Unsteady free convection and mass transfer flow past an impulsively started vertical plate with Soret and Dufour effects: an analytical approach

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Abstract

This research presents an analytical solution of unsteady free convection and mass transfer flow past a vertical plate with Soret and Dufour effects. The dimensionless system of governing equations is solved analytically with appropriate initial and boundary conditions. The accuracy of the analytical method is ensured by obtaining numerical solutions with PDEPE of MATLAB and comparing with the analytical results. Perturbation method is first adopted to decouple the system of equations that arise as a result of coupling Soret and Dufour effects. Laplace Transform Technique is then applied to solve the system. The expressions for velocity, temperature, concentration, Skin-friction, Nusselt and Sherwood numbers are obtained. In the course of discussions, the effects of main parameters are described. It is observed that increase in Soret number reduces the temperature while increasing the velocity and concentration. Moreover, Soret effect is more significant on the concentration than on the temperature. Similarly, the Dufour parameter causes the temperature and velocity to increase while the concentration decreases and the effect is more significant on the temperature than on the concentration. However, there is no significant difference on the effects of Dufour and Soret parameters on the velocity. The velocity, temperature and concentration profiles are presented graphically for $Pr = 0.71$ and $Sc = 0.78$ as well as for arbitrary values of other parameters.

Keywords Soret effect · Dufour effect · Free convection · Mass transfer · Perturbation method · Laplace Transform Technique (LTT)

List of symbols

C'	Fluid specie concentration	Sh	Sherwood number
x'	Vertical axis along $y' = 0$	l	Distance between the two walls (m)
C'_0	Concentration of the fluid near $y' = 0$	S^*	Dimensional Soret parameter
y'	Axis perpendicular to the walls	Pr	Prandtl parameter
C'_l	Concentration of the fluid near $y' = l$	Sc	Schmidt parameter
y	Dimensionless axis in y' direction	$erfc$	Complementary error function
Gr	Grashof number	D_f	Dimensionless Dufour number
Gm	Modified Grashof number	exp	Exponential function
D_m	Mass diffusivity coefficient (m^2/s)	t'	Dimensional time (s)
C_f	Skin-friction coefficient	S_r	Dimensionless Soret number
g	Gravitational acceleration (m/s^2)	t	Dimensionless time
Nu	Nusselt number	T'	Fluid temperature (K)
N	Buoyancy ratio	T'_0	Wall temperature at $y' = 0$ (K)
		T'_l	Wall temperature at $y' = l$ (K)

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D^*	Dimensional Dufour parameter
u'	Fluid velocity (m/s ²)
u	Dimensionless fluid velocity

Greek letters

ω	Laplace Transform parameter
β	Coefficient of thermal expansion (K ⁻¹)
β^*	Coefficient of concentration expansion
α	Coefficient of thermal diffusivity (m ² /s)
θ	Dimensionless temperature
ϕ	Dimensionless concentration
ν	Kinematic viscosity (m ² /s)
ρ	Density (kg/m ³)

1 Introduction

Free convection and mass transfer flow are highly essential in several engineering systems with direct applications in processing of materials, petrology, solar collectors, cooling of nuclear reactors, and so on [1–10]. A lot of investigations have been carried out on convection flows caused by temperature and concentration gradients [11–18]. The authors in [19] investigated the heat absorption or non-uniform generation effects and heterogeneous-homogeneous reactions on nanofluid flow of a stretching sheet having boundary condition with convective heat transfer. Ethylene–glycol was selected as the base fluid in the study while the nanoparticle was titanium-dioxide. The technique of Runge–Kutta–Fehlberg shooting was employed in solving the problem. Mixed convection on MHD flow of casson nanofluid over a non-linearly permeable stretching sheet was investigated in [20]. The effects of suction and Joule heating, chemical reaction, heat absorption/generation, thermal radiation and viscous dissipation were considered. Moreover, the effects of different parameters on Nusselt number, Sherwood number and coefficient of skin friction were investigated.

