Unsteady MHD Casson fluid flow through vertical plate in the presence of Hall current

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Abstract

The problem of unsteady magnetohydrodynamic flow of non-Newtonian fluid through a vertical plate in the presence of Hall current is studied in this paper. Using similarity transformations, the governing coupled partial differential equations of the defined problem are transformed as nonlinear ordinary differential equations which are solved analytically by employing perturbation technique. The core objective of this research is to examine the impact of pertinent physical constraints like magnetic parameter, thermal radiation, and Dufour parameter over the velocity, temperature, and concentration profiles of the fluid. It is noted that Casson fluid has superior heat transfer characteristics compared to Newtonian fluid. Lorentz force which is determined from magnetic field has a proclivity to diminish the flow velocity.

Keywords Hall current · MHD · Casson fluid · Dufour effect

1 Introduction

Continuously flowing material in the presence of additional shear stress is termed as fluid. Flow of fluid caused by infinite vertical pervious plate is a recapitulating topic for researchers, as it has a wide range of applications in many technological and industrial processes. The implications of vertical plate were predominantly analysed by Huang [1]. He explored the outgrowths of non-Darcy and magnetohydrodynamic influence on non-Newtonian fluids with vertical plate in porous medium in the presence of thermal diffusion with diffusion thermoeffects. The study of flow of magnetic nanoparticles was carried out analytically by Ashwinkumar et al. [2] and found that volume friction of magnetic nanoparticles controls the heat transfer rate and wall friction and also deduced that heat transfer rate and flow are maximum for aligned magnetic field than the transverse one.

Basically fluids are categorized as Newtonian and non-Newtonian. Non-Newtonian fluids have viscosity varying according to applied stress or force. In recent days, the

study on non-Newtonian fluids has gained the interest of many sundry researchers owing to its considerable implications in mechanical and chemical engineering areas. Casson fluid is the most desired fluid among all non-Newtonian fluids. In 1959, N. Casson investigated the Casson fluid type to forecast the behaviour of flow of pigment oil in printing oil; Casson fluid is considered as the maximum favoured non-Newtonian fluid from its rheological properties, which can be used to examine the rheological character of materials like ketchup, blood, honey, shampoos, flow of plasma as well as mercury amalgams. Raju et al. [3] considered the Casson fluid to examine the significance of magnetic field through a stretching sheet and perceived that the induced magnetic parameter has the propensity to raise the heat transfer rate. Reddy et al. [4] gave detailed description on combined effects of frictional and irregular heat over Casson and Maxwell fluids and concluded that velocity profiles for Casson fluid are maximum than for Maxwell fluid. Numerical results for Casson fluid with the combined influence of heat source and magnetic field over different geometries were carried out by [5–7] and

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revealed that the fluid temperature is controlled by Casson parameter. Further, the combined investigation of heat and mass transfer of MHD Casson fluid under the effect of Brownian motion with thermophoresis was carried out by Kumar et al. [8] and concluded that energy and concentration fields of Maxwell fluid are affected by appropriate parameter as compared to Casson fluid.

The interpretation of electrically conducting fluids with magnetic effect is termed as magnetohydrodynamics. The term magnetohydrodynamics was originated by Hannes Alfvén in 1942. Flow of fluid towards magnetic field produces the electricity that affects the magnetic flux, and the effect of magnetic intensity upon the electric current implies tensile strength that changes the fluid motion. In present days, working on MHD flow became a topic of great interest as it has several implications in engineering, astrophysics, and geophysics. Recently, Kataria and Patel [9] have accomplished the outflow, thermal, and mass transfer features of magnetohydrodynamic Casson fluid and observed that increased magnetic field declines the fluid velocity as well as boundary layer thickness. Further, the problem of investigating the flow, thermal, and mass transfer performance of flow of MHD past a vertical rotating cone with the impact of radiation, chemical reaction, and thermal diffusion was examined numerically by Sulochana et al. [10]; effect of magnetohydrodynamic flow with heat transfer over distinct materialistic cases like thermal radiation, heat absorption/generation, Joule heating, Hartman number past various geometries was carried out by [11–13]. Similar study was carried out by Khan et al. [14] with the consideration of Sisko nanomaterial passed over a stretching sheet.

Numerical solutions for heat transfer in ferrofluid with applied magnetic field were illustrated by Javed and Siddiqui [15]. The study of heat transfer holds an imperative role due to its countless applications in environmental, industrial, and engineering processes.

Natural convection motion takes place as buoyancy-induced flow obtained like design of many devices like radiators, solar collectors, various components of power plants, space craft, and many more. Theoretical approach over the motion of Carreau fluid has been carried out by Kumaran et al. [16] and revealed that in parabolic motion, the melting heat transfer rate with buoyancy effect and external heat source have property to increase the thermal energy transfer. Comparatively, Sheikholeslami et al. [17] studied the heat transfer properties of refrigerant-based nanofluid and observed the conduction as well as micro-convection in fluid and obtained results. In a while, heat transfer of non-Newtonian fluid using various geometries was examined by [18–21].

