Application of mathematical modeling value-at-risk (VaR) to optimize decision making in distribution networks

E. Khorshidi¹ · V. R. Ghezavati¹

Received: 24 July 2019 / Accepted: 16 November 2019 / Published online: 25 November 2019
© Springer Nature Switzerland AG 2019

Abstract

Managers and capital masters of companies and factories try to adopt methods that maximize their profit and minimize their costs. A way for increasing profit is risk management. The risk management and the type of risk are defined in the literature of financial management. For measurement of the risk, many methods are defined which are all created in the recent century and it means the risk and risk management are rather new concepts. Among the newest tools for measuring the risk is value-at-risk (VaR), which was modified by Morgan (Riskmetrics technical document, Morgan Guaranty Trust Company, New York, 1996). VaR represents the maximum expected loss over a certain period of time and at a given confidence level. Many parametric, semi-parametric and nonparametric methods for VaR estimation have been developed. In this article using the mean–variance method one of the parametric techniques of VaR, has been tried to minimize the cost arisen due to locating the supply chain and minimize the maximum level of capital losses, to optimize decision making in the distribution network. The calculated model is tested with numerical examples by MATLAB and Lingo software, and this example supported the resulting model.

Keywords Value at risk · Supply chain · Location-allocation · Location · Integer programming

1 Introduction

Site selection is one of the most important issues in industrial engineering, which can lead to the reduction in costs and successfulness of industrial units. Site selection for service centers is known as a selection of a position for one or more service centers considering other centers and also existing limitations so that a particular goal would be optimized. Site selection problem is one of the network design problems which has been solved as one of the strategic decision making processes. In site selection problems, there are a set of nodes that certain demands and they must be fulfilled by a set of service centers through some transportation modes. The term site selection refers to modeling, formulation and solving those problems which are seeking the best location for the establishment of the

service center and facilities [1]. The first scientific framework of this theory was introduced formally by Weber [2]. He was one of the theorists who theorized the site selection and cost minimization which constitute major transportation costs [3]. The site selection-allocation problem is also used as a combination of supply chain problems. A provided model which is a combination of the site-selection-allocation, path determination, and control problems developed by Kheybari et al. [4] is one of these problems.

Distribution refers to a stage in which a product is stored and transported from a supplier to a customer in a supply chain. For this purpose, raw materials, components and elements are transferred from a supplier to manufacturers and then final products are transferred to end customers. Performance of a distribution network should be measured through two aspects:

✉ V. R. Ghezavati, v_ghezavati@azad.ac.ir; vrghezavati@gmail.com | ¹School of Industrial Engineering, Islamic Azad University, South Tehran Branch, Tehran, Iran.

1. Customers' demands must be met.
2. Cost of satisfying these needs.

Decisions regarded to the design of a supply chain network are classified as follow:

1. The role of facilities: What is the role of each facility? In addition, what is processed and run in each facility?
2. Location of facilities: Where should be the location of each facility?
3. Capacity allocation: How much capacity should be allocated to each facility?

Among the effective factors on the distribution networks, one can refer to taxes, tariffs, currency rate, demand risk etc. This research is going to consider the effect of the risk factors on the demand parameter. Site selection of the facilities was gradually proposed to supply chain which investigates the models of site selection in the supply chain [5]. Usually, in management of a supply chain, there are three levels of planning depending on time horizon: strategic, tactical and operational. Simchi-Levi et al. [6] stated that the strategic level deals with decisions which would have long-term impacts on the firm.

Sometimes, the meaning of application of supply chain network design is the same as strategic planning [7, 8]. Designing a bi-level and single period distribution network is also studied for managing a supply chain. This problem includes site selection for factories and distributing stockpiles. Moreover, the capacity of these facilities is determined from the possible selected choices, the related decisions on how to distribute products from a factory to distribution piles and also from stockpiles to customers [9].

Lejarza and Baldea [10] considered the issue of the supply chain in which the number and site of the distribution center are determined. Customers are facing random demands and each distribution center maintains a certain amount of guaranteed storage in order to achieve a certain service level for customers.

