Effect of magnetite nanoparticles on couple stress fluid between two parallel squeezing and expanding surfaces

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Received: 8 August 2019 / Accepted: 3 December 2019 / Published online: 8 April 2020
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Abstract

This article investigates the analysis of hydromagnetic flow of a couple stress fluid through two parallel squeezing surfaces where magnetite nanoparticles are suspended in it. Here ethylene glycol is considered as the base fluid. The dissipations due to magnetic field, viscous and couple stress fluid are considered in the energy equation. The modelled governing differential equations are transformed into dimensionless form of nonlinear coupled ordinary differential equations with the aid of similarity transformations. A shooting method with Runge–Kutta 4th order is utilized to solve the system of reduced flow field equations. The influence of various parameters on the velocity (axial and radial) components and temperature are studied for squeezing and expanding cases for distinct physical parameter through graphs. The results indicate that the axial velocity and temperature show increasing trend for augmented values of couple stress fluid parameter. Further, radial velocity and temperature also depict augmenting trend for all increasing values of Hartmann number.

Keywords Couple stress fluid · Squeezing flow · Ethylene glycol · Magnetite nanoparticles · Shooting method

PACS Nos. 47.15.-x · 47.10.ad · 44.90.+c · 47.60.+i · 47.65.-d · 02.60.cb

1 Introduction

Nowadays the analysis of nanofluid research is a popular area and this interest is escalating due to their essential applicability in modern industries. One of the major complications in improving the heat transfer of industrial systems; for example, electronic cooling and solar stations, is the restriction over the thermal properties of the cooling fluids. The conventional base fluids such as water, ethylene glycol, engine oil, mineral oils and glycerol have less thermal conductivity and they alone are insufficient for achieving requirements of high cooling rate. To solve these kinds of problems, the pioneers like Choi and Eastman [1] introduced new category of fluids and termed as 'nanofluid'. Nanofluids are synthesized by dispersing solid

nanoparticles in the base fluids. These fluids have more thermal conductivity and heat transfer efficiency due to the suspension of nanometer-sized particles in the above-mentioned conventional fluids [2, 3]. Some nanometer-sized particles, such as metals (Au, Cu, Ag), carbide ceramics, carbon oxide ceramics (Al_2O_3 , CuO), metal nitrides (AlN, SiN), TiO_2 , Fe, and SiO_2 , can be used as main components of nanofluid due to its applicability in medical and engineering field [4]. The use of nanofluids in some industrial applications like engines cooling systems, magnetic field sensors, nuclear reactors, magnetic storage media, transportation, and the cooling systems of electronics, could be a promising solution in attaining the required heat transfer capabilities. This improved thermal performance of nanofluids can give a betterment in a number of

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industrialized sectors including fuel cells, transportation, microelectronics, micro-manufacturing, pharmaceutical processes, chemical and metallurgical sectors, along with cancer therapy and it is also a safer option in surgery by cooling, biomedicine, and many more [5–7]. However, in the literature there are various number of nanoparticles and base fluids have been explored to analyse different characteristics of nanofluids. Among those Fe_3O_4 nanofluids, is an essential one and it has been extensively studied and applied into various fields of research. Fe_3O_4 nanofluid has the capability to retort to an external applied magnetic field and it reveals magnetic and liquid properties that are developed in a diverse of applications in cooling systems of micro devices [8].