Edwin Hall revealed Hall current in 1879. This phenomenon reports the nature of electrons across the conductor

under electric and magnetic field effects due to Lorentz force due to an electric potential difference among both sides of plate. It is observed that if the current across the plate is applied, the electrons move in a direction opposite to that of implemented magnetic field. Again, if enforced flux field is at right angle to the movement of electrons, the motion of electrons takes a curved path, and hence, electrons in motion gather along a side of the plate. It results in voltage development towards both the sides of plate; such a voltage is called ‘Hall voltage’, normal to the flow of magnetic as well as electric current. Hall current is employed in power generators, magnetometers, automotive fuel level indicators, planetary fluid dynamics, etc. In view of a wide range of applications, Biswas and Ahmed [22] examined the effect of radiant heat and chemical reaction with Hall current on variable Casson nanofluid and reported that velocity fall of with growing Casson parameter and as temperature profile decreases the heat generation raises. Further, [23–26] explored the Hall current effect on MHD.

Radiative heat transfer plays a vital role in the initiation of excessive temperature and hence has gained prominence due its usage in nuclear power plants, aircraft propulsion, space vehicles, and gas turbines. For instance, Sulochana et al. [26] elaborated the study of 3D Casson fluid flow with the influence of thermal radiation and thermodiffusion with unsteady heat source/sink. In one more attempt, Gupta et al. [27] addressed the effect of Brownian motion and thermophoresis in non-Newtonian nanofluid. Further, Hayat et al. [28] considered analytically the outcome of Joule heating and thermal radiation with chemical reaction of first order by considering flow of Maxwell nanofluid. Similar studies related to thermal radiation can be seen in [29].

From the above noted studies towards the boundary of the flow, we further consider the 2D motion of non-Newtonian Casson fluid with Dufour effect and chemical reaction. Sharma et al. [30] elaborated numerically by considering thermal diffusion and diffusion thermoreactions on free convective, heat absorption radiative nanofluid. The stagnation point of Casson fluid was presented by Shaw et al. [29] under the influence of radiation, thermal diffusion, diffusion thermoeffects with chemical reaction.

The study of heat and mass transfer including chemical reaction has influential aspect in various operations and hence gained substantial importance in present days, like evaporation of water body, polymer production, formulation as well as dispersion of fog, heat transfer in moist cooling tower, etc. The flow of magnetic nanofluid using ferrous nanoparticles with an elongated sheet was addressed by Poojari et al. [31] while Ibrahim et al. [32] addressed the combined influence of heat and mass transfer in view of Casson fluid with influence of thermal radiation with Soret and Dufour

effects and concluded that strength of Dufour number boosts with raise in density of thermal boundary layer.

To the best extent of our knowledge, no work has been carried by the researchers to note the effect of Hall current over an incompressible boundary layer slip motion of Casson fluid over an infinite vertical plate together with heat suction, thermal radiation, and chemical reaction. We made the comparative study for Newtonian and non-Newtonian fluids. Hence, by making use of the above-mentioned results, we attempted to bridge the gaps by extending El-Aziz and Yahya [33] problem. To workout the ordinary differential equations, we speculated an analytical perturbation technique and obtained graphical illustrations with the aid of MATLAB package.

2 Formulation of the problem

The consequence of heat and mass transfer with unstable slip motion of Casson fluid over an unbounded plane pervious plate is considered. Here $(\bar{x}, \bar{y}, \bar{z})$ represents the cartesian geometry, where $\bar{x}$ -axis is considered upwards to the direction of vertical plate, direction of $\bar{y}$ -axis is chosen perpendicular to the flat surface of the plate towards the fluid zone, and also $\bar{z}$ -axis is directed perpendicular to $\bar{x}$ $\bar{y}$ -axis. When plate owns the plane $\bar{y} = 0$ to incessant term, every substantial term confines only on $\bar{y}$ and $\bar{t}$. When a powerful stable transverse magnetic flux of strength B_0 is enforced in the direction of $\bar{y}$ -axis, the effect of Hall current influences an electrical phenomenon which flows orthogonal to flux field as well as electric effect that instigates a transverse fluid flow. Hence, the additional flow is generated by the Hall current, and thus, there are two elements of velocity. Again, it is expressed that magnetic flux of the flow is imperceptible compared to the enforced one which implies negligible magnetic Reynolds number given as $B = (0, B_0, 0)$. If the Hall the term is confined, then from the Ohm's generalized law the below expression holds:

$$J + \frac{m}{B_0}(J \times B) = \sigma(E + V \times B), \quad (1)$$

where $m = w_e \tau_e$ is Hall variable, in which w_e is the frequency of electron and τ_e is the time collision of electrons; $J = (J_{\bar{x}}, J_{\bar{y}}, J_{\bar{z}})$ represents the vector for direction of electrical density, whereas $V = (\bar{u}, \bar{v}, \bar{w})$ represents direction of momentum vector with σ as electrical potential. From the above-mentioned considerations, Eq. (1) takes the form:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{x}} + \nu \left(1 + \frac{1}{\xi} \right) \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2}{\rho(1+m^2)} (\bar{u} - \bar{U}_\infty + m\bar{w}) + g\beta(\bar{T} - \bar{T}_\infty), \quad (12)$$

$$J_{\bar{x}} - mJ_{\bar{z}} = \sigma(E_{\bar{x}} - B_0\bar{w}), \quad (2)$$

$$J_{\bar{y}} = 0, \quad (3)$$

$$J_{\bar{z}} + mJ_{\bar{x}} = \sigma(E_{\bar{z}} + B_0\bar{u}). \quad (4)$$

Hence, there is no electrical current in free flow as magnetic flux remains unaltered.

