n'')^2 f_n''' + A_2\left(\frac{2}{3}f_n f_n'' - \frac{1}{3}f_n'^2\right) \quad (33)$$

$$\Omega_\theta = (A_3 + A_4(1 + \Delta\theta_n)^{\frac{3}{2}} - A_5(1 + \Delta\theta_n)^{\frac{1}{2}} + \text{Nr}(1 + \Delta\theta_n)^3)\theta_n'' + \Delta\left(\frac{3A_4}{2}(1 + \Delta\theta_n)^{\frac{1}{2}} - \frac{A_5}{2}(1 + \Delta\theta_n)^{-\frac{1}{2}} + 3\text{Nr}(1 + \Delta\theta_n)^2\right)\theta_n'^2 + \frac{2}{3}\text{Pr}_v f_n \theta_n' (A_3 + A_4(1 + \Delta\theta_n)^{\frac{3}{2}} - A_5(1 + \Delta\theta_n)^{\frac{1}{2}}) \exp(-A_1 - \alpha(1 + \Delta\theta_n)) = 0; \quad (34)$$

Eqs. (33) and (34) can be decoupled according to the following algorithm:

1. From Eq. (33) Ω_f , solve f_{n+1} assuming that θ_n is known from a previous iteration.
2. Solve θ_{n+1} from Eq. (34) Ω_θ using the updated solution of f_n .
3. Subsequent iterative solutions are obtained by repeating steps 1 and 2.

In the framework of the SLLM, the following iterative scheme is obtained

$$a_{1,n}f_{n+1}''' + a_{2,n}f_{n+1}'' + a_{3,n}f_{n+1}' + a_{4,n}f_{n+1} = R^f, \quad (35)$$

$$a_{5,n}\theta_{n+1}'' + a_{6,n}\theta_{n+1}' + a_{7,n}\theta_{n+1} = R^\theta, \quad (36)$$

$$f_{n+1}(0) = F_w, \quad f_{n+1}'(0) = 1, \quad f_{n+1}'(\infty) = 0, \quad (37)$$

$$\theta_{n+1}(0) = 1, \quad \theta_{n+1}(\infty) = 0.$$

The coefficients in Eqs. (35) and (36) along with their right-hand sides are defined as follows:

$$\begin{aligned} a_{1,n} &= \frac{\partial \Omega_f}{\partial f_n'''} = A_1 \exp(A_1 + \alpha(1 + \Delta\theta_n)) \\ &\quad + \lambda\left(1 - \delta f_n''^2\right), \quad a_{3,n} = \frac{\partial \Omega_f}{\partial f_n'} \\ &= -2/3 A_2 f_n', \\ a_{2,n} &= \frac{\partial \Omega_f}{\partial f_n''} = \alpha\Delta \exp(A_1 + \alpha(1 + \Delta\theta_n))\theta_n' - 2\delta\lambda f_n'' f_n''' \\ &\quad + \frac{2}{3} A_2 f_n', \quad a_{4,n} = \frac{\partial \Omega_f}{\partial f_n} = \frac{2}{3} A_2 f_n'', \\ a_{5,n} &= \frac{\partial \Omega_\theta}{\partial \theta_n''} = A_3 + A_4(\Delta\theta_n + 1)^{\frac{3}{2}} - A_5(\Delta\theta_n + 1)^{\frac{1}{2}} \\ &\quad + \text{Nr}(\Delta\theta_n + 1)^3 \\ a_{6,n} &= \frac{\partial \Omega_\theta}{\partial \theta_n'} = 2\Delta\left(\frac{3}{2}A_4(\Delta\theta_n + 1)^{\frac{1}{2}} - \frac{1}{2}A_5(\Delta\theta_n + 1)^{-\frac{1}{2}}\right. \\ &\quad \left.+ 3\text{Nr}(\Delta\theta_n + 1)^2\right)\theta_n' + \frac{2}{3}\text{Pr}_v f_n (A_3 + A_4(\Delta\theta_n + 1)^{\frac{3}{2}} \\ &\quad \left.- A_5(\Delta\theta_n + 1)^{\frac{1}{2}}\right) \exp(-A_1 - \alpha(1 + \Delta\theta_n)) \\ a_{7,n} &= \frac{\partial \Omega_\theta}{\partial \theta_n} = \Delta\left(\frac{3}{2}A_4(1 + \Delta\theta_n)^{\frac{1}{2}} - \frac{1}{2}A_5(\Delta\theta_n + 1)^{-\frac{1}{2}}\right. \\ &\quad \left.+ 3\text{Nr}(\Delta\theta_n + 1)^2\right)\theta_n'' + \Delta^2\left(\frac{3}{4}A_4(1 + \Delta\theta_n)^{-\frac{1}{2}}\right. \\ &\quad \left.+ \frac{1}{4}A_5(\Delta\theta_n + 1)^{-\frac{3}{2}} + 6\text{Nr}(\Delta\theta_n + 1)\right)\theta_n'^2 \\ &\quad \left.+ \frac{1}{3}\text{Pr}_v f_n \theta_n' \Delta \exp(-A_1 - \alpha(1 + \Delta\theta_n))\left[3A_4(\Delta\theta_n + 1)^{\frac{1}{2}}\right. \right. \\ &\quad \left. \left.- A_5(\Delta\theta_n + 1)^{-\frac{1}{2}}\right. \right. \\ &\quad \left. \left.- 2\alpha\left(A_3 + A_4(\Delta\theta_n + 1)^{\frac{3}{2}} - A_5(\Delta\theta_n + 1)^{\frac{1}{2}}\right)\right]\right], \\ R^f &= a_{1,n}f_n''' + a_{2,n}f_n'' + a_{3,n}f_n' + a_{4,n}f_n - \Omega_f, \\ R^\theta &= a_{5,n}\theta_n'' + a_{6,n}\theta_n' + a_{7,n}\theta_n - \Omega_\theta. \end{aligned} \quad (38)$$

