Research Article

Analyzing a socially responsible closed-loop distribution channel with recycling facility

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Abstract

This paper deals with a closed-loop distribution channel consisting of a socially responsible manufacturer, multiple retailers and a third party collector. In reality, collection of used products (plastic, glass, metal) by a third party collector is more common than the collection through retailers. This is because retailers generally faces difficulties such as lack of space and manpower. Aligned with many closed loop supply chains, this paper assumes that the third party operates the reverse channel by collecting the used products. The third party collects used products, segregates recyclable items and sends them to the manufacturer for further use. The manufacturer not only shows social responsibility to the stakeholders and shareholders, but also collects the used products from the third party and recycles them to new products. Considering profit maximizing motives of the channel members, the paper examines the effect of manufacturer's degree of social responsibility on the collection activity of the third party. Under manufacturer Stackelberg game setting, it is found that product recycling is directly proportional to the manufacturer's corporate social responsibility (CSR) concerns and there must be a threshold of recycling for the optimal benefit that can be acquired through CSR practice. The proposed model is illustrated by a numerical example and a sensitivity analysis reveals nature of the parameters.

Keywords Closed-loop supply chain · Recycling · Corporate social responsibility · Game theory · Supply chain coordination

1 Introduction

In the modern corporate world, the concept of traditional business has undergone multiple changes. Production houses have to take greater responsibility in addition to making profit, and need to shoulder more responsibility for the welfare of the society. For instance, many companies are nowadays running hospitals for people, or extending their hands in community learning programs by establishing educational institutes. Some other companies are conducting projects to uplift socially and economically backward classes people. To do so, companies tend to share their profit with the members of supply chain and the society, which, in turn, makes them a responsible entity in a society. Companies are also bearing responsibilities in making the society we live in, healthier where the so-called term 'corporate Social Responsibility (CSR)' is used to entitle this set of activities. On the other hand, the matter of scarce resources on the earth pushes companies to sustain some resources for the next generation. As for example, Tata Motors , the largest Indian automotive manufacturer, is trying to actively take part in CSR programs providing support for the enhancement of schedule caste and schedule tribe communities by Tata Affirmative Action Program (TAAP). Furthermore, Infosys supports non

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profitable organizations to eradicate hunger, poverty and malnutrition.

Closed Loop Supply Chain (CLSC), a more environmental friendly type of supply chain, has gained increasing attention with the growing concern of protecting environment and preserving natural resources. In fact, the importance of CLSC has been intensified by the pressure from environmental activist groups and regulations set up by authorities. CLSC includes all forward logistics flows (such as procurement of new raw materials, production and distribution) along with the reverse logistics (collection of used products and reuse). Recycling is a key element of CLSC aims at turning the activities of a forward supply more sustainable, which indeed helps in maintaining ecological balance through eco-friendly activities. Recycling is defined as the set of activities for breaking down or reusing the consumed products (such as papers, glasses, plastics, metals, batteries etc) that would otherwise be disposed of as waste. However, collecting the used products for the purpose of recycling from the waste has always been a challenging activity in the reverse supply chain. It is evident that customers cannot return the used product due to the non-availability of the collection centers and the lack of awareness of the evil effect of their disposal. They rather distort or waste the used items. So, establishment of awareness campaigns among the end users is of crucial importance and the corporate houses as well as retailers have to come forward to make consumers aware of the eco-friendly effect of recycling. Therefore, introduction/consideration of collectors of recyclable items in supply chains is very essential. Doing so not only benefits the environment, but also helps companies to become more socially responsible as increasing the number of collection centers can proportionally enhance the number of employments.

In the present paper, we develop a two-echelon CLSC comprising of a manufacturer, multiple retailers and a third party collector. The third party collects the used items from the customers and separates recyclable items through screening. He then makes an arrangement to deliver the recyclable items to the manufacturer for further use. The process of recycling thus lessens the crisis of natural resources and maintains sustainability, which is the crying need of the hour for our successors. The paper uses the concept of the Vicker's [40] classic principle stating that non-profit maximizing motives may earn higher profits than the pure profit-maximizers. Here the objectives of the manufacturer is to engage in socially responsible activities and to explore its effects on recycling and profit making. To analyze different aspects of the manufacturer's contribution in socially responsible activities, we discuss the individual profit of the channel members separately in the decentralized channel as well as the joint profit in the centralized channel. Channel members in the present model are contributing not only in CSR activities but also in making an eco-friendly environment, and thus information sharing among them will be more easier than those firms which have only profit maximizing motive. Thus, to establish a more efficient channel, we propose a profit sharing mechanism in order to implement centralized decisions. Outcomes of the model are also investigated through numerical examples and a comprehensive sensitivity analysis is presented to demonstrate effect of parameters.

2 Literature review

The alarming rate at which resources are currently being exploited calls for reshaping the extraction and consumption of materials. Businesses are central part of the revolutionary movement toward a greener society, and therefore they are trying to incorporate sustainable business practices in all stages of their supply chain. This has led to creation of CLSC, which has been around for years, but it did not attract much attention in the business world until the last decade since then companies started to realize the importance of their contribution in building more sustainable operations. CLSC is a revolutionized business model that has emerged through integrating the conventional supply chain flows with the reverse one, dealing with collecting and processing End of Life (EOL) products. A very important element of CLSC is traceability of elements flowing along the supply chain. Products, production residues, part of the products must be identified in the original chain (tracking) and in the reverse flow (tracing). CLSC also seeks to maximize profit [13], through making a trade-off between the economic benefits of the supply chain, and its social and environmental effects.

The literature on mathematical modeling of reverse logistics and CLSC has advanced over the years since the seminal paper of Schrady [37], who developed an Economic Order Quantity (EOQ) model with joint determination of production and recovery process. Since then, numerous numbers of studies have been published to analyze the structure of CLSCM and to examine it under different business scenarios. For instance, through a case study and survey research, Cruz [6] studied an environmentally responsible supply chain network using a multi-criteria decision making approach. Yi [43] studied CLSC models under different market powers. Recycling and remanufacturing of waste products have become an important strategic operation of a modern enterprise, and it may bring produce significant economic and social benefits [11]. In this regard, Zheng et al. [45] presented a comparison between a conventional and a reverse supply chain in the Betrand Competition. Pan et al. [26] analyzed reverse logistics costs and benefits to explore the advantage of reverse logistics over the traditional supply chain. Ma [17] developed a model to investigate the strategy of optimal recycling channels for manufacturers under the case of multiple recycling channels. A challenge for manufacturers is to convince customers (regardless of whether they are the manager of the store, or the end users), that it is worth to return the used products to a collection point (gate keeping point). Decision on the destination of the product depends on the value that can be recovered, and also on the determination of reusable items [39]. Wei and Wei [42] suggested that a third-party logistics service as a collection center can lessen the capital investment, differentiate management risk, and advance service level. Ramos et al. [33] proved that a collection center in a reverse logistics can reduce environmental and social impacts on the environment. Zheng and Wu [46] made a deep study on CLSC with the third party recycler based on consumer preference. Aligned with the literature, in this paper a third party collector has been included so as it facilitates the collection and screening of reusable processes for the manufacturer. Qualified items after screening are shipped to the manufacturer for further use. However, disqualified items have been destroyed. In our model recycling is considered.