The concept of risk is arisen from uncertainty for input parameters in a decision making process and shows the possible loss in the problem. Taymaz et al. [11] introduced the risk as an unwanted result for a decision making problem. Xu et al. [12] introduced the sustainability of risks in the logistic network design. Value at risk is a risk assessment and diagnosis that uses standard statistical techniques that are used every day in many tactical and operational problems. Value at risk indicates the maximum expected loss in a given planning horizon. In addition, there are other value-at-risk metrics that have been published independently and have been introduced as tools to maximize the portfolio of capital in order to maximize returns for a given level of risk [13]. VaR applied

the confidence α in which the most values of VaR would not be exceeded to lose during the next h days [14, 15]. In the framework of parametric approaches, the first model is proposed by Morgan [16]. One major disadvantage of the mentioned model is the assumption of the financial yield normality while the experimental evidence indicates that the financial yield does not follow a normal distribution. The second disadvantage is that in order to estimate the fluctuation, the financial yield should be used; and the third one is that the parametric approach assumes independency of the yields. There are significant experimental evident on the normality of financial yield distribution [17–21]. The VaR also includes point estimation, but deviates from the median that requires some probability level details, and then provides a better expected point to occur in a better probability. VaR calculation method is divided into parametric and non-parametric approaches [22].

The reason that we considered risk in our model is uncertainty in the environment of decision space. In a decision making problem, there are some input parameters that fluctuates the results and they are not under decision maker control such as customers' demand. Therefore, the decision maker should create the mathematical model in which the model is able to handle such uncertainty due to prevent future costs. Thus, we considered value at risk (VaR) as a new framework of controlling the risk in our problem.

In this research, the implementation of VaR regarded with the demands of customers for the distribution network site selection model under uncertainty is performed and the parametric method is used to solve this problem. In this paper, the parametric type of VaR (mean–variance) methods are applied to formulate the uncertainty.

2 Problem definition

In this research, the problem of the supply chain is considered regarding the demand under uncertainty with a normal distribution with minimization of costs such as fixed costs of locating the facility in a candidate site and costs of the services delivered to the customers by the facility as well as minimization of value at risk. The problem faced here includes a bi-level supply chain considering suppliers level and customers level. In different periods, the demand for different products is delivered probabilistically to the suppliers in most scenarios. The supplier is responsible for meeting the demands and he should minimize the costs of fulfilling and services to the customer.

VaR is a tool for controlling the risk in financial aspects. There are other tools such as robust optimization for controlling the risk but an expert tool for financial risk is VaR. Since in our paper, the total cost is based on the financial

risk, therefore, we used VaR in this way. It should be noted this tool very recently is used in the optimization problems such as facility location. In the literature, we focused on the application of VaR in optimization problems due to its major impact in our model [23].

2.1 Problem assumptions

1. In site selection for a supply chain network, nodes are candidates for the locations of both customers and facilities.
2. Locating a facility in a candidate site has fixed and certain costs.
3. Only one system delivers services to each customer.
4. The amount of flow of each facility depends on its capacity.
5. The cost of delivering a service to a customer is fixed.
6. The amount of flow between a facility and each customer is determined by a standard normal distribution using variance and mean indexes.
7. The sum of demands is considered as the lost value which is not supplied considering the capacity of a facility and demands of a customer.

2.2 Proposed mathematical model

The site selection model of the supply chain is defined by implementing the VaR.

2.2.1 Indexes, parameters and decision variables

I	Set of customers' sites; $i = 1, 2, \dots, m$
J	Set of candidate sites for facilities; $j = 1, 2, \dots, n$
h_i	Average of demand for customer i
σ_i	Amount of variance of demand for customer i
σ_j	Total Amount of variance of demands assigned to candidate facility j
f_j	Fixed cost of locating facility in candidate site j
b_j	Total Amount of mean of demands assigned to candidate facility j
c_{ij}	Cost of each transportation unit between customer i and candidate site j
p_j	Amount of capacity for candidate facility j
Y_{ij}	A binary variable that equals 1, if demand of customer i is supplied by candidate facility j , otherwise it is equal to zero
X_j	A binary variable that equals 1, if a facility is located in site j otherwise it is equal to zero
VaR_j	The highest expected loss in a given time horizon with the level of reliability at site j

2.2.2 Objective functions and limitations of the model

$$\text{Min } Z_1 = \sum_{j \in J} VaR_j \quad (1)$$

$$\text{Min } Z_2 = \sum_{j \in J} f_j X_j + \sum_{j \in J} \sum_{i \in I} h_i c_{ij} Y_{ij} \quad (2)$$

s.t:

$$\sum_{j \in J} Y_{ij} = 1 \quad \forall i \in I \quad (3)$$

$$\sum_{i \in I} h_i Y_{ij} - p_j X_j \leq 0 \quad \forall j \in J \quad (4)$$

$$pr(b_j - p_j \leq VaR_j) \geq \alpha \quad \forall j \in J \quad (5)$$

$$b_j = \sum_{i \in I} h_i Y_{ij} \quad \forall j \in J \quad (6)$$

$$\sigma_j = \sum_{i \in I} \sigma_i Y_{ij} \quad \forall j \in J \quad (7)$$

$$VaR_j \geq 0 \quad \forall j \in J \quad (8)$$

$$X_j \in \{0, 1\} \quad \forall j \in J \quad (9)$$

$$Y_{ij} \in \{0, 1\} \quad \forall i \in I \quad \forall j \in J \quad (10)$$

The first objective function minimizes the total amount of VaR for all activated facilities. The second objective function minimizes total costs including the cost of establishing facilities and also cost of transportation between facilities and customers.