Equations (35)–(37) are integrated numerically using the Chebyshev pseudo-spectral technique (see Mondal et al. [31], Motsa et al. [32] and Maleki et al. [33]). In order to apply this technique, the semi-infinite domain $\eta \in [0, \infty)$ is replaced with a truncated domain $\eta \in [0, \varpi_\infty]$, where $\varpi_\infty \in \mathbb{Z}^+$ is a mapping parameter. Using the transformation $\eta = \frac{1}{2}(\xi + 1)\varpi_\infty$, the interval $[0, \varpi_\infty]$ is mapped to $[-1, 1]$ which the Chebyshev pseudo-spectral technique can be used. The unknown functions $f(\eta)$ and $\theta(\eta)$ are

discretized using the Chebyshev–Gauss–Lobatto collocation points:

$$\xi_k = \cos\left(\frac{\pi k}{\bar{N}}\right), \quad k = 0, 1, \dots, \bar{N}; \quad -1 \leq \xi \leq 1. \quad (39)$$

The derivatives of $f(\eta)$ and $\theta(\eta)$ are computed using the Chebyshev differentiation matrix D , at the collocation points as a matrix vector product, that is:

$$\frac{df}{d\eta} = \sum_{i=0}^{\bar{N}} D_{ij} f(\xi_i) = \mathbf{D}F, \quad j = 0, 1, 2, \dots, \bar{N}, \quad (40)$$

where $\bar{N} + 1$ is the number of collocation points, $\mathbf{D} = 2D/\varpi_\infty$ and $F = [f(\xi_0), f(\xi_1), \dots, f(\xi_{\bar{N}})]^T$ is a vector function at the collocation point.

Let Θ be a similarity vector function representing θ . The higher-order derivatives of f and θ are evaluated as powers of $\mathbf{D}$, that is

$$f^s(\eta) = \mathbf{D}^s F, \quad \theta^s(\eta) = \mathbf{D}^s \Theta. \quad (41)$$

Substituting Eqs. (39)–(41) into Eqs. (35)–(37) yields the following decoupled matrices

$$\begin{bmatrix} \mathbf{D}_{1,1} & \dots & \mathbf{D}_{1,\bar{N}+1} \\ \text{diag}[a_{1,n}]\mathbf{D}^3 + \text{diag}[a_{2,n}]\mathbf{D}^2 + \text{diag}[a_{3,n}]\mathbf{D} + \text{diag}[a_{4,n}]\mathbf{I} & & \\ \mathbf{D}_{\bar{N},1} & \dots & \mathbf{D}_{\bar{N},\bar{N}+1} \\ 0 & \dots & 1 \end{bmatrix} \times \begin{bmatrix} f_{n+1}(\xi_0) \\ f_{n+1}(\xi_1) \\ \vdots \\ f_{n+1}(\xi_{\bar{N}-1}) \\ f_{n+1}(\xi_{\bar{N}}) \end{bmatrix} = \begin{bmatrix} 0 \\ R_{n+1}^f(\xi_1) \\ \vdots \\ 1 \\ F_w \end{bmatrix}, \quad (42)$$

$$\begin{bmatrix} 1 & 0 & \dots & 0 \\ \text{diag}[a_{5,n}]\mathbf{D}^2 + \text{diag}[a_{6,n}]\mathbf{D} + \text{diag}[a_{7,n}]\mathbf{I} & & & \\ 0 & \dots & 1 & \end{bmatrix} \begin{bmatrix} \theta_{n+1}(\xi_0) \\ \vdots \\ \theta_{n+1}(\xi_{\bar{N}}) \end{bmatrix} = \begin{bmatrix} 0 \\ R_{n+1}^\theta(\xi_1) \\ \vdots \\ 1 \end{bmatrix} \quad (43)$$

