Entropy generation in MHD nanofluid flow with heat source/sink

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Abstract

This work investigates the generation of entropy in the presence of a heat source/sink in a sloping channel filled with porous medium in magnetohydrodynamic nanofluid flow. The regulating equations are nonlinear and coupled thermal and hydrodynamic equations. Homotopy analysis method is used in the handling of equations. Comparisons with existing literature have been produced and were discovered to be in excellent accord, which are a particular situation of the present issue. The impact on entropy generation, Bejan number, Nusselt number and skin friction of pertinent fluid parameters is addressed, developed and displayed graphically. Entropy generation was found to be minimum just above the center of the channel throughout the study. Skin friction and Nusselt number were found to be higher for the case of heat generation than heat absorption.

Keywords Entropy · Inclined channel · Mixed convection · Nanofluid · Heat source · MHD · Homotopy analysis method

1 Introduction

High performing supercomputers compute at extremely high speeds and generate high amount of heat. As a result, ultra-fast heat transportation is required to cool the supercomputers. This is one of the many problems the current generation of academicians and industrial technologies are facing. However, the current method of cooling machines need to be upgraded along with the advancement in technologies. Nanofluids were pioneered by Choi and Eastman [1] to aid in the cooling of high heat generating machines and consist of base fluid (either water, alcohol or oil) with suspended nanoparticles in them. It is a current niche research topic and has been tested for various applications. Application of nanofluid in real-life processes is still in its infancy though many applications have been tested including solar collectors, boiling processes, machining processes, heat exchangers, microelectromechanical systems (MEMS), refrigerators, etc. [2–15]

The magnetohydrodynamic (MHD) effect is an important effect that acts on nanofluid flow. Fakour et al. [16]

found that with the increment of magnetic field effects, the maximum velocity and temperature decreased in the horizontal porous canal flow. In their inquiry of MHD nanofluid stream in a tube packed with a porous medium, Aaiza et al. [17] accounted for the form of nanoparticles. They noted that there was reduced velocity in elongated particle form relative to other particle shapes. Sheikholeslami et al. [18] regarded a flow in a semi-porous channel with an MHD impact. It was found that the velocity boundary layer thickness decreased with the increase in the Reynolds number. In their analysis for the MHD nanofluid flow in a rotating channel, Raza et al. [19] considered slip effects. When Azimi and Riazi [20] increased the Reynolds number, skin frictions and Nusselt numbers were found to be enhanced. Dogonchi et al. [21] analysis of MHD nanofluid flow in a porous canal took the thermal radiation effect into account and found an inverse relationship between the radiation and the temperature profile. Noreen et al. [22] found that velocity increases in the center of the channel where electroosmotic parameters have been reduced

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during a microchannel MHD nanofluid flow study. Ma et al. [23] studied the hybrid nanofluid flow on a channel and found that the Reynolds number significantly affected the velocity distribution. In the study performed by Selimefendigil and Öztop [24] for a pulsating MHD nanofluid flow in a channel with blocks, it was found that Nusselt number increases with an increased magnetic field angle. Chamkha et al. [25] conducted a further, but not exhaustive, literature review on the MHD nanofluid flow which could be referred for further insight by the reader.

A numerical study of flow of a nanofluid along a vertical plate with heat source and sink was carried out by Rana and Bhargava [26], observing that both the momentum and the thermal boundary layer decreased for the heat sink. Alsaedi et al. [27] noted that nanofluid flow suction and injection had opposite effects over a heat generated/absorbed surface. Hassan [28] showed the average Nusselt number to be increased when the amount of Cu nanoparticles in an enclosure with a heat source and sink was increased. In their analysis of the fluid flow in tapered channels with thermal source, radiation and magnetic field, Kothandapani and Prakash [29] noted that the mass transfer of nanoparticles and temperature distribution had opposite behavior. Uma-vathi and Sheremet [30] noted that heat generation has increased flow field velocity, while heat absorption has reduced flow rate for nanofluid flow in a vertical channel with heat source and sink. Reddy et al. [31] has investigated the impact of radiation and heat generation/absorption on the flow of nanofluids over a tilted porous plate and found that increasing the tilt angles has enhanced the distribution of temperature and concentration. Ali et al. [32] found the heat source to have a positive effect on the overall efficiency for a nanofluid flow in a grooved channel.

Recently, the importance of entropy analysis has picked up researcher's attention for heat transfer phenomena. Bejan [33, 34] has brought it to the spotlight as a matter of fundamental importance. Li and Kleinstreuer [35] concluded it was recommended to use microchannels in nanofluid flows to reduce entropy. López et al. [36] demonstrated that entropy generation has intensified in an upright microchannel with radiation, slip and convective–radiative conditions at its boundaries with an increased nanofluid flow buoyancy forces. The study by Chamkha [37] found that an increase in nanoparticles led to a decline in entropy generation. Decrease in the temperature gradients in the wall, in the analysis carried out by Abbaszadeh et al. [38], was observed to cause decrease in the entropy generation. In their study of entropy generation in a porous square enclosure, Chamkha et al. [39] considered partial slip. The following papers can provide

further reviews and discussions about entropy generation [40–43].

With regard to the above-mentioned literature, the present research analyzes a MHD nanofluid flow in saturated porous medium in an angled channel with heat source/sink while considering hydrodynamic slip and convection at the boundary. The governing equations are solved using the homotopy analysis method (HAM) [44]. For clarity, the impacts of relevant flow parameters are graphically displayed and explained. To our best knowledge, no researchers have tried the current work.