Set constraint (3) indicates that each customer must be assigned to one facility in order to be served. Set constraint (4) ensures that each customer can be assigned to a facility once the facility is activated. In addition, each facility can cover total demands up to its capacity. Set constraint (5) which is given in a non-linear form indicates that the probability of losing demand up to the amount of VAR for each facility must be at least α . Considering the fact that a non-linear term can lead to a remarkable increase in the solution time, then this limitation is first rewritten and then converted to a linear model using a linear integer model. Set constraints (6) and (7) determine the total amount of mean and variance of demands assigned to candidate facility j , respectively. The stated probability in set constraint (5) can be converted to a standard normal function probability considering that α

is the reliability value, then one can extract the accumulative distribution function value from the table of standard normal function which is equal to Z_α . These descriptions are indicated in the following calculations:

$$\begin{aligned} pr(b_j - p_j \leq VaR_j) &\geq \alpha \rightarrow pr\left(Z \leq \frac{VaR_j + p_j - b_j}{\sqrt{\sigma_j}}\right) \\ &\geq \alpha \rightarrow \frac{VaR_j + p_j - b_j}{\sqrt{\sigma_j}} \geq Z_\alpha \rightarrow VaR_j + p_j - b_j \geq Z_\alpha * \sqrt{\sigma_j} \end{aligned}$$

In order to increase the speed of solving the problem, one can make the limitation as follows by changing the variable $\sqrt{\sigma_j} = w_j$:

$$VaR_j + p_j - b_j \geq Z_\alpha w_j \quad \forall j \in J \quad (11)$$

Now, set constraint (5) is substituted by constraint (11). Considering the integrals, this limitation is non-linear. Therefore, the fragmentation model can be used to make it linear. Since the limit of variable σ_j considering the problem parameters can be determined, in order to increase the speed of problem solving one can approximate w_j . Assume that the maximum amount of w_j be a . To do so, the interval $[0, a]$ is divided into L parts. The more the L is, the more accurate the approximation would be achieved. The limitations 12–30 are added to the model in order to make it linear. In this paper, we assume that the maximum value for w_j be 120. Thus, interval $[0, 120]$ are separated into 12 sub-intervals where break points are: 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110 and 120. Therefore, the mathematical model in order to approximate linearization is as follows:

$$\begin{aligned} w_j = &\lambda_{j0} \sqrt{0} + \lambda_{j1} \sqrt{10} + \lambda_{j2} \sqrt{20} + \lambda_{j3} \sqrt{30} + \lambda_{j4} \sqrt{40} \\ &+ \lambda_{j5} \sqrt{50} + \lambda_{j6} \sqrt{60} + \lambda_{j7} \sqrt{70} + \lambda_{j8} \sqrt{80} + \lambda_{j9} \sqrt{90} \\ &+ \lambda_{j10} \sqrt{100} + \lambda_{j11} \sqrt{110} + \lambda_{j12} \sqrt{120} \quad \forall j \in J \end{aligned} \quad (12)$$

$$\begin{aligned} \sigma_j = &\lambda_{j0} 0 + \lambda_{j1} 10 + \lambda_{j2} 20 + \lambda_{j3} 30 + \lambda_{j4} 40 \\ &+ \lambda_{j5} 50 + \lambda_{j6} 60 + \lambda_{j7} 70 + \lambda_{j8} 80 + \lambda_{j9} 90 \\ &+ \lambda_{j10} 100 + \lambda_{j11} 110 + \lambda_{j12} 120 \quad \forall j \in J \end{aligned} \quad (13)$$

$$\sum_{l=0}^{12} \lambda_{jl} = 1 \quad \forall j \in J \quad (14)$$

$$\lambda_{jl} \geq 0 \quad \forall j \in J, \quad l = 0, 1, \dots, 12 \quad (15)$$

