Effects of slip conditions and compressibility on the peristaltic flow of particulate suspension in a planar channel

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Abstract

This paper concerns with the peristaltic flow of a particle–fluid suspension (as blood model) through asymmetric two-dimensional channel with slip conditions. The problem formulation is presented for a duct filled with a compressible viscous fluid mixed with rigid spherical particles. The resulting problems are solved by the perturbation expansion method. The effects of different physical parameters such as particles concentration, compressibility parameter, and slip parameter have a significant effect on velocity distributions and are presented and discussed graphically. It is found that the mean axial velocity increases as the slip parameter increases at a minimal compressibility parameter, and it decreases at a higher value of compressibility parameter with increasing slip parameter. This paper gives a scope for estimating the effects of the slip conditions of two-phase flow characteristics with compressible fluid problems and helps in understanding the role of the engineering applications of pumping solid–fluid mixture by peristaltically driven motion.

Keywords Peristaltic · Two-phase flow · Compressible liquid · Slip conditions · Perturbation method

List of symbols

$2d$	The mean width $2d$
a	The amplitude
a_0	The radius of the particle
C	The volume fraction density of the particles
c	The wave speed
c_0	The speed of sound in the oil
$J_z(y)$	The mean-velocity perturbation function
k^*	The compressibility of the liquid
Kn	The Knudsen number
M and N	The suspension parameters
p	The pressure
R	The suspension Reynolds number
S	The drag coefficient of interaction for the force exerted by one phase on the other
T	The absolute temperature (°K)
u_f and v_f	The liquid-phase velocities
u_p and v_p	The particulate-phase velocities
$\langle V_{fx} \rangle$	The net time axial velocity of the fluid phase

V_{fx}	The dimensional net axial velocity of the fluid phase
x and y	The Cartesian coordinates

Greek symbol

α	The wavenumber
ε	The amplitude ratio
η	The transverse displacement of the wall
λ	The wavelength
μ	The viscosity of the liquid
μ_0	The fluid viscosity
$\mu_s(C)$	The particle–liquid mixture viscosity (also the effective viscosity of the suspension)
ρ_0	A constant density at the reference pressure
ρ_0	The actual density of the materials constituting liquid
$(1 - C)\rho_f$	The liquid-phase density
ρ_p	The actual density of the particulate phase, respectively

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$C\rho_p$	The particulate-phase density
χ	The compressibility number

1 Introduction

Peristalsis is considered as a kind of fluid transport induced with a released wave of area expansion or contraction through the walls on the distensible channel carrying some materials. On physiology, peristaltic transportation is asymmetrical radial relaxation and contraction of muscles which propagate through a wave in a tube into a retrograde and sometimes anterograde path. In the digestive canals, the smooth muscle shrinks in sequence to make a peristaltic wave which propels the bolus of food through the track. Similarly, peristaltic pumping happens in many useful applications including biomechanical systems like a finger, roller pumps, and heart–lung machine. There are attempts to introduce the problem, and studying peristaltic transport characteristics has been discussed. Latham's study [1] was the initial release of a system of peristaltic movement. A listing of the majority of the examinations has been offered specifically by Rath [2], Srivastava and Srivastava [3–5], and Srivastava and Saxena [6]. The study of peristaltic blood flow inside the stenosis vessel under the effect of solenoid magnetic field using ferrohydrodynamics principles was discussed by Abbas et al. [7]. Mohammadein and Abu-Nab [8] studied the peristaltic flow of a Newtonian fluid through a porous medium surrounding vapor bubble in a curved channel. Bhatti and Sara [9] studied the peristaltic transport of a non-Newtonian fluid with suspended nanoparticles through a tube, while Bhatti et al. [10] discussed the heat and mass transfer effects on the transporting of an electrically conducting particle–fluid suspension through a channel in the presence of a transverse magnetic field. Mekheimer et al. [11] studied the peristaltic movement of an incompressible Newtonian fluid with nonlinear variable viscosity through a tube. Bhatti and Sara [12] studied the variable magnetic field and endoscope effects on peristaltic blood flow of nanofluid containing TiO₂ nanoparticles (NPs) through a porous annulus. Mekheimer and Sara [13] studied the peristaltic motion of a couple stress fluid in a rotating channel, whereas Afifi et al. [14] studied the influence of magnetic field on the peristaltic transport of a Jeffery fluid in asymmetric channel through a porous medium.

The modeling concerning two-phase flows has a variety of scientific and engineering applications, such as fluidization procedures, pollution dispersion through the atmosphere, and aerosol deposition within aerosol medication. The sedimentation of contaminants in a fluid is important in many processes of chemical engineering where

erythrocyte sedimentation is rolling out in the standard specific medical analysis. A number of research results in this problem, with and without peristalsis even, have been looked into by several researchers; Hung and Brown [15] executed the first attempt for investigating particulate suspension during peristalsis. They started experimental studies on the particle movement through 2D vertical ducts having various geometries within peristalsis. Srivastava and Srivastava [5] discussed the act of peristaltic pumping to the particle–fluid composition through a 2D channel when a perturbation solution is obtained mathematically. The peristaltic pumping of the particle–fluid suspension was investigated within a planar duct by Mekheimer et al. [16]. In addition, El Misery et al. [17] discussed the peristaltic transport of the incompressible generalized Newtonian liquid through a planar duct. The flow analysis of particulate suspension on an asymmetric peristaltic motion in a curved configuration with heat and mass transfer was studied by Bhatti et al. [18], and Sara and Vafai [19] discussed particulate suspension effect on peristaltically induced unsteady pulsatile flow in a narrow artery. Elmaoud et al. [20] studied the properties of the electromagnetic movement for two-layer of immiscible fluids through an inclined channel.

Although liquids are considered incompressible usually, Aarts and Ooms [21] have proven that the compressibility has a great effect on the liquid flow. They looked into the impact associated with ultrasonic rays within an axisymmetric cylindrical pore with the flow of the compressible fluid within a porous medium. They recognized that the effects of ultrasonic rays on the liquid are similar to the peristaltic pumping. The peristaltic movements of the compressible liquid were analyzed through a finite tube by Antanovskii and Ramkissoon [22] subject to a time-dependent pressure drop under the lubrication hypothesis approach. These types of flow are relevant to the peristalsis of the bloodstream because of the heart pumping within the expansion synchronic to constricted vessels. Tsiklauri and Beresenev [23] researched the non-Newtonian results on the peristaltic movement of the compressible Maxwell fluid. Castulic [24] discussed the Boltzmann formula for the compressible boundary layers, while Kawashita [25] studied the global solution regarding Cauchy problems for compressible Navier–Stokes equations. The peristaltic movement of the Newtonian and also non-Newtonian compressible liquid in a cylindrical tube was considered by Eldesoky and Mousa [26] by using the perturbation technique. However, Eldesoky [27] analyzed the main impact involving slip concern of the peristaltic movement on a compressible Maxwell liquid within porous media in a duct. Sara and Vafai [28] studied the peristaltic movement of a compressible and electrically conducting Jeffrey fluid