2 Mathematical formulation

A laminar, fully developed incompressible MHD nanofluid flow in an inclined parallel plate channel filled with a saturated porous medium and a heat source/sink is taken into consideration. The x -axis is taken along the walls of the channel in the ascending direction. The y -axis is taken perpendicular to the walls of the channel. The walls of the channel are parallel to each other and at distance of L apart. The walls of the channel are assumed to be under same heat convection, as depicted in Fig. 1. The Boussinesq approximation [45] and viscous dissipation for the flow are taken into consideration, and the channel is kept inclined at an angle ψ . Under these assumptions, the nonlinear coupled equations for momentum and temperature distribution in the channel are as follows:

$$\mu_{nf} \frac{\partial^2 u}{\partial y^2} + g(\rho\beta)_{nf} \sin \psi (T - T_0) - \frac{\mu_{nf}}{K} u - \sigma_{nf} B_0^2 u = \frac{dp}{dx}, \quad (1)$$

$$\kappa_{nf} \frac{\partial^2 T}{\partial y^2} + \mu_{nf} \left(\frac{\partial u}{\partial y} \right)^2 + \bar{Q}(T - T_0) + \sigma_{nf} B_0^2 u^2 = 0, \quad (2)$$

Fig. 1 Problem configuration

with the associated boundary equations:

$$\begin{cases} u = -\alpha \frac{\partial u}{\partial y} \\ \kappa_{nf} \frac{\partial T}{\partial y} = -h_f(T_1 - T) \end{cases} \text{ at } y = 0' \quad (3a)$$

$$\begin{cases} u = -\alpha \frac{\partial u}{\partial y} \\ \kappa_{nf} \frac{\partial T}{\partial y} = h_f(T_2 - T) \end{cases} \text{ at } y = L. \quad (3b)$$

In this study, we use the following relations for the physical properties of nanofluids and fluids [46]:

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad (4)$$

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \quad (5)$$

$$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s, \quad (6)$$

$$\frac{\kappa_{nf}}{\kappa_f} = \frac{(\kappa_s + 2\kappa_f) + 2\phi(\kappa_f - \kappa_s)}{(\kappa_s + 2\kappa_f) + \phi(\kappa_f - \kappa_s)}. \quad (7)$$

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \left(\frac{\sigma_s}{\sigma_f} - 1\right)\phi} \quad (8)$$

Introducing the following dimensionless variables in Eqs. (1) and (2):

$$\begin{aligned} Y = \frac{y}{L}; U = \frac{u}{U_0}; \theta = \frac{T - T_0}{\Delta T}; \epsilon = \frac{Gr}{Re}; Q = \frac{4\tilde{Q}L^2}{\kappa_f}; \\ Gr = \frac{8\rho_f^2 g \beta_f \Delta T L^3}{\mu_f^2}; Re = \frac{2U_0 \rho_f L}{\mu_f}; Br = \frac{\mu_f U_0^2}{\kappa_f \Delta T}; \\ T_0 = \frac{T_1 + T_2}{2}; \Delta T = T_2 - T_1; r_T = \frac{T_2 - T_1}{\Delta T}; P = -\frac{L^2}{U_0 \mu_f} \frac{dp}{dx}; \\ U_0 = -\frac{L^2}{12\mu_f} \frac{dp}{dx}; Ha^2 = \frac{4\sigma B_0^2 L^2}{\mu_f}; \Lambda_1 = \frac{1}{(1 - \phi)^{2.5}}; \\ \Lambda_2 = (1 - \phi) + \phi \frac{(\rho\beta)_s}{(\rho\beta)_f}; \Lambda_3 = \frac{\kappa_{nf}}{\kappa_f}; \Lambda_4 = \frac{\sigma_{nf}}{\sigma_f} \end{aligned} \quad (9)$$

we have:

$$\Lambda_1 \frac{d^2 U}{dY^2} + \frac{\Lambda_2}{4} \epsilon \theta \sin \psi + P - \Lambda_1 \frac{U}{Da} - \frac{\Lambda_4}{4} Ha^2 U = 0, \quad (10)$$

$$\frac{d^2 \theta}{dY^2} + Br \frac{\Lambda_1}{\Lambda_3} \left(\frac{dU}{dY} \right)^2 + \frac{Q\theta}{4\Lambda_3} + \frac{\Lambda_4}{\Lambda_3} \frac{Ha^2 Br}{4} U^2 = 0, \quad (11)$$

with the dimensionless boundary conditions:

$$\begin{cases} U = -\gamma \frac{dU}{dY} \\ \frac{d\theta}{dY} = \frac{Bi}{2} \left(\frac{r_T}{2} + \theta \right) \end{cases} \text{ at } Y = 0' \quad (12a)$$

$$\begin{cases} U = -\gamma \frac{dU}{dY} \\ \frac{d\theta}{dY} = \frac{Bi}{2} \left(\frac{r_T}{2} - \theta \right) \end{cases} \text{ at } Y = 1. \quad (12b)$$

where Gr is the Grashof number, Re is the Reynolds number, Br is the Brinkman number, Da is the Darcy number, Ha is the Hartmann number, ϵ is the mixed convection parameter, Q is the heat source/sink parameter, P is the dimensionless pressure gradient, and U_0 is the mean fluid velocity. r_T is the dimensionless temperature difference ratio where $r_T = 0$ denotes symmetric wall temperature; $T_1 = T_2$ and $r_T = 1$ denote asymmetric wall temperature, $T_1 < T_2$.

2.1 Skin friction and Nusselt number

Quantities which are of great interest to engineering applications are the skin friction and Nusselt numbers of the flow. On a rough basis, the skin friction is to the velocity boundary layer what the Nusselt number is to the thermal boundary layer. The skin friction quantifies the resistance offered by the surface over which the fluid is flowing. It is mainly caused due to the viscosity of the fluid. On the other hand, the Nusselt number is basically a ratio of the heat transfer due to convection to the heat transfer due to conduction. Hence, Nusselt number tells us the amount of enhancement in heat transfer due to movement of fluid. Thus, skin friction and Nusselt number are defined, respectively, as:

$$Cf = \frac{\tau_w}{\rho_f U_0^2}, Nu = \frac{2Lq_w}{\kappa_f \Delta T} \quad (13)$$

where the shear stress τ and heat transfer coefficient q_w are defined, respectively, as:

$$\tau_w = \pm \mu_{nf} \left(\frac{\partial u}{\partial y} \right)_{y=0,L}, q_w = \pm \kappa_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=0,L} \quad (14)$$

Using the dimensionless transformation Eq. (9), the dimensionless skin friction and Nusselt number can be expressed as:

$$ReCf_1 = \Lambda_1 \left(\frac{dU}{dY} \right)_{Y=0}, ReCf_2 = -\Lambda_1 \left(\frac{dU}{dY} \right)_{Y=1} \quad (15)$$

$$Nu_1 = 2\Lambda_3 \left(\frac{d\theta}{dY} \right)_{Y=0}, Nu_2 = -2\Lambda_3 \left(\frac{d\theta}{dY} \right)_{Y=1} \quad (16)$$