$$\lambda_{j0} \leq \delta_{j0} \quad \forall j \in J \quad (16)$$

$$\lambda_{j1} \leq \delta_{j0} + \delta_{j1} \quad \forall j \in J \quad (17)$$

$$\lambda_{j2} \leq \delta_{j1} + \delta_{j2} \quad \forall j \in J \quad (18)$$

$$\lambda_{j3} \leq \delta_{j2} + \delta_{j3} \quad \forall j \in J \quad (19)$$

$$\lambda_{j4} \leq \delta_{j3} + \delta_{j4} \quad \forall j \in J \quad (20)$$

$$\lambda_{j5} \leq \delta_{j4} + \delta_{j5} \quad \forall j \in J \quad (21)$$

$$\lambda_{j6} \leq \delta_{j5} + \delta_{j6} \quad \forall j \in J \quad (22)$$

$$\lambda_{j7} \leq \delta_{j6} + \delta_{j7} \quad \forall j \in J \quad (23)$$

$$\lambda_{j8} \leq \delta_{j7} + \delta_{j8} \quad \forall j \in J \quad (24)$$

$$\lambda_{j9} \leq \delta_{j8} + \delta_{j9} \quad \forall j \in J \quad (25)$$

$$\lambda_{j10} \leq \delta_{j9} + \delta_{j10} \quad \forall j \in J \quad (26)$$

$$\lambda_{j11} \leq \delta_{j10} + \delta_{j11} \quad \forall j \in J \quad (27)$$

$$\lambda_{j12} \leq \delta_{j11} \quad \forall j \in J \quad (28)$$

$$\sum_{l=1}^{12} \delta_{jl} = 1 \quad \forall j \in J \quad (29)$$

$$\delta_{jl} \in \{0, 1\} \quad \forall j \in J, \quad l = 0, 1, \dots, \quad (30)$$

Above procedure says that in order to compute linear approximation of w_j , firstly, It should be clear that the point is like w_j , which lies between two consecutive extremes of defined distances.

Then, the value of $\sqrt{w_j}$ is computed exactly according to the same convex linear combination which was used for w_j (for more information, we can refer to Ghezavati et. al. [24]).

2.3 Implementation of genetic algorithm for the proposed model

In this paper, we use Lingo solver for solving the proposed model for small-sized examples and we aim to use a genetic algorithm for solving large-sized examples. For this purpose, we need to code the genetic algorithm in a language programming. In this way, we choose MATLAB software to code the genetic algorithm.

The primary population is matrix Y_{ij} ; i.e. chromosomes are elements of matrix Y_{ij} . The primary population is the same as the size of the population from random matrices

Y_{ij} . For example, if the population involves 200 individuals, the random matrix Y_{ij} will be the matrix to meet the limitations. The cross-over used for solving the model is a uni-point cross-section in a way that we have for example 2 parents namely Y_{ij} and Y_{ij} . Y_1 and Y_2 are two $I \times J$ matrices. A random number is selected between 1 and I and using row r , 2 new children are made; i.e. matrixes Y_1 and Y_2 are substituted in rows $1 - r$. Doing so maintains the limitation of the problem in the new children:

$$\sum_j Y_{ij} = 1 \quad \forall i \in I$$

In order to solve the model, a multi-point mutation is used. In this kind of mutation, some genes are selected randomly in the chromosomes and their values would be changed. In this problem, a row is selected randomly and then new values are allocated to this row based on limitation $\sum_j Y_{ij} = 1$. In fact, it tries to make a situation in which the limitations of the problem would not be contradicted in the new child. Therefore, for row R the new value allocation would be as follows:

$$\sum_j Y_{ij} = 1$$

The steps of the proposed genetic algorithm are described as follows:

Step 1 Initialize the input parameters of the algorithm including: the probability of the crossover and mutation, population size, the maximum number of iterations.

Step 2 The required number of initial solutions are generated randomly.

Step 3 Feasibility of the initial solutions is checked.

Step 4 The required solutions (called parents) are selected for crossover operation according to the crossover probability.

Step 5 The crossover operation is performed on parents to generate new solutions (called offspring).

Step 6 Feasibility of the offspring solutions is checked.

Step 7 The required solutions (called parents) are selected for mutation operation according to the mutation probability.

Step 8 The mutation operation is performed on parents to generate new neighborhood solutions.

Step 9 Feasibility of the neighborhood solutions is checked.

Step 10 Create the pool of solutions including the current solutions, offspring solutions, and neighborhood solutions.

Step 11 The fitness function for all solutions in the pool is computed according to the objective function in the Lp-metric method.

Step 12 According to the population size, the required number of solutions are selected according to the roulette cycle.

Step 13 Check the stopping criterion. If it is satisfied to stop and report the best solution, otherwise, go to step 4.

2.4 Implementation of Lp-metric approach

Since the model function has two objectives, the problem is solved at first with the first objective function and solution Z_1^* is obtained. Then it is solved with the second objective function to give solution Z_2^* . Moreover, in order to consider both objective functions simultaneously, the Lp-metric technique is used [25]:

$$\min z = \left[w_1 \frac{(z_1 - z_1^*)^p}{z_1^*} + w_2 \frac{(z_2 - z_2^*)^p}{z_2^*} \right]^{1/p}$$

where $w_1 = 0.5$, $w_2 = 5$ and $p = 1$.