CSR intends to ensure that companies show their responsibilities toward their stakeholders such as channel members, consumers and society. CSR is nowadays a key factor in consumer and client's decisions to choose a company to work with. In 2002, a survey depicted that, 94 per cent of companies believe that the implementation of a CSR strategy can produce real business benefits [10]. The reputable companies such as GAP, Adidas, Walmart, Nike are among the famous brands that have successfully implemented various forms of CSR in their supply chains [2]. As a result, a large number of principle organizations in different supply chain networks are taking CSR and using it as a tool for profit enhancement. On the other hand, customers keen to pay more price for CSR promised products [3, 5]. There have been a couple of studies addressing CSR issue in the literature. Cruz [7] developed a decision support system for modeling and analyzing of a CSR supply chain network. Cruz [8] studied the effect of CSR activities in the alleviation of risks in supply chain. Agan et al. [1] identified that CSR affects environmental performance of a supplier in supply chain. In recent years, supply chain operations and coordination under the consideration of CSR activities become an essential issue for every business entities [14, 19, 22–24, 28, 29].

Successful implementation of a CLSC depends heavily on construction of a well-performed coordination, required to take into account the interest of different entities of the supply chain. The coordination should be designed so that it triggers the enthusiasm of members to attain the defined objective of the CLSC. The issue of CLSC coordination has extensively been studied in the literature of supply chain management, aiming at improving the performance of members individually and jointly. Among them, Mafakheri and Nasiri [18] considered post-consumption activities like reusing, refurbishing, remanufacturing, and recycling in a dynamic setting and depicted that a higher environmental performance can be achieved through the coordination using the revenue sharing mechanism. Yoo et al. [44] examined the effectiveness of wholesale price, buy-back and quantity discount contracts to coordinate a CLSC. Panda et al. [30] introduced two different sequential bargaining procedures for benefit sharing among the channel members of a distribution channel. Dutta et al. [9] showed that the buy-back offer along with revenue sharing mechanism can generate better channel performance in a CLSC. Considering demand and collection disruptions, a different type of all unit quantity discount contract is suggested by Wang and Wang [41] to coordinate a closed-loop supply chain. Modak et al. [21] used profit sharing mechanism to coordinate a two-echelon CLSC having deopolies retailers. Saha et al. [34] proposed a three-way discount mechanism to coordinate a closed-loop dual-channel supply chain. Modak et al. [20] investigated different structures of CLSC and used the alternative offer bargaining strategy for channel coordination. An overview of contract mechanism applied to coordinate CLSC in the literature of reverse supply chain can be found in the review of Govindan et al. [12], Aydin et al. [4] and Kazemi et al. [15].

This paper studies a scenario in a CLSC consisting of a manufacturer, multiple retailers where a third party logistics is responsible for collecting the used items. The paper focuses on used product recycling in a socially responsible distribution channel with the purpose to explore connection between social and environment responsibilities. This study considers generalized channel structure with a third party who operates the reverse channel. In real supply chains, it is difficult for the retailer to collect the used products along with its retail sale. This is due to several reasons, such as lack of space, types of the used products (plastic, glass, metal etc.), and lack of manpower. That is why, third party collection of used plastic, glass and metal products is more common. The manufacturer not only shows social responsibility to the stakeholders and shareholders, but also collects the used products from the third party and recycles them to new products. Considering profit maximizing motives of the channel members, the paper examines the effect of manufacturer's degree of social responsibility on the collection activity of the third party. Furthermore, the optimal solutions are determined analytically for both centralized and decentralized models and it is additionally examined how profit sharing mechanism enhances the channel's performance.

3 Notations and assumptions

For model formulation we used the following notations.

p_i Selling price of ith retailer (i = 1, 2, … n), a decision variable.

r	Recycling factor of the third party, a decision variable.
I	Investment in collection activities of the third party.
$C(r)$	Total cost to collect the used product.
w	Wholesale price of the manufacturer.
c_m	Marginal cost of the manufacturer.
c_0	Amount paid by the third party to the customer for returning a used product.
c_1	Amount paid by the manufacturer to the third party for returning a recyclable used product.
c_2	Inspection cost of a used product for the third party.
c_3	Disposal cost a non-recyclable product for the third party.
c_4	Value of each recyclable item for the manufacturer.
x	Probability of recyclable item.
$(1-x)$	Probability of non recyclable item.
π_{ri}	Profit function of the ith the retailer (i = 1, 2, … n).
π_t	Profit function of the third party.
π_m	Pure profit function of the manufacturer.
v_m	Total profit function of the manufacturer.
p_c	Retail price of the product in centralized decision
π_c	Pure profit function of centralized channel
v_c	Total profit function of centralized channel
$E(.)$	Expectation of '.'

Assumption Before the analytical discussion of the mathematical model, we state our key assumptions that are particularly defined for the model.

Assumptions 1 The manufacturer is socially responsible as well as Stackelberg leader of the channel. Retailers are non-competitive. Demand of the product at ith retailer's end is assumed to be linearly price sensitive, that is, $D_i = a_i - b_i p_i$. Where, $a_i > 0$ is the market potential and $b_i > 0$ is the customer's price sensitivity factor. $p_i \in (0, a_i/b_i)$ ensures that under any circumstance the demand at the ith retailer's end is non-negative. Linear price dependent demand is common and well-established in supply chain literature [27, 31].

Assumptions 2 Lead time is equal to zero, i.e., the product flows from the upstream to the downstream channel without any delay when demand is placed. The manufacturer follows lot-for-lot policy. The system running cost of each channel member is normalized to zero because in such case the qualitative results will not be altered.

Assumptions 3 The third party collects the used products at a rate $r, (0 < r < 1)$ whose functional form is given by $r = \sqrt{I/\beta}$, where I is the investment of the retailer in collection activity and β is a scaling parameter. This assumption is well established [36] in CLSC literature. The total cost of collection of the used products is then a function of the return rate (r) of the used products and is given by $C(r) = I + c_0 r \sum_{i=1}^{n} D_i = \beta + c_0 r \sum_{i=1}^{n} D_i$, i.e., the sum of investment in collection and the total number of used products collected from customers. For non-negative profits of the third party it is assumed that $(c_1 + c_3)E(x) - (c_0 + c_2 + c_3) > 0$ and $c_m > c_4 > c_1 > c_0$.