The heat transfer study of time-dependent squeezing flow between parallel plates is one among utmost significant research areas owing to its widespread in engineering and scientific applications, such as power transmission systems, aerodynamic heating, compression, polymer processing, modelling the lubrication equipment, hydrodynamical machines, injection moulding, animal joints, bio-lubrication joints, destruction of crops caused by freezing, synovial joints, thrust bearing, chemical processing, automotive components, lubrication system etc. These squeezing flows are made by employing vertical velocities or normal stresses via moving boundary, which could lead to different hydro-dynamical technologies and tools. The earliest analysis on the squeezing flow was described by Stefan [9]. Following Stefan, many investigators have considered squeezing flow concept for theoretical and experimental study [10–18]. These investigations are restricted to Newtonian fluid. However, in the evolution of modern engineering, the role of non-Newtonian squeeze-flow is vital. Numerous scholars have analysed the squeezing flow problem with various non-Newtonian fluid models [19–25]. Amongst of these non-Newtonian models, couple stress fluid model proposed by Stoke's [26] has been given considerable attention by various investigators in field of fluid mechanics. This model sufficiently describes the fluid-flow behaviour of some polymer additives, animal blood and liquid crystals substructure. The stresses in this fluid model are formed due to mechanical interaction of in the medium of fluid. Colloidal fluids, blood, polymer thickened oils, polymer suspensions, synthetic fluids, liquid crystals, lubrications and additive electro-rheological fluids, etc. are some of the examples and importance of this fluid model can be widely seen in the analysing axisymmetric flow problems [27], lubrication theory [28], thin film [29], peristaltic transport [30], squeeze film [31] and many more.

Another topic portrayed in this article is the analysis of magnetohydrodynamic (MHD). This theory explores dynamics of electrically conducting fluids, for instance, salt water, molten metal liquid metals, and plasma. MHD is a

flux concept which can be expressed in terms of parameters of flow-field variables, pressure, density and applied magnetic fields. MHD spectroscopy, microfluidic devices, electromagnetic stirring, Jet printer, MHD pump, microelectronics, MHD generators, MHD dynamo and electromagnetic valves, etc. are some of the applications. Also, the squeezing flow with MHD effect leads to significant applications particularly in the field of chemical industry, liquid–metal lubrications, and fusion reactors for cooling of nuclear reactor, etc.,. The effect of MHD on squeezing flow for various geometries and explored the importance of these flows [32–38]. Recently, Qayyum et al. [39] examined the MHD squeezing flow of non-Newtonian fluid in a porous medium. Ghadikolaei et al. [40] investigated the magnetohydrodynamic squeezing flow, heat and mass transfer of Eyring-Powell fluid between two parallel plates. Muhammad et al. [41] analysed the hydromagnetic transient squeezing flow of viscoelastic fluid between walls. Similar unsteady MHD squeezing flow problem for radiative non-Newtonian fluid in a channel was examined by Adesanya [42]. Hayat et al. [43] considered the squeezing flow of couple stress fluid in a parallel plate channel where the effective electrical conductivity is not considered. Nagler [44] examined the Jeffrey-Hamel flow of nanofluid inside a wedge analytically. Recently, Ahmad et al. [45] explored the comparative study on MHD squeezing flow problem with entropy generation. He considered water as base fluid with five different nanoparticles. Aforementioned literature shows that no endeavour has been made on time-dependent hydromagnetic flow of couple stress fluid in a squeezing and expanding channel with magnetite (Fe_3O_4) nanoparticles.

2 Physical model

Consider an unsteady 2-D hydromagnetic squeezing flow of couple stress fluid with suspended Fe_3O_4 nanoparticles in ethylene glycol in a parallel plate channel as depicted in Fig. 1. The distance between two plates is

Fig. 1 Geometry of the problem

considered to be function of time $h(t) = \sqrt{\frac{v(1-\gamma t)}{a}}$. The lower plate is considered as permeable stretching surface and upper plate is moving with velocity $\frac{\gamma}{2} \sqrt{\frac{v}{a(1-\gamma t)}}$ towards the lower plate. The fluid flow is presumed to be incompressible and it is electrically conductive in nature. The presence time dependent magnetic field $\frac{B_0}{(1-\gamma t)}$ which applied in the Z-direction has an effect on the momentum and temperature across the channel. The governing conservation equations are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\rho_{nf} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} \right) = -\frac{\partial p}{\partial x} + \mu_{nf} \nabla^2 u - \eta_1 \nabla^4 u - \frac{\sigma_{nf} B_0^2 u}{(1-\gamma t)} \quad (2)$$

$$\rho_{nf} \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial v}{\partial t} \right) = -\frac{\partial p}{\partial y} + \mu_{nf} \nabla^2 v - \eta_1 \nabla^4 v - \frac{\sigma_{nf} B_0^2 v}{(1-\gamma t)} \quad (3)$$

$$\begin{aligned} \rho_{nf} c_{nf} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \frac{\partial T}{\partial t} \right) &= k_{nf} \nabla^2 T + \eta_1 \left((\nabla^2 u)^2 + (\nabla^2 v)^2 \right) \\ &+ 4\mu_{nf} \left(\frac{\partial u}{\partial x} \right)^2 + \mu_{nf} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \\ &+ \frac{\sigma_{nf} B_0^2 (u^2 + v^2)}{(1-\gamma t)} \end{aligned} \quad (4)$$

with associated boundary conditions are given by:

