Fluid transport with suspended particles by means of dilating peristaltic waves

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Abstract

In the present study, transport of a Newtonian fluid containing uniformly distributed tiny particles moving in a cylindrical tube is investigated when the peristaltic wave amplitude dilates exponentially. Long wavelength approximation is used to get rid of non-linear inertia terms. It is observed that dilation of wave-amplitude during propagation enhances pressure but reduces it with rising concentration of particulate suspension. Pressure rises with the amplitude dilation parameter in case solid particles are suspended in the fluid. Pressure is also observed to increase in the distal part of the tube.

Keywords Peristalsis · Waves of dilating amplitude · Suspended particles–fluid mixture · Volume fraction of solid particles

1 Introduction

In order to obtain qualitative and quantitative insight of the motion of suspended solid and rigid particles distributed uniformly in a homogeneous medium due to the passage of discrete peristaltic waves, Hung and Brown [1] employed a two-dimensional experimental model to investigate various geometries and dynamic effects on peristaltic transport. It was followed by Kaimal [2] who investigated peristaltic pumping of a Newtonian fluid with rigid solid particles suspended in it at low Reynolds number using long wavelength approximations. He discussed reflux and trapping but noted that the presence of the suspended particles does not lead to any significant disturbance to the flow field of the fluid. The governing equations had an additional term associated with the viscous part in terms of the difference of fluid and particulate velocities only. It was meant for modelling peristaltic pumping due to muscular contractions and also for engineering application of pumping solid–fluid mixtures.

Drew [3] studied the stability of a Stokes' layer of a particle–fluid mixture. He obtained a set of coupled Orr–Sommerfeld equations governing the evolution of infinitesimal disturbances. He further found that for low concentration of coarse dust, flow can be stabilized by decreasing the particle relaxation time, which is otherwise unstable. The model suggested was potentially different from those used by Kaimal [2].

Using Drew's model [3], Srivastava and Srivastava [4] investigated peristaltic transport of a particle–fluid suspension in a channel, which was further analysed by Misra and Pandey [5] for axi-symmetric flow. The contribution of the nonlinear convective acceleration terms in the flow equations were duly accounted for in the theoretical analyses. A perturbation technique was used to analyse the problem. The analysis was carried out for the situation in which the amplitude ratio was small. In order to illustrate the applicability of the theoretical analysis, numerical values of various physical quantities were computed for the flow of urine through the ureter. They further observed reversal of flow at a certain critical value of the pressure

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gradient. That was favoured by the presence of particles. Moreover, contrary to the two-dimensional flow, the mean flow in the axi-symmetric case was found to exhibit reversal of flow at the boundaries also (Misra and Pandey [5]).

Mekheimer et al. [6] studied peristaltic motion of a particle–fluid suspension in a planar channel. Mekheimer and Abd elmaboudb [7] studied the influence of an endoscope on the peristaltic flow of a particle–fluid suspension modelling blood flow through tubes by using long wavelength approximation through a uniform and non-uniform infinite annulus filled with an incompressible viscous and Newtonian fluid mixed with rigid spherical particles of identical size. The inner tube was considered uniform, rigid and moving with a constant velocity while the outer non-uniform tube had a sinusoidal wave travelling down its wall.

Lozano and Sen [8] investigated particle dispersion in a two-dimensional peristaltic flow with zero pressure rise. They observed that particle motion can be forward or retrograde depending on the degree of occlusion of peristalsis. They further concluded that the degree of occlusion of the wave can be used to promote dispersion of particles and aggregation in the form of a traveling bolus; and the trapped particles arrange themselves in a self-similar manner using the fractal dimension.

Gaididei et al. [9] recently reported pattern formation in flows of asymmetrically interacting particles with peristaltic pedestrian dynamics as a case study.