4 Mathematical modelling

Consider a scenario where n retailers source a single product from a manufacturer. The supply chain is a closed loop type which is equipped with the third party collector and recycling facilities. The third party collects the used products from customers. After collecting the used products, the third party inspects them to differentiate recyclable and non-recyclable items. It then sends recyclable items to the manufacturer and disposes non-recyclable items. Channel structure of the proposed model is presented in Fig. 1.

Many leading brands face intense pressure to involve in socially responsible activities [2]. A commonly noted response to this pressure is that the primary firm introduces code of conduct to its business partners that motivates or forces them to be more socially responsible [32]. Thus, we assume that the manufacturer invests in CSR and aligns its CSR goal with the channel performance. The cost associated with the CSR is shared among all the channel members through a transfer pricing [6]. In modeling and analysis, we only consider effects of CSR in the form of consumer surplus rather than the CSR activities. It is well established that a firm's CSR is accounted through consumer surplus of it's stakeholders [16, 25]. Consumer surplus is the difference between the maximum price that the consumers are willing to pay for a product and the market price that they actually pay for the product. Thus, the consumer surplus is

$$\sum_{i=1}^{n} \left[\int_{p_{i/mtk}}^{p_{i/max}} D_i(p_i) dp_i \right] = \sum_{i=1}^{n} [(D_i^2/2b_i)] \quad (1)$$

Fig. 1 Channel structure of the model under study

where $p_{i/max}$ and $p_{i/mtk}$ respectively denotes the maximum price that the consumers are willing to pay for a product and the market price that they actually pay for the product at ith retailer's end. Suppose, $\theta \in [0, 1]$ is the fraction of CSR that the socially responsible manufacturer is concerned, then it incorporates $\theta \sum_{i=1}^{n} [(D_i^2/2b_i)]$ as consumer surplus in its profit. $\theta = 0$ implies that the channel is the pure profit maximizer and $\theta = 1$ represents that the manufacturer is the perfect welfare maximizer.

4.1 Decentralized decision

In decentralized decision making, the channel members operate independently and maximize their individual profit functions. The manufacturer is the leader of the channel and other members are its follower. The manufacturer initiates the first move and enforces its own strategy on the third party and retailers. Based on the manufacturer's strategy, the third party and retailers find their move that maximizes their own profit. In decentralized decision making context, profit function of the ith retailer is

$$\pi_{ri}(p_i) = (p_i - w)D_i \tag{2}$$

Profit function of the third party is

$$\pi_t = xc_1 r \sum_{i=1}^{n} D_i - (1-x)c_3 r \sum_{i=1}^{n} D_i - (c_0 + c_2) r \sum_{i=1}^{n} D_i - \beta r^2 \tag{3}$$

Pure profit function of the manufacturer is given by

$$\pi_m = (w - c_m)\sum_{i=1}^{n} D_i + (c_4 - c_1)xr \sum_{i=1}^{n} D_i \tag{4}$$

As a result, the total profit function of the manufacturer is calculated as

$$v_m(w) = (w - c_m)\sum_{i=1}^{n} D_i + (c_4 - c_1)xr \sum_{i=1}^{n} D_i + \theta \sum_{i=1}^{n}[(D_i^2/2b_i)] \tag{5}$$

The expected profits of the third party and the expected pure and the total profit of the manufacturer are

$$E(\pi_t) = E(x)c_1 r \sum_{i=1}^{n} D_i - (1-E(x))c_3 r \sum_{i=1}^{n} D_i - (c_0 + c_2) r \sum_{i=1}^{n} D_i - \beta r^2 \tag{6}$$

$$E(\pi_m) = (w - c_m)\sum_{i=1}^{n} D_i + (c_4 - c_1)E(x) r \sum_{i=1}^{n} D_i \tag{7}$$

$$E(v_m) = (w - c_m)\sum_{i=1}^{n} D_i + (c_4 - c_1)E(x) r \sum_{i=1}^{n} D_i + \theta \sum_{i=1}^{n}[(D_i^2/2b_i)] \tag{8}$$

Using backward induction, the Stackelberg equilibrium solution in the decentralized decision making process can be found as follows. First, each of the n retailers will announce their individual selling price to maximize their respective profits for a given wholesale price, w of the manufacturer. Solving the necessary condition $d\pi_{ri}(p_i)/dp_i = 0$, to optimize the profit function of the ith retailer (i = 1, 2, …, n) yields

$$p_i = \frac{a_i + b_i w}{2b_i}, i = 1, 2, 3, ..n \tag{9}$$

Note that, $d^2\pi_{ri}/dp_i^2 = -2b_i < 0$, for all $i = 1, 2, , n$. That is, all π_{ri} are concave function of $p_i\, i = 1, 2, \ldots, n$. Following n retailers reaction, the third party will optimize its expected profit function and will determine the optimum level of recycling facility. Now solving the necessary condition $dE(\pi_t)/dr = 0$, we get

$$r = \frac{(\sum_{i=1}^{n} a_i - (\sum_{i=1}^{n} b_i)w)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3))}{4\beta} \tag{10}$$

Moreover, note that, $d^2E(\pi_t)/dr^2 = -\beta < 0$ that is, $E(\pi_t)$ is a concave function of r and so the value of r given in Eq. (10) will maximize profit function of the third party. Following decisions of the n retailers and the third party, finally the manufacturer makes its decision. Since the manufacturer is involved in CSR, there exist two parts (pure profit and consumer surplus) in the manufacturer's total expected profit. As a result. the socially responsible manufacturer will maximize its expected total profit and the necessary condition of optimization i.e., $dE(v_m)/dw = 0$ yields

$$w^* = \frac{(\sum_{i=1}^{n} a_i(\beta(2-\theta) - \sum_{i=1}^{n} b_i(c_4 - c_1)E(x)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3))) + 2\sum_{i=1}^{n} b_i c_m \beta)}{\sum_{i=1}^{n} b_i(\beta(4-\theta) - \sum_{i=1}^{n} b_i(c_4 - c_1)E(x)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3)))} \tag{11}$$

Moreover, note that $d^2E(v_m)/dw^2 = -\frac{\sum_{i=1}^{n} b_i}{4\beta}(\beta(4-\theta) - \sum_{i=1}^{n} b_i(c_4 - c_1)E(x)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3)))$. Thus the expected total profit of the manufacturer ($E(v_m)$) will be a concave function of w if $(\beta(4-\theta) - \sum_{i=1}^{n} b_i(c_4 - c_1)E(x)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3))) > 0$. From the above discussion we have the following proposition.

Proposition 1 *The total expected profit function of the manufacturer ($E(v_m)$) is a concave function of w if* $(\beta(4-\theta) - \sum_{i=1}^{n} b_i(c_4 - c_1)E(x)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3))) > 0$.

