Effects of radiation and magnetohydrodynamics on heat transfer of nanofluid flow over a plate

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Abstract

The purpose of this study is to investigate the effects of radiation and magnetohydrodynamics on heat transfer of a nanofluid flow over a plate under constant heat flux or constant temperature. The effects of magnetohydrodynamics and radiation on the wall are assumed to be uniform and the plate is static. Momentum and energy equations are solved using the Crank–Nicholson finite difference method. In this paper, the effect of parameters such as Prandtl number, radiation parameter, magnetohydrodynamic parameter on velocity, temperature, Nusselt number and friction coefficients is investigated. The results demonstrated that an increase in magnetohydrodynamic parameter results in an increase in fluid surface penetration in the vicinity of the wall. The temperature gradient increases with the radiation parameter, leading to an increase in the fluid motion.

Keywords Static flow · Nanofluid · Magnetohydrodynamics · Radiation effects · Vertical plate

1 Introduction

One of the topics of interest in new technologies is heat transfer in the Magnetohydrodynamics (MHD) systems. MHD describes the interaction between fluid and magnetic field. The problems related to nanofluids on the plate and heat transfer in the presence of the magnetic field have many applications in engineering sciences, including in the aerospace industry, turbomachines, MHD pumps, chemical and petrochemical engineering, MHD power generators, heat exchangers, flow meters, electronics, chemistry, nuclear reactors, and combustion systems. Numerous investigations have been done on the laminar flow over a vertical plate. These studies can be divided into natural convection, forced convection, and mixed convection with symmetric or asymmetric heating when the plates have constant temperature or heat flux. In general, fully developed flows have been studied numerically. Thermo-MHD flow over different geometries

has many practical applications due to that the magnetic field improves the control of heat transfer rate. Thus, thermo-MHD is commonly employed in technological process such as metallurgical processes, transpiration cooling of hypersonic airplanes, production of semiconductors, etc. Heat transfer can be controlled by thermal MHD flows: these flows can increase or suppress the heat transfer.

Sattar et al. [1] studied free convection heat and mass transfer on a vertical porous plate in a porous medium with variable suction rate. Soundalgekar et al. [2] investigated free convective heat transfer of an unstable flow passing through a vertical plate with constant suction and mass transfer. They used a finite difference numerical method to discrete the dimensionless equations.

Raptis and Kafousia [3] studied heat transfer in porous media with a uniform MHD flow on a vertical plate. Ghokhale [4] investigated free convection heat transfer with a uniform MHD flow on a vertical porous plate under constant heat flux conditions. Ahmad [5] studied the free

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convection heat transfer coupled with the uniform MHD effect of a non-Newtonian power law fluid on a cold plate using constant temperature boundary condition. He did not consider the porosity effects and solved the governing equations numerically.

Naby et al. [6] studied the effects of radiation on free convection heat transfer with uniform MHD on a vertical porous plate. Kim [7] studied free convection heat transfer with the effect of MHD for the power law fluid flow on a vertical porous plate in a porous medium. Sherman and Singh [8] investigated free convection heat transfer coupled with mass transfer on a vertical porous plate with variable suction and heat source. After obtaining the similarity parameters, they were placed in momentum, energy, and mass transfer equations. They obtained a series of dimensionless equations and solved them using the Rang Kuta numerical method and Shooting method. Takhar et al. [9] studied free convection heat transfer on a vertical plate with unlimited length and variable surface temperature. Suundalgekar et al. [10] investigated the effects of free convection heat transfer and mass transfer along a vertical plate on the uniform magnetohydrodynamic flow applied to the plate. Sacheti et al. [11] studied an exact solution for the free convection flow coupled with the magnetohydrodynamic effect

Isman et al. [17] for the first time used the concept of nanotechnology as an obstacle in earlier centuries using the unique properties of nanoparticles. Solid nanoparticles are also ultrafine (100 nm) and can be considered as fluid.