through a porous medium in a microchannel under the effect of magnetic field. Mekheimer et al. [29] studied the MHD peristaltic transport of a compressible and electrically conducting Maxwell fluid in a microchannel through a porous medium. The analytical solution for the net flow rate and mean axial velocity of the fluid was being constructed throughout closed form with a perturbation evaluation in another investigation. A brief review of the relevant literature with momentum equation along with the features responsible for a liquid to be able to conduct compression solved through the perturbation way is further offered. The net annulus flow for the compressible fluid through peristalsis was studied by Mekheimer and Abdel-Wahab [30]. Felderhof [31] discussed the dissipation within the compressible fluid due to peristaltic pumping and demonstrated it as an identical resonance. The axisymmetric peristaltic movement of the viscous compressible fluid was studied by Elsawahy et al. [32] within a tapered pore. The computation demonstrated that the compressibility of the liquid has a great impact on the movement of the flow induced.

The slip state plays an essential part in the spurt, shear pores and skin, and hysteresis results. The characteristic of the flowing fluids has a great impact by boundary conditions which have an extremely essential relevance to forecasting fluid moving through many applications. Kwang et al. [33] analyzed the peristaltic movement of the Newtonian fluid within a 2D microchannel in which the slip impact exists. The fluids that show boundary slip possess important technical applications; many of these are, for example, polishing valves for the artificial heart. Elsawahy et al. [34] investigated the slip impacts on the peristaltic movement of the non-Newtonian Maxwellian fluid, and Mekheimer et al. [35] studied the particulate suspension slip flow induced by peristaltic waves in a rectangular duct with effect of lateral walls. The study of non-Newtonian nanofluid with hall and ion slip effects on peristaltically induced motion in a non-uniform channel was discussed by Bhatti and Sara [36]. The previous applications of a 2D peristaltic movement

of the solid–fluid structure were discussed by Srivastava and Srivastava [5], Fung and Yih [37], and Eldesoky et al. [38].

The goal of the present study is to investigate the effect of slip conditions on the peristaltic movement of the compressible fluid carrying suspended particles through a planar duct. The governing fluid motion equations are modeled and reduced to a simple form using perturbation method; then, the analytical calculation is performed to solve it. The analyses for the velocities and the pressure gradient have been obtained for different values of the current problem parameters which are shown with discussions.

2 Formulation of the problem

A two-dimensional infinite duct of mean width $2d$ contains a mixture of small rigid particles through a *compressible* Newtonian viscous liquid as demonstrated in Fig. 1 The walls of the duct are flexible and enforced by non-conducting moving waves of the sinusoidal form of small amplitude.

The equations managing the linear momentum and conservation of mass designed for both liquid and particle phases using a continuum procedure are suggested as follows:

Fluid phase:

Momentum

$$\begin{aligned} (1-C)\rho_f \left[\frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial x} + v_f \frac{\partial u_f}{\partial y} \right] \\ = -(1-C) \frac{\partial p}{\partial x} + (1-C)\mu_s(C) \left(\frac{\partial^2 u_f}{\partial x^2} + \frac{\partial^2 u_f}{\partial y^2} \right) \\ + \frac{(1-C)\mu_s(C)}{3} \frac{\partial}{\partial x} \left(\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} \right) + CS(u_p - u_f) \quad (1) \end{aligned}$$

Fig. 1 Schematic diagram of the fluid flow

$$\begin{aligned}
 (1-C)\rho_f \left[\frac{\partial v_f}{\partial t} + u_f \frac{\partial v_f}{\partial x} + v_f \frac{\partial v_f}{\partial y} \right] \\
 = -(1-C) \frac{\partial p}{\partial y} + (1-C)\mu_s(C) \left(\frac{\partial^2 v_f}{\partial x^2} + \frac{\partial^2 v_f}{\partial y^2} \right) \\
 + \frac{(1-C)\mu_s(C)}{3} \frac{\partial}{\partial y} \left(\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} \right) + CS(v_p - v_f) \quad (2)
 \end{aligned}$$

Continuity

$$\begin{aligned}
 \frac{\partial(1-C)\rho_f}{\partial t} + u_f \frac{\partial(1-C)\rho_f}{\partial x} + v_f \frac{\partial(1-C)\rho_f}{\partial y} \\
 + \rho_f \left(\frac{\partial(1-C)u_f}{\partial x} + \frac{\partial(1-C)v_f}{\partial y} \right) = 0 \quad (3)
 \end{aligned}$$

Here, C is the quantity portion denoting the particles that is recognized as a constant (Srivastava and Srivastava [5], Eldesoky et al. [39]). Thus, (3) becomes

$$\frac{\partial \rho_f}{\partial t} + u_f \frac{\partial \rho_f}{\partial x} + v_f \frac{\partial \rho_f}{\partial y} + \rho_f \left(\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} \right) = 0 \quad (4)$$

Particulate phase:

Momentum

$$C\rho_p \left(\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} \right) = -C \frac{\partial p}{\partial x} + CS(u_f - u_p) \quad (5)$$

$$C\rho_p \left(\frac{\partial v_p}{\partial t} + u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} \right) = -C \frac{\partial p}{\partial y} + CS(v_f - v_p) \quad (6)$$

Continuity

$$\frac{\partial}{\partial x}(Cu_p) + \frac{\partial}{\partial y}(Cv_p) = 0 \quad (7)$$

The constitutive equation [39] introduced an equation of state for compressible fluid as follows:

$$\frac{1}{\rho} \frac{\partial \rho}{\partial(1-C)p} = k^* \quad (8)$$

By solving (8), the density as a function of pressure is described by:

$$\rho_f = \rho_o e^{((1-C)k^*(p-p_o))} \quad (9)$$

The particle concentration may be neglected because the field interaction between particles is so small. Therefore, the diffusivity conditions which could model the effects of particle–particle impacts because of the Brownian motion are neglected. Batchelor [40, 41] and others considered the effect of Brownian motion.

The drag coefficient expression for the current problem is given by

$$S = \frac{9}{2} \frac{\mu_o}{a_o^2} \lambda'(C) \quad (10a)$$

$$\lambda'(C) = \frac{4 + 3[8C - 3C^2]^{1/2} + 3C}{[2 - 3C]^2} \quad (10b)$$

Relation (10) was obtained by Tam [42].