$y = 0$:

$$T = T_0, \quad v = \frac{-V_0}{1-\gamma t}, \quad u = U_0 = \frac{ax}{1-\gamma t}, \quad \frac{\partial u}{\partial y} = 0$$

$y = h(t)$:

$$T = T_0 + \frac{T_0}{1-\gamma t}, \quad v = -\frac{\gamma}{2} \sqrt{\frac{v}{a(1-\gamma t)}}, \quad u = 0, \quad \frac{\partial u}{\partial y} = 0 \quad (5)$$

In the above equation u and v are the velocities along the horizontal and vertical directions respectively, p is the pressure, T is the temperature and η_1 is the couple stress viscosity parameter, T_0 denotes lower stretching porous surface temperature, 'a' stands for lower surface stretching rate, $V_0 > 0$ and $V_0 < 0$ shows the suction

and injection velocity, respectively. The parameters namely, μ_{nf} , ρ_{nf} , $(\rho c_p)_{nf}$, k_{nf} and σ_{nf} respectively denote nanofluid viscosity, density, specific heat capacity, thermal conductivity and electrical conductivity.

The relation between the viscosity of the nanofluid and the base fluid as given by Brinkman [46]

$$\mu_{nf} = \frac{\mu_f}{\sqrt{(1-\phi)^{2.5}}} \quad (6)$$

Following Tiwari and Das [47], the effective heat capacity and effective density are expressed as

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \quad (\rho c)_{nf} = (1-\phi)(\rho c)_f + \phi(\rho c)_s \quad (7)$$

The effective thermal conductivity of nanofluid as (Maxwell–Garnett model by following Khanafer et al. [48])

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)} \quad (8)$$

Following Maxwell [49], the effective electrical conductivity of nanofluid is given by

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\frac{\sigma_s}{\sigma_f} + 2 - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi} \quad (9)$$

ϕ denotes the nanoparticle volume fraction. ρ_s , c_s , k_s , σ_s and ρ_f , c_f , k_f , σ_f are density, specific heat, thermal conductivity, electric conductivity of nanoparticles and base fluid respectively.

Introducing following similarity transformations [43] leads to the study in one variable (η).

$$\begin{aligned} T &= T_0 + \frac{T_0}{1-\gamma t} \theta(\eta), \quad u = U_0 f'(\eta), \\ v &= -\sqrt{\frac{av}{1-\gamma t}} f(\eta) \quad \text{where} \quad \eta = \frac{y}{h(t)} \end{aligned} \quad (10)$$

Substituting (6)–(10) into (2), (3) and (4), we obtain

$$\begin{aligned} f^{vi} &= \frac{1}{\beta(1-\phi)^{2.5}} (f^{iv} - A_2 Ha^2 (1-\phi)^{2.5} f'') \\ &- \frac{A_1}{\beta} \left(\frac{S_q}{2} (3f'' + \eta f''') + f' f'' - f f''' \right) \end{aligned} \quad (11)$$

$$\begin{aligned} \theta'' &= \frac{\text{Pr} A_3}{A_4} \left(S_q \left(\theta + \frac{\eta \theta'}{2} \right) - f \theta' \right) - \frac{\text{Pr} Ec}{A_4 \sqrt{(1-\phi)^5}} (4f'^2 + \delta^2 f''^2) - \frac{\beta \text{Pr} Ec}{A_4} (f''^2 + \delta^2 f'''^2) \\ &- \frac{A_2 \text{Pr} Ec Ha^2}{A_4} (f^2 + \delta^2 f'^2) \end{aligned} \quad (12)$$