Here $\mathbf{I}$ is an $(\bar{N} + 1) \times (\bar{N} + 1)$ identity matrix, and $\text{diag}[]$ denotes a diagonal matrix. A suitable initial approximation for the SLLM scheme is

$$f_0(\eta) = F_w + 1 - \exp(-\eta), \quad \theta_0(\eta) = \exp(-\eta), \quad (44)$$

5 Numerical validation

The cases $\lambda = A_1 = A_4 = A_5 = 0$, $\theta_w = A_2 = 1$ and $A_3 = 1$ correspond to a Newtonian fluid with constant viscosity, thermal conductivity and linear thermal radiation which has been studied by Jalil and Asghar [16] using the Keller-box method. To validate the correctness of the numerical results obtained from the iterative scheme given by Eqs. (35–37), the skin friction coefficient $f''(0)$ at the seventh iterates with $N = 80$ is compared with Jalil and Asghar [16] in Table 3. It is apparent that there is a good agreement between the two results.

6 Results and discussion

In this section, we give the results of the numerical simulation and the effects of the nanoparticle volume fraction, ϕ , fluid parameter, λ , suction/injection parameter, F_w , thermal radiation parameter, Nr , and temperature ratio parameter, θ_w on the nanofluid velocity profile, $f'(\eta)$, temperature profile, $\theta(\eta)$, skin friction coefficient and Nusselt number are performed and discussed. The range of parameters is as follows: $1\% \leq \phi \leq 10\%$, $0 \leq \lambda \leq 5$, $0 \leq \delta \leq 3$, $1.2 \leq \theta_w \leq 2.1$ and $-1 \leq F_w \leq 1$. (See Jalil and Asghar [16] and Sandeep and Gnaneswara [7]). The default parameter values used in simulating the velocity and temperature profiles are $\lambda = \delta = Nr = Fw = 1$, $\phi = 0.1$, $\theta_w = 1.5$ and $Pr_v = 6.96$. Therefore, where these parameter values are not explicitly stated, it will be understood that such a parameter is assigned the default value.

The nanoparticle volume fraction quantifies the amount of the Al_2O_3 nanoparticles contained in the synthesized nanofluid. The nanofluid velocity and temperature profiles for different values of nanoparticle volume fraction are displayed in Fig. 1. We observed that both the velocity and temperature profiles are enhanced with an increase in the value of nanoparticle volume fraction. Also, the

Table 3 Comparison of the SLLM results for $-f''(0)$ with Jalil and Asghar [16] for distinct values of F_w when $\lambda = A_1 = A_4 = A_5 = 0$, $\theta_w = A_2 = 1$ and $A_3 = 1$

F_w	$f''(0)$		
	Jalil and Asghar [16]	SLLM	Relative error
0.75	0.984436	0.98443940	0
0.50	0.873640	0.87364290	0
0	0.677647	0.67764824	0
−0.50	0.518869	0.51887049	0
−0.75	0.453523	0.45352500	0

Fig. 1 Effect of the nanoparticle volume fraction on the velocity and temperature profiles

momentum and thermal boundary layers become thicker with an increase in the nanoparticle volume fraction. Figure 2 shows the effect of the fluid parameter on the nanofluid velocity profiles. It is seen that with an increase in the fluid parameter, the nanofluid velocity profiles and

the momentum boundary layer thickness are enhanced. Physically, this is correct since the fluid parameter has an inverse relation with the nanofluid dynamic viscosity; thus, the fluid becomes less viscous with large value of the fluid parameter. Hence, the velocity profiles are enhanced. This

Fig. 2 Effect of the fluid parameter on the velocity profile

Fig. 3 Effect of the nanoparticle volume fraction on the temperature profile

Fig. 4 Effect of the suction/injection parameter on the velocity and temperature profile

finding is consistent with Javed et al. [23] for the case of a pure fluid.