For the current problem, the next empirical relation of the viscosity for the suspension, given by Charm and Kurland [43], will be applied in the preceding analysis:

$$\mu_s(C) = \mu_o \frac{1}{1 - qC} \quad (11a)$$

$$q = 0.07 \exp \left[2.49C + \frac{1107}{T} \exp(-1.69C) \right] \quad (11b)$$

The suspension viscosity given by this relation is getting to be reasonably accurate at $C=0.6$.

The channel walls are supposed to be flexible and extensible with a moving sinusoidal wave, and the displacement in the duct walls is only in transverse direction.

Boundary conditions: for the current model physical description, walls of channel are considered with slip conditions, non-permeability, where the wavy movement is resulting from elastic features of the walls and only vertical displacement of walls occurs. So, these conditions must be achieved at the channel walls as follows:

$$y = \pm(d + \eta) = \pm \left(d + a \cos \frac{2\pi}{\lambda}(x - ct) \right) \quad (12)$$

(a) The slip boundary condition is:

$$u_f = \mp A \frac{\partial u_f}{\partial y} \quad (13a)$$

(b) The non-permeability boundary condition is:

$$v_f = v_p = \pm \frac{\partial \eta}{\partial t} \quad (13b)$$

The following dimensionless variables and dimensionless parameters are:

$$\left. \begin{aligned} \bar{x} &= \frac{x}{d}, \quad \bar{y} = \frac{y}{d}, \quad \bar{u}_f = \frac{u_f}{c}, \quad \bar{v}_f = \frac{v_f}{c} \\ \bar{u}_p &= \frac{u_p}{c}, \quad \bar{v}_p = \frac{v_p}{c}, \quad \bar{\rho} = \frac{\rho_f}{\rho_o}, \quad \bar{\eta} = \frac{\eta}{d} \\ \bar{t} &= \frac{ct}{d}, \quad \bar{p} = \frac{p}{\rho_o c^2} \quad \text{and} \quad \bar{p}_o = \frac{p_o}{\rho_o c^2} \end{aligned} \right\} \quad (14a)$$

$$\left. \begin{aligned} v_o &= \frac{\mu}{\rho_o}, \quad R = \frac{cd\rho_o}{(1-C)\mu_s}, \quad \alpha = \frac{2\pi d}{\lambda} \\ \chi &= K^*\rho_o c^2, \quad \varepsilon = \frac{a}{d}, \quad Kn = \frac{A}{d} \\ M &= \frac{Sd^2}{(1-C)\mu_s}, \quad N = \frac{Sd^2\rho_o}{(1-C)\rho\mu_s} \end{aligned} \right\} \quad (14b)$$

Equations (1), (2), (4–7), (9), and (12–14) are rewritten for the previous assumption of non-dimensional analysis after dropping the primes:

$$\frac{\partial \rho_f}{\partial t} + u_f \frac{\partial \rho_f}{\partial x} + v_f \frac{\partial \rho_f}{\partial y} + \rho_f \left(\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} \right) = 0 \quad (15)$$

$$\begin{aligned} (1-C)R\rho_f \left[\frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial x} + v_f \frac{\partial u_f}{\partial y} \right] \\ = -(1-C)R \frac{\partial p}{\partial x} + \left[\frac{\partial^2 u_f}{\partial x^2} + \frac{\partial^2 u_f}{\partial y^2} \right] \\ + \frac{1}{3} \frac{\partial}{\partial x} \left[\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} \right] + CM(u_p - u_f) \end{aligned} \quad (16)$$

$$\begin{aligned} (1-C)R\rho_f \left[\frac{\partial v_f}{\partial t} + u_f \frac{\partial v_f}{\partial x} + v_f \frac{\partial v_f}{\partial y} \right] \\ = -(1-C)R \frac{\partial p}{\partial y} + \left[\frac{\partial^2 v_f}{\partial x^2} + \frac{\partial^2 v_f}{\partial y^2} \right] \\ + \frac{1}{3} \frac{\partial}{\partial y} \left[\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} \right] + CM(v_p - v_f) \end{aligned} \quad (17)$$

$$CR \left[\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} \right] = -CR \frac{\partial p}{\partial x} + CN(u_f - u_p) \quad (18)$$

$$CR \left[\frac{\partial v_p}{\partial t} + u_p \frac{\partial v_p}{\partial x} + v_p \frac{\partial v_p}{\partial y} \right] = -CR \frac{\partial p}{\partial y} + CN(v_f - v_p) \quad (19)$$

$$\rho_f = e^{(1-C)\chi(p-p_o)} \quad (20)$$

$$\eta = \varepsilon \cos \alpha(x-t) \quad (21)$$

Also, boundary conditions at $y = \pm(1+\eta)$ become:

$$u_f = \mp Kn \frac{\partial u_f}{\partial y} \quad (22a)$$

$$v_f = v_p = \pm \alpha \varepsilon \sin \alpha(x-t) \quad (22b)$$

3 Perturbation technique solution

The flow equations and the boundary conditions (15–22) are expanded by applying the perturbation technique which handling the equations as a power series, with small

parameter “ $\varepsilon \ll 1$.” Assume that the fluid transport will take place only due to peristaltic wave so at the entrance, the zero orders of velocities are assumed to be zero. So, the equations of $u_f, v_f, u_p, v_p, \rho_f$, and p are presented in the following form:

$$p = p_o + \varepsilon p_1(x, y, t) + \varepsilon^2 p_2(x, y, t) + \varepsilon^3 p_3(x, y, t) + \dots \quad (23)$$

$$u_f = \varepsilon u_{f1}(x, y, t) + \varepsilon^2 u_{f2}(x, y, t) + \varepsilon^3 u_{f3}(x, y, t) + \dots \quad (24)$$

$$v_f = \varepsilon v_{f1}(x, y, t) + \varepsilon^2 v_{f2}(x, y, t) + \varepsilon^3 v_{f3}(x, y, t) + \dots \quad (25)$$

$$u_p = \varepsilon u_{p1}(x, y, t) + \varepsilon^2 u_{p2}(x, y, t) + \varepsilon^3 u_{p3}(x, y, t) + \dots \quad (26)$$

$$v_p = \varepsilon v_{p1}(x, y, t) + \varepsilon^2 v_{p2}(x, y, t) + \varepsilon^3 v_{p3}(x, y, t) + \dots \quad (27)$$

$$\rho_f = 1 + \varepsilon \rho_{f1}(x, y, t) + \varepsilon^2 \rho_{f2}(x, y, t) + \varepsilon^3 \rho_{f3}(x, y, t) + \dots \quad (28)$$