3 Homotopy analysis method

We solve the above coupled Eqs. (10) and (11) with the associated boundary conditions Eq. (12) using HAM. We define a set of base functions to express the solution of the velocity and temperature as

$$\{Y^n | n = 0, 1, 2, 3, \dots\} \quad (17)$$

Therefore, the final expression for velocity and temperature can be defined as

$$U(Y) = \sum_{n=0}^{\infty} \zeta_n Y^n \text{ and } \theta(Y) = \sum_{n=0}^{\infty} \xi_n Y^n \quad (18)$$

where ζ_n and ξ_n are unknown coefficients to be determined. We take the linear auxiliary operators as:

$$\mathfrak{L}_i = \frac{d^2}{dY^2} \quad \text{where } i = 1, 2 \quad (19)$$

Initial guess approximations of $U(Y)$ and $\theta(Y)$ are now chosen using boundary conditions (Eq. 12) as:

$$U_0 = 0, \quad \theta_0 = \frac{\text{Bi}_T}{\text{Bi} + 4} \left(Y - \frac{1}{2} \right), \quad (20)$$

such that

$$\mathfrak{L}_1(\Xi_1 + \Xi_2 Y) = 0, \quad \mathfrak{L}_2(\Xi_3 + \Xi_4 Y) = 0, \quad (21)$$

where Ξ_i ($i = 1, 2, 3, 4$) are constants.

With nonzero auxiliary parameters h_1, h_2 , we develop the deformation equations of zeroth order as:

$$(1 - \delta)\mathfrak{L}_1[U(Y; \delta) - U_0(Y)] = \delta h_1 \mathfrak{N}_1[U(Y; \delta)], \quad (22)$$

$$(1 - \delta)\mathfrak{L}_2[\theta(Y; \delta) - \theta_0(Y)] = \delta h_2 \mathfrak{N}_2[\theta(Y; \delta)], \quad (23)$$

subject to the boundary conditions

$$U(0; \delta) = -\gamma \frac{dU}{dY} \Big|_{Y=0} \quad (24a)$$

$$A_3 \frac{d\theta}{dY} \Big|_{Y=0} = \frac{\text{Bi}}{2} \left(\frac{r_T}{2} + \theta(0; p) \right)$$

$$U(1; \delta) = -\gamma \frac{dU}{dY} \Big|_{Y=1} \quad (24b)$$

$$\frac{d\theta}{dY} \Big|_{Y=1} = \frac{\text{Bi}}{2} \left(\frac{r_T}{2} - \theta(1; p) \right)$$

where $\delta \in [0, 1]$ is the embedding parameter and the nonlinear operators $\mathfrak{N}_1$ and $\mathfrak{N}_2$ are defined as

$$\begin{aligned} \mathfrak{N}_1[U(Y, \delta), \theta(Y, \delta)] \\ = \Lambda_1 U'' + \Lambda_2 \epsilon \theta \sin \psi + P - \Lambda_1 \frac{U}{\text{Da}} - \frac{\Lambda_4}{4} \text{Ha}^2 U, \end{aligned} \quad (25)$$

$$\begin{aligned} \mathfrak{N}_2[U(Y, \delta), \theta(Y, \delta)] \\ = \theta'' + \frac{Q}{4\Lambda_3} \theta + \text{Br} \frac{\Lambda_1}{\Lambda_3} U'^2 + \frac{\Lambda_4}{\Lambda_3} \frac{\text{Ha}^2 \text{Br}}{4} U^2. \end{aligned} \quad (26)$$

For $\delta = 0$ we have the initial guess approximations

$$U(Y; 0) = U_0(Y), \quad \theta(Y; 0) = \theta_0(Y), \quad (27)$$

and for $\delta = 1$ we get the final solutions

$$U(Y; 1) = U(Y), \quad \theta(Y; 1) = \theta(Y). \quad (28)$$

Hence, as δ varies from 0 to 1, $U(Y; \delta)$ and $\theta(Y; \delta)$ vary continuously from the initial guesses $U_0(Y)$ and $\theta_0(Y)$ to the final solutions $U(Y)$ and $\theta(Y)$, respectively.

The m th-order deformation equations are written as

$$\mathfrak{L}_1[U_m(Y) - \mathfrak{T}_m U_{m-1}(Y)] = h_1 O_m^U(Y), \quad (29)$$

$$\mathfrak{L}_2[\theta_m(Y) - \mathfrak{T}_m \theta_{m-1}(Y)] = h_2 O_m^\theta(Y), \quad (30)$$

with the boundary conditions

$$\begin{aligned} U(0; \delta)_{m-1} = -\gamma \frac{dU_{m-1}}{dY} \Big|_{Y=0}, \\ \frac{d\theta_{m-1}}{dY} \Big|_{Y=0} - \frac{\text{Bi}}{2} \theta(0; p)_{m-1} = 0, \end{aligned} \quad (31a)$$

$$\begin{aligned} U(1; \delta)_{m-1} = -\gamma \frac{dU_{m-1}}{dY} \Big|_{Y=1}, \\ \frac{d\theta_{m-1}}{dY} \Big|_{Y=1} + \frac{\text{Bi}}{2} \theta(0; p)_{m-1} = 0, \end{aligned} \quad (31b)$$

where

$$O_m^U(Y) = \Lambda_1 U_{m-1}'' + \Lambda_2 \epsilon \theta_{m-1} \sin \psi - \Lambda_1 \frac{U_{m-1}}{\text{Da}} - \frac{\Lambda_4}{4} \text{Ha}^2 U_{m-1}, \quad (32)$$

$$\begin{aligned} O_m^N(Y) = \theta_{m-1}'' + \text{Br} \frac{\Lambda_1}{\Lambda_3} \left(\sum_{n=0}^{m-1} U'_{m-1-n} U'_n \right) + \frac{Q}{4\Lambda_3} \theta_{m-1} \\ + \frac{\Lambda_4}{\Lambda_3} \frac{\text{Ha}^2 \text{Br}}{4} \left(\sum_{n=0}^{m-1} U_{m-1-n} U_n \right), \end{aligned} \quad (33)$$

where m is an integer and

$$\mathfrak{T}_m = \begin{cases} 0, & \text{for } m \leq 1 \\ 1, & \text{for } m > 1. \end{cases} \quad (34)$$