Although some drinks we consume contain some edible suspended particles in them and the swallowing of anything in oesophagus is peristaltically driven, peristaltic transport with reference to swallowing has not been seriously taken up for investigation with a single exception of Pandey and Singh [10]. Pandey and Singh [10] have recently published an article on peristaltic swallowing of a fluid with suspended solid particles. The article has perturbation techniques to solve the problem with additional consideration of small wave number which is the ratio of the tube radius and the wavelength. The analysis has been restricted up to the first order of the wave number due to the appearance of highly involved expressions. A very practical result they obtained is that the fluid moves axially faster than the solid particles almost everywhere. However, in the proximity of the wall, the axial velocity of the fluid is slower due to the fact that the no-slip condition is imposed on the wall but the suspended particulate material has non-zero positive axial velocity. Thus, the suspended particles move faster near the tube wall than the fluid velocity. However, they did not mention anything about the pressure distribution along the axis of the oesophagus.

We intend to theoretically model swallowing of such a drink with suspended particles in oesophagus in order

to investigate the impact of presence and concentration of the suspended rigid solid particles in the fluid on the pressure distribution along the length of the oesophagus.

In order to formulate peristaltic waves propagating along oesophageal wall, Pandey et al. [11] analyzed the experimental data of Kahrilas et al. [12] and Xia et al. [13] and concluded that the upper oesophagus is thicker in the dilated state and the lower oesophagus is thicker in the contracted state. They presented a modified model with exponentially increasing amplitude of peristaltic waves and the results of Pandey et al. [11] endorses the experimental results of Kahrilas et al. [12].

The oesophagus is therefore supposed to be a circular cylindrical tube of finite length as the dimensions of the length and diameter are comparable. At a time swallowing of only one bolus will be considered for investigation. The peristaltic waves are supposed to be those suggested by Pandey et al. [11] which help generate higher pressure in the distal part of the oesophagus as reported by Kahrilas et al. [12].

For particle–fluid suspension, the governing equations we use are those suggested by Drew [3]. Since the flow in oesophagus is creeping flow and occlusions are comparable with the radius, we won't go for any perturbation technique for getting the solution of the problem, nor shall we use any recently developed technique for solution [14].

2 Mathematical model

We consider a single transverse progressive peristaltic wave to propagate along the wall of the tube which contains a fluid with suspended solid particles in it. The model for the peristaltic wave is considered as a dilating wave travelling along oesophageal walls hypothesized by Pandey et al. [11] and given by

$$H'(z', t') = a - \varnothing' e^{k'z'} \cos^2 \frac{\pi}{\lambda} (z' - ct'), \quad (1)$$

where H' , a , $\varnothing'$, k' , λ , c are the radial distances of walls, the radius of the tube, the amplitude of the wave, the amplitude dilation parameter, the wavelength, the wave velocity respectively.

The wave is considered to propagate along the tubular wall to generate an unsteady axisymmetric flow in a uniform circular cylindrical tube of an incompressible Newtonian fluid in which are suspended extremely small circular solid particles in a Newtonian fluid. Applying the continuum mechanics approach and using the Drew's model [3], the governing equations for the motion

described above are given, for the fluid and the particulate phases, by

Fluid phase:

$$\frac{\partial((1-C)u'_f)}{\partial z'} + \frac{1}{r'} \frac{\partial((1-C)r'v'_f)}{\partial r'} = 0, \quad (2)$$

$$(1-C)\rho_f \left(\frac{\partial u'_f}{\partial t'} + u'_f \frac{\partial u'_f}{\partial z'} + v'_f \frac{\partial u'_f}{\partial r'} \right) = -(1-C) \frac{\partial p'}{\partial z'} + (1-C)\mu_s(C) \left\{ \frac{1}{r'} \frac{\partial}{\partial r'} \left(r' \frac{\partial u'_f}{\partial r'} \right) + \frac{\partial^2 u'_f}{\partial z'^2} \right\} + CS(u'_p u'_f), \quad (3)$$

$$(1-C)\rho_f \left(\frac{\partial v'_f}{\partial t'} + u'_f \frac{\partial v'_f}{\partial z'} + v'_f \frac{\partial v'_f}{\partial r'} \right) = -(1-C) \frac{\partial p'}{\partial r'} + (1-C)\mu_s(C) \left\{ \frac{\partial}{\partial r'} \left(\frac{1}{r'} \frac{\partial(r'v'_f)}{\partial r'} \right) + \frac{\partial^2 v'_f}{\partial z'^2} \right\} + CS(v'_p - v'_f). \quad (4)$$