Convection becomes complicated when concentration and temperature interact simultaneously. In 1879, Charles Soret [21] found out that the composition of a mixture of salt contained in a tube having different temperatures at its ends was non-uniform—the concentration of the salt close to the edge of the tube having lower temperature was more than near the end with higher temperature. His conclusion was that a transfer of salt took place as a result of temperature change. This phenomenon is called Soret effect or thermophoresis. However, when heat transfer is generated by concentration gradients, then the phenomenon is termed as Dufour effect. The two effects are applicable in flow systems in which density variations exist such as in the separation of isotopes, chemical processing and so on [21–28]. Several studies reveal that heat

and mass transfer flow can be affected by Soret effect, Dufour effect or the combination of the two. However, when the two effects are combined, the resulting system of governing equations becomes highly complicated and the analytical solutions have been avoided by scholars for years. All analytical studies on the combined effects that exist in literatures are limited to steady-state case.

Investigating fluid flows past a vertical plate has applications in engineering such as in designing solar collectors, spaceship and so forth. The work in [29] focused on investigating Soret effect, the effect of Lewis parameter and the effect of Brownian motion on Magnetohydrodynamics and heat transfer flow of a nanofluid through two parallel plates. The system of equations governing the flow was transformed to a set of ODEs using an appropriate transformation. It was observed that when the Brownian motion parameter increased, the temperature profile increased while the concentration profile decreased and vice versa. On the other hand, increase in Soret parameter decreased the temperature profile and increased the concentration. The authors in [30] investigated the radiation reaction, chemical reaction and Dufour and Soret effects on unsteady magnetohydrodynamics flow past an inclined moving plate. The governing equations were solved numerically only. Kafoussias and Williams [31] observed that when heat and mass transfer take place simultaneously in a fluid that has been set to motion, then there would always exist a complex connection between the fluxes and the driving force. This result was obtained after investigating Soret and Dufour effects on a steady heat and mass transfer near the boundary layer having mixed free-forced convection. Their conclusion was that concentration gradients can also produce an energy flux. Using Homotopy Analysis Method, the authors in [32] solved the system of governing equations from the steady 2-D flow over a long moving porous wall containing an electrically conducting fluid in a permeable medium. The authors in [33] presented an analytical study on Soret and Dufour effects of a second grade fluid flow along a stretching cylinder taking into account the effect of thermal radiation. It was anticipated that when Soret and Dufour parameters varies concurrently, the heat and mass transfer rate would have an inverse relation on one another. Increasing the Dufour parameter and decreasing the Soret parameter decreased the rate of heat transfer and increased the rate of mass transfer. The authors in [34] presented numerical and analytical investigations of a transient magnetohydrodynamics natural convection flow through a channel in the presence of magnetic field, Soret effect, Dufour effect and thermal radiation. However, the analytical results obtained were only for steady-state case. More numerical investigations carried out on the combined Soret and Dufour effects can be found in [35–38].

Studying Soret effect or Dufour effect (separately) on free convection flows is usually straightforward and there are many literatures on such studies with constant boundary conditions. But Jha and Gambo [39] recently carried out an analytical study on unsteady free convection and mass transfer flow through a vertical channel with Dufour effect and non-constant boundary conditions (ramped wall temperature and concentration). The novelty of the work was incorporating ramped boundary conditions. However, when the two effects (Soret and Dufour) are combined in unsteady (time dependent) flows, then finding analytical solutions for the model becomes very difficult and has remained an open problem for years. As a result of coupling heat and mass transfer, previous analytical studies were only for steady flows.

The purpose of this research is to present a complete analytical solutions for unsteady free convection and mass transfer flow over a vertical plate that has been impulsively set to motion in the presence of Soret and Dufour effects. The main benefits of the analytical method is a transparent dependency of the quantity of interest of the governing parameters. The accuracy of the analytical solutions is ensured by making a comparison with the numerical solutions. Furthermore, the results presented here are compared with existing literatures where Soret and Dufour effects are absent. Solutions presented by Soundalgekar [40] have been derived as a particular case when Soret and Dufour effects are absent. This is one of the validity checks for our results. The results in this paper will be relevant in designing geothermal systems, chemical processing equipment, electronic equipment and solar energy collectors.

2 Mathematical analysis

The physical situation considered here corresponds to the generalization of the work of Soundalgekar [40] in which theoretical results have been reported on natural convection and mass transfer flow past a vertical plate that has been impulsively set to motion in the absence of Soret and Dufour effects. The y axis is normal to the plate and the x axis is parallel to it (see Fig. 1).