The initial approximations, linear operators and auxiliary parameters are chosen such that Eqs. (22) and (23) have a solution at each point of $\delta \in [0, 1]$. Using Taylor's series and Eq. (27), the solutions expression can be written as

$$U(Y; \delta) = U_0(Y) + \sum_{m=1}^{\infty} U_m(Y) \delta^m, \quad (35)$$

$$\theta(Y; \delta) = \theta_0(Y) + \sum_{m=1}^{\infty} \theta_m(Y) \delta^m, \quad (36)$$

where h_1 and h_2 are chosen such that the series Eqs. (35) and (36) are convergent at $\delta = 1$. Hence, using Eq. (28), we have

$$U(Y) = U_0(Y) + \sum_{m=1}^{\infty} U_m(Y), \quad (37)$$

$$\theta(Y) = \theta_0(Y) + \sum_{m=1}^{\infty} \theta_m(Y), \quad (38)$$

with

$$U_m(Y) = \frac{1}{m!} \left. \frac{\partial^m U(Y; \delta)}{\partial \delta^m} \right|_{\delta=1},$$

$$\theta_m(Y) = \frac{1}{m!} \left. \frac{\partial^m \theta(Y; \delta)}{\partial \delta^m} \right|_{\delta=1}.$$

4 Convergence and validation

As the HAM solution is strongly dependent on the convergence control parameter h , h_1 and h_2 curves are plotted for various order of approximation so as to assure convergence as shown in Figs. 2 and 3, respectively, for $P = 12$, $Br = 0.05$, $\psi = \pi/4$, $\phi = 5\%$, $Da = 0.5$, $Ha = 2$, $\epsilon = 100$. It is clear from the figures that the admissible range for h_1 is $-0.75 \leq h_1 \leq -0.25$ and that for h_2 is $-2 \leq h_2 \leq -1$. To choose the optimal value of the convergence control parameter, the average residual errors are defined as [51]:

$$E_{U,m} = \frac{1}{K} \sum_{i=0}^K \left[\aleph_1 \left(\sum_{j=0}^m U_j(i \Delta t) \right) \right]^2, \quad (39)$$

$$E_{\theta,m} = \frac{1}{K} \sum_{i=0}^K \left[\aleph_2 \left(\sum_{j=0}^m \theta_j(i \Delta t) \right) \right]^2, \quad (40)$$

where $\Delta t = 1/K$ and $K = 20$. The minimum average residual errors are shown in Table 1 for different order of

Fig. 2 h -curve for h_1

Fig. 3 h -curve for h_2

Table 1 h_1 and h_2 optimum values for different approximation orders

Order (m)	h_1	Minimum error	h_2	Minimum error
14	-0.62	6.33×10^{-6}	-1.16	3.24×10^{-6}
16	-0.62	2.92×10^{-6}	-1.16	1.63×10^{-6}
18	-0.62	8.11×10^{-7}	-1.16	3.55×10^{-8}

approximations. Thus, the minimum average residual error for U is taken at $h_1 = -0.62$ and for θ is taken at $h_2 = -1.16$.

Using the technique of homotopy analysis method, the velocity and temperature characteristics of the issue

Table 2 Velocity at different points on the channel for $P = 12$; $\epsilon = 100$; $r_T = 1$; $Ha = 2$; $\psi = \pi/2$; $Da = \infty$; $Bi = 10^{10}$; $Q = \phi = \gamma = 0$

Y	Liu and Lo (DTM) [47]			Umavathi and Malashetty (Perturbation) [48]			Present (HAM)		
	$\epsilon = -100$	$\epsilon = 100$	$\epsilon = 100$	$\epsilon = -100$	$\epsilon = 100$	$\epsilon = 100$	$\epsilon = -100$	$\epsilon = 100$	$\epsilon = 100$
	Br = 0.08	Br = 0	Br = 0.08	Br = 0.08	Br = 0	Br = 0.08	Br = 0.08	Br = 0	Br = 0.08
0.2	0.965	0.681	0.822	0.985	0.69	0.805	0.982	0.681	0.821
0.4	1.23	1.21	1.432	1.235	1.22	1.395	1.266	1.208	1.428
0.6	1.04	1.4	1.63	1.1	1.392	1.55	1.073	1.402	1.626
0.8	0.571	1.071	1.22	0.585	1.075	1.18	0.596	1.071	1.216

Table 3 Temperature at different points on the channel for $P = 12$; $\epsilon = 100$; $r_T = 1$; $Ha = 2$; $\psi = \pi/2$; $Da = \infty$; $Bi = 10^{10}$; $Q = \phi = \gamma = 0$

Y	Liu and Lo (DTM) [47]			Umavathi and Malashetty (Perturbation) [48]			Present (HAM)		
	$\epsilon = -100$	$\epsilon = 100$	$\epsilon = 100$	$\epsilon = -100$	$\epsilon = 100$	$\epsilon = 100$	$\epsilon = -100$	$\epsilon = 100$	$\epsilon = 100$
	Br = 0.08	Br = 0	Br = 0.08	Br = 0.08	Br = 0	Br = 0.08	Br = 0.08	Br = 0	Br = 0.08
0.2	-0.249	-0.3	-0.231	-0.25	-0.3	-0.233	-0.253	-0.3	-0.232
0.4	-0.035	-0.1	-0.008	-0.04	-0.1	-0.01	-0.045	-0.1	-0.009
0.6	0.171	0.1	0.1908	0.158	0.1	0.178	0.155	0.1	0.191
0.8	0.36	0.3	0.376	0.34	0.3	0.355	0.34	0.3	0.377

Table 4 Comparison of Nusselt numbers for $P = 12$; $\epsilon = 100$; $\psi = \pi/2$; $Bi = 10^{10}$; $Da = \infty$; $Q = Ha = r_T = \phi = \gamma = 0$

Br	Barletta [49]	Zanchini [50]	Present
-0.04	-43.211	-43.211	-43.212
-0.03	-33.219	-33.219	-33.218
-0.02	-22.722	-22.722	-22.722
-0.01	-11.669	-11.669	-11.669
-0.005	-5.916	-5.916	-5.916
0	0	0	0
0.005	6.087	6.087	6.087
0.01	12.356	12.356	12.354
0.02	25.482	25.482	25.469
0.03	39.478	39.478	39.427
0.04	54.463	54.464	54.322

were determined. Tables 2 and 3 show the numerical values of velocity and temperature field, respectively, obtained by HAM and compared with the results obtained by Liu and Lo [47], who used differential transformation method (DTM) and Umavathi and Malashetty [48], who used perturbation method. It can be seen that the answers agree with each other exceptionally. Moreover, Nusselt numbers were calculated for different values of Br and compared with existing results presented by Barletta [49] and Zanchini [50]. They were found to be in excellent agreement and shown in Table 4. The thermophysical properties of water and copper can be seen in Table 5.