Also, expanding the boundary conditions by using Taylor expansion at $y = \pm 1$:

$$\begin{aligned} u_f(x, \pm\eta, t) &= u_f(x, \pm 1, t) \pm \eta(x, t) \frac{\partial u_f(x, \pm 1, t)}{\partial y} \\ &+ \frac{\eta^2(x, t)}{2} \frac{\partial^2 u_f(x, \pm 1, t)}{\partial y^2} + \dots \end{aligned} \quad (29a)$$

$$\begin{aligned} v_f(x, \pm\eta, t) &= v_f(x, \pm 1, t) \pm \eta(x, t) \frac{\partial v_f(x, \pm 1, t)}{\partial y} \\ &+ \frac{\eta^2(x, t)}{2} \frac{\partial^2 v_f(x, \pm 1, t)}{\partial y^2} + \dots \end{aligned} \quad (29b)$$

$$\begin{aligned} v_p(x, \pm\eta, t) &= v_p(x, \pm 1, t) \pm \eta(x, t) \frac{\partial v_p(x, \pm 1, t)}{\partial y} \\ &+ \frac{\eta^2(x, t)}{2} \frac{\partial^2 v_p(x, \pm 1, t)}{\partial y^2} + \dots \end{aligned} \quad (29c)$$

For obtaining the problem solution, the procedures are started by substituting (23–29) into (15–22) then collecting similar power terms of ε and obtained only two sets of differential equations in terms of ε which are sufficient to get the problem solution.

For first set of ε

$$\frac{\partial \rho_{f1}}{\partial t} + \frac{\partial u_{f1}}{\partial x} + \frac{\partial v_{f1}}{\partial y} = 0 \quad (30)$$

$$\begin{aligned} (1-C)R \frac{\partial u_{f1}}{\partial t} &= -(1-C)R \frac{\partial p_1}{\partial x} + \left[\frac{\partial^2 u_{f1}}{\partial x^2} + \frac{\partial^2 u_{f1}}{\partial y^2} \right] \\ &+ \frac{1}{3} \frac{\partial}{\partial x} \left[\frac{\partial u_{f1}}{\partial x} + \frac{\partial v_{f1}}{\partial y} \right] + CM(u_{p1} - u_{f1}) \end{aligned} \quad (31)$$

$$(1-C)R \frac{\partial v_{f1}}{\partial t} = -(1-C)R \frac{\partial p_1}{\partial y} + \left[\frac{\partial^2 v_{f1}}{\partial x^2} + \frac{\partial^2 v_{f1}}{\partial y^2} \right] + \frac{1}{3} \frac{\partial}{\partial y} \left[\frac{\partial u_{f1}}{\partial x} + \frac{\partial v_{f1}}{\partial y} \right] + CM(v_{p1} - v_{f1}) \quad (32)$$

$$\rho_{f1} = (1-C)\chi p_1 \quad (33)$$

$$R \frac{\partial u_{p1}}{\partial t} = -R \frac{\partial p_1}{\partial x} + N(u_{f1} - u_{p1}) \quad (34)$$

$$R \frac{\partial v_{p1}}{\partial t} = -R \frac{\partial p_1}{\partial y} + N(v_{f1} - v_{p1}) \quad (35)$$

For second set of ε^2 ,

$$\frac{\partial \rho_{f2}}{\partial t} + u_{f1} \frac{\partial \rho_{f1}}{\partial x} + v_{f1} \frac{\partial \rho_{f1}}{\partial y} + \frac{\partial u_{f2}}{\partial x} + \frac{\partial v_{f2}}{\partial y} + \rho_{f1} \left(\frac{\partial u_{f1}}{\partial x} + \frac{\partial v_{f1}}{\partial y} \right) = 0 \quad (36)$$

$$(1-C)R \left[\frac{\partial u_{f2}}{\partial t} + \rho_{f1} \frac{\partial u_{f1}}{\partial t} + u_{f1} \frac{\partial u_{f1}}{\partial x} + v_{f1} \frac{\partial u_{f1}}{\partial y} \right] = -(1-C)R \frac{\partial p_2}{\partial x} + \left[\frac{\partial^2 u_{f2}}{\partial x^2} + \frac{\partial^2 u_{f2}}{\partial y^2} \right] + \frac{1}{3} \frac{\partial}{\partial x} \left[\frac{\partial u_{f2}}{\partial x} + \frac{\partial v_{f2}}{\partial y} \right] + CM(u_{p2} - u_{f2}) \quad (37)$$

$$(1-C)R \left[\frac{\partial v_{f2}}{\partial t} + \rho_{f1} \frac{\partial v_{f1}}{\partial t} + u_{f1} \frac{\partial v_{f1}}{\partial x} + v_{f1} \frac{\partial v_{f1}}{\partial y} \right] = -(1-C)R \frac{\partial p_2}{\partial y} + \left[\frac{\partial^2 v_{f2}}{\partial x^2} + \frac{\partial^2 v_{f2}}{\partial y^2} \right] + \frac{1}{3} \frac{\partial}{\partial y} \left[\frac{\partial u_{f2}}{\partial x} + \frac{\partial v_{f2}}{\partial y} \right] + CM(v_{p2} - v_{f2}) \quad (38)$$

$$\rho_{f2} = (1-C)\chi p_2 + \frac{(1-C)^2 \chi^2 p_1^2}{2} \quad (39)$$

$$R \left[\frac{\partial u_{p2}}{\partial t} + u_{p1} \frac{\partial u_{p1}}{\partial x} + v_{p1} \frac{\partial u_{p1}}{\partial y} \right] = -R \frac{\partial p_2}{\partial x} + N(u_{f2} - u_{p2}) \quad (40)$$

$$R \left[\frac{\partial v_{p2}}{\partial t} + u_{p1} \frac{\partial v_{p1}}{\partial x} + v_{p1} \frac{\partial v_{p1}}{\partial y} \right] = -R \frac{\partial p_2}{\partial y} + N(v_{f2} - v_{p2}) \quad (41)$$

In addition, the boundary conditions are:

The first-order term ε is:

$$u_{f1}(\pm 1) = \mp Kn \frac{\partial u_{f1}(\pm 1)}{\partial y} \quad (42a)$$

$$v_{f1}(\pm 1) = v_{p1}(\pm 1) = \mp \frac{i\alpha}{2} (e^{i\alpha(x-t)} - e^{-i\alpha(x-t)}) \quad (42b)$$

The second-order term ε^2 is:

$$u_{f2}(\pm 1) \pm \frac{1}{2} (e^{i\alpha(x-t)} + e^{-i\alpha(x-t)}) \frac{\partial u_{f1}(\pm 1)}{\partial y} = \mp Kn \frac{\partial u_{f2}(\pm 1)}{\partial y} - \frac{Kn}{2} (e^{i\alpha(x-t)} + e^{-i\alpha(x-t)}) \frac{\partial^2 u_{f1}(\pm 1)}{\partial y^2} \quad (43a)$$

$$v_{f2}(\pm 1) \pm \frac{1}{2} (e^{i\alpha(x-t)} + e^{-i\alpha(x-t)}) \frac{\partial v_{f1}(\pm 1)}{\partial y} = 0 \quad (43b)$$

$$v_{p2}(\pm 1) \pm \frac{1}{2} (e^{i\alpha(x-t)} + e^{-i\alpha(x-t)}) \frac{\partial v_{p1}(\pm 1)}{\partial y} = 0 \quad (43c)$$