Initially, at time $t' \leq 0$, both the fluid and the plate are at rest having the same constant temperature (T'_∞) and concentration (C'_∞). At $t' > 0$, the plate begins to move with a

Fig. 1 Geometry of the problem

velocity U_0 in vertical direction and the temperature as well as the concentration are maintained at T'_0 and C'_0 respectively. All physical quantities depend on t' and y' only since the plate that occupies the plane $y' = 0$ is assumed to have an infinite length. The equations governing the flow under such assumptions and Boussinesq approximation in the presence of Soret and Dufour effects are:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \alpha \frac{\partial^2 T'}{\partial y'^2} + D^* \frac{\partial^2 C'}{\partial y'^2} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} + S^* \frac{\partial^2 T'}{\partial y'^2} \quad (3)$$

Subject to

$$\begin{aligned} t' \leq 0 : & \quad u' = 0, T' = T'_\infty, C' = C'_\infty \quad \text{for } y' \geq 0 \\ t' > 0 : & \quad \begin{cases} u' = U_0, T' = T'_0, C' = C'_0 & \text{at } y' = 0 \\ u' \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty & \text{as } y' \rightarrow \infty \end{cases} \end{aligned} \quad (4)$$

It is worth noting that when Soret and Dufour effects are absent ($S^* = D^* = 0$), the mathematical model described by Eqs. (1)–(4) is identical to that in Soundalgekar [40].

When the following non-dimensional quantities are introduced

$$\begin{aligned} t &= \frac{t' U_0^2}{\nu}, \quad y = \frac{y' U_0}{\nu}, \quad Pr = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{D} \\ \theta &= \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad u = \frac{u'}{U_0}, \quad S_r = \frac{S^*(T'_w - T'_\infty)}{C'_w - C'_\infty} \\ D_f &= \frac{D^*(C'_w - C'_\infty)}{T'_w - T'_\infty}, \quad Gr = \frac{\nu g \beta (T'_w - T'_\infty)}{U_0^3}, \quad Gm = \frac{\nu g \beta^* (C'_w - C'_\infty)}{U_0^3} \end{aligned} \quad (5)$$

(Pr is the Prandtl number, Sc is the Schmidt number, Gr is the Grashof number, Gm is the modified Grashof number, S_r is the dimensionless Soret parameter and D_f is the dimensionless Dufour parameter) in Eqs. (1)–(3), we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gm\phi \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + \frac{D_f}{Pr} \frac{\partial^2 \phi}{\partial y^2} \quad (7)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + \frac{S_r}{Sc} \frac{\partial^2 \theta}{\partial y^2} \quad (8)$$

with the following initial and boundary conditions

$$\begin{aligned} t \leq 0 : & \quad u(y, t) = 0, \quad \theta(y, t) = 0, \quad \phi(y, t) = 0 \quad \text{for } y \geq 0 \\ t > 0 : & \quad \begin{cases} u(y, t) = 1, \quad \theta(y, t) = 1, \quad \phi(y, t) = 1 & \text{at } y = 0 \\ u(y, t) \rightarrow 0, \quad \theta(y, t) \rightarrow 0, \quad \phi(y, t) \rightarrow 0 & \text{as } y \rightarrow \infty \end{cases} \end{aligned} \quad (9)$$

3 Analytical solutions

In order to derive analytical solutions to the coupled Eqs. (6)–(8), we use a small non-zero parameter ε to decouple the equations.

$$\theta = \theta_0 + \varepsilon \theta_1 + \dots \quad (10)$$

$$\phi = \phi_0 + \varepsilon \phi_1 + \dots \quad (11)$$

$$u = u_0 + \varepsilon u_1 + \dots \quad (12)$$

Here

$$D_f = k\varepsilon, \quad S_r = \lambda\varepsilon \quad (13)$$

and k and λ are constants of order $O(1)$. Then substituting (10)–(13) into (6)–(8) gives:

3.1 Order ε^0

$$\frac{\partial u_0}{\partial t} = \frac{\partial^2 u_0}{\partial y^2} + Gr\theta_0 + Gm\phi_0 \quad (14)$$

$$\frac{\partial \theta_0}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta_0}{\partial y^2} \quad (15)$$

$$\frac{\partial \phi_0}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi_0}{\partial y^2} \quad (16)$$

$$\begin{aligned} u_0(y, 0) = 0, \quad \theta_0(y, 0) = 0, \quad \phi_0(y, 0) = 0; & \quad \text{for } y \geq 0 \\ u_0(y, t) = 1, \quad \theta_0(y, t) = 1, \quad \phi_0(y, t) = 1 & \quad \text{at } y = 0 \\ u_0(y, t) = 0, \quad \theta_0(y, t) = 0, \quad \phi_0(y, t) = 0 & \quad \text{as } y \rightarrow \infty \end{aligned} \quad (17)$$