Table 1 Thermophysical properties of base fluids and nanoparticles

	ρ (kg m ⁻³)	c_p (J kg ⁻¹ K ⁻¹)	k (W m ⁻¹ K ⁻¹)	σ (Ω m) ⁻¹
Ethylene glycol	1132	2349	0.258	3.3×10^{-4}
Fe ₃ O ₄	5180	670	9.7	25,000

where $S_q = \frac{\gamma}{a}$ is the squeezing number which mainly depends on the motion of the plates, $\beta = \frac{\eta_1 a}{v_f^2 \rho_f (1 - \gamma t)}$ is the couple stress fluid parameter, $Pr = \frac{\mu_f c_f}{k_f}$ is Prandtl number, $Ha^2 = \frac{\sigma_f B_0^2}{\rho_f a}$ is Hartmann number, $Ec = \frac{v_f a}{c_f T_0}$ is Eckert number. A_1 , A_2 , A_3 and A_4 are nanofluid parameters corresponding to density, electrical conductivity, specific heat capacity and they are defined as

$$A_1 = (1 - \varphi) + \varphi \frac{\rho_s}{\rho_f}, \quad A_2 = 1 + \frac{3 \left(\frac{\sigma_s}{\sigma_f} - 1 \right) \varphi}{\frac{\sigma_s}{\sigma_f} + 2 - \left(\frac{\sigma_s}{\sigma_f} - 1 \right) \varphi},$$

$$A_3 = (1 - \varphi) + \varphi \frac{(\rho c)_s}{(\rho c)_f}, \quad A_4 = \frac{k_s + 2k_f - 2\varphi(k_f - k_s)}{k_s + 2k_f + 2\varphi(k_f - k_s)}$$

The values for A_1 , A_2 , A_3 and A_4 are computed by using the values in Table 1.

Non-dimensional form of boundary conditions can be written as

$$f(0) = S, \quad f'(0) = 1, \quad f''(0) = 0, \quad \theta(0) = 0,$$

$$f(1) = \frac{S_q}{2}, \quad f'(1) = 0, \quad f''(1) = 0, \quad \theta(1) = 1 \quad (13)$$

where $S = \frac{V_0}{ah}$ for suction/blowing parameter.

Skin friction

The shear stress for couple stress fluid is given by

$$\tau_w = \mu_{nf} \frac{\partial u}{\partial y} - \eta_1 \frac{\partial^3 u}{\partial y^3} \quad (14)$$

The local skin-friction coefficient C_f at lower and upper surfaces is given by

$$C_f = \frac{2\tau_w}{\rho v_w^2} = \left(\frac{1}{(1 - \varphi)^{2.5}} f''(\eta) - \beta f^{iv}(\eta) \right)_{\eta=0,1} \quad (15)$$

Nusselt number

The Nusselt number at lower and upper surfaces defined as follows

$$Nu = -(\theta'(\eta))_{\eta=0,1} \quad (16)$$

3 Mathematical solution

The Eqs. (11) and (12) in η are integrated with a shooting technique using the boundary conditions at 0 and 1. In order to carry the integration, the coupled system of

non-linear differential equations is set in a system form by taking $f^{(i)}(\eta) = y_{i+1}$ ($i = 1, 2, 3, 4, 5$), $\theta(\eta)$ and $\theta'(\eta)$ are y_7 and y_8 .

$$y'_6 = \frac{1}{\beta(1 - \varphi)^{2.5}} (y_5 - A_2 Ha^2 (1 - \varphi)^{2.5} y_3) - \frac{A_1}{\beta} \left(\frac{S_q}{2} (3y_3 + \eta y_4) + y_2 y_3 - y_1 y_4 \right) \quad (17)$$

$$y'_8 = \frac{Pr A_3}{A_4} \left(S_q \left(y_7 + \frac{\eta y_8}{2} \right) - y_1 y_8 \right) - \frac{Pr Ec}{A_4 \sqrt{(1 - \varphi)^5}} (4y_2^2 + \delta^2 y_3^2) - \frac{\beta Pr Ec}{A_4} (y_3^2 + \delta^2 y_4^2) - \frac{A_2 Pr Ec Ha^2}{A_4} (y_1^2 + \delta^2 y_2^2) \quad (18)$$