Figure 3 shows the effect of the temperature ratio parameter on the nanofluid temperature profiles. We observed that the temperature profiles, as well as the

thermal boundary layer thickness, are enhanced with an increase in the value of the temperature ratio parameter. This trend is in agreement with Sandeep and Gnaneswara [7].

Fig. 5 Effect of the thermal radiation parameter on the temperature profile

Fig. 6 Effect of the fluid parameter, λ , on the skin friction coefficient for different values of the fluid parameter, δ

Fig. 7 Effect of the temperature ratio parameter on the Nusselt number for different values of nanoparticle volume fraction

Figure 4 shows the effect of the suction ($f_w > 0$) and injection ($f_w < 0$) parameter on the nanofluid velocity and temperature profiles. From Fig. 4a, it is seen that the

Fig. 8 Effect of the thermal radiation parameter on the Nusselt number for different values of nanoparticle volume fraction

velocity profiles, as well as the thickness of the momentum boundary layer, decrease with an increase in the suction parameter. However, the velocity and momentum

Table 4 The computed results for skin friction coefficient, C_f , and Nusselt number, Nu, for $\text{Al}_2\text{O}_3/\text{water}$ nanofluid for varying the parameters: ϕ , λ , δ , fw , θ_w and Nr

ϕ	λ	δ	fw	θ_w	Nr	C_f	Nu
0.01	1	1	1	1.5	1	-1.31566819	4.04218773
0.04						-1.53640389	2.69425801
0.07						-1.83309969	1.82571968
0.1						-2.23626267	1.25732143
	0					-2.05499683	1.24567815
	1					-2.23626267	1.25732143
	3					-2.55833356	1.27519931
	5					-2.84185954	1.28832024
		1				-2.23626267	1.25732143
		1.5				-2.23360279	1.25715057
		2				-2.23091741	1.25697829
		2.5				-2.22820586	1.25680457
			-1			-1.47135446	0.51499994
			-0.5			-1.64056501	0.67145981
			0.5			-2.02355151	1.04510263
			1			-2.23626267	1.25732143
				1.2		-2.03895232	1.11810439
				1.5		-2.23626267	1.25732143
				1.8		-2.43248202	1.41661565
				2.1		-2.62786066	1.59104731
					0	-2.27747031	0.97952469
					0.5	-2.25163761	1.14019626
					1	-2.23626267	1.25732143
					1.5	-2.22583993	1.34955641

boundary layers are enhanced with an increase in the injection parameter. Similarly, the temperature profiles and the thermal boundary layer thickness reduce with an increase in the suction parameter, whereas an opposite trend is observed for the case of injection as shown in Fig. 4b. This result is in agreement with the findings of Jalil and Asghar [16].

The impact of the thermal radiation parameter on the nanofluid temperature profiles is presented in Fig. 5. From this plot, the nanofluid temperature profile is an increasing function of the thermal radiation parameter. The physical reason for this observed trend is that, for a higher value of the radiation parameter, more heat is transferred to the nanofluid since the mean absorption coefficient k_m reduces with an increase in the radiation parameter. This temperature profile is similar to Ramzan et al. [27] for the case of pure fluid.

Figure 6 displays the effect of the fluid parameter, λ , on the skin friction for distinct values of the fluid parameter, δ . Clearly, it is seen that the skin friction coefficient increases with an increase δ . The influence of the temperature ratio parameter and the thermal radiation parameter on the Nusselt number is displayed in Figs. 7 and 8. It is evident from these plots that the Nusselt number increases with an increase in the values of θ_w and N_r (Table 4).

7 Conclusions

The flow and heat transfer in a Powell–Eyring nanofluid flow past a stretching surface have been studied. The similarity solution to the model describing the nanofluid flow and energy balance was found using the Lie group analysis. An iterative spectral local linearization method was used to solve the conservation equations. The effects of nanoparticles, thermal radiation and suction/injection have been considered in the problem. The effects of these parameters on the nanofluid velocity and temperature profiles, as well as the skin friction coefficient and Nusselt number, are determined and discussed. A summary of the results of the study is as follows:

1. The velocity profiles are enhanced with an increase in the fluid parameter, λ and nanoparticle volume fraction.
2. The velocity and temperature profiles are decreasing functions of the suction parameter, while injection shows an opposite trend.
3. An increase in the nanoparticle volume fraction, thermal radiation parameter and temperature ratio parameter enhances the thermal boundary layer thickness as well as the temperature profiles.

4. The skin friction coefficient increases with an increase in the fluid parameter, δ .
5. Increasing the temperature ratio parameter and thermal radiation parameter increases the Nusselt number.

Funding The authors are grateful to the University of KwaZulu-Natal, South Africa, for financial support.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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