Using LTT [41–43], the solutions of (14)–(16) with initial and boundary conditions (17) are obtained as

$$\theta_0(y, t) = \operatorname{erfc}\left(\frac{y}{2} \sqrt{\frac{Pr}{t}}\right) \quad (18)$$

$$\phi_0(y, t) = \operatorname{erfc}\left(\frac{y}{2} \sqrt{\frac{Sc}{t}}\right) \quad (19)$$

$$\begin{aligned} u_0(y, t) = & \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) \\ & + \left(\frac{Gr}{Pr-1} + \frac{Gc}{Sc-1}\right) \left[\left(t + \frac{y^2}{2}\right) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) - y\sqrt{\frac{t}{\pi}} \exp\left(-\frac{y^2}{4t}\right) \right] \\ & - \frac{Gr}{Pr-1} \left[\left(t + \frac{y^2 Pr}{2}\right) \operatorname{erfc}\left(\frac{y}{2} \sqrt{\frac{Pr}{t}}\right) - y\sqrt{\frac{t Pr}{\pi}} \exp\left(-\frac{y^2 Pr}{4t}\right) \right] \\ & - \frac{Gc}{Sc-1} \left[\left(t + \frac{y^2 Sc}{2}\right) \operatorname{erfc}\left(\frac{y}{2} \sqrt{\frac{Sc}{t}}\right) - y\sqrt{\frac{t Sc}{\pi}} \exp\left(-\frac{y^2 Sc}{4t}\right) \right] \end{aligned} \quad (20)$$

It is important to mention here that (18)–(20) are exactly the same as discussed by Soundalgekar [40] as the Soret and Dufour effects are absent.

3.2 Order ε^1

$$\frac{\partial u_1}{\partial t} = \frac{\partial^2 u_1}{\partial y^2} + Gr\theta_1 + Gm\phi_1 \quad (21)$$

$$\frac{\partial \theta_1}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta_1}{\partial y^2} + \frac{k}{Pr} \frac{\partial^2 \phi_0}{\partial y^2} \quad (22)$$

Substituting Eqs. (13), (18)–(20) and (25)–(27) in (10)–(12) gives respectively the following analytical solutions for the temperature, concentration and velocity:

$$\theta = \operatorname{erfc}(\eta\sqrt{Pr}) + \frac{D_f Sc}{Pr - Sc} \left[\operatorname{erfc}(\eta\sqrt{Sc}) - \operatorname{erfc}(\eta\sqrt{Pr}) \right] \quad (28)$$

$$\phi = \operatorname{erfc}(\eta\sqrt{Sc}) + \frac{S_r Pr}{Pr - Sc} \left[\operatorname{erfc}(\eta\sqrt{Sc}) - \operatorname{erfc}(\eta\sqrt{Pr}) \right] \quad (29)$$

$$\begin{aligned} u = \operatorname{erfc}(\eta) + t \left(\frac{Gr(Sc - 1) + Gm(Pr - 1) + D_f Sc Gr + S_r Pr Gm}{(Pr - 1)(Sc - 1)} \right) & \left[(1 + 2\eta^2) \operatorname{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}} e^{-\eta^2} \right] \\ + \frac{t}{Pr - 1} \left(\frac{D_f Sc Gr + S_r Pr Gm}{Pr - Sc} - Gr \right) & \left[(1 + 2\eta^2 Pr) \operatorname{erfc}(\eta\sqrt{Pr}) - 2\eta \sqrt{\frac{Pr}{\pi}} e^{-\eta^2 Pr} \right] \\ - \frac{t}{Sc - 1} \left(\frac{D_f Sc Gr + S_r Pr Gm}{Pr - Sc} + Gm \right) & \left[(1 + 2\eta^2 Sc) \operatorname{erfc}(\eta\sqrt{Sc}) - 2\eta \sqrt{\frac{Sc}{\pi}} e^{-\eta^2 Sc} \right] \end{aligned} \quad (30)$$