The integrating can be done by any standard numerical procedure. In this study, we have implemented RK4 algorithm (4th order Runge–Kutta method) due to its fast convergence nature in most of the highly non linear cases. But RK4 solves an initial value problem and the system (17) and (18) is a boundary value problem. In order to solve this, additional slopes are assumed at the stretching sheet surface, i.e. at $\eta = 0$.

$$y_1(0) = S, \quad y_2(0) = 1, \quad y_3(0) = 0, \quad y_4(0) = a_1,$$

$$y_5(0) = a_2, \quad y_6(0) = a_3, \quad y_7(0) = 0, \quad y_8(0) = a_4$$

Boundary conditions

$$y_1(1) = \frac{S_q}{2}, \quad y_2(1) = 0, \quad y_3(1) = 0, \quad y_7(1) = 1$$

Now, we enforce these initial values in the 4th order Runge–Kutta method to compute the values at the upper wall $\eta = 1$. The initial values consist four unknowns and the boundary conditions are specified for 4 variables. Thus an algebraic 4×4 system of nonlinear equations has been formulated. This system is solved by employing a Newton Raphson method which has a convergence rate 2.

In order to validate the results, the numerical values of non-dimensional shear stress at lower and upper surfaces, for different values of S and Ha for Newtonian fluid are tabulated in the Table 2. From this, one can observe that the present results are very much agreeing with Hayat et al. [43].

Table 2 Comparative values of non-dimensional shear stress at both surfaces for different values of S and Ha when $S_q = 0$, $\beta = 0$

S	Ha	Hayat et al. [43]		Present	
		$f''(0)$	$f''(1)$	$f''(0)$	$f''(1)$
0.5	0.0	-7.411153	4.713303	-7.411145	4.713291
	1.0	-7.591618	4.739017	-7.591607	4.739007
	2.0	-8.110334	4.820251	-8.110315	4.820231
	3.0	-8.910096	4.964870	-8.910067	4.964837
0.0	2.0	-4.587891	1.842447	-4.587883	1.842438
0.3		-6.665662	3.653695	-6.665648	3.653682
0.6		-8.851444	5.391248	-8.851422	5.391224
1.0		-11.948584	7.593426	-11.948548	7.593391

4 Results and discussions

This section addresses the physical analysis and discussion on Fe_3O_4 -ethylene glycol nanofluid velocity (f' , f) and temperature (θ) profiles in two different situations with respect to the movement of the plates ($S_q > 0$) and ($S_q < 0$). The results are obtained for various physical parameter values such as couple stress parameter β , Hartmann number Ha , volume fraction φ and squeezing parameter S_q .

4.1 Velocity profile

The variation of couple stress parameter (β) on axial and radial velocity profiles of Fe_3O_4 -ethylene glycol nanofluid is displayed in Figs. 2 and 3. The positive and negative situations of wall motion have distinct effect on velocity components. Figure 2a, b reveal that as β augments, the axial velocity shows decreasing trend initially i.e., $0 < \eta < 0.5$ and a reverse trend for $\eta > 0.5$ in both modes of S_q .

Fig. 2 Axial velocity profile for different values of couple stress parameter

Fig. 3 Radial velocity profile for different values of couple stress parameter

Physically, the effect of S_q is insignificant in the initial values of η (the time and space variable). As the upper plate moves in downward direction, an extra force is instigated on the liquid particles which result in a boost in the axial velocity. The radial velocity profile is depicted in Fig. 3a, b. From Fig. 3a, b it is clear that the radial velocity decreases for increasing β values. The reason is obvious, as $S_q < 0$ indicates the upper plate is moving towards lower plate, the plate movement results in a force in the downward direction of the fluid and hence velocity reduces. Similarly, when plate is moving away i.e., $S_q > 0$ the radial velocity of fluid decreases in the negative direction as the volume occupied by the fluid in the channel increases.