Now, by inserting the optimal value of the manufacturer's wholesale price into (10) and (9), we have the optimal values of recycling rate and selling price as follows

$$r^* = \min\left\{\frac{(\sum_{i=1}^n a_i - \sum_{i=1}^n b_i c_m)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3))}{2(\beta(4-\theta) - \sum_{i=1}^n b_i(c_4 - c_1)E(x)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3)))}, 1\right\} \tag{12}$$

$$p_i^* = \frac{a_i}{2b_i} + \frac{(\sum_{i=1}^n a_i(\beta(2-\theta) - \sum_{i=1}^n b_i(c_4 - c_1)E(x)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3))) + 2\sum_{i=1}^n b_i c_m \beta)}{2\sum_{i=1}^n b_i(\beta(4-\theta) - \sum_{i=1}^n b_i(c_4 - c_1)E(x)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3)))}, i = 1, 2, 3, ..n \tag{13}$$

Note that, $dr^*/dE(x) = (\sum_{i=1}^n a_i - \sum_{i=1}^n b_i c_m)(\sum_{i=1}^n b_i(c_4 - c_1)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3))^2 + (c_1 + c_3)\beta(4-\theta))/(2(2(\beta(4-\theta) - \sum_{i=1}^n b_i(c_4 - c_1)E(x)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3))))^2) > 0$.It indicates that, the optimal recycling rate of the third party increases with the increase of the expected value of the recyclable items. The result is straightforward because the higher value of the expected number of recyclable items from the collected used products provides more profit to the third party, which an incentive that motivates it to collect more used items. Substituting optimal values from (11)–(13) in (2), (6)–(8) we get optimal profits of the channel members.

From Eq. (12), it can be concluded that if $(\sum_{i=1}^n a_i - \sum_{i=1}^n b_i c_m)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3)) > 2(\beta(4-\theta) - \sum_{i=1}^n b_i(c_4 - c_1)E(x)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3)))$ then $r^* = 1$, entailing that the third party collects 100% of the used products for recycling. In this case, the modified expected profit of the third party, expected pure and the expected total profit of the manufacturer can be found from Eq. (6)–(8) respectively by just replacing $r = 1$. In this special situation, the optimal wholesale and selling prices of the decentralized channel are as follows

$$w_{sp}^* = \frac{(2-\theta)\sum_{i=1}^n a_i + 2\sum_{i=1}^n b_i c_m - 2\sum_{i=1}^n b_i(c_4 - c_1)E(x)}{\sum_{i=1}^n b_i(4-\theta)} \tag{14}$$

$$p_{i/sp}^* = \frac{a_i}{2b_i} + \frac{(2-\theta)\sum_{i=1}^n a_i + 2\sum_{i=1}^n b_i c_m - 2\sum_{i=1}^n b_i(c_4 - c_1)E(x)}{2\sum_{i=1}^n b_i(4-\theta)}, \quad i = 1, 2, , n \tag{15}$$

Hence, substituting $p_i = p_{i/sp}^*$, $w = w_{sp}^*$ and $r = 1$ in (2), (6)–(8), we get the optimal expected pure and total profit of the integrated channel in this special case.

4.2 Centralized decision

In centralized decision making, the channel members operate jointly, i.e., supply chain makes integrated decision. A single decision maker took all decisions to maximize the total channel profit. Under the centralized scenario, the expected pure profit and the total expected profit of the channel are respectively given by

$$E(\pi_c) = \sum_{i=1}^n (p_{ci} - c_m)D_i + (c_4 - c_1)E(x)r\sum_{i=1}^n D_i - \beta r^2 \tag{16}$$

$$E(v_c) = \sum_{i=1}^n (p_{ci} - c_m)D_i + (c_4 - c_1)E(x)r\sum_{i=1}^n D_i + \theta\sum_{i=1}^n [(D_i^2/2b_i)] - \beta r^2 \tag{17}$$

Solving the necessary conditions to optimize the total expected profit function of the channel i.e., solving $\partial E(v_c)/\partial p_{ci} = 0$ i = 1, 2, …, n and $\partial E(v_c)/\partial r = 0$ gives optimal selling prices and recycling rate as follows.

$$p_{ci}^* = \frac{(1-\theta)a_i + b_i c_m}{b_i(2-\theta)} - \frac{b_i(\sum_{i=1}^n a_i - c\sum_{i=1}^n b_i)(c_0 + c_2 + c_3 - (c_3 + c_4)E(x))^2}{b_i(2-\theta)\left[2\beta(2-\theta) - \sum_{i=1}^n b_i(c_0 + c_2 + c_3 - (c_3 + c_4)E(x))^2\right]} \tag{18}$$

$$r_c^* = \min\left\{\frac{(\sum_{i=1}^n a_i - c_m\sum_{i=1}^n b_i)((c_3 + c_4)E(x) - c_0 - c_2 - c_3)}{2\beta(2-\theta) - B(c_0 + c_2 + c_3 - (c_3 + c_4)E(x))^2}, 1\right\} \tag{19}$$

The optimal selling prices (p_{ci}^*) and recycling rate (r_c^*) will maximize the total expected profit function of the channel if $E(v_c)$ satisfies concavity conditions. The following proposition depicts concavity conditions of $E(v_c)$.

Proposition 2 *The total expected profit function of the channel ($E(v_c)$) is a concave function of the decision variables (p_1, p_2, …, $p_n$and r) if*

$2b_1b_2\dots b_n\beta(2-\theta)^n - b_1b_2\dots b_nT^2(2-\theta)^{(n-1)}(\sum_{i=1}^n b_i) > 0$ *where* $n = 1, 2, \dots$ *denotes number of retail outlets.*

Proof Please see "Appendix 1".

By substituting the optimal values from (18), and (19) in (16) and (17), we obtain the optimal expected pure and the total profit of the integrated channel. Note that, if $(\sum_{i=1}^n a_i - c_m\sum_{i=1}^n b_i)((c_3+c_4)E(x) - c_0 - c_2 - c_3) > [2\beta(2-\theta) - \sum_{i=1}^n b_i(c_0 + c_2 + c_3 - (c_3+c_4)E(x))^2]$ then $r_c^* = 1$. In such a situation modified expected pure profit and total expected profit of the channel can be found from Eqs. (14) and (15) respectively by replacing $r = 1$. In this case, the optimal selling price of the centralized channel at ith retailer's end is given by

$$p_{ci/sp}^* = \frac{a_i(1-\theta) + b_ic_m - b_1((c_3+c_4)E(x) - c_0 - c_2 - c_3)}{b_1(2-\theta)}, \quad i = 1, 2, , n \tag{20}$$

Hence, substituting $p_{ci} = p_{ci/sp}^*$ and $r = 1$ in (16) and (17) we drive the optimal expected pure and total profit of the integrated channel in this special case. We will now discuss the effect of the degree of social concerns of the manufacturer on the optimal decisions.