Particulate phase:

$$\frac{\partial(Cu'_p)}{\partial z'} + \frac{1}{r'} \frac{\partial(Cr'v'_p)}{\partial r'} = 0, \quad (5)$$

$$C\rho_p \left(\frac{\partial u'_p}{\partial t'} + u'_p \frac{\partial u'_p}{\partial z'} + v'_p \frac{\partial u'_p}{\partial r'} \right) = -C \frac{\partial p'}{\partial z'} + CS(u'_f - u'_p), \quad (6)$$

$$C\rho_p \left(\frac{\partial v'_p}{\partial t'} + u'_p \frac{\partial v'_p}{\partial z'} + v'_p \frac{\partial v'_p}{\partial r'} \right) = -C \frac{\partial p'}{\partial r'} + CS(v'_f - v'_p), \quad (7)$$

where $z', r', t', u'_f, v'_f, C, \rho_f, (1-C)\rho_f, u'_p, v'_p, \rho_p, C\rho_p, p, S, \mu_s(C)$ represent respectively the axial, the radial and the temporal coordinates, the axial and the radial velocities of the fluid phase, the volume fraction of the particles in the mixture, the density of the pure fluid, the fluid phase density, the axial and the radial velocities of the particulate phase, the density of the particle material, the particle phase density, the pressure, the drag coefficient of interaction for the force exerted by one phase on the other, the effective viscosity of the suspension.

The drag coefficient for this problem is chosen as

$$S = \frac{9}{2} \frac{\mu_0}{b^2} \lambda'(C), \quad \lambda'(C) = \frac{4 + 3[8C - 3C^2]^{1/2} + 3C}{[2 - 3C]^2}. \quad (8)$$

where μ_0 is the fluid viscosity, and b is the radius of the suspended solid particle. The relation (8) represents classical Stokes' drag for small particle Reynolds number and

has been modified by Tam [15] to account for the finite particulate fractional volume through the function $\lambda'(C)$.

For the present problem, the viscosity of suspension is given as:

$$\mu_s(C) = \frac{\mu_0}{1 - \alpha C}, \quad (9)$$

where $\alpha = 0.07 \exp\left(2.49C + \frac{1107}{T} \exp(-1.69C)\right)$ and T represents absolute temperature in K. The formula (9) was tested by Charm and Kurland [16] and found reasonable up to $C = 0.6$. Einstein's formula $\mu_s(C) = \mu_0/(1 - 2.5C)$ is good for the viscosity of the suspension only when C is less than 0.05 (cf. Srivastava and Srivastava, [4]).

For further analysis of the problem, we non-dimensionalise the various parameters in the following way:

$$\left. \begin{aligned} z &= \frac{z'}{\lambda}, & r &= \frac{r'}{a}, & u_f &= \frac{u'_f}{c}, & u_p &= \frac{u'_p}{c}, & v_f &= \frac{\lambda v'_f}{ca}, \\ v_p &= \frac{\lambda v'_p}{ca}, & t &= \frac{ct'}{\lambda}, & H &= \frac{H'}{a}, & I &= \frac{I'}{\lambda}, & \rho &= \frac{\rho_p}{\rho_f}, & \delta &= \frac{a}{\lambda}, \\ \emptyset &= \frac{\emptyset'}{a}, & p &= \frac{p'a^2}{\mu_0 c \lambda}, & Q &= \frac{Q'}{\pi a^2 c}, & \bar{\mu} &= \frac{\mu_s}{\mu_0}, & Re_f &= \frac{ca\rho_f}{\mu_0} \end{aligned} \right\}, \quad (10)$$

with the suspension parameters $M = \frac{CSa^2}{\mu_s(1-C)}N = \frac{Sa^2}{\mu_s}$.