$$\therefore J_{\bar{x}} \rightarrow 0, J_{\bar{z}} \rightarrow 0 \text{ when } \bar{y} \rightarrow \infty. \quad (5)$$

Considering $\bar{u} \rightarrow \bar{U}_\infty$, $\bar{w} \rightarrow 0$ at $\bar{y} \rightarrow \infty$, Eqs. (2) as well as (4):

$$E_{\bar{x}} = 0 \text{ and } E_{\bar{z}} = -B_0\bar{U}_\infty \quad (6)$$

$$\therefore J_{\bar{x}} - mJ_{\bar{z}} = -\sigma B_0\bar{w}, \quad (7)$$

$$J_{\bar{z}} + mJ_{\bar{x}} = \sigma B_0(\bar{u} - \bar{U}_\infty). \quad (8)$$

From Eqs. (7) and (8), we get:

$$J_{\bar{x}} = \frac{\sigma B_0}{1+m^2} (m(\bar{u} - \bar{U}_\infty) - \bar{w}), \quad (9)$$

$$J_{\bar{z}} = \frac{\sigma B_0}{1+m^2} (\bar{u} - \bar{U}_\infty + m\bar{w}), \quad (10)$$

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \frac{\tau_0}{\sqrt{2\pi}}\right)e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + \frac{\tau_0}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi < \pi_c \end{cases}$$

where π represents the amount of deformation rate with $\pi = e_{ij}e_{ij}$ where e_{ij} represents (i, j) th element of measure of deformation, critical point of the product depend upon non-Newtonian fluid model is, the plastic absolute viscosity of non-Newtonian fluid is μ_B , and resultant fluid stress is τ_0 .

From the above assumptions, equations of flow for the present work with the effect of Hall current from Boussinesq approximation are given as represented in [33]:

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0, \quad (11)$$

$$\frac{\partial \bar{w}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{w}}{\partial \bar{y}} = v \left(1 + \frac{1}{\xi} \right) \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{\sigma B_0^2}{\rho(1+m^2)} [m(\bar{u} - \bar{U}_\infty) - \bar{w}], \quad (13)$$

slip velocity component. When $\chi = 0$, non-slip condition can be found. Equation (11) proves that the suction velocity at the plate is invariant of time. Hence, considering the suction velocity as assimilatory, therefore Eq. (24) can be

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k}{\rho c_p} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{1}{\rho c_p} \frac{16\sigma T_\infty^3}{3K} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 \bar{c}}{\partial \bar{y}^2} - \frac{Q_0}{\rho c_p} (\bar{T} - \bar{T}_\infty), \quad (14)$$

$$\frac{\partial \bar{c}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{c}}{\partial \bar{y}} = D_B \frac{\partial^2 \bar{c}}{\partial \bar{y}^2} - K_r (\bar{c} - \bar{c}_\infty). \quad (15)$$

written as:

$$\bar{v} = -V_0 [1 + \varepsilon A \exp(i\bar{\omega}\bar{t})]. \quad (17)$$

Here $\bar{t}$ is the spatial time, g is the gravitational acceleration, $\bar{T}$ is the spatial fluid temperature close to the plate, $\bar{T}_\infty$ is the free flow spatial temperature, β is the heat expansion factor, μ is the fluid viscosity, ρ is the density of fluid, k is the thermal conductivity, $v = \frac{\mu}{\rho}$ is the kinematic viscosity of fluid, ξ is the Casson fluid constant, Q_0 represents the spatial heat exhaustion factor with c_p as definite temperature of the fluid with steady force. From the above

Here V_0 expresses the average velocity absorption, A is a real absolute constant, ε as well as εA is insignificant terms not more than one. Here minus symbol expresses the absorption of velocity about the plate. Away from interfacial layer, Eq. (12) implies:

$$-\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{x}} = \frac{d\bar{U}_\infty}{d\bar{t}}. \quad (18)$$

Using (17) in (12), we get:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{d\bar{U}_\infty}{d\bar{t}} + v \left(1 + \frac{1}{\xi} \right) \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2}{\rho(1+m^2)} (\bar{u} - \bar{U}_\infty + m\bar{w}) + g\beta(\bar{T} - \bar{T}_\infty). \quad (19)$$

suppositions, suitable preconditions for rate of velocity as well as thermal fields are delimited as (24).