4.3 Analyzing the effect of the degree of CSR

As discussed before, the manufacturer acts as a Stackelberg leader of the channel and shows CSR. The manufacturer can enhance its CSR program by highlighting recycling activity. Thus, there exists an intra-relationship between the recycling rate and degree of the manufacturer's CSR activity. Note that,

$$\frac{dr^*}{d\theta} = \frac{(\sum_{i=1}^n a_i - \sum_{i=1}^n b_ic_m)((c_1+c_3)E(x) - (c_0+c_2+c_3))\beta}{2(\beta(4-\theta) - (c_4-c_1)E(x)((c_1+c_3)E(x) - (c_0+c_2+c_3))\sum_{i=1}^n b_i)^2} \tag{21}$$

Clearly, $\frac{dr^*}{d\theta} > 0$ that is optimal recycling rate increases with the increasing degree of CSR activity of the manufacturer. If the manufacturer is more concerned about the CSR, then the third party will collect more used products for recycling. As a consequence, the recycling factor attains the minimum value when the manufacturer is purely profit maximizer i.e., $\theta = 0$ and it attains maximum value when the manufacturer is the perfect welfare maximizer, i.e., $\theta = 1$, for which the recycling factor may be higher than one. This is quite obvious because the retailer has no obligation in participating in the reverse supply chain through collection of the used products when the manufacturer is not taking up CSR. Nevertheless, the recycling factor is bounded by 0 and 1. Thus, the maximum and the minimum value of the recycling factor in decentralized scenario are respectively as follows.

$$r_{dmin}^* = \min\left\{\frac{(\sum_{i=1}^n a_i - \sum_{i=1}^n b_ic_m)((c_1+c_3)E(x) - (c_0+c_2+c_3))}{2(4\beta - \sum_{i=1}^n b_i(c_4-c_1)E(x)((c_1+c_3)E(x) - (c_0+c_2+c_3)))}, 1\right\} \tag{22}$$

$$r_{dmax}^* = \min\left\{\frac{(\sum_{i=1}^n a_i - \sum_{i=1}^n b_ic_m)((c_1+c_3)E(x) - (c_0+c_2+c_3))}{2(3\beta - \sum_{i=1}^n b_i(c_4-c_1)E(x)((c_1+c_3)E(x) - (c_0+c_2+c_3)))}, 1\right\} \tag{23}$$

On the other hand, differentiating the centralized optimal recycling rate with respect to θ, we get

$$\frac{dr_c^*}{d\theta} = \frac{2\beta(\sum_{i=1}^n a_i - c_m\sum_{i=1}^n b_i)((c_3+c_4)E(x) - c_0 - c_2 - c_3)}{[2\beta(2-\theta) - B(c_0+c_2+c_3 - (c_3+c_4)E(x))^2]^2} > 0 \tag{24}$$

From Eq. (24) it can be understood that the optimal recycling rate in the centralized scenario also behaves in similar manner as in the decentralized scenario with respect to θ. The maximum and the minimum value of the recycling factor in the decentralized scenario are respectively as follows.

$$r_{cmin}^* = \min\left\{\frac{(\sum_{i=1}^n a_i - c_m\sum_{i=1}^n b_i)((c_3+c_4)E(x) - c_0 - c_2 - c_3)}{4\beta - B(c_0+c_2+c_3 - (c_3+c_4)E(x))^2}, 1\right\} \tag{25}$$

$$r_{cmax}^* = \min\left\{\frac{(\sum_{i=1}^n a_i - c_m\sum_{i=1}^n b_i)((c_3+c_4)E(x) - c_0 - c_2 - c_3)}{2\beta - B(c_0+c_2+c_3 - (c_3+c_4)E(x))^2}, 1\right\} \tag{26}$$

A graphical representation of r^* and r_c^* with respect to θ is demonstrated in Fig. 2. Numerical value of the parameters are considered the same as the numerical section.

From the above discussion, we can derive the following proposition.

Proposition 3 *In both centralized and decentralized scenarios the optimal recycling rate increases with the increasing concerns of the CSR activity and it lies in the range* (r^*_{dmin}, r^*_{dmax}) *and* (r^*_{cmin}, r^*_{cmax}) *respectively for the centralized and decentralized scenarios.*

Figure 2 reveals that, the decentralized recycling rate increases with the growth in the value of θ. On the other hand, recycling rate in the centralized model is equal to 1 for all values of $\theta \in [0, 1]$ because $r^*_{cmin} = \min\{2.91137, 1\} = 1$ and $r^*_{cmax} = \min\{17.1588, 1\} = 1$. Moreover, as $dw^*/d\theta < 0$ and $p_i^*/d\theta < 0$ (please see "Appendix 2"), the wholesale price of the manufacturer decreases if value of θ elevates. It leads us to conclude that the selling price decreases but demand increases with the increasing value of θ. Figure 3 demonstrates the effect of θ on individual profits, as well as the pure and the total profit of the manufacturer.

From Fig. 3 it is clear that retailers and the third party obtain more profits for higher value of θ. They will receive the minimum profits when $\theta = 0$ and the maximum profits for $\theta = 1$. Thus, purely profit maximizing motive of the manufacturer provides the minimum profit to the other channel members while totally welfare maximizing motive gives them the opportunity to realize the maximum profits. Furthermore, the pure profit of the manufacturer decreases with the increasing value of θ, yet the total profit of the manufacturer increases with respect to θ.

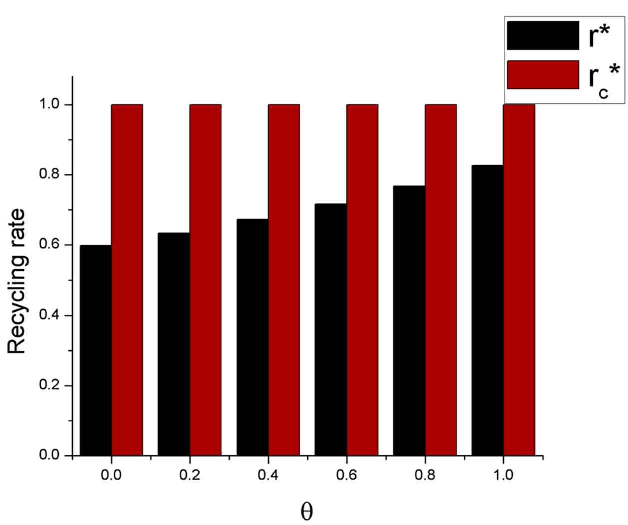

Fig. 2 Effect of θ on optimal recycling rates

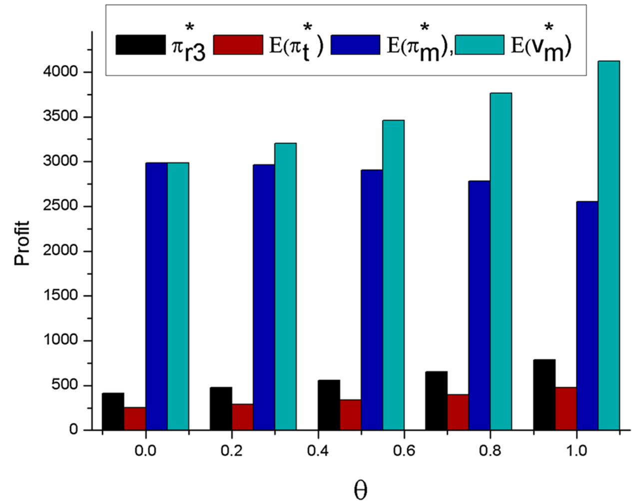

Fig. 3 Effect of θ on profits

These results show that increment in the consumer surplus overestimates the loss in the pure profit.