Using the dimensionless parameters, defined in Eq. (10), in Eqs. (1–7), we get the corresponding dimensionless forms of the governing Eqs. (2–7) as follows:

$$H(z, t) = 1 - \emptyset e^{kz} \cos^2 \pi(z - ct) \quad (11)$$

Fluid phase:

$$\frac{\partial((1-C)u_f)}{\partial z} + \frac{1}{r} \frac{\partial((1-C)rv_f)}{\partial r} = 0, \quad (12)$$

$$\begin{aligned} Re_f \delta \left(\frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial z} + v_f \frac{\partial u_f}{\partial r} \right) &= -\frac{\partial p}{\partial z} + \bar{\mu} \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_f}{\partial r} \right) + \delta^2 \frac{\partial^2 u_f}{\partial z^2} \right\} + M(u_p - u_f), \end{aligned} \quad (13)$$

$$\begin{aligned} Re_f \delta^3 \left(\frac{\partial v_f}{\partial t} + u_f \frac{\partial v_f}{\partial z} + v_f \frac{\partial v_f}{\partial r} \right) &= -\frac{\partial p}{\partial r} + \bar{\mu} \delta^2 \left\{ \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial(rv_f)}{\partial r} \right) + \delta^2 \frac{\partial^2 v_f}{\partial z^2} \right\} + M\delta^2(v_p - v_f). \end{aligned} \quad (14)$$

Particulate phase:

$$\frac{\partial(Cu_p)}{\partial z} + \frac{1}{r} \frac{\partial(Crv_p)}{\partial r} = 0, \quad (15)$$

$$\rho Re_f \delta \left(\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial z} + v_p \frac{\partial u_p}{\partial r} \right) = -\frac{\partial p}{\partial z} + N(u_f - u_p), \quad (16)$$

$$\rho Re_f \delta^3 \left(\frac{\partial v_p}{\partial t} + u_p \frac{\partial v_p}{\partial z} + v_p \frac{\partial v_p}{\partial r} \right) = -\frac{\partial p}{\partial r} + N\delta^2(v_f - v_p). \quad (17)$$

Now using long wavelength and low Reynolds number approximations, we get Eqs. (12–17) reduced to

$$\frac{\partial(1-C)u_f}{\partial z} + \frac{1}{r} \frac{\partial((1-C)rv_f)}{\partial r} = 0, \quad (18)$$

$$\frac{\partial p}{\partial z} = \frac{\bar{\mu}}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_f}{\partial r} \right) + M(u_p - u_f) \quad (19)$$

$$\frac{\partial p}{\partial r} = 0, \quad (20)$$

$$\frac{\partial(Cu_p)}{\partial z} + \frac{1}{r} \frac{\partial(Crv_p)}{\partial r} = 0, \quad (21)$$

$$\frac{\partial p}{\partial z} = N(u_f - u_p). \quad (22)$$

The various boundary and regularity conditions imposed are as given below:

$$\left. \begin{aligned} u_f &= 0, \text{ at } r = h \text{ (i.e., no-slip condition for the fluid particles);} \\ v_f &= \frac{\partial H}{\partial t}, \text{ at } r = h \text{ (i.e., radial fluid velocity at the wall);} \\ u_f &= \text{finite, at } r = 0 \text{ (i.e., regularity condition);} \\ v_f &= 0 \text{ at } r = 0. \end{aligned} \right\} \quad (23)$$