$$\frac{\partial \phi_1}{\partial t} = \frac{1}{Sc} \frac{\partial^2 \phi_1}{\partial y^2} + \frac{\lambda}{Sc} \frac{\partial^2 \theta_0}{\partial y^2} \quad (23)$$

$$\begin{aligned} u_1(y, 0) = 0, \quad \theta_1(y, 0) = 0, \quad \phi_1(y, 0) = 0; \quad & \text{for } y \geq 0 \\ u_1(y, t) = 0, \quad \theta_1(y, t) = 0, \quad \phi_1(y, t) = 0 \quad & \text{at } y = 0 \\ u_1(y, t) \rightarrow 0, \quad \theta_1(y, t) \rightarrow 0, \quad \phi_1(y, t) \rightarrow 0 \quad & \text{as } y \rightarrow \infty \end{aligned} \quad (24)$$

Using LTT with relevant initial and boundary conditions (24), the solutions to (21)–(23) are given by

$$\theta_1(y, t) = \frac{kSc}{Pr - Sc} \left[\operatorname{erfc}\left(\frac{y}{2} \sqrt{\frac{Sc}{t}}\right) - \operatorname{erfc}\left(\frac{y}{2} \sqrt{\frac{Pr}{t}}\right) \right] \quad (25)$$

$$\phi_1(y, t) = \frac{\lambda Pr}{Pr - Sc} \left[\operatorname{erfc}\left(\frac{y}{2} \sqrt{\frac{Sc}{t}}\right) - \operatorname{erfc}\left(\frac{y}{2} \sqrt{\frac{Pr}{t}}\right) \right] \quad (26)$$

where $\eta = \frac{y}{2\sqrt{t}}$.

Nuselt number, Sherwood number and Skin friction coefficient are important parameters in heat and mass transfer process which are respectively given in non-dimensional form as

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -\frac{1}{2\sqrt{t}} \left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} \quad (31)$$

$$Sh = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = -\frac{1}{2\sqrt{t}} \left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=0} \quad (32)$$

$$C_f = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = -\frac{1}{2\sqrt{t}} \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} \quad (33)$$

$$\begin{aligned} u_1(y, t) = \frac{kScGr + \lambda PrGc}{(Pr - 1)(Sc - 1)} & \left[\left(t + \frac{y^2}{2}\right) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) - y \sqrt{\frac{t}{\pi}} \exp\left(-\frac{y^2}{4t}\right) \right] \\ + \frac{kScGr + \lambda PrGc}{Sc - Pr} & \left\{ \frac{1}{Sc - 1} \left[\left(t + \frac{y^2 Sc}{2}\right) \operatorname{erfc}\left(\frac{y}{2} \sqrt{\frac{Sc}{t}}\right) - y \sqrt{\frac{t Sc}{\pi}} \exp\left(-\frac{y^2 Sc}{4t}\right) \right] \right. \\ & \left. - \frac{1}{Pr - 1} \left[\left(t + \frac{y^2 Pr}{2}\right) \operatorname{erfc}\left(\frac{y}{2} \sqrt{\frac{Pr}{t}}\right) - y \sqrt{\frac{t Pr}{\pi}} \exp\left(-\frac{y^2 Pr}{4t}\right) \right] \right\} \end{aligned} \quad (27)$$

Table 1 Comparing analytical and numerical solutions for velocity (u), temperature (θ) and concentration (ϕ) profiles for $Gr = 2$, $Gm = 3$, $Pr = 0.71$, $Sc = 0.78$, $S_r = 0.01$, $D_f = 0.01$, $t = 0.6$, $y = 7$

y	Numerical solution			Analytical solution		
	θ	ϕ	u	θ	ϕ	u
0.00	1	1	1	1	1	1
0.25	0.18051	0.16032	0.42387	0.18050	0.16030	0.42338
0.50	0.00738	0.00503	0.01098	0.00736	0.00501	0.01088
0.75	0.00006	0.00003	0.00004	0.00006	0.00003	0.00004
1.00	0	0	0	0	0	0

Fig. 2 Temperature profile for different values of S_r . $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Sc = 0.78$, $t = 0.4$, $D_f = 0.15$ **Fig. 4** Concentration profile for different values of S_r . $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Sc = 0.78$, $t = 0.4$, $D_f = 0.15$ **Fig. 3** Temperature profile for different values of D_f . $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Sc = 0.78$, $t = 0.4$, $S_r = 0.15$ **Fig. 5** Concentration profile for different values of D_f . $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Sc = 0.78$, $t = 0.4$, $S_r = 0.15$