5 Cooperative solution and profit sharing mechanism

Cooperative integrated decision is more profitable than the non-cooperative decision due to non occurrence of the well known 'double marginalization' problem. This section demonstrates an approach to adopt the centralized decision through a profit sharing mechanism. For accepting the centralized order quantity, which is more than the decentralized one, the retailer's cost will increase and there is no reason that the retailer will adopt the centralized policy unless it has proper incentive. As an incentive, the manufacturer can offer the retailers and the third party to share the surplus profit if they adopt the centralized decisions. Under the profit sharing mechanisms, the system performance is first optimized and the resultant benefit is then shared among the channel members. Note that, the total expected profit in the decentralized scenario is equal to $E(v_m)^* + E(\pi_t)^* + \sum_{i=1}^{n} \pi^*_{ri}$. Thus, if the channel members adopt the centralized decision then the channel generates surplus profit, $\Delta_{sp} = E(v_c)^* - [E(v_m)^* + E(\pi_t)^* + \sum_{i=1}^{n} \pi^*_{ri}]$. Under the proposed profit sharing mechanism, the manufacturer offers the channel members to share surplus profit according to their decentralized profit if they adopt centralized decisions. Hence, under the proposed contract, profit of the ith retailer, the third party and the manufacturer can be found as follows

$$\pi^{sp}_{ri} = \pi^*_{ri} + \frac{\pi^*_{ri}\Delta_{sp}}{E(v_m)^* + E(\pi_t)^* + \sum_{i=1}^{n} \pi^*_{ri}} i = 1, 2, , n \quad (27)$$

Table 1 The optimal values in decentralized and centralized model

Optimal	Decentralized channel						Centralized channel
	R_1	R_2	R_3	R_4	Third party	Manufacturer	
Wholesale price	–	–	–	–	–	139.5	–
Selling price	169.7	174.7	171.7	170.7	–	–	122.7, 126.5, 124.2, 123.5
Recycling rate	–	–	–	–	0.6727	–	1
Demand	15.1	17.6	16.1	15.6	–	–	38.6, 41.8, 39.9, 39.2
Profit	457.6	621.4	520.1	488.4	316.8	3355.6	9484.8

$$\pi_t^{sp} = E(\pi_t)^* + \frac{E(\pi_t)^* \Delta_{sp}}{E(v_m)^* + E(\pi_t)^* + \sum_{i=1}^n \pi_{ri}^*} \tag{28}$$

$$\pi_m^{sp} = E(v_m)^* + \frac{E(v_m)^* \Delta_{sp}}{E(v_m)^* + E(\pi_t)^* + \sum_{i=1}^n \pi_{ri}^*} \tag{29}$$

Note that, $\pi_m^{sp} + \pi_t^{sp} + \sum_{i=1}^n \pi_{ri}^{sp} = E(v_c)^*$. That is, the profit sharing scheme not only provides the best channel outcomes, but also assures the win–win profit for all the channel members. The next section illustrates a numerical example of the proposed model, followed by a sensitivity analysis.

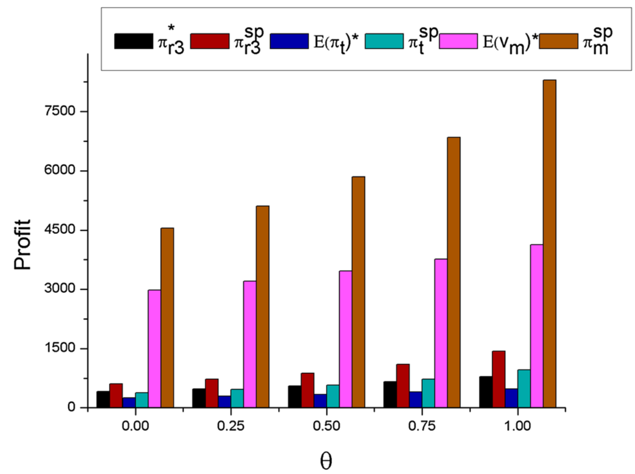

Fig. 4 Decentralized versus coordinated profit for different θ

6 Numerical illustration

Consider a distribution channel consisting of a manufacturer (M), and four retailers ($R_1, R_2, R_3, andR_4$). The parameter values are considered as $a_1 = 100, a_2 = 105, a_3 = 102, a_4 = 101, b_1 = b_2 = b_3 = b_4 = 0.5, c_0 = 10, c_1 = 30, c_2 = 1, c_3 = 14, c_4 = 40, c_m = 100, \beta = 700\ \theta = 0.4$, Suppose the random variable x follows the uniform distribution over the interval [0.80, 1.0], then $E(x) = 0.90$. Based on the present parameter setting the optimal solutions are calculated and presented in Table 1.

The optimal solutions of the centralized channel displayed in the Table 1 are determined from the special case because $(\sum_{i=1}^n a_i - c_m \sum_{i=1}^n b_i)((c_3 + c_4)E(x) - c_0 - c_2 - c_3) = 4908.8 > [2\beta(2-\theta) - \sum_{i=1}^n b_i(c_0 + c_2 + c_3 - (c_3 + c_4)E(x))^2] = 1126.08$. Owing to the fact that $r_c^* = 1 > r^* = 0.6727$, the centralized channel is more environmental friendly than the decentralized supply chain. Consumers also prefer the centralized model as they have to pay less as a consequence demand in the centralized channel is much higher than the decentralized channel. The expected pure profit and consumer surplus of the manufacturer in the decentralized scenario is equal to 2938.16 and 417.44. While in the centralized scenario, the expected pure profit and consumer surplus of the channel is equal to 6938.6 and 2546.2. The expected total profit in the decentralized channel is equal to 5759.91, which is 64% less than the centralized channel's expected profit. Thus, channel coordination is imperative to improve its performance. Under the proposed profit sharing mechanism channel members have the opportunity to improve the channel's profit equal to 3724.9. Profit of the four retailers will be 753.53, 803.71, 672.69, and 804.24 respectively if they adopt cooperative decisions. Profit of the third party and the manufacturer under this mechanism are 521.67 and 5525.65 respectively.