Solving Eq. (19) under the applications of the first and the third condition of (23), we obtain the axial velocity of fluid as

$$u_f(z, r, t) = \frac{1}{4(1-C)\bar{\mu}} (r^2 - H^2) \frac{\partial p}{\partial z}, \quad (24)$$

and the radial velocity of fluid as

$$v_f(z, r, t) = -\frac{1}{4(1-C)\bar{\mu}} r \left\{ \frac{1}{4} (r^2 - 2H^2) \frac{\partial^2 p}{\partial z^2} - H \frac{\partial H}{\partial z} \frac{\partial p}{\partial z} \right\}. \quad (25)$$

Using (24) and (25) in Eqs. (21) and (22) together with the imposition of the condition $v_f = 0$ at $r=0$ on the solid

particles, the axial and radial components of particle velocity are obtained respectively as

$$\left. \begin{aligned} u_p(z, r, t) &= \frac{\partial p}{\partial z} \left\{ \frac{1}{4(1-C)\bar{\mu}} (r^2 - H^2) - \frac{1}{N} \right\} \\ v_p(z, r, t) &= -\frac{1}{4(1-C)\bar{\mu}} r \left\{ \left(\frac{r^2}{4} - \frac{H^2}{2} - \frac{2(1-C)\bar{\mu}}{M} \right) \frac{\partial^2 p}{\partial z^2} - H \frac{\partial H}{\partial z} \frac{\partial p}{\partial z} \right\} \end{aligned} \right\} \quad (26)$$

For $C=0$, Eqs. (24) and (25) reduce to the corresponding velocities for pure Newtonian fluids given in Pandey et al. [11], where $H(z, t)$ is given in Eq. (11). This is to be noted that the forms of the velocities obtained here are different from those derived by Pandey and Singh [10]. Therefore theoretical comparison is not possible. Qualitatively they may be compared in the discussion section.

Using the second boundary condition of (23), we get

$$\frac{\partial H}{\partial t} = \frac{1}{4(1-C)\bar{\mu}} \left\{ \frac{H^3}{4} \frac{\partial^2 p}{\partial z^2} + H^2 \frac{\partial H}{\partial z} \frac{\partial p}{\partial z} \right\}, \quad (27)$$

from which we obtain pressure gradient as

$$\frac{\partial p}{\partial z} = \frac{\gamma(t) + 16(1-C)\bar{\mu} \int_0^z H \frac{\partial H}{\partial t} dz}{h^4}, \quad (28)$$

where

$$\gamma(t) = \frac{\Delta p(t) - 16(1-C)\bar{\mu} \int_0^l \int_0^z H \frac{\partial H}{\partial t} dz ds}{\int_0^l \frac{1}{H^4} dz}. \quad (29)$$

Integrating Eq. (28) from 0 to z , we get pressure formulated as

$$p(z, t) = p_0(t) + \int_0^z \frac{\gamma(t) + 16(1-C)\bar{\mu} \int_0^z H \frac{\partial H}{\partial t} dz}{H^4} ds. \quad (30)$$

The volume flow rate is given by

$$Q(z, t) = Q_f(z, t) + Q_p(z, t), \quad (31)$$

where

$$Q_p(z, t) = C \int_0^H 2ru_p dr,$$

$$Q_f(z, t) = (1-C) \int_0^H 2ru_f dr,$$

so that the volume flow rate is given by

$$Q(z, t) = -\frac{\partial p}{\partial z} \left\{ \frac{1}{8(1-C)\bar{\mu}} H^4 + \frac{C}{N} H^2 \right\}, \quad (32)$$

From the above expression, with some manipulations, we get the pressure gradient in terms of $Q(z, t)$ as

$$\frac{\partial p}{\partial z} = -\frac{Q(z, t)}{\left(\frac{1}{8(1-C)\bar{\mu}}H^2 + \frac{C}{N}\right)H^2}. \quad (33)$$

Integrating Eq. (33), from 0 to z , pressure, in terms of the volume flow rate, may be given by

$$p(z, t) = p_0(t) - \int_0^z \frac{Q(s, t)}{H^2 \left\{ \frac{1}{8(1-C)\bar{\mu}}H^2 + \frac{C}{N} \right\}} ds, \quad (34)$$

where s is a dummy variable created for the purpose of integration.