Table 5 Thermophysical properties of base fluid and nanoparticles [46]

Property	Base fluid (water)	Nanoparticle (copper)
C_p (J/kg K)	4179	383
σ (Ωm) ⁻¹	0.05	5.96×10^7
ρ (kg/m ³)	997.1	8954
k (W/mK)	0.6	400
β (K ⁻¹)	2.1×10^{-4}	1.67×10^{-5}

5 Second law analysis

The local rate of generation of entropy for MHD nano-fluid flow in a porous medium is given by [52, 53]

$$S_G = \frac{\kappa_{nf}}{T_0^2} \left(\frac{\partial T}{\partial y} \right)^2 + \frac{\mu_{nf}}{T_0} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\mu_{nf}}{KT_0} u^2 + \sigma_{nf} B^2 \frac{u^2}{T_0}. \quad (41)$$

The entropy generation for the flow in consideration is due mainly to four components. The first term on the right-hand side is the entropy generation due to heat transfer, the second due to fluid friction/viscous dissipation, the third term due to Darcy viscous dissipation as a result of the presence of porous medium and the fourth term due to the presence of magnetic field.

We apply the non-dimensional variables defined in Eq. (9) to the above equation, and using the non-dimensional entropy generation formula

$$N_s = \frac{L^2 T_0^2}{\kappa_f \Delta T^2} S_G, \quad (42)$$

we get the following equation

$$N_s = \Lambda_3 \left(\frac{\partial \theta}{\partial Y} \right)^2 + \Lambda_1 \frac{Br}{\Omega} \left[\left(\frac{\partial U}{\partial Y} \right)^2 + \frac{U^2}{Da} \right] + \Lambda_4 Ha^2 \frac{Br}{\Omega} U^2. \quad (43)$$

where $\Omega = \Delta T/T_0$. The entropy generation rate can be roughly separated into two categories, namely entropy due to heat transfer and entropy due to viscous dissipation, and we write it as

$$N_s = N_h|_{\text{heat transfer}} + N_v|_{\text{fluid friction}}. \quad (44)$$

such that

$$N_h|_{\text{heat transfer}} = \Lambda_3 \left(\frac{\partial \theta}{\partial Y} \right)^2$$

$$N_v|_{\text{fluid friction}} = \Lambda_1 \frac{Br}{\Omega} \left[\left(\frac{\partial U}{\partial Y} \right)^2 + \frac{U^2}{Da} \right] + \Lambda_4 Ha^2 \frac{Br}{\Omega} U^2.$$

Another important aspect of thermodynamic analysis of heat transfer problems is the Bejan number. The Bejan number is defined as the ratio of entropy generation due to heat transfer to the total entropy generation, written as:

$$Be = \frac{\text{Generation of Entropy by heat transfer}}{\text{Total Entropy generated}}. \quad (45)$$

The Bejan number indicates the dominance of irreversibility in the process and ranges from 0 to 1. For $Be > 0.5$, the irreversibility due to heat transfer dominates, while for $Be < 0.5$, the irreversibility due to fluid friction dominates. At $Be = 0.5$, the contribution to irreversibility from both heat transfer and fluid friction is equal.

6 Results and discussion

So as to conduct a thorough analysis of the various parameters governing the flow, computations were conducted and graphs were plotted accordingly. Unless mentioned otherwise, we have taken $P = 12$, $Br = 0.05$, $\phi = 5\%$, $Da = 0.5$, $\varepsilon = 100$, $\psi = \pi/4$, $\Omega = 0.5$, $Bi = 10$, $\gamma = 0.1$, $Ha = 2$, $r_T = 1$ throughout the study. In Figs. 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, the entropy generation profile and Bejan number profiles are shown graphically. The dotted lines denote the case of heat absorption, while the solid lines denote heat generation case.

Fig. 4 Graph of entropy generation with the influence of Brinkman number. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

Fig. 5 Graph of Bejan number with the influence of Brinkman number. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

6.1 Entropy generation and Bejan number

6.1.1 Brinkman number

Figure 4 shows the Brinkman number influence on the entropy generation profile. Increased Brinkman numbers can be found to lead to reduced entropy generation in the scheme. It may be noted that the rise in the Brinkman number shows that the viscous dissipation increases, and this reduces the fluid velocity. Entropy in the scheme therefore decreases. Figure 5 also shows the effect of rising

Fig. 6 Graph of entropy generation with the influence of Biot number. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

Fig. 8 Graph of entropy generation with the influence of Darcy number. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

Fig. 7 Graph of Bejan number with the influence of Biot number. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

Fig. 9 Graph of Bejan number with the influence of Darcy number. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

Brinkman number on Bejan number. With the Brinkman increase, it may be observed that the Bejan number is declining. This shows the increasing impact of viscous dissipation on irreversibility.

6.1.2 Biot number

An increase in Biot number and its impact on the generation of entropy is shown in Fig. 6. An increase in the Biot number can be seen as leading to enhanced entropy.

The increase in the Biot number indicates a higher convection on the channel walls that causes a greater temperature at the channel walls. As a result, entropy generation is on the rise. Bejan numbers also rise with the rise of the Biot number, a result shown in Fig. 7. As already stated, the rise in Biot number results in greater wall temperatures, leading in turn to a greater effect of the irreversibility by heat transfer. Therefore, the Bejan number is increasing.

Fig. 10 Graph of entropy generation with the influence of Hartmann number. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

Fig. 12 Graph of entropy generation with the influence of slip parameter. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

Fig. 11 Graph of Bejan number with the influence of Hartmann number. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

Fig. 13 Graph of Bejan number with the influence of slip parameter. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

6.1.3 Darcy number

An increase in Darcy number leads to a complete increase in entropy generation in the system, as illustrated in Fig. 8. Increased permeability of the porous material increases the Darcy number. This contributes to a rise in the liquid temperature which in turn increases heat production in the scheme. Figure 9 shows the influence on the Bejan number of the increment of Darcy number. The increase in the Darcy number leads to a reduction in the Bejan number near the channel walls.

But Bejan number just above the middle of the channel is slightly rising.