Fig. 6 Velocity profile for different values of S_r . $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Sc = 0.78$, $t = 0.4$, $D_f = 0.15$

Fig. 7 Velocity profile for different values of D_f . $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Sc = 0.78$, $t = 0.4$, $S_r = 0.15$

Therefore, substituting (28)–(30) into (31)–(33) we obtain

$$Nu = \frac{D_f Sc}{Pr - Sc} \sqrt{\frac{Sc}{\pi t}} + \left(1 - \frac{D_f Sc}{Pr - Sc}\right) \sqrt{\frac{Pr}{\pi t}} \quad (34)$$

$$Sh = \left(1 + \frac{S_r Pr}{Pr - Sc}\right) \sqrt{\frac{Sc}{\pi t}} - \frac{S_r Pr}{Pr - Sc} \sqrt{\frac{Pr}{\pi t}} \quad (35)$$

$$C_f = \frac{1}{\sqrt{\pi t}} \left[1 + 2t \left(\frac{Gr(Sc - 1) + Gm(Pr - 1) + D_f Sc Gr + S_r Pr Gm}{(Pr - 1)(Sc - 1)} \right) + \frac{2t \sqrt{Pr}}{Pr - 1} \left(\frac{D_f Sc Gr + S_r Pr Gm}{Pr - Sc} - Gr \right) - \frac{2t \sqrt{Sc}}{Sc - 1} \left(\frac{D_f Sc Gr + S_r Pr Gm}{Pr - Sc} + Gm \right) \right] \quad (36)$$

Fig. 8 Contour temperature profile in the presence of Soret and Dufour effects. $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Sc = 0.78$, $S_r = 0.3$, $D_f = 0.3$

Fig. 9 Contour temperature profile in the absence of Soret and Dufour effects. $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Sc = 0.78$, $S_r = 0.0$, $D_f = 0.0$

Observe that when Soret and Dufour effects are absent, that is when $S_r = D_f = 0$, the results in (28)–(30) correspond exactly to the work in [40]. That is,

$$\theta = \operatorname{erfc}(\eta \sqrt{Pr}) \quad (37)$$

$$\phi = \operatorname{erfc}(\eta \sqrt{Sc}) \quad (38)$$

Fig. 10 Contour concentration profile in the presence of Soret and Dufour effects. $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Sc = 0.78$, $S_r = 0.3$, $D_f = 0.3$

Fig. 12 Contour velocity profile in the presence of Soret and Dufour effects. $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Sc = 0.78$, $S_r = 0.3$, $D_f = 0.3$

Fig. 11 Contour concentration profile in the absence of Soret and Dufour effects. $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Sc = 0.78$, $S_r = 0.0$, $D_f = 0.0$

Fig. 13 Contour velocity profile in the absence of Soret and Dufour effects. $Gr = 5$, $Gm = 5$, $Pr = 0.71$, $Sc = 0.78$, $S_r = 0.0$, $D_f = 0.0$

$$u = \operatorname{erfc}(\eta) + \frac{tGr}{Pr-1} \left[(1+2\eta^2)\operatorname{erfc}(\eta) - (1+2\eta^2Pr)\operatorname{erfc}(\eta\sqrt{Pr}) - \frac{2\eta}{\sqrt{\pi}}e^{-\eta^2} - 2\eta\sqrt{\frac{Pr}{\pi}}e^{-\eta^2Pr} \right] - \frac{tGm}{Sc-1} \left[(1+2\eta^2)\operatorname{erfc}(\eta) - (1+2\eta^2Sc)\operatorname{erfc}(\eta\sqrt{Sc}) - \frac{2\eta}{\sqrt{\pi}}e^{-\eta^2} + 2\eta\sqrt{\frac{Sc}{\pi}}e^{-\eta^2Sc} \right] \quad (39)$$

Moreover, setting $S_r = D_f = 0$ in (34)–(36) or substituting (37)–(39) into (31)–(33), we obtain

$$Nu = \sqrt{\frac{Pr}{\pi t}} \quad (40)$$

$$Sh = \sqrt{\frac{Sc}{\pi t}} \quad (41)$$

$$C_f = \frac{1}{\sqrt{\pi t}} \left[1 + \frac{2tGr}{Pr-1} (1 - \sqrt{Pr}) + \frac{2tGm}{Sc-1} (1 - \sqrt{Sc}) \right] \quad (42)$$

However, observe that Eq. (12) in [40] is wrongly computed from the velocity expression given by (10) in the same paper. The correct expression is given by Eq. (42) above.