The system by which code writing and implementing is done is involves 4 GB RAM, CPU: Core i5, 230 GHz. Once the proposed model is transferred to a single objective function, it is solved by Lingo 8 software for small sized examples and also it is solved by the proposed genetic algorithm for large sized examples that are coded by MATLAB 2011.

3 Numerical example

In order to show the effect of VaR on the mathematical model, a problem with 10 customers and 8 facility sites from 5 data sets and different input sets are considered. Later, a problem with 20 customers sites, 15 facilities sites and 5 data sets and different input sets, in addition to a problem with 50 customers sites and 40 facilities sites, and finally a problem with 70 customers site and 50 facilities sites are considered. Furthermore, it is attempted to consider the cost of each transportation unit between location j and customer i , fixed cost of locating the facility for site j (f), mean customer's demand (h) and capacity of facility j (p) as being different. Therefore, in all problems σ_i is assumed to be the normal distribution with a mean of 12 and variance of 2. Other parameters are also considered as $\alpha = 95\%$ and $a = 120$ and $L = 12$.

Created input data for solving the model by "MATLAB" and "Lingo" are shown in Table 1.

Results of input data from Table 1 for solving model considering just the first objective function is shown in Table 2. These results have gotten by "MATLAB" and "lingo". In Fig. 1, the solution run time for the proposed genetic algorithm and Lingo solver is illustrated. As can be seen,

Table 1 Input data

Data	I	J	c	f	h	P
<i>Sets and inputs</i>						
1	10	8	[5–15]	[10–20]	[20–25]	[20–40]
2	10	8	[5–15]	[15–50]	[100–250]	[150–300]
3	10	8	[5]	[10]	[10–25]	[15–35]
4	10	8	[5]	[10]	[10–25]	[20–50]
5	10	8	[5]	[10–20]	[20–25]	[20–40]
6	20	15	[5–15]	[10–20]	[20–25]	[20–40]
7	20	15	[5–15]	[10–20]	[20–25]	[20–50]
8	20	15	[5–15]	[10–20]	[20–25]	[40–50]
9	20	15	[5–15]	[10–20]	[20–30]	[20–60]
10	20	15	[5–15]	[10–20]	[20–30]	[20–50]
11	50	40	[5–15]	[10–20]	[10–30]	[20–60]
12	70	60	[5–15]	[10–20]	[20–30]	[20–50]

Table 2 Comparison between results of MATLAB and LINGO

MATLAB			LINGO	
Data	Result	Time (s)	Result	Time (s)
<i>In the first function</i>				
1	9.057	137	9.041503	945
2	9.6325	88	9.6288	38
3	56.5394	110	56.5213	192
4	3.1173	156	3.1088	199
5	34.3619	62	34.3438	465
6	92.221	151	92.4037	6480
7	49.0423	248	49.041	7800
8	36.7346	257	36.724	8640
9	1.4371	142	1.4213	7560
10	99.9337	196	99.9124	5200
11	39.144	355	221.617	34 h
12	51.6566	450	Infeasible	–

due to the Np-hard type of the mathematical model, the solution run time for Lingo has an exponential pattern and therefore, by increasing the size of the model actually Lingo is unable to solve the model. In this way, the solution run time of the genetic algorithm by MATLAB software has a linear form and this makes possible to solve the large-sized examples by this algorithm.

Results of input data from Table 1 for solving model considering just second objective function are shown in Table 3. These results have gotten by “MATLAB” and “Lingo”.

Results of input data from Table 1 for solving model considering both of objective functions are shown in Table 4. These results have gotten by “MATLAB” and “lingo”.

Now testing will be done to show the effect of increasing each input on the value of objective functions. The first 5 numerical examples of Table 4 are used in the test. Considering the importance of the objective function results, it

Fig. 1 Comparison of solution runs time of genetic algorithm and Lingo**Table 3** Comparison between results of MATLAB and LINGO

MATLAB			LINGO	
Data	Result	Time (s)	Result	Time (s)
<i>In second function</i>				
1	1286	92	1286	1
2	10640	68	10,640	1
3	985	95	985	1
4	875	91	875	1
5	1297	53	1297	1
6	2504	71	2504	1
7	2628	46	2628	1
8	2509	48	2509	1
9	2737	62	2737	1
10	2856	58	2856	1
11	4615	150	4615	43
12	6663	310	6663	190