Figures 4 and 5 illustrate the influence of Hartmann number Ha on velocity profiles. Figure 4a, b describes the variation in axial velocity profiles (f') for different values of Ha . At the outset, f' increases when $0 < \eta < 0.5$ whereas velocity reduces when $\eta < 0.5$ for all augmenting values Ha and $S_q > 0$. However reverse trend is noticed for the case, $S_q < 0$. Since in a MHD flow, the reduction in the velocity of the fluid at the neighboring surface of the plates would be balanced by an upsurge in the fluid velocity adjacent to the central region and this develops a cross-flow behavior. Figure 5a, b describe the behavior of radial velocity f for boosting values of Ha . The radial velocity increases in the both cases of wall motion for all augmented values of Ha . This is due to the magnetic resistive forces on the nanoparticles and the base fluid velocity field near the lower sheet.

Figures 6 and 7 depicts the impact of nanoparticle volume fraction (φ) on velocity profiles f' and f . From Fig. 6a, b

it is observed that axial velocity profile f' changes abruptly for augmenting values of φ . Also, boundary layer thickness increases when $S_q > 0$ whereas it decreases for $S_q < 0$. Similarly, radial velocity f increases for all augmenting values of φ in both mode of S_q as depicted in Fig. 7a and b.

4.2 Temperature profile

Effect of couple stress parameter (β), Hartmann number (Ha) and volume fraction (φ) on temperature profile θ are depicted in Fig. 8, 9 and 10. Figure 8a, b denotes the variation of β values on temperature profile. When $S_q > 0$, the temperature has a decreasing trend (as thermal boundary increases) for augmented values of β . Since the movement of upper plate is in downward direction, the friction between the surface of the boundary plate and nanofluid reduces and hence temperature of the couple stress nanofluid decreases. On the other hand, the temperature shows a complete reverse trend for the case $S_q < 0$. Figure 9a, b represents the temperature profile distribution with respect to Ha . Almost similar trend is observed for all values of Ha in both cases of S_q . The temperature profile θ is an increasing function for all augmenting values of Ha . Physically increasing Hartmann number results in Joule heating and thus more heat is created. It is also noted that when $S_q > 0$, thermal boundary layer thickness is more as compared to $S_q < 0$. For both the cases of S_q , and various values of nanoparticle volume fraction, Fig. 10a, b show the effect on temperature profile.

Fig. 4 Axial velocity profile for different values of Hartmann number

Fig. 5 Radial velocity profile for different values of Hartmann number

Fig. 6 Axial velocity profile for different values of volume fraction

Fig. 7 Radial velocity profile for different values of volume fraction

Fig. 8 Temperature profile for different values of couple stress parameter

Fig. 9 Temperature profile for different values of Hartmann number

Fig. 10 Temperature profile for different values of volume fraction

$\theta(\eta)$ shows a decreasing trend for augmented values of φ . Also, the thermal boundary layer thickness is more for all increasing values of φ when compared to previous temperature profiles. This is due to variation in specific heat and thermal conductivity of Fe_3O_4 nanofluid.

5 Conclusions

In the present article the analysis of transient hydro-magnetic squeezing flow of couple stress Fe_3O_4 -ethylene glycol nanofluid between two parallel plates has been carried out. The coupled governing non-linear PDE's are transformed to non-linear coupled ODE's by suitable similarity variable and then resolved by using well-known shooting numerical technique. The numerical analysis has been done via graphs and tables to demonstrate the details of the flow features. Based on above discussion few main outcomes of present study are catalogued below.

1. Axial velocity upsurges for augmenting values of couple stress parameter whereas reverse trend is noticed for radial velocity profile.
2. For all augmenting values of Ha the axial velocity shows the oscillating trend for both case of S_q . However radial velocity increases in both cases of S_q for all increasing values of Ha .
3. For all boosting values of volume fraction the axial velocity changes abruptly (greater boundary layer thickness) and radial velocity shows the increasing trend.
4. When $S_q > 0$ the temperature upsurges for augmenting values of β . However reverse tendency is noticed when $S_q < 0$.
5. For both the cases of S_q the temperature is increased with Ha whereas scenario is reverse for increasing of volume fraction.

Acknowledgements The authors are thankful to the referees their valuable suggestions which led to improvement of the paper.