Combining Eqs. (13) into (18) and using compound variable

At $\bar{y} = 0$:

$$\bar{q} \text{ so that } \bar{q} = \bar{u} + i\bar{w} \quad (20)$$

$$\bar{u} = \bar{u}_{\text{slip}} = \chi \left(1 + \frac{1}{\xi} \right) \frac{\partial \bar{u}}{\partial \bar{y}}, \quad \bar{w} = \bar{w}_{\text{slip}} = \chi \left(1 + \frac{1}{\xi} \right) \frac{\partial \bar{w}}{\partial \bar{y}}, \quad \bar{T} = \bar{T}_w + \varepsilon (\bar{T}_w - \bar{T}_\infty) \exp(i\bar{\omega}\bar{t}).$$

$$\frac{\partial \bar{q}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{q}}{\partial \bar{y}} = v \left(1 + \frac{1}{\xi} \right) \frac{\partial^2 \bar{q}}{\partial \bar{y}^2} - \frac{\sigma B_0^2}{\rho(1+m^2)} (1 - im)(\bar{q} - \bar{U}_\infty) + g\beta(\bar{T} - \bar{T}_\infty) + \frac{d\bar{U}_\infty}{d\bar{t}}. \quad (21)$$

And the equation of energy takes the form:

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k}{\rho c_p} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{1}{\rho c_p} \frac{16\sigma T_\infty^3}{3K} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{D_m k_T}{c_s c_p} \frac{\partial^2 \bar{c}}{\partial \bar{y}^2} - \frac{Q_0}{\rho c_p} (\bar{T} - \bar{T}_\infty). \quad (22)$$

As

Also, the equation for mass exchange is given as:

$$\bar{y} \rightarrow \infty : \bar{u} \rightarrow \bar{U}_\infty = U_0 [1 + \varepsilon \exp(i\bar{\omega}\bar{t})] \quad \bar{w} \rightarrow 0, \quad \bar{T} \rightarrow \bar{T}_\infty. \quad (16)$$

Here $\bar{T}_w$ is the dimensional temperature of wall, $\bar{U}_\infty$ is the dimensional velocity of free flow, $\bar{\omega}$ is the dimensional frequency of vibration, U_0 is the invariant term, and χ is the

$$\frac{\partial \bar{c}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{c}}{\partial \bar{y}} = D_B \frac{\partial^2 \bar{c}}{\partial \bar{y}^2} - K_r (\bar{c} - \bar{c}_\infty). \quad (23)$$

Then, the suitable boundary conditions pertinent to given problem imply:

At $\bar{y} = 0$:

$$\bar{q}_{\text{slip}} = \chi \left(1 + \frac{1}{\xi} \right) \frac{\partial \bar{q}}{\partial \bar{y}}, \quad \bar{T} = \bar{T}_w + \varepsilon (\bar{T}_w - \bar{T}_\infty) \exp(i\omega \bar{t}).$$

As $\bar{y} \rightarrow \infty$:

$$\bar{q} \rightarrow \bar{U}_\infty = U_0 (1 + \varepsilon \exp(i\omega \bar{t})), \quad \bar{T} \rightarrow \bar{T}_\infty. \quad (24)$$

We make use of the succeeding dimensionless variables:

$$\begin{aligned} q &= \frac{\bar{q}}{U_0}, v = \frac{\bar{v}}{V_0}, y = \frac{V_0 \bar{y}}{v}, U_\infty = \frac{\bar{U}_\infty}{U_0}, t = \frac{V_0^2 \bar{t}}{v}, \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, \omega = \frac{\bar{\omega} v}{V_0^2}, \\ Gr &= \frac{g v \beta (\bar{T}_w - \bar{T}_\infty)}{U_0 V_0^2}, M = \frac{\sigma B_0^2 v}{\rho V_0^2}, Pr = \frac{v \rho c_p}{k}, Q_H = \frac{Q_0 v}{\rho c_p V_0^2}, \phi = \frac{\bar{c} - \bar{c}_\infty}{\bar{c}_w - \bar{c}_\infty}, \\ R &= \frac{16 \bar{\sigma} T_\infty^3}{3 k \bar{k}}, Sc = \frac{v}{D_B}, K = \frac{k v}{V_0^2}, Du = \frac{D_m k_T (\bar{c}_w - \bar{c}_\infty)}{v c_s c_p (\bar{T}_w - \bar{T}_\infty)}. \end{aligned} \quad (25)$$

Taking into consideration of (25), dimensionless mode of Eq. (20) can be revealed as:

$$\frac{\partial q}{\partial t} - (1 + A \varepsilon e^{i\omega t}) \frac{\partial q}{\partial y} = \left(1 + \frac{1}{\xi} \right) \frac{\partial^2 q}{\partial y^2} + \frac{dU_\infty}{dt} + Gr \theta + \frac{M(1 - im)}{1 + m^2} (U_\infty - q). \quad (26)$$

Using Eqs. (25), (22) becomes dimensionless form, given as:

$$\frac{\partial \theta}{\partial t} - (1 + A \varepsilon e^{i\omega t}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} = \left(\frac{\partial^2 \theta}{\partial y^2} + R \frac{\partial^2 \theta}{\partial y^2} \right) + Du \frac{\partial^2 \phi}{\partial y^2} - Q_H \theta. \quad (27)$$

Similarly, using (24), in (23), we get, i.e.

$$\begin{aligned} \varepsilon e^{i\omega t} \left[f_1'' + \frac{1}{a} f_1' - \left(\frac{i\omega}{a} + \frac{M(1 - im)}{a(1 + m^2)} \right) f_1 + \frac{A}{a} f_0' + \frac{Gr}{a} g_1 + \frac{M(1 - im)}{a(1 + m^2)} + \frac{i\omega}{a} \right] &= -f_0'' - \frac{1}{a} f_0' \\ + \frac{M(1 - im)}{a(1 + m^2)} f_0 - \frac{Gr}{a} g_0 - \frac{M(1 - im)}{a(1 + m^2)}. \end{aligned} \quad (33)$$

$$\frac{\partial \phi}{\partial t} - (1 + A \varepsilon e^{i\omega t}) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - Kr \phi. \quad (28)$$