Table 4 Comparison between results of MATLAB and LINGO

MATLAB			LINGO	
Data	Result	Time (s)	Result	Time (s)
<i>After implementing LP-metric</i>				
1	0.1844	115	0.184307	1983
2	0.0918	12	0.0918	106
3	0.0355	65	0.0332	1690
4	0.04	134	0.04	394
5	0.01045	118	1034	186
6	0.2015	137	0.2011	7095
7	0.2159	243	0.2153	9300
8	0.2758	218	0.2742	9660
9	1.174	155	1.1622	9480
10	0.1459	130	0.145	6900
11	0.4516	328	Not computed	–
12	0.3498	1537	Infeasible	–

Table 5 Model inputs by increase “c”

Data	I	J	f	h	p
1	10	8	[10–20]	[20–25]	[20–40]
2	10	8	[15–50]	[100–250]	[150–300]
3	10	8	[10]	[10–25]	[15–35]
4	10	8	[10]	[10–25]	[20–50]
5	10	8	[10–20]	[20–25]	[20–40]

is decided to ignore the calculated time. It should be noted that each test has done by Lingo.

3.1 First example

In the first test, 5 units will be added to the value of “c” in each example and the result has been shown in the following table. Input data for this test are shown in Table 5.

We use input data of Table 5 to perform this test and achieve effect modality of increase “c” on the objective functions, results by “MATLAB” and “Lingo” are illustrated in Table 6.

Table 6 Result of increase “c”

Data	New first objective function	Old first objective function	New second objective function	Old second objective function	New general objective function	Old general objective function
1	9.0415	9.057	2276	1286	0.121	0.1844
2	9.6288	9.6325	18,685	10,640	0.0522	0.0918
3	56.5213	56.5394	1960	985	0.18293	0.0355
4	3.1088	3.1173	1740	875	0.0201	0.04
5	34.3619	34.3619	2238	1297	0.0138	0.01045

Considering Table 6, it is obvious that by increasing the value of “c”, the first objective function does not change significantly but the second objective function increases while the second objective function has a descending trend due to a decrease in the total costs. Figure 2 shows the comparison between the old and new values of the second objective function. In addition, with respect to the Lp-metric method, it does not have a special effect on the general objective function.

3.2 Second example

In this test, the value of parameter “f” has increased for 2 units. Input data for this test are shown in Table 7.

We use input data of Table 7 to perform this test and achieve effect modality of increase “f” on the objective functions, results by “MATLAB” and “Lingo” are illustrated in Table 8.

As can be seen, the increasing the value of parameter “f” does not show a distinguished effect on the first objective function and causes just an insignificant reduction. However, its effect on the second objective function is obvious and increases the cost, but it is still not as much effective as parameter “c”.

3.3 Third example

In this section, the effect of increasing 2 units for parameter “h” on the objective functions will be checked. Input data for this test are shown in Table 9.

We use input data of Table 9 to perform this test and achieve effect modality of increasing parameter “h” on the objective functions, results by “MATLAB” and “Lingo” are reported in Table 10.

According to the obtained results, increasing “h” has a significant impact on increasing the first and second objective functions and therefore increases the general objective function.

Figure 3 shows the comparison between the values for the first objective function. As can be seen, by increasing parameter “h” in the model, the value of the first objective

Fig. 2 The comparison between the old and new values of the second objective function

Table 7 Model inputs by increase “f”

Data	I	J	c	h	P
1	10	8	[5–15]	[20–25]	[20–40]
2	10	8	[5–15]	[100–250]	[150–300]
3	10	8	[5]	[10–25]	[15–35]
4	10	8	[5]	[10–25]	[20–50]
5	10	8	[5]	[20–25]	[20–40]

Table 8 Result of increase “f”

Data	New first objective function	Old first objective function	New second objective function	Old second objective function	New general objective function	Old general objective function
1	9.0415	9.057	1296	1286	0.1852	0.1844
2	9.6288	9.6325	10,652	10,640	0.0919	0.0918
3	56.5121	56.5394	987	985	0.0325	0.0355
4	3.1088	3.1173	877	875	0.0478	0.04
5	34.2438	34.3619	1307	1297	0.1038	0.01045

Table 9 Model inputs by increase “h”

Data	I	J	c	f	p
1	10	8	[5–15]	[10–20]	[20–40]
2	10	8	[5–15]	[15–50]	[150–300]
3	10	8	[5]	[10]	[15–35]
4	10	8	[5]	[10]	[20–50]
5	10	8	[5]	[10–20]	[20–40]

increases. This is pattern is because of the fact that by increasing the holding cost, the model tries to minimize the number of products in the warehouse and this leads to increasing the value at risk for satisfying the demands. So, the level of VaR increases in this regard. However, as can

be seen from Fig. 4, in this analysis the model tries to minimize holding cost and this leads to decrease total costs of the model and therefore, the total cost has a descending trend.