In this paper, two-dimensional free convection heat transfer of nanofluids is considered. In order to increase the heat transfer, a uniform external magnetic field is applied perpendicular to the flow. In this problem, the governing equations are non-dimensionalized and solved numerically. The main goal of the present paper is to find the influence of MHD and radiation on velocity, temperature, Nusselt number, and friction coefficient

2 Governing equations

The governing equations for incompressible fluid are continuity, momentum and energy equations [18, 19]:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

Momentum equation:

$$\begin{aligned} \rho_{nf} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \mu_{nf} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ \rho_{nf} \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \mu_{nf} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\ \rho_{nf} \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \mu_{nf} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \end{aligned} \quad (2)$$

under constant wall heat flux conditions. Shanker and Kishan [12] investigated the effects of mass transfer on magnetohydrodynamic flow on a vertical plate with variable wall temperature or variable wall heat flux as the plate moves suddenly. Elbashbeshy [13] studied heat and mass transfer on a vertical plate with variable surface temperature under magnetohydrodynamic effect. Helmy [14] investigated the unsteady free convection heat transfer on a vertical porous plate. Takhar et al. [15] studied unsteady mixed convection heat transfer in a rotating vertical cone with magnetohydrodynamic flow. Ganesan and Palani [16] studied numerically the unsteady free convection flow on a vertical surface with a variable surface temperature under a magnetohydrodynamic effect. Although Maxwell's theory is outdated, it can be upgraded using the idea of nanoparticles to provide more basic fluid stability. This is a great opportunity to apply nanotechnology to thermal engineering.

Energy equation:

$$(\rho C_p)_{nf} \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k_{nf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (3)$$

Table 1 shows the thermophysical properties of water and nanofluids. There are several papers about the prediction of thermophysical properties of nanofluids [20–22].

Density, dynamic viscosity, and thermal conductivity for nanofluids based on the properties of base fluid and nanoparticles are expressed as follows [23]:

$$\begin{aligned} \rho_{nf} &= (1 - \phi)\rho_f + \phi\rho_s, & (\rho C_p)_{nf} &= (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \\ \mu_{nf} &= \frac{\mu_f}{(1 - \phi)^{2.5}}, & \frac{k_{nf}}{k_f} &= \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)} \end{aligned} \quad (4)$$

Table 1 Therophysical properties of water and nanofluids

Property	Test instruments	Water	Cu	Ag	Al ₂ O ₃	TiO ₂
C_p (J/Kg K)	Hot disk—Dynalene—differential scanning calorimeter (DSC)—Laser Flash Technique (LFT)	4179	385	235	765	686.2
ρ (Kg/m ³)	Hydrometer (for water)—Eq. 4 is used to calculate the nanofluid density	997.1	8933	10,500	3970	4250
k (W/m K)	Thermal conductivity meters: TLS-100, THW-L2, TPS-EFF, GFHM-02 and TPS-M1, Light-Flash analyzer	0.613	401	429	40	9.9538
$\beta \times 10^{-5}$ (K ⁻¹)	Time domain thermoreflectance—laser flash/xenon flash—transient hot bridge	21	1.67	1.89	0.85	0.9

where Φ is the volume fraction of nanoparticles. The subtypes f and s refer to the solid and fluid, respectively.

3 Governing equations under magnetohydrodynamic and radiation effects

The governing equations under magnetohydrodynamic and radiation effects are presented in this section. It should be noted that the viscosity dissipation is considered in the energy equation. Further simplifications are: (A) first- and second-order derivatives of u , v , and T are zero and.

(B) the derivative of pressure relative to x , y , and z are assumed to be zero.