Further, for linear problem solution, following the approach introduced by Aarts and Ooms [21],

$$u_{f1}(x, y, t) = U_{f1}(y) e^{i\alpha(x-t)} + \bar{U}_{f1}(y) e^{-i\alpha(x-t)} \quad (44a)$$

$$v_{f1}(x, y, t) = V_{f1}(y) e^{i\alpha(x-t)} + \bar{V}_{f1}(y) e^{-i\alpha(x-t)} \quad (44b)$$

$$u_{p1}(x, y, t) = U_{p1}(y) e^{i\alpha(x-t)} + \bar{U}_{p1}(y) e^{-i\alpha(x-t)} \quad (44c)$$

$$v_{p1}(x, y, t) = V_{p1}(y) e^{i\alpha(x-t)} + \bar{V}_{p1}(y) e^{-i\alpha(x-t)} \quad (44d)$$

$$p_1(x, y, t) = P_1(y) e^{i\alpha(x-t)} + \bar{P}_1(y) e^{-i\alpha(x-t)} \quad (44e)$$

$$\rho_{f1}(x, y, t) = (1-C)\chi P_1(y) e^{i\alpha(x-t)} + (1-C)\chi \bar{P}_1(y) e^{-i\alpha(x-t)} \quad (44f)$$

In addition,

$$u_{f2}(x, y, t) = U_{f20}(y) + U_{f2}(y) e^{2i\alpha(x-t)} + \bar{U}_{f2}(y) e^{-2i\alpha(x-t)} \quad (45a)$$

$$v_{f2}(x, y, t) = V_{f20}(y) + V_{f2}(y) e^{2i\alpha(x-t)} + \bar{V}_{f2}(y) e^{-2i\alpha(x-t)} \quad (45b)$$

$$u_{p2}(x, y, t) = U_{p20}(y) + U_{p2}(y) e^{2i\alpha(x-t)} + \bar{U}_{p2}(y) e^{-2i\alpha(x-t)} \quad (45c)$$

$$v_{p2}(x, y, t) = V_{p20}(y) + V_{p2}(y) e^{2i\alpha(x-t)} + \bar{V}_{p2}(y) e^{-2i\alpha(x-t)} \quad (45d)$$

$$p_2(x, y, t) = P_{20}(y) + P_2(y) e^{2i\alpha(x-t)} + \bar{P}_2(y) e^{-2i\alpha(x-t)} \quad (45e)$$

$$\rho_{f2}(x, y, t) = D_{20}(y) + D_2(y) e^{2i\alpha(x-t)} + \bar{D}_2(y) e^{-2i\alpha(x-t)} \quad (45f)$$

where the over-bars denote to a conjugate of complex variables.

Inserting a non-oscillatory term in the first order provides only a trivial solution. So, non-oscillatory terms can be added only in the second and higher orders such as $U_{f20}(y)$, $V_{f20}(y)$, $U_{p20}(y)$, $V_{p20}(y)$, $P_{20}(y)$, and $D_{20}(y)$. Substituting Eq. (44) into Eqs. (30–35) and (42) obtains the first system of equations.

The first-order system is as follows:

$$V'_{f1} + i\alpha U_{f1} = (1-C)i\alpha \chi P_1 \quad (46)$$

$$-i\alpha(1-C)RU_{f1} = -i\alpha(1-C)RP_1 + U_{f1}'' - \alpha^2 U_{f1} + \frac{1}{3}i\alpha[V_{f1}' + i\alpha U_{f1}] + CM[U_{p1} - U_{f1}] \quad (47)$$

$$-i\alpha(1-C)RV_{f1} = -(1-C)RP_1' + V_{f1}'' - \alpha^2 V_{f1} + \frac{1}{3}\frac{d}{dy}[V_{f1}' + i\alpha U_{f1}] + CM[V_{p1} - V_{f1}] \quad (48)$$

$$-i\alpha RU_{p1} = -i\alpha RP_1 + N[U_{f1} - U_{p1}] \quad (49)$$

$$-i\alpha RV_{p1} = -RP_1' + N[V_{f1} - V_{p1}] \quad (50)$$

The boundary conditions of the first set of ε are:

$$U_{f1}(\pm 1) = \mp Kn \frac{\partial U_{f1}(\pm 1)}{\partial y} \quad (51a)$$

$$V_{f1}(\pm 1) = V_{p1}(\pm 1) = \mp \frac{i\alpha}{2} \quad (51b)$$

Firstly, substituting Eq. (46) into Eqs. (47) and (48) yields:

$$-\gamma P_1 - \frac{i}{\alpha} \left[\frac{d^2}{dy^2} - \beta^2 \right] U_{f1} = 0 \quad (52)$$

$$-\gamma \frac{dP_1}{dy} + \left[\frac{d^2}{dy^2} - \beta^2 \right] V_{f1} = 0 \quad (53)$$

where the complex parameters γ and β are given by:

$$\gamma = (1-C)R - \frac{i(1-C)\alpha\chi}{3} + \frac{RCM}{N - i\alpha R} \quad (54a)$$

$$\beta^2 = \alpha^2 - i\alpha R \left[1 - C + \frac{CM}{N - i\alpha R} \right] \quad (54b)$$

By using Eq. (46), Eq. (52) is rewritten as when eliminating U_{f1} :

$$\frac{1}{\alpha^2} \frac{d}{dy} \left[\frac{d^2}{dy^2} - \beta^2 \right] V_{f1} - \frac{i(1-C)\chi}{\alpha} \left[\frac{d^2}{dy^2} - \left(\beta^2 + \frac{i\alpha\gamma}{(1-C)\chi} \right) \right] P_1 = 0 \quad (55)$$

Differentiating Eq. (55) with respect to y and using Eq. (53):

$$\frac{1}{\alpha^2} \frac{d^2}{dy^2} \left[\frac{d^2}{dy^2} - \beta^2 \right] V_{f1} - \frac{i(1-C)\chi}{\alpha\gamma} \times \left[\frac{d^2}{dy^2} - \left(\beta^2 + \frac{i\alpha\gamma}{(1-C)\chi} \right) \right] \left[\frac{d^2}{dy^2} - \beta^2 \right] V_{f1} = 0 \quad (56)$$

This equation is rewritten as:

$$B \left(\frac{d^2}{dy^2} - v^2 \right) \left(\frac{d^2}{dy^2} - \beta^2 \right) V_{f1} = 0 \quad (57)$$

where the complex parameters B and v are given by:

$$B = 1 - \frac{i(1-C)\alpha\chi}{\gamma} \quad (58a)$$

$$v^2 = \beta^2 - \frac{\beta^2 - \alpha^2}{B} \quad (58b)$$

Clearly, the boundary conditions of Eq. (51) demand that $V_{f1}(y)$ be odd in y , and hence, the solution for $V_{f1}(y)$ is given by:

$$V_{f1}(y) = C_1 \sinh(vy) + C_2 \sinh(\beta y) \quad (59)$$

where C_1 and C_2 are the complex constants.