3.4 Forth example

In this analysis, 2 units are added to parameter “p” and the outputs are gained again. Input data for this test are shown in Table 11 and then the outputs are reported.

We use input data of Table 11 to perform this test and achieve effect modality of increasing parameter “p” on the objective functions, results by “MATLAB” and “Lingo” are reported in Table 12.

Above results show that increasing of parameter “p” does not influence the second objective function but considerably reduces the first objective function with an insignificant effect on the general objective function.

4 Results and discussion

It should be noted that increasing the capacity of facility j leads to a reduction of VaR in the first objective function. Considering that during solving the model by software,

the objective function is simulated individually. Also, it can be observed that in the second objective function the values obtained from Lingo and MATLAB are the same, but the run time in Lingo for small dimensions is 1 s and as the dimensions increase, this time increases as well. Meanwhile, change of the value of parameters and problem inputs affects the amount of cost value (second objective function) more than the first one.

In addition, it can be seen that any increase in the value of fixed costs (f) leads to a small decrease in the first objective function of the proposed model. Once the value of capacities of the facilities increased in the model, the first objective function reduces more than previous saturations, however, such an event cannot change the global

Table 10 Result of increase “h”

Data	New first objective function	Old first objective function	New second objective function	Old second objective function	New general objective function	Old general objective function
1	12.0415	9.057	1391	1286	0.1866	0.1844
2	11.6288	9.6325	10,770	10,640	0.0948	0.0918
3	75.8962	56.5394	1085	985	0.0360	0.0355
4	17.6843	3.1173	975	875	0.0416	0.04
5	49.6171	34.3619	1403	1297	0.1066	0.01045

Fig. 3 Comparison between the values for the first objective function by increasing parameter “h” **Fig. 4** Comparison between the values for the second objective function by increasing parameter “h”**Table 11** Model inputs by increasing “p”

Data	I	J	c	f	h
1	10	8	[5–15]	[10–20]	[20–25]
2	10	8	[5–15]	[15–50]	[100–250]
3	10	8	[5]	[10]	[10–25]
4	10	8	[5]	[10]	[10–25]
5	10	8	[5]	[10–20]	[20–25]

objective function. According to the definition of VaR in previous sections, the value of the objective function improves by the length of the interval of demand once the technique of VaR is applied.

Table 13 illustrates the consequence of changing each parameter individually on first, second, and global objective functions.

5 Conclusion

In this study, a framework for evaluating and managing the investment in facility location problem regarded with site selection by applying a general site selection problems framework with respect to risk concepts is presented using the VaR approach. The best advantage of VaR is the creation of a structured approach to making precise risk decisions. The process of achieving the right VaR depends on its numerical value. In this article, the decision maker tries to optimize the value of VaR in the site selection problem and the costs associated with fixed costs where the manager would be able to forecast the total risks which can occur in the future.

It was also worked in this research to create a certain and linear method by implementing the VaR and linearization of the model by fragmentation linear approximation method. Also, by considering the fact that the obtained mathematical model is bi-objective, each objective function was solved separately by lingo and MATLAB using Lp-metric and the obtained results are compared. Meanwhile, it is proved that applying the VaR, one can minimize the costs and because of the profit by predicting the maximum loss. In this paper, demand was considered uncertain and it followed a normal distribution function. In this condition, even if the demand was considered as a risk factor and the amount of customer’s demand was not determined, one can manage the risk and control the maximum loss using the criteria of VaR measurement to be able to minimize the total costs.

For future researches, authors can consider the main parameters regarded to VaR as a fuzzy numbers since

Table 12 Result of increase “p”

Data	New first objective function	Old first objective function	New second objective function	Old second objective function	New general objective function	Old general objective function
1	1.6284	9.057	1286	1286	0.1955	0.1844
2	7.6288	9.6325	10,640	10,640	0.1859	0.0918
3	43.3129	56.5394	985	985	0.0549	0.0355
4	0.6886	3.1173	875	875	0.04	0.04
5	26.3811	34.3619	1297	1297	0.1182	0.01045

Table 13 Sensitivity analysis for the value of parameters on the VaR and costs

Parameter	Value at risk	Total cost	General
c	Reduce	Exceeded	Indeterminate
f	Reduce	Exceeded	Indeterminate
h	Exceeded	Exceeded	Increase
p	Reduce	Indifferent	Increase

tactical financial parameters are almost uncertain. In addition, implementing the proposed method for designing distribution networks and reverse logistics is necessary. Furthermore, developing for VaR concept using the fuzzy theory can be a suitable research in this area. Finally, applying multi objective metaheuristic methods such as multi objective particle swarm optimization (MOPSO) method will be another interesting improvement.