Figure 4 depicts a comparison of individual profit of the channel members between decentralized and coordinated situation (in Fig. 4, without loss of generality we have considered one retailer instead of all retailers because all the retailers have similar output in comparison). Clearly, profit sharing mechanism provides win–win opportunity for all the channel members. Next subsection provides a sensitivity analysis on the model parameters.

6.1 Sensitivity analysis

Sensitivity analysis of the parameters is presented in Table 2. From Table 2, it can be observed that the market potentiality and price sensitivity parameters have high effect on the profits of the channel members. The parameter c_0 highly affects recycling rate and as a result it has

Table 2 Sensitivity of the key parameters

	Changes (%)	w	r	p	D	π_{ri}	$E(\pi_t)$	$E(\pi_m)$	$E(v_m)$	p_c	D_c	$E(\pi_c)$	$E(v_c)$
a_i	− 40	− 5.45	− 19.4	− 25.81	− 119.2	− 96.13	− 34.76	− 34.76	− 30.69	− 24.37	− 14.76	− 22.59	− 22.04
	− 20	− 2.72	− 8.95	− 12.9	− 59.6	− 83.87	− 42.18	− 18.31	− 17.16	− 12.14	− 32.38	− 13.99	− 15.81
	+ 20	+ 2.72	10.45	12.9	60.26	155.47	16.79	20.16	20.8	12.3	32.38	19.4	18.93
	+ 40	5.45	19.4	25.81	119.87	227.07	42.18	42.16	45.3	24.53	64.77	44.21	43.12
b_i	− 40	6.81	8.96	42.13	82.78	456.86	17.43	32.18	38.05	40.83	24.87	41.10	40.08
	− 20	3.15	4.48	16.09	40.4	145.61	8.59	15.05	0.98	141.5	12.44	14.92	14.55
	20	− 2.95	− 4.48	− 10.96	− 37.75	− 68.09	− 8.3	− 13.27	− 12.24	− 10.19	− 12.44	− 9.28	− 9.05
	40	− 5.52	− 7.46	− 19.09	− 74.17	− 95.34	− 16.36	− 24.98	− 22.08	17.44	− 24.61	− 15.96	− 14.98
c_0	− 40	− 1.43	31.34	− 0.59	3.31	7.14	73.19	2.81	3.3	− 2.04	3.37	7.0	6.83
	− 20	− 0.79	16.42	− 0.29	1.99	3.5	33.53	1.38	1.63	− 0.98	1.56	3.48	3.39
	20	0.72	− 14.92	0.29	− 1.32	− 3.32	− 27.82	− 1.35	− 1.57	1.06	− 1.56	− 3.42	− 3.34
	40	1.43	− 29.85	0.59	− 3.31	− 6.49	− 50.49	− 2.67	− 3.09	2.04	− 3.11	− 6.69	− 6.62
c_1	− 40	2.14	− 303.33	0.88	− 4.97	− 10.20	− 1526.74	− 4.28	− 4.97	–	–	–	–
	− 20	− 0.04	− 58.54	− 0.02	0.10	0.19	− 151.3	0.08	0.10	–	–	–	–
	20	2.26	23.16	0.93	− 5.26	− 10.6	40.95	− 4.52	− 5.26	–	–	–	–
	30	3.97	29.36	1.65	− 9.83	− 20.63	50.1	− 8.45	− 9.83	–	–	–	–
c_2	− 40	− 0.14	2.98	− 0.06	0.66	0.68	6.25	0.27	0.32	− 0.16	0.52	0.69	0.67
	− 20	− 0.07	1.49	0	0.66	0.35	3.09	0.14	0.16	− 0.08	0.26	0.34	0.34
	20	0.07	− 1.49	0.06	0	− 0.33	− 2.99	− 0.14	− 0.16	0.08	0	− 0.34	− 0.34
	40	0.14	− 2.98	0.06	0	− 0.68	− 5.97	− 0.28	− 0.32	0.24	− 0.26	− 0.69	− 0.67
c_3	− 40	− 0.22	4.48	− 0.06	0.46	0.94	8.81	0.38	0.45	− 0.24	0.51	0.97	0.94
	− 20	− 0.07	2.98	0	0.26	0.48	4.36	0.19	0.23	− 0.08	0.26	0.48	0.47
	20	0.71	− 1.49	0.06	− 0.26	− 0.48	− 4.2	− 0.19	− 0.22	0.16	− 0.26	− 0.48	− 0.47
	40	0.21	− 4.47	− 0.12	− 0.53	− 0.94	− 8.33	− 0.38	− 0.44	0.32	− 0.52	− 0.96	− 0.94
c_4	− 20	3.94	− 7.46	1.65	− 8.6	− 17.35	− 16.29	− 7.42	− 8.52	3.67	− 5.7	− 12.06	− 11.77
	− 10	2.01	− 4.66	0.82	− 4.66	− 9.53	− 9.53	− 4.00	− 4.65	1.87	− 3.17	− 6.86	− 6.37
	20	− 4.73	10.45	− 1.94	11.26	23.1	21.63	8.66	10.27	− 3.67	5.96	12.76	12.45
	40	− 10.61	23.88	− 4.3	24.5	54.76	51.06	18.91	22.88	− 7.25	11.66	26.23	25.58
c_m	− 40	− 17.78	38.81	− 7.31	41.06	98.82	91.76	91.71	91.68	− 29	32.38	79.77	77.81
	− 20	− 8.89	19.4	− 3.65	20.53	45.21	42.18	42.16	42.14	− 10.19	16.32	37.18	36.27
	20	8.89	− 19.4	3.65	− 20.4	− 36.8	− 34.76	− 34.76	− 34.75	10.19	− 16.06	− 31.78	− 30.97
	40	17.78	− 38.8	7.36	− 41.06	− 65.19	− 62.14	− 62.13	− 62.10	20.46	− 32.38	− 58.15	− 56.72

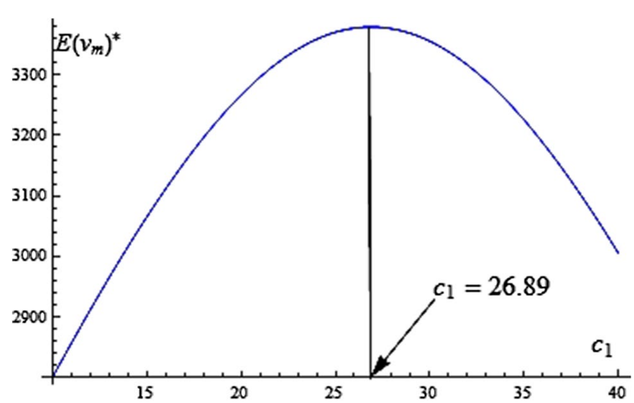

Fig. 5 Nature of the total expected profit of the manufacturer with respect to c_1

a high impact on the expected profit of the third party. However, the wholesale price, the selling price, profit of retailers, and the expected pure and the total profits of the manufacturer are less sensitive on c_0. We have set up 30% increment of c_1 instead of 40%, and 10%, 20% decrement of c_4 because of the assumption $c_4 > c_1$. c_1 is not involved in the centralized policy and so it has no impact on the centralized decisions.