6.1.4 Hartmann number

The rise in the Hartmann number causes a lower entropy generation near the walls of the canal, but around the center of the canal was seen a significant increase in entropy generation, as shown in Fig. 10. Hartmann number rise results in a rise in liquid temperature, which increases entropy generation. This rise in fluid temperature

Fig. 14 Graph of entropy generation with the influence of nanoparticle volume fraction. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

Fig. 16 Graph of entropy generation with the influence of Brinkman number. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

Fig. 15 Graph of Bejan number with the influence of nanoparticle volume fraction. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

Fig. 17 Graph of Bejan number with the influence of channel inclination angle. Dotted lines denote heat sink, $Q = -1$, and solid lines denote heat source, $Q = 1$

also results in an increase in Bejan number near the channel walls, as shown in Fig. 11. The boost in the Hartmann number results in a decrease in the fluid velocity that contributes to the decrease in the Bejan number.

6.1.5 Slip parameter

The rise in the slip parameter resulted to a higher entropy generation in the bottom of the channel, while the entropy generated at the top wall reduced as shown in Fig. 12. The increase in slip parameter decreased the Bejan number

on the bottom of the channel, while the Bejan number increased on the top of the channel, as shown in Fig. 13.

6.1.6 Nanoparticle volume fraction

Figure 14 shows an increase in the amount of nanoparticles in the fluid and its consequence on entropy generation. The thermal conductivity of the liquid also improves with a rise in nanoparticles, and energy is transmitted more quickly through the fluid. Consequently, entropy produced increases. The irreversibility of the liquid also

progressively depends on the heat transfer as the thermal conductivity of the liquid increases, an attribute that is demonstrated in Fig. 15.

6.1.7 Channel inclination angle

Figure 16 shows the increase in the channel's tilt angle and its influence on entropy generation. The entropy generation at the bottom plate decreased with an increase in tilt angle but gradually increased from around the channel widths' quarter point. On the other hand, as shown in Fig. 17, the increase in tilt angle initially increased the Bejan number. The Bejan number was found to be decreasing at around the channel's quarter width with an increase in tilt angle.

It may be noted that in all the cases, entropy generated near the walls of the channel was found to be higher for heat absorption case than heat generation cases, but the entropy generated near the center of the channel was lower for heat absorption case than heat generation case. The same can be observed for Bejan number.

6.2 Nusselt number and skin friction

Figures 18, 19, 20, 21, 22, 23 show the effect of various flow parameters on the Nusselt number and local skin friction, respectively. Figures 18 and 19 depict the influence of increasing nanoparticles in the fluid on Nusselt number and skin friction, respectively. It may be observed from Fig. 18 that increasing the amount of nanoparticles in the fluid increases the Nusselt number on the lower wall of the channel. This is due to the fact that nanoparticles in

Fig. 19 Graph of local skin friction with the influence of nanoparticle volume fraction. Solid lines denote lower channel wall, $ReCf_1$, and dotted lines denote upper channel wall, $ReCf_2$

the fluid increase the thermal conductivity of the fluid. The local skin friction was observed to decrease with increasing the nanoparticles in the fluid. This is attributed to the fact that increasing the amount of nanoparticles in the fluid increases the density of the fluid and hence the velocity decreases. Thus, skin friction decreases. Increasing the Hartmann number and its impact on Nusselt number and skin friction are shown in Figs. 20 and 21, respectively. Increasing the Hartmann number leads to an increase in Lorentz forces and suppresses the heat transfer capability of the fluid, leading to declining Nusselt number as

Fig. 18 Graph of Nusselt number with the influence of nanoparticle volume fraction. Solid lines denote lower channel wall, Nu_1 , and dotted lines denote upper channel wall, Nu_2

Fig. 20 Graph of Nusselt number with the influence of Hartmann number. Solid lines denote lower channel wall, Nu_1 , and dotted lines denote upper channel wall, Nu_2

Fig. 21 Graph of local skin friction with the influence of Hartmann number. Solid lines denote lower channel wall, $ReCf_1$, and dotted lines denote upper channel wall, $ReCf_2$

Fig. 23 Graph of local skin friction with the influence of channel inclination angle. Solid lines denote lower channel wall, $ReCf_1$, and dotted lines denote upper channel wall, $ReCf_2$

Fig. 22 Graph of Nusselt number with the influence of channel inclination angle. Solid lines denote lower channel wall, Nu_1 , and dotted lines denote upper channel wall, Nu_2

shown in Fig. 20. The same Lorentz force also retards the fluid flow velocity, and hence, the skin friction decreases, as depicted in Fig. 21. Nusselt number and skin friction as a function of channel tilt angle are shown in Figs. 22 and 23. Nusselt number was observed to increase with increasing the channel tilt angle, as demonstrated in Fig. 22. As the fluid flow increases, the heat transfer capability of the fluid also surges and hence, Nusselt number increases. The skin friction on the upper wall of the channel increased, while the skin friction on the lower wall of the channel decreased with increasing the channel angle of inclination,

as depicted in Fig. 23. As the channel becomes increasingly perpendicular, the amount of fluid in contact with the lower wall of the channel depletes which leads to a decrease in the skin friction. On the other hand, the amount of fluid in contact with the upper wall of the channel increases which leads to more skin friction.

In the ongoing section, Nusselt number and skin friction were taken as a function of the heat source/sink parameter. It may be observed that both Nusselt number and skin friction are increasing functions of the heat source/sink parameter.

7 Conclusion

The current work investigated the entropy generation for a nanofluid flow in a tilted channel with heat source/sink filled with a saturated porous medium with asymmetric wall temperatures while also considering Navier slip and convection at the walls of the channel. The governing equations were reduced to non-dimensional form and solved using HAM. Special case of the problem was compared with existing literature and found to be in excellent agreement. Throughout the study, entropy generation was found to be highest at the lower wall of the channel and minimum just above the center of the channel width. At the center of the channel, irreversibility process was found to be dominated by heat transfer irreversibility. It may also be noted that Bejan number and entropy generation were lower for heat absorption case than heat generation case near the center of the channel for all cases in the study. However, both Nusselt number and skin friction

were observed to be higher for heat generation than heat absorption for all cases.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