Using the above simplifications, the equations of continuity, momentum, and energy are as follows, respectively:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)} \frac{k_f}{[(1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s]} \left[\frac{\partial^2 T}{\partial z^2} - \frac{16\sigma^* T_\infty^3}{3k^*} \left(\frac{\partial^2 T}{\partial z^2} \right) \right] \quad (11)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (5)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = v_{nf} \left(\frac{\partial^2 u}{\partial z^2} \right) - \frac{\sigma B_0^2}{\rho_{nf}} u \quad (6)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = v_{nf} \left(\frac{\partial^2 w}{\partial z^2} \right) \quad (7)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \bar{\alpha}_{nf} \frac{\partial^2 T}{\partial z^2} - \frac{16\sigma^* T_\infty^3}{3(\rho C_p)_{nf} k^*} \left(\frac{\partial^2 T}{\partial z^2} \right) \quad (8)$$

where u and v are velocity components in the direction x and y . T is the fluid temperature, t is time, B_0 is magnetic

field strength, σ is electrical conductivity coefficient, C_p is heat transfer coefficient at constant pressure, q_r is thermal radiation, σ^* is Stephen Boltzmann constant, k^* is Rutland absorption coefficient, v_{nf} is nanofluid kinematic viscosity, β is heat expansion coefficient of Nanofluids, ρ is fluid density, T_∞ is free-flow temperature, and g is gravity acceleration.

Also, using the nanofluid model, thermal conductivity of nanofluid can be written as follows:

$$k_{nf} = k_f \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)} \quad (9)$$

And the thermo-physical properties of nanofluid can be described as:

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \quad (10)$$

Thus, the energy equation is:

The governing boundary conditions for velocity are as follows:

$$z = 0 : t \leq 0, u = 0, v = 0, w = 0$$

$$z = 0 : w = 0, v = 0, u = 0, t > 0$$

$$x = 0 : w = 0, v = 0, u = 0$$

$$z \rightarrow \infty : u \rightarrow ax, v = 0, w = -2az$$

The governing boundary conditions for temperature are as follows:

$$z = 0 : T = T_\infty, t \leq 0$$

$$z = 0 : T = T_w, t > 0$$

$$x = 0 : T = T_\infty$$

Fig. 1 Computational domain

Fig. 2 Velocity profile for different Reynolds numbers

$$z \rightarrow \infty : T \rightarrow T_{\infty}$$

$$z = 0 : T = T_{\infty}, t \leq 0$$

$$z = 0 : \frac{\partial T}{\partial z} = -\frac{q_w}{k_f}, t > 0$$

$$x = 0 : T = T_{\infty}$$

$$z \rightarrow \infty : T \rightarrow T_{\infty}$$

Non-dimensional governing parameters are Prandtl number, Reynolds number, and Nusselt number that are defined as follows, respectively:

$$Pr = \frac{v_f}{\alpha_f} = \frac{(\mu/\rho)_f}{(k/\rho C_p)_f} = \frac{C_p \mu_f}{k_f} \quad (12)$$

$$Re = \frac{aL^2}{2v_f} \quad (13)$$

$$Nu = \frac{hL}{2k} \quad (14)$$

Fig. 3 Dimensionless temperature profile for different Reynolds numbers when $T_w = \text{constant}$

Fig. 4 Dimensionless temperature profile for different Reynolds numbers when $q_w = \text{constant}$

4 Results

The effects of radiation and magnetohydrodynamics on static flow and nanofluid heat transfer on a plate under constant wall flux conditions and constant wall temperature are studied.

The Crank–Nicholson finite difference method is used to solve the governing equations. The effects of parameters such as Prandtl number (Pr), Reynolds number (Re), magnetohydrodynamic parameter ($M = \sigma B_0^2 / \rho C_p$), radiation parameter ($R_d = \frac{16\sigma^* T_\infty^3}{3k^*}$), volume fraction of nanoparticles and fluid surface area (S) on static flow and nanofluid heat transfer are investigated.

Fig. 5 Dimensionless temperature profile for different volume fractions when $T_w = \text{constant}$

Fig. 7 Dimensionless temperature profile for different magnetohydrodynamic parameters

Fig. 6 Velocity profile for different magnetohydrodynamic parameters

Fig. 8 Velocity profile for different surface wall penetrations

Since finite difference method does not need symmetric grid and the temperature and velocity gradients near the wall are high, finer grid is used in left and bottom surfaces (Fig. 1).