Here Gr is the Grashof number, Pr is the Prandtl number, M is the constant of magnetic flux, with Q_H as the parameter of heat absorption. The non-dimensional mode preconditions of the boundary (24) turn into:

$$\text{At } y = 0; \quad q_{\text{slip}} = \delta \left(1 + \frac{1}{\xi} \right) \frac{\partial q}{\partial y}, \quad (29)$$

$$\text{As } y \rightarrow \infty : \quad q \rightarrow U_\infty = 1 + \varepsilon e^{i\omega t}, \quad \theta = 0.$$

Here $\delta = \frac{(\chi V_0)}{v}$ is the slip parameter. Equations (26), (27) and (28) are PDE's and cannot be solved directly; however, the set of PDE's may be reduced into set of ODE's in non-dimensional form and the solutions can be found analytically. In this,

'q' is the velocity, 'θ' is the temperature, and 'φ' is the concentration, which are given as follows:

$$q = f_0(y) + \varepsilon e^{i\omega t} f_1(y) + o(\varepsilon^2), \quad (30)$$

$$\theta = g_0(y) + \varepsilon e^{i\omega t} g_1(y) + o(\varepsilon^2), \quad (31)$$

$$\phi = h_0(y) + \varepsilon e^{i\omega t} h_1(y) + o(\varepsilon^2). \quad (32)$$

Using value of 'q' from Eq. (30) in (26), we get:

Here $a = \left(1 + \frac{1}{\xi} \right)$. Comparing harmonic as well as non-harmonic terms and also ignoring greater order terms of $o(\varepsilon^2)$ in Eq. (33) imply:

$$f_1'' + \frac{1}{a}f_1' - \frac{1}{a}\left(i\omega + \frac{M(1-im)}{1+m^2}\right)f_1 = -\frac{1}{a}\left(i\omega + \frac{M(1-im)}{1+m^2}\right) - \frac{A}{a}f_0' - \frac{Gr}{a}g_1 \quad (34)$$

and

$$f_0'' + \frac{1}{a}f_0' - \left[\frac{M(1-im)}{a(1+m^2)}\right]f_0 = -\frac{Gr}{a}g_0 - \frac{M(1-im)}{a(1+m^2)}. \quad (35)$$

Also, substituting value of ' θ' ' in Eq. (27), we obtain:

$$\begin{aligned} \varepsilon e^{i\omega t} \left[-\left(\frac{1+R}{Pr}\right)g_1'' - g_1' + (Q_H + i\omega)g_1 - Ag_0' - Duh_1'' \right] \\ = \left(\frac{1+R}{Pr}\right)g_0'' + g_0' - Q_H g_0 + Duh_0''. \end{aligned} \quad (36)$$

Equating harmonic and non-harmonic terms and ignoring greater order terms of ε in Eq. (36), we obtain:

$$g_1'' + \left(\frac{Pr}{1+R}\right)g_1' - \left(\frac{Pr}{1+R}\right)(Q_H + i\omega)g_1 = \left(-\frac{Pr}{1+R}\right)Ag_0' - \left(\frac{Pr}{1+R}\right)Duh_1'' \quad (37)$$

and

$$g_0'' + \left(\frac{Pr}{1+R}\right)g_0' - \left(\frac{Pr}{1+R}\right)Q_H g_0 = -\left(\frac{Pr}{1+R}\right)Duh_0''. \quad (38)$$

Similarly, substituting value of ϕ in Eq. (28) we get:

$$\varepsilon e^{i\omega t} \left[-\frac{1}{Sc}h_1'' - h_1' + (Kr + i\omega)h_1 - Ah_0' \right] = \frac{1}{Sc}h_0'' + h_0' - Krh_0. \quad (39)$$

Comparing harmonic and non-harmonic terms, we get:

$$h_1'' + Sch_1' - (ScKr + Sci\omega)h_1 = -ScAh_0' \quad (40)$$

and

$$h_0'' + Sch_0' - ScKrh_0 = 0. \quad (41)$$

First solving Eq. (41) analytically, we acquire the solution as:

$$h_0(y) = c_1 e^{\left(\frac{-Sc + \sqrt{(Sc)^2 + 4ScKr}}{2}\right)y} + c_2 e^{m_1 y}. \quad (42)$$