Substituting Eq. (59) into Eq. (53):

$$P_1'(y) = C_1 \frac{v^2 - \beta^2}{\gamma} \sinh(vy) \quad (60)$$

Then, integration Eq. (60) as:

$$P_1(y) = C_1 \frac{v^2 - \beta^2}{v\gamma} \cosh(vy) + C_3 \quad (61)$$

where C_3 is a complex constant. Substituting Eqs. (59) and (61) into Eq. (46), we obtain:

$$U_{f1}(y) = \frac{i\alpha}{v} C_1 \cosh(vy) + \frac{i\beta}{\alpha} C_2 \cosh(\beta y) + (1-C)\chi C_3 \quad (62)$$

Then, substituting Eqs. (61) and (62) into Eq. (52) deduces that $C_3 = 0.0$. Then, Eqs. (61) and (62) become:

$$P_1(y) = C_1 \frac{v^2 - \beta^2}{v\gamma} \cosh(vy) \quad (63)$$

$$U_{f1}(y) = \frac{i\alpha}{v} C_1 \cosh(vy) + \frac{i\beta}{\alpha} C_2 \cosh(\beta y) \quad (64)$$

Substituting the boundary conditions (51), we obtain the complex constants C_1 and C_2 as follows:

$$C_1 = \frac{i\alpha\beta v [\cosh(\beta) + Kn\beta \sinh(\beta)]}{\mathfrak{M}} \quad (65a)$$

$$C_2 = \frac{-i\alpha^3 [\cosh(v) + Knv \sinh(v)]}{\mathfrak{M}} \quad (65b)$$

where

$$\mathfrak{M} = 2[\alpha^2 \cosh(v) \sinh(\beta) - \beta v \sinh(v) \cosh(\beta) + Knv(\alpha^2 - \beta^2) \sinh(\beta) \sinh(v)] \quad (66)$$

Substituting Eqs. (63) and (64) into Eq. (49), $U_{p1}(y)$ is obtained as:

$$U_{p1}(y) = i\alpha C_4 \left[\frac{C_5}{v} C_1 \cosh(vy) + \frac{\beta}{\alpha^2} C_2 \cosh(\beta y) \right] \quad (67)$$

Then, substituting Eqs. (60) and (61) into Eq. (51), $V_{p1}(y)$ is obtained as:

$$V_{p1}(y) = C_4 [C_1 C_5 \sinh(vy) + C_2 \sinh(\beta y)] \quad (68)$$

Finally, the complex constants C_4 and C_5 are defined by:

$$C_4 = \frac{N}{N - i\alpha R} \quad (69a)$$

$$C_5 = \frac{-R(v^2 - \beta^2) + \gamma N}{\gamma N} \quad (69b)$$

The second system of equations is obtained by substituting (45) into Eqs. (36–41) and (43), and the following set of equations is obtained as:

The second-order system equations is:

$$V'_{f20}(y) = -(1 - C)\chi \left[P_1(y) \overline{V'_{f1}}(y) + \overline{P_1}(y) V'_{f1}(y) + V_{f1}(y) \overline{P'_1}(y) + \overline{V_{f1}}(y) P'_1(y) \right] \quad (70)$$

$$U''_{f20}(y) = (1 - C)R \left[(1 - C)i\alpha\chi P_1(y) \overline{U'_{f1}}(y) - (1 - C)i\alpha\chi \overline{P_1}(y) U'_{f1}(y) + V_{f1}(y) \overline{U'_{f1}}(y) + \overline{V_{f1}}(y) U'_{f1}(y) \right] + CM[U_{f20}(y) - U_{p20}(y)] \quad (71)$$

$$\begin{aligned} & - (1 - C)RP'_{20}(y) + \frac{4}{3}V''_{f20}(y) \\ & = (1 - C)R \left[(1 - C)i\alpha\chi P_1(y) \overline{V'_{f1}}(y) - (1 - C)i\alpha\chi \overline{P_1}(y) V'_{f1}(y) - i\alpha U_{f1}(y) \overline{V'_{f1}}(y) + i\alpha \overline{U_{f1}}(y) V'_{f1}(y) + V_{f1}(y) \overline{V'_{f1}}(y) + \overline{V_{f1}}(y) V'_{f1}(y) \right] \\ & + CM[V_{f20}(y) - V_{p20}(y)] \end{aligned} \quad (72)$$

$$D_{20}(y) = (1 - C)\chi P_{20}(y) + (1 - C)^2 \chi^2 P(y)_1 \overline{P_1}(y) \quad (73)$$

$$R \left[V_{p1}(y) \overline{U'_{p1}}(y) + \overline{V_{p1}}(y) U'_{p1}(y) \right] = N[U_{f20}(y) - U_{p20}(y)] \quad (74)$$

$$\begin{aligned} & R \left[-i\alpha \overline{V_{p1}}(y) U_{p1}(y) + i\alpha V_{p1}(y) \overline{U_{p1}}(y) + V_{p1}(y) \overline{V'_{p1}}(y) + \overline{V_{p1}}(y) V'_{p1}(y) \right] \\ & = -RP'_{20}(y) + N[V_{f20}(y) - V_{p20}(y)] \end{aligned} \quad (75)$$

Also, their boundary conditions are:

$$\begin{aligned} U_{f20}(\pm 1) \pm \frac{1}{2} \left[\overline{U'_{f1}}(\pm 1) + U'_{f1}(\pm 1) \right] \\ = \mp Kn U'_{f20}(\pm 1) - \frac{Kn}{2} \left[\overline{U'_{f1}}(\pm 1) + U'_{f1}(\pm 1) \right] \end{aligned} \quad (76a)$$

$$V_{f20}(\pm 1) \pm \frac{1}{2} \left[V'_{f1}(\pm 1) + \overline{V'_{f1}}(\pm 1) \right] = 0 \quad (76b)$$

$$V_{p20}(\pm 1) \pm \frac{1}{2} \left[V'_{p1}(\pm 1) + \overline{V'_{p1}}(\pm 1) \right] = 0 \quad (76c)$$