References

- Choi S, Eastman J (1995) Enhancing thermal conductivity of fluids with nanoparticles. *ASME Int Mech Eng Congr Expos* 66:99. <https://doi.org/10.1115/1.1532008>
- Das SK, Choi SU, Patel HE (2006) Heat transfer in nanofluids—a review. *Heat Transf Eng* 27(10):3. <https://doi.org/10.1080/01457630600904593>
- Wang XQ, Mujumdar AS (2007) Heat transfer characteristics of nanofluids: a review. *Int J Therm Sci* 46(1):1. <https://doi.org/10.1016/j.jthermalsci.2006.06.010>
- Wen D, Lin G, Vafaei S, Zhang K (2009) Review of nanofluids for heat transfer applications. *Particuology* 7(2):141. <https://doi.org/10.1016/j.partic.2009.01.007>
- Saidur R, Leong KY, Mohammad HA (2011) A review on applications and challenges of nanofluids. *Renew Sustain Energy Rev* 15(3):1646. <https://doi.org/10.1016/j.rser.2010.11.035>
- Huminic G, Huminic A (2012) Application of nanofluids in heat exchangers: a review. *Renew Sustain Energy Rev* 16(8):5625. <https://doi.org/10.1016/j.rser.2012.05.023>
- Taylor R, Coulombe S, Otanicar T, Phelan P, Gunawan A, Lv W, Rosengarten G, Prasher R, Tyagi H (2013) Small particles, big impacts: a review of the diverse applications of nanofluids. *J Appl Phys* 113:1. <https://doi.org/10.1063/1.4754271>
- Mahian O, Kianifar A, Kalogirou SA, Pop I, Wongwises S (2013) Small particles, big impacts: a review of the diverse applications of nanofluids. *Int J Heat Mass Transf* 57(2):582. <https://doi.org/10.1016/j.jheatmasstransfer.2012.10.037>
- Sharma AK, Tiwari AK, Dixit AR (2015) Progress of nanofluid application in machining: a review. *Mater Manuf Process* 30(7):813. <https://doi.org/10.1080/10426914.2014.973583>
- Kasaeian A, Eshghi AT, Sameti M (2015) A review on the applications of nanofluids in solar energy systems. *Renew Sustain Energy Rev* 43:584. <https://doi.org/10.1016/j.rser.2014.11.020>
- Fang X, Chen Y, Zhang H, Chen W, Dong A, Wang R (2016) Heat transfer and critical heat flux of nanofluid boiling: a comprehensive review. *Renew Sustain Energy Rev* 62:924. <https://doi.org/10.1016/j.rser.2016.05.047>
- Tawfik MM (2017) Experimental studies of nanofluid thermal conductivity enhancement and applications: a review. *Renew Sustain Energy Rev* 75(August):1239. <https://doi.org/10.1016/j.rser.2016.11.111>
- Sidik NAC, Samion S, Ghaderian J, Yazid MNAWM (2017) Recent progress on the application of nanofluids in minimum quantity lubrication machining: a review. *Int J Heat Mass Transf* 108:79. <https://doi.org/10.1016/j.jheatmasstransfer.2016.11.105>
- Rashidi S, Mahian O, Languri EM (2018) Applications of nanofluids in condensing and evaporating systems: a review. *J Therm Anal Calorim* 131(3):2027. <https://doi.org/10.1007/s10973-017-6773-7>
- Zhang Z, Cai J, Chen F, Li H, Zhang W, Qi W (2018) Progress in enhancement of CO₂ absorption by nanofluids: a mini review of mechanisms and current status. *Renew Energy* 118(June):527. <https://doi.org/10.1016/j.renene.2017.11.031>
- Fakour M, Ganji DD, Abbasi M (2015) Investigation of nanofluid MHD flow and heat transfer in a channel. *J Adv Phys* 4(1):46. <https://doi.org/10.1166/jap.2015.1153>
- Aaiza G, Khan I, Shafie S (2015) Energy transfer in mixed convection MHD flow of nanofluid containing different shapes of nanoparticles in a channel filled with saturated porous medium. *Nanoscale Res Lett* 10(1):1. <https://doi.org/10.1186/s11671-015-1144-4>
- Sheikholeslami M, Hatami M, Ganji DD (2013) Analytical investigation of MHD nanofluid flow in a semi-porous channel. *Powder Technol* 246:327. <https://doi.org/10.1016/j.powtec.2013.05.030>
- Raza J, Rohni AM, Omar Z, Awais M (2016) Heat and mass transfer analysis of MHD nanofluid flow in a rotating channel with slip effects. *J Mol Liq* 219:703. <https://doi.org/10.1016/j.molliq.2016.04.003>
- Azimi M, Riazi R (2016) MHD copper-water nanofluid flow and heat transfer through convergent-divergent channel. *J Mech Sci Technol* 30(10):4679. <https://doi.org/10.1007/s12206-016-0938-3>
- Dogonchi AS, Alizadeh M, Ganji DD (2017) Advanced powder technology (May). <https://doi.org/10.1016/j.appt.2017.04.022>
- Noreen S, Waheed S, Hussanan A (2019) Bound value problem. <https://doi.org/10.1186/s13661-019-1118-z>
- Ma Y, Mohebbi R, Rashidi MM, Yang Z (2019) MHD convective heat transfer of Ag-MgO/water hybrid nanofluid in a channel with active heaters and coolers. *Int J Heat Mass Transf* 137:714. <https://doi.org/10.1016/j.jheatmasstransfer.2019.03.169>
- Selimefendigil F, Öztürk HF (2019) MHD Pulsating forced convection of nanofluid over parallel plates with blocks in a channel. *Int J Mech Sci* 157–158(April):726. <https://doi.org/10.1016/j.ijmecsci.2019.04.048>
- Chamkha AJ, Jena SK, Mahapatra SK (2015) MHD convection of nanofluids: a review. *J Nanofluids* 4(3):271. <https://doi.org/10.1166/jon.2015.1166>
- Rana P, Bhargava R (2011) Numerical study of heat transfer enhancement in mixed convection flow along a vertical plate with heat source/sink utilizing nanofluids. *Commun Nonlinear Sci Numer Simul* 16(11):4318. <https://doi.org/10.1016/j.cnsns.2011.03.014>
- Alsaedi A, Awais M, Hayat T (2012) Effects of heat generation/absorption on stagnation point flow of nanofluid over a surface with convective boundary conditions. *Commun Nonlinear Sci Numer Simul* 17(11):4210. <https://doi.org/10.1016/j.cnsns.2012.03.008>
- Hassan H (2014) Heat transfer of Cu-water nanofluid in an enclosure with a heat sink and discrete heat source. *Eur J Mech B/Fluids* 45:72. <https://doi.org/10.1016/j.euromechflu.2013.12.003>
- Kothandapani M, Prakash J (2015) Influence of heat source, thermal radiation, and inclined magnetic field on peristaltic flow of a hyperbolic tangent nanofluid in a tapered asymmetric channel. *IEEE Trans Nanobiosci* 14(4):385. <https://doi.org/10.1109/TNB.2014.2363673>
- Umavathi JC, Sheremet MA (2016) Mixed convection flow of an electrically conducting fluid in a vertical channel using Robin boundary conditions with heat source/sink. *Eur J Mech B/Fluids* 55:132. <https://doi.org/10.1016/j.euromechflu.2015.08.013>
- Reddy PS, Chamkha AJ, Al-Mudhaf A (2017) MHD heat and mass transfer flow of a nanofluid over an inclined vertical. *Adv*