Applying boundary conditions

$$h_0 = 1 \quad \text{at } y = 0$$

$$h_0 = 0 \quad \text{as } y \rightarrow \infty$$

implies

$$h_0(y) = e^{m_1 y}. \quad (43)$$

Next solving Eq. (40), the auxiliary equation is:

$$m^2 + Scm - (ScKr + Sci\omega)h_1 = 0$$

$$C.F = c_1 e^{\frac{-Sc + \sqrt{(Sc)^2 + 4(ScKr + Sci\omega)}}{2}} + c_3 e^{m_2 y} \quad (44)$$

$$P.I = B_1 e^{m_1 y}$$

$$h_1(y) = C.F + P.I$$

$$h_1(y) = C_1 e^{Ay} + C_3 e^{m_2 y} + B_1 e^{m_1 y}.$$

Applying the boundary conditions

$$\begin{aligned} h_1 &= 1 \quad \text{at } y = 0 \\ h_1 &= 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (45)$$

and applying the boundary conditions (44), Eq. (43) implies

$$h_1(y) = C_3 e^{m_2 y} + B_1 e^{m_1 y}. \quad (46)$$

Now solving Eq. (38), the characteristic equation is:

$$m^2 + \left(\frac{Pr}{1+R}\right)m - \left(\frac{Pr}{1+R}\right)Q_H g_0 = 0.$$

The complementary function is:

$$C.F = c_1 e^{\frac{\left(-\frac{Pr}{1+R}\right) + \left(\frac{Pr}{1+R}\right)^2 + 4\left(\frac{Pr}{1+R}\right)Q_H}{2}}y} + c_4 e^{m_3 y}$$

$$P.I = B_2 e^{m_1 y}$$

$$g_0(y) = c_1 e^{\left[\frac{\left(-\frac{Pr}{1+R}\right) + \sqrt{\left(\frac{Pr}{1+R}\right)^2 + 4\left(\frac{Pr}{1+R}\right)Q_H}}{2}\right]y} + c_4 e^{m_3 y} + B_2 e^{m_1 y}. \quad (47)$$

The suitable boundary conditions are:

$$\begin{aligned} g_0 &= 1 \quad \text{at } y = 0 \\ g_0 &= 0 \quad \text{as } y \rightarrow \infty. \end{aligned} \quad (48)$$

Applying the boundary conditions (47) into Eq. (46) and now solving Eq. (37), the characteristic equation is:

$$m^2 + \left(\frac{Pr}{1+R}\right)m - \left(\frac{Pr}{1+R}\right)(Q_H + i\omega) = 0$$

$$C.F = C_1 e^{\left[\left(\frac{\left(\frac{-Pr}{1+R}\right) + \sqrt{\left(\frac{Pr}{1+R}\right)^2 + 4\left(\frac{Pr}{1+R}\right)(Q_H + i\omega)}}{2}\right)y\right]} + C_5 e^{m_4 y}, \quad (49)$$

$$P.I = B_3 e^{m_1 y} + B_4 e^{m_2 y} + B_5 e^{m_3 y},$$

$$g_1(y) = C.F + P.I$$

$$g_1(y) = C_1 e^{\left[\left(\frac{\left(\frac{-Pr}{1+R}\right) + \sqrt{\left(\frac{Pr}{1+R}\right)^2 + 4\left(\frac{Pr}{1+R}\right)(Q_H + i\omega)}}{2}\right)y\right]} + C_5 e^{m_4 y} + B_3 e^{m_1 y} + B_4 e^{m_2 y} + B_5 e^{m_3 y}. \quad (50)$$

The boundary conditions are:

$$g_1 = 1 \quad \text{at} \quad y = 0$$

$$g_1 = 0 \quad \text{at} \quad y \rightarrow \infty \quad (51)$$

$$\therefore g_1(y) = C_5 e^{m_4 y} + B_3 e^{m_1 y} + B_4 e^{m_2 y} + B_5 e^{m_3 y}. \quad (52)$$

Solving Eq. (35) analytically,
the characteristic equation is:

$$m^2 + \frac{1}{a}m - \frac{M(1-im)}{a(1+m^2)} = 0,$$

$$C.F = C_1 \left[e^{\left(\frac{-\frac{1}{a} + \sqrt{\frac{1}{a^2} + \frac{4M(1-im)}{a(1+m^2)}}}{2}\right)y} \right] + C_6 e^{m_5 y}$$

$$P.I = B_6 e^{m_3 y} + B_7 e^{m_1 y} + 1$$

$$\therefore f_0(y) = C_1 e^{\left(\frac{-\frac{1}{a} + \sqrt{\frac{1}{a^2} + \frac{4M(1-im)}{a(1+m^2)}}}{2}\right)y} + C_6 e^{m_5 y} + B_6 e^{m_3 y} + B_7 e^{m_1 y} + 1. \quad (53)$$

Applying the boundary conditions,

$$f_0 = \delta a f'_0 \quad \text{at} \quad y = 0$$

$$f_0 = 1 \quad \text{as} \quad y \rightarrow \infty \quad (54)$$

$$f_0(y) = C_6 e^{m_5 y} + B_6 e^{m_3 y} + B_7 e^{m_1 y} + 1. \quad (55)$$

Now solving Eq. (34), we get:

$$C.F = C_1 e^{\frac{-\frac{1}{a} + \sqrt{\frac{1}{a^2} + \frac{4}{a}\left(i\omega + \frac{M(1-im)}{1+m^2}\right)y}}{2}} + C_7 e^{m_6 y}, \quad (56)$$

$$\therefore f_1(y) = C_1 e^{\frac{-\frac{1}{a} + \sqrt{\frac{1}{a^2} + \frac{4}{a}\left(i\omega + \frac{M(1-im)}{1+m^2}\right)y}}{2}} + C_7 e^{m_6 y} + B_8 e^{m_1 y} + B_9 e^{m_2 y} + B_{10} e^{m_3 y} + B_{11} e^{m_4 y} + B_{12} e^{m_5 y} + 1. \quad (57)$$