Figures 2, 3 and 4 show the temperature and velocity profiles at different Reynolds numbers for water/copper nanofluid and pure water, where constant surface temperature and constant heat flux conditions are assumed. Other parameters are: $Pr = 1$, $S = 1$, and $\phi = 0.02$. As the Reynolds number increases, the velocity gradient increases, leading to an increase in the acceleration of fluid flow. Therefore, the heat transfer increases. But, the temperature decreases by increasing the Reynolds number. This is due to that as the Reynolds number increases, the temperature gradient between the surface and the fluid

decreases. As a result, the temperature distribution of the fluid decreases.

Figure 5 shows the temperature profile for different volume fractions of nanoparticles for water/copper nanofluids in which constant surface temperature condition is assumed. Other parameters are: $Pr = 1$, $S = 1$ and $Re = 10$. As the volume fraction of nanoparticles increases, the fluid velocity decreases. But, increasing the volume fraction of the nanoparticles leads to an increase in the thermal penetration of the fluid near the wall as a result of increasing the temperature value.

Figures 6 and 7 show non-dimensional temperature and velocity profiles for different magnetohydrodynamic parameters. Here, constant surface temperature and constant heat flux are considered for the simulations. Also,

Fig. 9 Dimensionless temperature profile for different surface wall penetrations

Fig. 11 Dimensionless temperature profile for different nanofluids when q_w = constant

Fig. 10 Dimensionless temperature profile for different nanofluids when T_w = constant

Fig. 12 Dimensionless temperature profile for different radiation parameter when T_w = constant

$Pr = 1$, $S = 1$ and $Re = 10$. Magnetohydrodynamics creates the Lorenz force opposed to the flow direction. Thus, the fluid velocity near the wall decreases, indicating a reduction in the fluid acceleration. As the magnetohydrodynamic parameter increases, the velocity decreases, resulting in a reduction in fluid motion and consequently the velocity distribution decreases. But, an increase in magnetohydrodynamic parameter results in an increase in fluid surface penetration in the vicinity of the wall. Hence, the temperature increases.

Figures 8 and 9 show the temperature profile for different surface wall penetrations of the fluid for the water/copper nanofluid, in which constant surface temperature

and constant heat flux are considered. Also, $Pr = 1$, $Re = 10$, and $\phi = 0.02$. The surface fluid penetration is assumed to be uniform. Thus, as the surface velocity increases, the temperature gradient near the wall increases.

Figures 10 and 11 show the temperature profile for different nanofluids, in which constant surface temperature and constant heat flux are considered. Also, $Pr = 1$, $Re = 10$, and $\phi = 0.02$. As can be seen here, silver nanoparticles have maximum velocity and alumina nanoparticles have minimum velocity. Also, silver nanoparticles have maximum heat transfer and titanium oxide nanoparticles have the minimum one.

Fig. 13 Dimensionless temperature profile for different radiation parameter when $q_w = \text{constant}$

Figures 12 and 13 show the temperature profile for different radiation parameters. As the radiation parameter increases, the temperature increases. This is due to that the temperature gradient increases with the radiation parameter, leading to an increase in the fluid motion. Thus, the temperature distribution is increased due to the contribution of the highly fluid heat absorption.

5 Conclusions

In this paper, the effects of radiation and magnetohydrodynamics on a static fluid and heat transfer of nanofluid on a plate were investigated. The main results are summarized as follows:

As the Reynolds number increases, the velocity gradient increases and the temperature gradient decreases. It was found that as the volume fraction of nanoparticles increases, the velocity and temperature gradients increase. Increasing the surface penetration of the fluid leads to an increase in the velocity and temperature of the fluid. Silver nanoparticles have the maximum heat transfer and the minimum velocity. The results demonstrated that an increase in magnetohydrodynamic parameter results in an increase in fluid surface penetration in the vicinity of the wall. The temperature gradient increases with the radiation parameter, leading to an increase in the fluid motion.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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