- Powder Technol 28(January):1008. <https://doi.org/10.1016/j.appt.2017.01.005>
32. Ali MM, Alim MA, Ahmed SS (2019) Oriented magnetic field effect on mixed convective flow of nanofluid in a grooved channel with internal rotating cylindrical heat source. *Int J Mech Sci* 151(November 2018):385. <https://doi.org/10.1016/j.ijmecsci.2018.11.027>
33. Bejan A (1979) A study of entropy generation in fundamental convective heat transfer. *J Heat Transf* 101(4):718. <https://doi.org/10.1115/1.3451063>
34. Bejan A (1980) Second law analysis in heat transfer. *Energy* 5(8–9):720. [https://doi.org/10.1016/0360-5442\(80\)90091-2](https://doi.org/10.1016/0360-5442(80)90091-2)
35. Li J, Kleinstreuer C (2010) Entropy generation analysis for nanofluid flow in microchannels. *J Heat Transf* 132(12):122401. <https://doi.org/10.1115/1.4002395>
36. López A, Ibáñez G, Pantoja J, Moreira J, Lastres O (2017) Entropy generation analysis of MHD nanofluid flow in a porous vertical microchannel with nonlinear thermal radiation, slip flow and convective-radiative boundary conditions. *Int J Heat Mass Transf* 107:982. <https://doi.org/10.1016/j.ijheatmasstransfer.2016.10.126>
37. Chamkha AJ (2017) Effects of partial slip on entropy generation and MHD combined convection in a lid-driven porous enclosure saturated with a Cu-water nanofluid. *J Therm Anal Calorim*. <https://doi.org/10.1007/s10973-017-6918-8>
38. Abbaszadeh M, Ababaei A, Arani AA, Sharifabadi AA (2017) MHD forced convection and entropy generation of CuO-water nanofluid in a microchannel considering slip velocity and temperature jump. *J Braz Soc Mech Sci Eng* 39(3):775. <https://doi.org/10.1007/s40430-016-0578-7>
39. Chamkha AJ, Rashad AM, Mansour MA, Armaghani T, Ghalambaz M (2017) Effects of heat sink and source and entropy generation on MHD mixed convection of a Cu-water nanofluid in a lid-driven square porous enclosure with partial slip. *Phys Fluids* 10(1063/1):4981911
40. Oztop HF, Al-Salem K (2012) A review on entropy generation in natural and mixed convection heat transfer for energy systems. *Renew Sustain Energy Rev* 16(1):911. <https://doi.org/10.1016/j.rser.2011.09.012>
41. Mahian O, Kianifar A, Kleinstreuer C, Al-Nimr MA, Pop I, Sahin AZ, Wongwises S (2013) A review of entropy generation in nanofluid flow. *Int J Heat Mass Transf* 65:514. <https://doi.org/10.1016/j.ijheatmasstransfer.2013.06.010>
42. Awad MM (2015) A review of entropy generation in microchannels. *Adv Mech Eng* 7(12):1. <https://doi.org/10.1177/1687814015590297>
43. Torabi M, Karimi N, Peterson GP, Yee S (2017) Challenges and progress on the modelling of entropy generation in porous media: a review. *Int J Heat Mass Transf* 114:31. <https://doi.org/10.1016/j.ijheatmasstransfer.2017.06.021>
44. Liao S (2004) Beyond perturbation: introduction to the homotopy analysis method. Chapman and Hall/CRC, Boca Raton
45. Barletta A, Zanchini E (1999) On the choice of the reference temperature for fully-developed mixed convection in a vertical channel. *Int J Heat Mass Transf* 42(16):3169. [https://doi.org/10.1016/S0017-9310\(99\)00011-3](https://doi.org/10.1016/S0017-9310(99)00011-3)
46. Tiwari RK, Das MK (2007) Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *Int J Heat Mass Transf* 50(9–10):2002. <https://doi.org/10.1016/j.ijheatmasstransfer.2006.09.034>
47. Liu CC, Lo CY (2012) Numerical analysis of entropy generation in mixed-convection MHD flow in vertical channel. *Int Commun Heat Mass Transf* 39(9):1354. <https://doi.org/10.1016/j.icheatmasstransfer.2012.08.001>
48. Umavathi JC, Malashetty MS (2005) Magnetohydrodynamic mixed convection in a vertical channel. *Int J Non-Linear Mech* 40(1):91. <https://doi.org/10.1016/j.jnonlinmec.2004.05.018>
49. Barletta A (1998) Laminar mixed convection with viscous dissipation in a vertical channel. *Int J Heat Mass Transf* 41(22):3501. [https://doi.org/10.1016/S0017-9310\(98\)00074-X](https://doi.org/10.1016/S0017-9310(98)00074-X)
50. Zanchini E (1998) Effect of viscous dissipation on mixed convection in a vertical channel with boundary conditions of the third kind. *Int J Heat Mass Transf* 41(23):3949. [https://doi.org/10.1016/S0017-9310\(98\)00114-8](https://doi.org/10.1016/S0017-9310(98)00114-8)
51. Liao S (2010) An optimal homotopy-analysis approach for strongly nonlinear differential equations. *Commun Nonlinear Sci Numer Simul* 15(8):2003. <https://doi.org/10.1016/j.cnsns.2009.09.002>
52. Butt AS, Ali A (2014) Investigation of entropy generation effects in magnetohydrodynamic three-dimensional flow and heat transfer of viscous fluid over a stretching surface. *J Braz Soc Mech Sci Eng* 37(1):211. <https://doi.org/10.1007/s40430-014-0163-x>
53. Jafari SS, Freidoonimehr N (2015) Second law of thermodynamics analysis of hydro-magnetic nano-fluid slip flow over a stretching permeable surface. *J Braz Soc Mech Sci Eng* 37(4):1245. <https://doi.org/10.1007/s40430-014-0250-z>

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