Substituting the suitable boundary conditions (58), in
Eq. (57), we get:

$$f_1 = \delta a f'_1 \quad \text{at} \quad y = 0$$

$$f_1 = 1 \quad \text{as} \quad y \rightarrow \infty$$

$$\therefore f_1(y) = C_7 e^{m_6 y} + B_8 e^{m_1 y} + B_9 e^{m_2 y} + B_{10} e^{m_3 y} + B_{11} e^{m_4 y} + B_{12} e^{m_5 y} + 1. \quad (58)$$

Substituting the values of f_0, f_1 in Eq. (30), g_0, g_1 in Eq. (31), h_0, h_1 in Eq. (32), to calculate the concluding values

Fig. 1 Geometry of fluid flow and physical model

Fig. 2 Influence of magnetic parameter ' M ' over velocity profiles

Fig. 4 Influence of slip parameter ' δ ' over velocity profiles

Fig. 3 Influence of Hall current parameter ' m ' over velocity profiles

Fig. 5 Influence of heat absorption parameter ' Q_H ' over velocity profiles

of velocity, thermal and concentration distributions in the boundary layer is given below:

$$q(y, t) = (C_6 e^{m_5 y} + B_6 e^{m_3 y} + B_7 e^{m_1 y} + 1) + \varepsilon e^{i\omega t} (C_7 e^{m_6 y} + B_8 e^{m_1 y} + B_9 e^{m_2 y} + B_{10} e^{m_3 y} + B_{11} e^{m_4 y} + B_{12} e^{m_5 y} + 1), \quad (59)$$

$$\theta(y, t) = C_4 e^{m_3 y} + B_2 e^{m_1 y} + \varepsilon e^{i\omega t} (C_5 e^{m_4 y} + B_5 e^{m_1 y} + B_4 e^{m_2 y} + B_5 e^{m_3 y}), \quad (60)$$

$$\phi(y, t) = e^{m_1 y} + \varepsilon e^{i\omega t} (C_3 e^{m_2 y} + B_1 e^{m_1 y}). \quad (61)$$

Skin friction

$$\tau = -[a(C_6 m_5 + B_6 m_3 + B_7 m_1) + \varepsilon e^{i\omega t} (C_7 m_6 + B_8 m_1 + B_9 m_2 + B_{10} m_3 + B_{11} m_4 + B_{12} m_5)]. \quad (62)$$

Nusselt number

Fig. 6 Influence of Grashof number 'Gr' over velocity profiles

Fig. 8 Influence of radiation parameter 'R' over velocity profiles

Fig. 7 Influence of Prandtl number 'Pr' over velocity profiles

Fig. 9 Influence of Dufour parameter 'Du' over velocity profiles

The measure of non-dimensional Heat transfer (Nusselt number) of current problem is as follows:

$$\begin{aligned} Nu &= -\frac{v}{V_0(\bar{T}_w - \bar{T}_\infty)} \frac{\partial \bar{T}}{\partial y} \Big|_{y=0} = -\frac{\partial \theta}{\partial y} \Big|_{y=0} \\ &= -C_4 m_3 + B_2 m_1 + \epsilon e^{i\omega t} (C_5 m_4 + B_5 m_1 + B_4 m_2 + B_5 m_3). \end{aligned} \quad (63)$$

Sherwood number

The Sherwood number coefficient is given below:

$$Sh = -\frac{v}{V_0(\bar{T}_w - \bar{T}_\infty)} \frac{\partial \bar{\phi}}{\partial y} \Big|_{y=0} = -\frac{\partial \phi}{\partial y} \Big|_{y=0} - m_1 + \epsilon e^{i\omega t} (C_3 m_2 + B_1 m_1). \quad (64)$$

3 Results and discussions

The perturbation method is performed to analyse the Hall current for translation of slip motion of Casson fluid through a vertical plate. To impart some suitable physical conditions of resultant values, the graphs of velocity, thermal and concentration profiles, skin friction, magnetic flux M , slip variable δ , Grashof number, Prandtl number,

Fig. 10 Influence of Prandtl number 'Pr' over temperature profiles

Fig. 12 Influence of heat absorption parameter ' Q_H ' over temperature profiles

Fig. 11 Influence of Dufour parameter 'Du' over temperature profiles

Fig. 13 Influence of radiation parameter ' R ' over temperature profiles

heat absorption parameter Q_H , for both non-Newtonian and Newtonian fluids, are drawn accordingly. We take into account the measures of non-dimensionalized parameters as $e=0.2$, $b=0.1$, $Pr=0.7$, $M=1$, $t=0.1$, $Gr=5$, $Du=2$, $Sc=0.6$, $Q_H=5$, $S=2$, $Kr=0.5$, $R=1$, $A=1$, $m=1$, $w=10$, as standard values in the entire study except indicated in the graphs (Fig. 1).

Figure 2 represents the influence of magnetic parameter M over momentum. It is noted that, for rising values of magnetic parameter, velocity profile diminishes, as magnetic parameter owns a propensity to accelerate the resistance which converses towards the flow. This energy

is known as Lorentz force that results in lowering the velocity.