From Eq. (71), by integration, the solution of V_{f20} is:

$$V_{f20}(y) = H_1 - (1 - C)\chi \left[P_1(y) \overline{V_{f1}}(y) + \overline{P_1}(y) V_{f1}(y) \right] \quad (77)$$

where H_1 is a complex constant:

$$H_1 = \frac{i\alpha Kn}{2} [\overline{U'_{f1}}(1) - U'_{f1}(1)] \quad (78)$$

From Eq. (72), it seen that the solution for U_{f20} is given by:

$$U_{f20}(y) = R[E(y) - E(1)] + H_2 \quad (79)$$

where

$$\begin{aligned} E(y) = & (1 - C) \left[\delta_1 \left(\frac{\cosh(\bar{v} + v)y}{(\bar{v} + v)^2} + \frac{\cosh(\bar{v} - v)y}{(\bar{v} - v)^2} \right) + \delta_2 \left(\frac{\cosh(\bar{v} + \beta)y}{(\bar{v} + \beta)^2} + \frac{\cosh(\bar{v} - \beta)y}{(\bar{v} - \beta)^2} \right) \right. \\ & + \delta_3 \left(\frac{\cosh(\bar{v} + \beta)y}{(\bar{v} + \beta)^2} - \frac{\cosh(\bar{v} - \beta)y}{(\bar{v} - \beta)^2} \right) + \delta_4 \left(\frac{\cosh(v + \bar{\beta})y}{(v + \bar{\beta})^2} + \frac{\cosh(v - \bar{\beta})y}{(v - \bar{\beta})^2} \right) \\ & + \delta_5 \left(\frac{\cosh(v + \bar{\beta})y}{(v + \bar{\beta})^2} - \frac{\cosh(v - \bar{\beta})y}{(v - \bar{\beta})^2} \right) + \delta_6 \left(\frac{\cosh(\beta + \bar{\beta})y}{(\beta + \bar{\beta})^2} - \frac{\cosh(\beta - \bar{\beta})y}{(\beta - \bar{\beta})^2} \right) \Bigg] \\ & + \frac{CM}{N} \left[\delta_7 \left(\frac{\cosh(v + \bar{\beta})y}{(v + \bar{\beta})^2} - \frac{\cosh(v - \bar{\beta})y}{(v - \bar{\beta})^2} \right) + \delta_8 \left(\frac{\cosh(\bar{v} + \beta)y}{(\bar{v} + \beta)^2} - \frac{\cosh(\bar{v} - \beta)y}{(\bar{v} - \beta)^2} \right) \right] \end{aligned} \quad (80)$$

and

where

$$\left. \begin{aligned} \delta_1 &= \frac{i\alpha C_1 \bar{C}_1}{2} \left(\frac{\bar{v}}{v} - \frac{v}{\bar{v}} \right), & \delta_2 &= \frac{i\beta \bar{C}_1 C_2}{2} \left(\frac{\bar{v}}{\alpha} - \frac{\alpha}{\bar{v}} \right), & \delta_3 &= \frac{i\bar{C}_1 C_2}{2} \left(\frac{\beta^2}{\alpha} - \alpha \right), \\ \delta_4 &= \frac{i\beta \bar{C}_1 \bar{C}_2}{2} \left(\frac{\alpha}{v} - \frac{v}{\alpha} \right), & \delta_5 &= \frac{iC_1 \bar{C}_2}{2} \left(\alpha - \frac{\bar{\beta}^2}{\alpha} \right), & \delta_6 &= \frac{iC_2 \bar{C}_2}{2\alpha} (\beta^2 - \bar{\beta}^2), \\ \delta_7 &= \frac{iC_1 \bar{C}_2 C_5 \bar{C}_4 C_5}{2\alpha} (\alpha^2 - \bar{\beta}^2), & \delta_8 &= \frac{i\bar{C}_1 C_2 C_4 \bar{C}_4 C_5}{2\alpha} (\beta^2 - \alpha^2) \end{aligned} \right\} \quad (81)$$

$$H_2 = U_{f20}(\pm 1) = W_1 - KnRE'(1) + KnW_2 \quad (82)$$

$$G(y) = E(y) - \frac{1}{N} \left[V_{p1} \bar{U}'_{p1} + \bar{V}_{p1} U'_{p1} \right] \quad (90)$$

$$W_1 = -\frac{1}{2} \left[i\alpha C_1 \sinh(v) + \frac{i\beta^2}{\alpha} C_2 \sinh(\beta) - i\alpha \bar{C}_1 \sinh(\bar{v}) - \frac{i\bar{\beta}^2}{\alpha} \bar{C}_2 \sinh(\bar{\beta}) \right] \quad (83a)$$

Finally, substituting Eqs. (77) into Eq. (75), $V_{p20}(y)$ is obtained as:

$$V_{p20}(y) = H_1 - (1 - C)\chi \left[P_1(y) \bar{V}_{f1}(y) + \bar{P}_1(y) V_{f1}(y) \right] - \frac{R}{N} [F_2(y) + P'_{20}(y)] \quad (91)$$

$$W_2 = -\frac{1}{2} \left[i\alpha C_1 v \cosh(v) + \frac{i\beta^3}{\alpha} C_2 \cosh(\beta) - i\alpha \bar{C}_1 \bar{v} \cosh(\bar{v}) - \frac{i\bar{\beta}^3}{\alpha} \bar{C}_2 \cosh(\bar{\beta}) \right] \quad (83b)$$

And the mean velocities can be written as

$$\langle V_{fx}(y) \rangle = \varepsilon^2 U_{f20}(y) \quad (92)$$

The solution of Eq. (72) is:

$$\langle V_{px}(y) \rangle = \varepsilon^2 U_{p20}(y) \quad (93)$$

$$P_{20}(y) = H_3 + \frac{4}{3R\mathbb{Z}} V'_{f20}(y) - \frac{(1-C)}{\mathbb{Z}} \int F_1(y) dy - \frac{CM}{N\mathbb{Z}} \int F_2(y) dy \quad (84)$$

where

$$F_1(y) = i(1-C)\alpha\chi P_1(y) \bar{V}_{f1}(y) - i(1-C)\alpha\chi \bar{P}_1(y) V_{f1}(y) - i\alpha \bar{V}_{f1}(y) U_{f1}(y) + i\alpha V_{f1}(y) \bar{U}_{f1}(y) + V_{f1}(y) \bar{V}'_{f1}(y) + \bar{V}_{f1}(y) V'_{f1}(y) \quad (85)$$

$$F_2(y) = -i\alpha \bar{V}_{p1}(y) U_{p1}(y) + i\alpha V_{p1}(y) \bar{U}_{p1}(y) + V_{p1}(y) \bar{V}'_{p1}(y) + \bar{V}_{p1}(y) V'_{p1}(y) \quad (86)$$