3 Numerical results and discussion

In this section, we plot graphs for local pressure against the axis with the consideration that the amplitude of peristaltic waves increases exponentially. We consider the case of free pumping to learn the effect of peristaltic waves on fluid transport. We also consider that a single peristaltic wave moves along the wall and consider four time instants

$t = 0.39, 0.89, 1.47, 1.79$ to observe the temporal effect on the flow pattern.

3.1 Effect of the concentration of suspended particles on flow

In order to examine the effects of the concentration of suspended particles on the flow dynamics, we set $\emptyset = 0.7, k = 0.01$ to examine the effects of concentration of suspended particles through computer simulation of the mathematical model. Temporal variations are randomly taken as $t = 0.39, 0.89, 1.47, 1.79$. $C=0$ is a special case to represent fluid without suspended particles. The wave amplitude dilation parameter is fixed as $k=0.01$. The plots are shown in Fig. 1.

Pressure increases in the beginning, to let the bolus move forward; and at the leading end of the bolus, it rises linearly from the trough, i.e., the lowest point to zero at the lower end of the tube. It is observed that pressure drops with rising concentration of the suspended particles. Once pressure drops, the Drew's model always gives that although the solid particles are dragged by the fluid, they move faster than the fluid (Srivastava and Srivastava [4], Misra and Pandey [5]). This is attributed to the fact that for a given pressure difference less fluid flows when

Fig. 1 a–d The diagrams display the motion of a single bolus and the corresponding pressure distribution along the oesophageal axis shown by the different dotted lines. The pressure distribution also corresponds to the time given in the diagram. Dashed curves symmetric about the horizontal axis represent wall positions. Other parameters for the diagrams are $L=3$, $\emptyset = 0.7, k = 0.01$

Fig. 2 a–d The diagrams display the motion of a single boluses and the corresponding pressure distribution along the oesophageal axis shown by the different dotted lines. The pressure distribution also corresponds to the time given in the diagram. Dashed curves symmetric about the horizontal axis represent wall positions. The other parameters for the diagrams are: $L = 3$, $\emptyset = 0.7$, $C = 0.1$

particulate suspension has greater concentration. The effective viscosity too is changed. Pandey and Singh [10], who computed velocities of fluid and solid particles separately for train wave propagation, observed that due to no-slip condition imposed on fluid, solid particles move faster near the wall but within the tube fluid moves faster. Comparing all the graphs in Fig. 1, we observe that pressure rises as the bolus moves towards the distal end of the tube.

3.2 Effect of amplitude dilation on flow

For studying the influence of dilating wave amplitude on fluid-particle suspension, pressure distributions, represented by pressure curves, have been plotted. Setting $\emptyset = 0.7$, $C = 0.1$, $l = 3$, at different time instants $t = 0.39, 0.89, 1.47, 1.79$ for the study. We obtained that pressure rises with dilating wave-amplitude substantiating the observation of Kahrilas et al. [11] regarding pressure. Similar was the observation of Pandey and Singh [10] with regard to dilation of the wave amplitude,

although their analysis is based on a perturbation technique and the models too are theoretically different (Fig. 2).

4 Conclusions

In the present study, we have analyzed the transport of a Newtonian fluid containing uniformly distributed tiny particles moving in a cylindrical tube when the peristaltic wave amplitude dilates exponentially. Solid particles are dragged by the fluid. It is inferred that the dilation of wave-amplitude during propagation enhances pressure but reduces it with rising concentration of particulate suspension. Pressure rises with the amplitude dilation parameter in case solid particles are suspended in the fluid. Similar was the observation for pure Newtonian fluids. Pressure is also observed to increase in the distal part of the oesophagus endorsing the observation of Kahrilas et al. [11].

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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