Figure 3 depicts the influence of Hall parameter m over the velocity field for both Newtonian and non-Newtonian fluids. From the graph, it is observed that rising values of Hall parameter lead to greater velocity. Since the effective conductivity decreases with the increase in Hall parameter and which intends decrease in magnetic damping, results the increase in velocity. Figure 4 shows the effect of slip parameters δ over momentum field. It is perceived that an escalation in velocity increases the slip parameter δ as

Fig. 14 Influence of Schmidt number 'Sc' over concentration profiles

Fig. 15 Influence of chemical reaction parameter 'Kr' over concentration profiles

it has a propensity of lowering the resistance forces that raises the velocity of fluids.

Figure 5 shows the result of heat absorption constant Q_H over momentum profiles. It is noticed that rising values of Q_H tend to drop the width of the boundaries since when energy is absorbed, lessening in buoyancy force takes place, and hence, the flow rate is influenced by negative effects and leads in depreciation in the values of velocity.

Figure 6 exhibits the Grashof number effect over velocity profile, and it is found that the momentum is a growth value for Grashof number. Since the transformation of

Fig. 16 Influence of magnetic parameter 'M' over skin friction coefficient

Fig. 17 Influence of Grashof number 'Gr' over skin friction coefficient

energy approximates the rate of energy, ratio of buoyancy to viscous force is a Grashof number. Hence, the larger buoyancy force tends to boost the strength of buoyancy which implies rise in momentum field. Prandtl number consequences over the velocity field are portrayed in Fig. 7, and it is found that gain in Prandtl count decreases the momentum. For rising up, the effectiveness of Prandtl number strengthens the kinematic viscosity; hence, the diffusivity of heat denigrates, which implies decreasing the velocity profile. The consequence of thermal radiation over the velocity field is displayed in sketch 8. The sketch concludes that velocity profile has a contraction for increasing values of radiation which implies in wide-ness of the boundary layer. Figures 9 and 10 depict the momentum as well as thermal fields for different values of

Fig. 18 Influence of radiation parameter 'R' over skin friction coefficient

Fig. 19 Influence of Prandtl number 'Pr' over skin friction coefficient

Dufour effect (Fig. 11). From the graph, we clearly observe that Dufour number is an increasing function of velocity, i.e. as Dufour number increases, velocity of the fluid also increases. Implications of temperature absorption variable Q_H over thermal profiles are plotted in Fig. 12. It is noticed that the thickness of the boundary layer declines when heat absorption parameter Q_H has been raised. Thermal radiation effects over temperature field are displayed in Fig. 13; from the graph it is estimated that strengthening the heat of the flow is a result of boost in radiation factor. Graphical representation for different plots of Sc is presented in Fig. 14. Also note that fluid concentration diminishes when Sc increases. An outcome of chemical

Fig. 20 Influence of magnetic parameter 'M' over Nusselt number

Fig. 21 Influence of Dufour parameter 'Du' over Sherwood number

reaction over concentration field is sketched in graph 15; it is visible that fluid concentration declines with growing chemical reaction parameter.

The graphical outcomes 16, 17, 18, 19 were sketched to represent the effect of magnetic constant M , Grashof number Gr , radiation constant R with Prandtl number Pr over skin friction parameter, respectively. Figure 16 demonstrates the change in friction parameter with respect to magnetic variable M . It is observed that magnetic field cuts down velocity of the fluid and raises the viscosity dominant to rise in skin friction factor. In Fig. 17, it is viewed that rise in Gr decreases the friction factor. The increase in the buoyancy force implies to reduce internal friction of fluid. Figure 18 shows the impact of friction

factor on radiation variable R . From the graph, it is concluded that growth of radiation parameter influences the fluid to diminish the temperature of the diffusive fluid which boosts the mean absorption coefficient which declines the skin friction that boosts the rate heat transfer as well as magnetic intensity at the surface. Influence of Prandtl number over Skin friction is represented in Fig. 19; it is viewed that skin friction raises gradually with raising Q_H .

Figure 20 demonstrates the outcome of Nusselt number over magnetic parameter. Figure 21 incorporates the effect of Dufour parameter on Sherwood number. The results reveal that rise in Dufour parameter tends to decline Sherwood number.

4 Conclusions

The motivation of the research is to procure accurate solutions for unsteady natural convective motion of Casson fluid over vertical penetrable plate with the existence of magnetic flux. The velocity, temperature as well as concentration expressions are obtained from analytical perturbation method. The outcomes of momentum, thermal, and concentration fields are represented figuratively. The most decisive findings of the research are summarized below.

- Fluid velocity rises with the rise in Hall parameter and slip parameter values, whereas it decreases with the increasing value of magnetic parameter.
- The velocity of fluid decreases with the rising heat absorption parameter and Prandtl number, whereas increasing Grashof number increases the fluid velocity.
- The radiation parameter and Dufour number help to strengthen the fluid velocity and temperatures.
- Increasing rates of Prandtl number and heat absorption variable diminish the rate of heat transfer.
- Concentration profiles increase due to Schmidt number and decrease due to chemical reaction parameter.
- Skin friction factor shows an increment against magnetic parameter and Prandtl number while it declines against Grashof number and radiation parameter.
- Nusselt number displays an increasing nature with growing values of magnetic parameter.
- Sherwood number decreases with rising Dufour numbers.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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