$$H_3 = P_{20}(y) - \frac{4}{3R\mathbb{Z}} V'_{f20}(y) + \frac{(1-C)}{\mathbb{Z}} \int F_1(y) dy + \frac{CM}{N\mathbb{Z}} \int F_2(y) dy \quad \text{at } y = -1 \quad (87)$$

$$\mathbb{Z} = \frac{(1-C)N + CM}{N} \quad (88)$$

Substituting Eq. (79) into Eq. (74), $U_{p20}(y)$ is obtained as:

$$U_{p20}(y) = R[G(y) - E(1)] + H_2 \quad (89)$$

4 Numerical results and discussion

Equation (92) discloses that the net axial velocity of the fluid phase V_{fx} is prevailed by the constant H_2 and the term of perturbation $R(E(y) - E(1))$ that varies across the duct. The value of constant H_2 that initially appears from the slip condition of the net axial velocity on the wall and the value of U_{f20} at the boundary is related to the net axial velocity at the boundaries of the channel ($y = \pm 1$) by $V_{fx} = \varepsilon^2 H_2$. This presents that the slip boundary condition pertains to the wavy wall and not to the mean position of the wall. It is worth remembering that the corresponding H_2 does not appear in the particulate-phase net axial velocity ($\bar{u}_p$) as the particulate-phase velocity at the walls is unspecified.

With regard to assessment, the mean-velocity perturbation function $\mathcal{J}_z(y)$ was determined by Fung and Yih [37], Srivastava and Srivastava [5], and Eldesoky et al. [38] in the form

$$\mathcal{J}_z(y) = -\frac{200}{\alpha^2 R} (E(y) - E(1)) \quad (94)$$

from which the net time-axial velocity of the fluid phase can be obtained as

Fig. 2 $\frac{\langle V_{fx} \rangle}{\epsilon^2}$ versus y for various values of particle concentration C at $Kn = 0$, $\alpha = 0.5$, $R = 1.0$, and $\chi = 0.1$

Fig. 3 $\frac{\langle V_{fx} \rangle}{\epsilon^2}$ versus y for various values of compressibility number χ and particle concentration C at $Kn = 0$, $\alpha = 0.5$, and $R = 1.0$

$$V_{f,x} = \frac{\epsilon^2}{2} \left[2H_2 - \frac{\alpha^2 R^2}{200} (2 \mathcal{J}_Z(y)) \right] \quad (95)$$

4.1 Validation of the problem

The calculation shows that the results of Fung and Yih [37] can be acquired by $\chi=0$, $C=0$, and $Kn = 0$ in the present problem. Choosing $\chi=0$ and $Kn = 0$ in the analysis indicates a well-agreed physical situation as discussed by Srivastava and Srivastava [5]. Furthermore, the results of Eldesoky et al. [38] can be reached by choosing $Kn = 0$ as shown in Figs. 2, 3.

4.2 Results

The parameters affecting the net axial velocity are shown in Figs. 4, 5, 6, 7, 8, 9, 10, 11, 12. The effect of the concentration of the particle C , for variation of the net axial velocity $\langle V_{fx} \rangle$, versus y is shown in Fig. 4. It is noticed that $\langle V_{fx} \rangle$ increases with an increase in concentration of the particle C at a minimal value of R . The influence of the parameter for compressibility χ on increasing net axial velocity $\langle V_{fx} \rangle$ versus y is represented in Figs. 5, 6. It is observed that χ reduces $\langle V_{fx} \rangle$ for $C=0$ and $C=0.3$. With the effect of Reynolds number R , the net axial velocity is represented in Figs. 7, 8. It is noticed that $\langle V_{fx} \rangle$ increases with the increase in R for different values of both χ and particle

Fig. 4 $\frac{\langle V_{fx} \rangle}{\epsilon^2}$ versus y for various values of particle concentration C at $Kn = 0.1$, $\alpha = 0.5$, $R = 1.0$, and $\chi = 0.1$

Fig. 5 $\frac{\langle V_{fx} \rangle}{\epsilon^2}$ versus y for various values of compressibility number χ at $Kn = 0.1$, $\alpha = 0.5$, $C = 0$, and $R = 1.0$

concentration C . The influence of Knudsen number Kn on the net axial velocity is represented in Figs. 9, 10, 11, and 12. It is noticed that $\langle V_{fx} \rangle$ raises with an increase in Kn when $(\chi > 0.105)$ as shown in Fig. 9 but when $\chi < 0.105$ (Figs. 10, 11, 12), the net axial velocity decreases with increasing the Knudsen number Kn for different values of both R and particle concentration C .

5 Conclusions

This article is focused on studying the slip effects of liquid compressibility on the peristaltic pumping of a particle–fluid composition for a two-dimensional planar duct. The study illustrated the influences of $[C, Kn, \chi, \text{ and } R]$ factors on the mean axial velocity profile, and the following was noticed.

Fig. 6 $\frac{\langle V_{fx} \rangle}{\epsilon^2}$ versus y for various values of compressibility number χ at $Kn = 0.1$, $\alpha = 0.5$, $C = 0.3$, and $R = 1.0$

Fig. 7 $\frac{\langle V_{fx} \rangle}{\epsilon^2}$ versus y for various values of Reynolds number R at $Kn = 0.1$, $\alpha = 0.5$, $C = 0$, and $\chi = 0.1$

Fig. 8 $\frac{\langle V_{fx} \rangle}{\epsilon^2}$ versus y for various values of Reynolds number R at $Kn = 0.1$, $\alpha = 0.5$, $C = 0.3$, and $\chi = 0.1$

Fig. 9 $\frac{\langle V_{fx} \rangle}{\epsilon^2}$ versus y for various values of Knudsen number Kn at $\chi = 0.3$, $\alpha = 0.5$, $C = 0.3$, and $R = 1.0$

Fig. 10 $\frac{\langle V_{fx} \rangle}{\epsilon^2}$ versus y for various values of Knudsen number Kn at $C = 0.3$, $\alpha = 0.5$, $R = 1.0$, and $\chi = 0.1$

Fig. 11 $\frac{\langle V_{fx} \rangle}{\epsilon^2}$ versus y for various values of Knudsen number Kn at $\alpha = 0.5$, $C = 0$, $R = 5.0$, and $\chi = 0.1$

Fig. 12 $\frac{\langle V_{fx} \rangle}{\epsilon^2}$ versus y for various values of Knudsen number Kn at $\alpha=0.5$, $C=0.3$, $R=5.0$, and $\chi=0.1$

With the presence of the Knudsen number, the net axial velocity increases with raising the particles concentration and Reynolds number. The compressibility parameter has a significant impact on the velocity distribution with increasing the slip parameter, where, at a higher value of compressibility parameter ($\chi > 0.105$), the net axial velocity increases and for minimal value ($\chi < 0.105$), the net axial velocity decreases.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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