Compliance with ethical standards

Conflict of interest The author(s) declare that they have no competing interests.

References

1. Choi SUS, Eastman JA (1995) Developments and applications of non-Newtonian flows. ASME Publ Fed 231:99
2. Wen D, Ding Y (2004) Experimental investigation into convective heat transfer of nanofluids at the entrance region under laminar flow conditions. *Int J Heat Mass Transf* 47:5181
3. Eastman JA, Choi SUS, Yu W, Thompson LJ (2001) Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles. *Appl Phys Lett* 78:718
4. Xie H, Lee H, Youn W, Choi M (2004) *J Appl Phys* 94:4967
5. Minkowycz WJ, Sparrow EM, Abraham JP (2013) Advances in numerical heat transfer. Nanoparticle Heat Transfer Fluid Flow
6. Wong KV, Leon OD (2010) Applications of nanofluids: current and future. *Adv Mech Eng* 2010:1
7. Saidur R, Leong KY, Mohammad HA (2011) A review on application and challenges of nanofluids. *Renew Sustain Energy Rev* 15:1646
8. Parekh K, Lee HS (2010) Magnetic field induced enhancement in thermal conductivity of magnetite nanofluid. *J Appl Phys* 107(9):09A310
9. Stefan MJ (1874) *Sitzungsberichte der Akademie der Wissenschaften in Wien. Mathematik- Naturwissen* 69:713
10. Tichy J, Wimmer WO (1970) Inertial considerations in parallel circular squeeze film bearings. *Trans ASME J Lub Technol* 92:588
11. Singh P, Radhakrishnan V, Narayan KA (1990) Squeezing flow between parallel plates. *Ingenieur-Archiv* 60:274
12. Hamdan MH, Baron RM (1992) Analysis of the squeezing flow of dusty fluids. *Appl Sci Res* 49:345
13. Hamza EA (1999) Suction and injection effects on a similar flow between parallel plates. *J Phys D Appl Phys* 32:656
14. Duwairi HM, Tashtoush B, Damseh R (2004) Squeezed flow and heat transfer over a porous surface for viscous fluid. *Heat Mass Transf* 41:112
15. Mahmood M, Asghar S, Hossain MA (2007) *Heat Mass Transf* 44:165
16. Siddiqui AM, Irum S, Ansari AR (2008) Unsteady squeezing flow of a viscous MHD fluid between parallel plates, a solution using the homotopy perturbation method. *Math Model Anal* 13(4):565
17. Mustafa M, Hayat T, Obaidat S (2012) On heat and mass transfer in the unsteady squeezing flow between parallel plates. *Mechanica* 47:1581
18. Petrov AG, Kharlamova IS (2014) The solutions of Navier–Stokes equations in squeezing flow between parallel plates. *Eur J Mech B* 48:40
19. Qayyum A, Awais M, Alsaedi A, Hayat T (2012) Unsteady squeezing flow of Jeffery fluid between two parallel disks. *Chin Phys Lett* 29(3):034701
20. Khan U, Khan SK, Ahmed N, Bano S, Mohyud-Din ST (2016) Heat transfer analysis for squeezing flow of a Casson fluid between parallel plates. *Ain Shams Eng J* 7:497
21. Vijayakumar B, Kesavan S (2017) Non-newtonian effects of incompressible second order fluid on equilateral triangular plates. *Int J Pure Appl Math* 113(13):301–309
22. Lin Jaw-Ren, Chu Li-Ming, Liang Long-Jin (2013) Effects of viscosity-pressure dependency on the non-Newtonian squeeze film of parallel circular plates. *Lubr Sci* 25:1
23. Naduvinnamani NB, Siddangouda A, Ayyappa GH (2016) *Tribology* 10(3):126
24. Sherwood JD (2011) Squeeze flow of a power-law fluid between non-parallel plates. *J Non-Newtonian Fluid Mech* 166:289
25. Kashyap KP, Kumar NN (2019) Squeezing flow of a chemically reacting upper convected Maxwell nanofluid with slip effects. *J Nanofluids* 8:509
26. Stokes VK (1966) Couple stresses in fluids. *Phys Fluids* 9:1709