Compliance with ethical standards

Conflict of interest The author declares that they have no conflict of interest.

References

- Mohamed IB, Klibi W, Vanderbeck F (2020) Designing a two-echelon distribution network under demand uncertainty. *Eur J Oper Res* 280(1):102–123
- Weber A (1929) “Über den Standort der Industrien”. Tübingen, 1909. Alfred Weber’s theory of the location of Industries, University of Chicago Press (English translation by CJ Friedrich, 1929)
- Chan Y (2001) Location theory and decision analysis. South-Western College, Nashville
- Kheybari S, Kazemi M, Rezaei J (2019) Bioethanol facility location selection using best-worst method. *Appl Energy* 242:612–623
- Golpira H (2020) Optimal integration of the facility location problem into the multi-project multi-supplier multi-resource construction supply chain network design under the vendor managed inventory strategy. *Exp Syst Appl* 139:112841
- Simchi-Levi D, Kaminsky P, Simchi-Levi E (2004) Managing the supply chain: the definitive guide for the business professional. McGraw-Hill Companies, New York
- Jonkman J, Barbosa-Póvoa AP, Bloemhof JM (2019) Integrating harvesting decisions in the design of agro-food supply chains. *Eur J Oper Res* 276(1):247–258
- Jia F, Gong Y, Brown S (2018) Multi-tier sustainable supply chain management: the role of supply chain leadership. *Int J Prod Econ*. <https://doi.org/10.1016/j.ijpe.2018.07.022>
- Cheraghalipour A, Paydar MM, Hajiaghahi-Keshteli M (2019) Designing and solving a bi-level model for rice supply chain using the evolutionary algorithms. *Comput Electron Agric* 162:651–668
- Lejarza F, Baldea M (2019) Closed-loop optimal operational planning of supply chains with product quality dynamics. *Comput Chem Eng* 132:106594
- Taymaz S, Iyigun C, Bayindir ZP, Dellaert NP (2019) A healthcare facility location problem for a multi-disease, multi-service environment under risk aversion. *Soc Econ Plan Sci*. <https://doi.org/10.1016/j.seps.2019.100755>
- Xu M, Cui Y, Hu M, Xu X, Zhang Z, Liang S, Qu S (2019) Supply chain sustainability risk and assessment. *J Clean Prod* 225:857–867
- Markowitz H (1952) Portfolio selection. *J Finance* 7(1):77–91
- Hull J, Treepongkaruna S, Colwell D, Heaney R, Pitt D (2013) Fundamentals of futures and options markets. Pearson Higher Education AU
- Mavronicolas M, Monien B (2019) Conditional value-at-risk: structure and complexity of equilibria. *Theoret Comput Sci*. <https://doi.org/10.1016/j.tcs.2019.09.015>
- Morgan JP (1996) Riskmetrics technical document, 4th edn. Morgan Guaranty Trust Company, New York
- Bali T, Weinbaum D (2007) A conditional extreme value volatility estimator based on high-frequency returns. *J Econ Dyn Control* 31:361–397
- Hansen B (1994) Autoregressive conditional density estimation. *Int Econ Rev* 35:705–730
- Harvey C, Siddique A (1999) Autoregressive conditional skewness. *J Financ Quant Anal* 34(4):465–487
- Nieto B, Novales A, Rubio G (2014) Variance swaps, non-normality and macroeconomic and financial risks. *Q Rev Econ Finance* 54(2):257–270
- Psaradakis Z, Vávra M (2017) A distance test of normality for a wide class of stationary processes. *Econ Stat* 2:50–60
- Mohammadi S, Nazemi A (2020) On portfolio management with value at risk and uncertain returns via an artificial neural network scheme. *Cognit Syst Res* 59:247–263
- Ghezavati V, Hosseini P (2018) Application of efficient metaheuristics to solve a new bi-objective optimization model for hub facility location problem considering value at risk criterion. *Soft Comput* 22(1):195–212

24. Ghezavati VR, Soltanzadeh F, Hafezalkotob A (2015) Optimization of reliability for a hierarchical facility location problem under disaster relief situations by a chance-constrained programming and robust optimization. *Proc Inst Mech Eng Part O J Risk Reliab* 229(6):542–555
25. Mardan E, Govindan K, Mina H, Gholami-Zanjani SM (2019) An accelerated benders decomposition algorithm for a bi-objective green closed loop supply chain network design problem. *J Clean Prod* 235:1499–1514

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.