Table 2 Variation of Nusselt number, Sherwood number and coefficients of Skin friction for distinct values of Soret and Dufour effects at $t = 0.4$

S_r	D_f	Nu	Sh	C_f
0.0	0.15	0.693274	0.788239	2.762960
0.15	0.15	0.699109	0.740555	2.815850
0.3	0.15	0.705117	0.691541	2.869490
0.15	0.0	0.752655	0.734120	2.756830
0.15	0.15	0.699109	0.740555	2.815850
0.15	0.3	0.644087	0.747219	2.875640

The variations of the parameter values are indicated in bold

4 Results and discussions

The system of PDEs (6)–(8) under relevant initial and boundary conditions (9) has been solved numerically using pdepe of MATLAB. The values of the main governing parameters are varied to obtain the solutions. The values of the Soret parameter (S_r), Dufour parameter (D_f), Grashof number (Gr), modified Grashof number (Gm) and time (t) are arbitrarily taken. In all computations, we choose $Pr = 0.71$ for air and $Sc = 0.78$ for ammonia [44].

The accuracy of the analytical method is ensured by comparing with the numerical results (see Table 1). It is apparent that the analytical and numerical solutions agree.

Figures 2, 3, 4, 5, 6 and 7 display the numerical solutions for the velocity, temperature and concentration equations by fixing a value for the Soret parameter and varying the Dufour parameter and vice versa. When the Soret parameter increases, the velocity and concentration also increase. Moreover, the concentration responds more significantly to the changes in Soret parameter than the temperature. This is in order considering the settings of Eqs. (7) and (8). Interestingly, increase in Soret parameter reduces the temperature distribution in the thermal boundary layer which leads to a reduction in the thickness of the thermal boundary layer (see Fig. 2).

Similarly in Fig. 3, observe that as the Dufour number increases, the temperature also increases leading to an increase in the thickness of thermal boundary layer. The velocity also increases when the Dufour parameter increases. Moreover, when the values of Soret and Dufour parameters increase, the peak value of the velocity generally increases. However, concentration profile decreases with increase in the Dufour number. This slightly decreases the thickness of the solutal boundary layer. As anticipated, the response in temperature to changes in the Dufour parameter is more significant compared to that of concentration. Moreover, in Fig. 4, it can be observed that for large values of Dufour parameter, the concentration increases from the surface of the plate

to its peak value in the boundary layer but then slowly decreases until it attains the minimum value of zero far away from the plate.

In Fig. 2, we see that Soret effect is lesser on the temperature than on the concentration given in Fig. 4. Similarly, Dufour effect is more significant on the temperature profile than on the concentration as seen in Figs. 3 and 5 respectively.

Figures 6 and 7 generally illustrate that the velocity increases with increase in Soret and Dufour parameters while decreases as it tends to free stream value. Moreover, there is a higher velocity gradient near the heated plate. It can also be observed that there is no significant difference on how the Soret and Dufour parameters affect the velocity.

Figures 8, 9, 10, 11, 12 and 13 present the effect of time on the temperature, concentration and velocity profiles. It can be observed that the temperature and concentration increase with time rapidly at first and then gradually until their free stream value values are attained. Figures 8, 9, 10 and 11 show that the duration for the concentration and temperature to attain their free stream value when Soret and Dufour effects are present is very short compared to when these effects are absent.

From Fig. 12 and 13, it can be observed that the transient flow formation occurs earlier in time when Soret and Dufour effects are present compared to when these effects are absent. Consequently, the effects cause delay in attaining the free stream velocity. Moreover, the velocity gradient close to the plate is higher.

The Nusselt number Nu , Sherwood number Sh and Skin-friction coefficient C_f are presented in Table 2 at $t = 0.4$. From the table, observe that if the Soret parameter increases, Nu and C_f also increase while Sh decreases. However, as the Dufour parameter increases, Sh and C_f increase but Nu decreases.

Compliance with ethical standards

Conflict of Interest The authors declare that they have no conflict of interest.

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