27. Lakshmana Rao SK, Iyengar TKV (1985) Analytical and computational studies in couple stress fluid flows. UGC Research Project: C-8-4/82 SR III
28. Naduvinamani NB, Hiremath PS, Gurubasavaraj G (2001) Squeeze film lubrication of a short porous journal bearing with couple stress fluids. *Tribol Int* 34(11):739
29. Adesanya Samuel O, Makinde Oluwale D (2015) Irreversibility analysis in a couple stress film flow along an inclined heated plate with adiabatic free surface. *Phys A* 432:222
30. Govindarajan A, Siva EP, Vidhya M (2015) Combined effect of heat and mass transfer on MHD Peristaltic transport of a couple stress fluid in a inclined asymmetric channel through a porous medium. *Int J Pure Appl Math* 105:685
31. Lin JR, Hung CR (2007) Combined effects of non-Newtonian couple stresses and fluid inertia on the squeeze film characteristics between a long cylinder and an infinite plate. *Fluid Dyn Res* 39:616
32. Grimm RJ (1976) Squeezing flows of Newtonian liquid films an analysis including fluid inertia. *Appl Sci Res* 32:149
33. Hamza EA (1988) Unsteady MHD squeezing flow between two parallel rotating discs. *J Tribol* 110:375
34. Bhattacharyya S, Pal A (1997) Unsteady MHD squeezing flow between two parallel rotating discs. *Mech Res Commun* 24:615
35. Ran XJ, Zhu QY, Li Y (2007) Approximate analysis of MHD squeeze flow between two parallel disks with suction or injection by homotopy perturbation method. *Commun Nonlinear Sci Numer Simul* 8:179
36. Domairry G, Aziz A (2009) Approximate analysis of MHD squeeze flow between two parallel disks with suction or injection by homotopy perturbation method. *Math Probl Eng* 2009:1
37. Hussain A, Mohyud-din ST, Cheema TA (2012) Analytical and numerical approaches to squeezing flow and heat transfer between two parallel disks with velocity slip and temperature jump. *Chin Phys Lett* 29:114705
38. Khan U, Ahmed N, Zaidi ZA, Asadullah S, Mohyud-Din ST (2014) MHD squeezing flow between two infinite plates. *Ain Shams Eng J* 5:187
39. Qayyum M, Khan H, Khan O (2017) Slip analysis at fluid-solid interface in MHD squeezing flow of Casson fluid through porous medium. *Result Phys* 7:732–750
40. Ghadikolaei SS, Hosseinzadeh K, Ganji DD (2017) Analysis of unsteady MHD Eyring-Powell squeezing flow in stretching channel with considering thermal radiation and Joule heating effect using AGM. *Case Stud Therm Eng* 10:579–594
41. Muhammad T, Hayat T, Alsaedi A, Qayyum A (2017) Hydromagnetic unsteady squeezing flow of Jeffrey fluid between two parallel plates. *Chin J Phys* 55:1511
42. Adesanya SO, Ogunseye HA, Jangili S (2018) Unsteady squeezing flow of a radiative Eyring-Powell fluid channel flow with chemical reactions. *Int J Therm Sci* 125:440
43. Hayat T, Sajjad R, Alsaedi A, Muhammad T, Ellahi R (2017) On squeezed flow of couple stress nanofluid between two parallel plates. *Result Phys* 7:553
44. Nagler J (2016) The electroosmotic magnetohydrodynamic nano flow. *J Nanofluids* 5(6):898
45. Ahmad S, Khan MI, Hayat T, Khan MI, Alsaedi A (2018) Entropy generation optimization and unsteady squeezing flow of viscous fluid with five different shapes of nanoparticles. *Colloids Surf A* 554:197
46. Brinkman HC (1952) The viscosity of concentrated suspensions and solutions. *J Chem Phys* 20:571
47. Tiwari RK, Das MK (2007) Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *Int J Heat Mass Trans* 50:2002
48. Khanafer K, Vafai K, Lightstone M (2003) Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. *Int J Heat Mass Trans* 46:3639
49. Maxwell JC (1904) A treatise on electricity and magnetism, 2nd edn. Oxford University Press, Cambridge

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