The sensitivity of the parameter c_1 is very interesting. If we consider c_1, the amount paid by the manufacturer to the third party for returning a recyclable used product as a decision variable then the total expected profit of the manufacturer will be maximum when it sets $c_1 = [c_0 + c_2 + (1 - E(x))c_3 + c_4E(x)]/2E(x)$. In the present illustration the total expected profit of the manufacturer will be maximum if $c_1 = 26.89$ (see Fig. 5). Wholesale and selling prices increase (or decrease) moderately with the increment (or decrement) of c_m but recycling rate behaves in an opposite manner. Both c_2 and c_3 have little impact on profit of the channel members except the third party. Total expected profit of the manufacturer in decentralized channel and total expected profit of the centralized channel are highly affected by the parameter c_4.

7 Concluding remarks

In this paper, we developed a socially responsible CLSC including a manufacturer, multiple retailers and a third party collector. The third party entity is responsible for collecting the used products from customers. The collector carries out inspection on the used products to differentiate recyclable and non-recyclable items, and then sends recyclable items to the manufacturer and disposes non-recyclable items. The developed model can be taken as a simulator of the supply chain of some specific products like plastic, glass and metals. In the proposed model, the manufacturer acts as a Stackelberg leader, who is willing to take part in corporate social responsibility schemes. The type of CSR which is taken in this study in the form of consumer surplus rather than any CSR activities. Hereby, the manufacturer has two motives which are pure profit and welfare maximization. To investigate the effect of the CSR activity of the manufacture, we analyzed the model to explore how manufacturer can set up its price and recycling rate to ensure maximizing the profit for both decentralized and centralized channels. It is observed that the expected total profit in the centralized system is much more than the one in the decentralized channel. It is further concluded that the optimal recycling rate increases with increasing the CSR activity of the manufacturer. Moreover, we found that the profit sharing mechanism provides best channel performance, giving a win–win profit to all the channel members. Finally, the numerical example showed that expected total profit function of the manufacturer is a concave function of c_1 and the manufacturer may treat it as decision variable if the third party agrees.

The paper at hand makes several contribution to the field, some of which are as follows. First, the paper is developed under a realistic closed-loop channel structure with multiple retailers and a third party collector. Second, the manufacturer shows social responsibility and non-profit maximization motive (i.e., social welfare motive), which are reflected through consumer surplus. Third, the model examines the effect of the degree of CSR on the decentralized and centralized channel structures. Fourth, the optimal selling prices in each retailer's end and recycling rate are determined analytically taking into account the restriction of recycling rate. Higher concerns about the CSR ensures more collection of used products. Finally, the optimal ranges of recycling rate are determined for both decentralized and centralized models.

The present work also reveals following insights. The value of each recyclable item for the manufacturer has noticeable impact on the total expected profit of the manufacturer in decentralized channel as well as total expected profit of the centralized channel. That is, value of the material extracted from used products is a key factor to operate the reverse channel supply chain. The amount paid by the manufacturer to the third party for returning a recyclable used product (c_1) is required to determine carefully to maximize the channel leader's expected profit and the present model provides value of c_1 in closed form. Market potential and marginal cost of the manufacturer have high impact on the total expected profits. The manufacturer's social welfare activity motivates the used product collection activity of the third party. Manufacturer's awareness activities to enhance recycling of used products can influence customers to return used products through third party. Awareness program can be consider as a part of CSR activity. The work also reveals that inefficiencies in decentralized channel can be possible to overcome using the profit sharing mechanism.

Although the proposed model provides some important insights to the field, it is not free of limitations. First, demand is assumed to be deterministic and dependent on price. Due to the variation of demand in real cases, it will be more realistic if the demand is assumed to be the probabilistic. Second, we assumed that there are n retailers in the channel who are non-competitive. As the retailers may be in competition in some scenarios, thus, the model can be extended to the case of competitive retailers. Third, this model can be extended in view of carbon emission reduction [35] and solid waste management [38]. Fourth, the model can be examined under different game theoretical aspects like evolutionary game, sub-game perfect equilibrium and strategic bargaining. Moreover, some other scenarios such as: manufacturer disruptions due to reliability of manufacturing system, imperfect quality items, proper cleaning and disinfections of the recycled items can be investigated in future research.

Compliance with ethical standards

Conflict of interest There is no conflict of interest in this article.

Appendix 1

To check the concavity of total expected profit function of the channel, we compute its corresponding Hessian matrix, H_c as follows

$$H_c = \begin{pmatrix} \frac{\partial^2 E(v_c)}{\partial p_1^2} & \frac{\partial^2 E(v_c)}{\partial p_1 \partial p_2} & \frac{\partial^2 E(v_c)}{\partial p_1 \partial p_3} & - & - & \frac{\partial^2 E(v_c)}{\partial p_1 \partial r} \\ \frac{\partial^2 E(v_c)}{\partial p_2 \partial p_1} & \frac{\partial^2 E(v_c)}{\partial p_2^2} & \frac{\partial^2 E(v_c)}{\partial p_2 \partial p_3} & - & - & \frac{\partial^2 E(v_c)}{\partial p_2 \partial r} \\ \frac{\partial^2 E(v_c)}{\partial p_3 \partial p_1} & \frac{\partial^2 E(v_c)}{\partial p_3 \partial p_2} & \frac{\partial^2 E(v_c)}{\partial p_3^2} & - & - & \frac{\partial^2 E(v_c)}{\partial p_3 \partial r} \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ \frac{\partial^2 E(v_c)}{\partial r \partial p_1} & \frac{\partial^2 E(v_c)}{\partial r \partial p_2} & \frac{\partial^2 E(v_c)}{\partial r \partial p_3} & - & - & \frac{\partial^2 E(v_c)}{\partial r^2} \end{pmatrix}$$

That is,

$$H_c = \begin{pmatrix} -b_1(2-\theta) & 0 & 0 & - & - & b_1 T \\ 0 & -b_2(2-\theta) & 0 & - & - & b_2 T \\ 0 & 0 & -b_3(2-\theta) & - & - & b_3 T \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ b_1 T & b_2 T & b_3 T & - & - & -2\beta \end{pmatrix}$$

where $T = (c_0 + c_2 + c_3 - (c_3 + c_4)E(x))$. If the principal minors are alternatively negative and positive, i.e., the kth order leading principal minor Δ_k follows the sign of $(-1)^k$, then the total expected profit function of the channel $(E(v_c))$ will be concave, i.e., maximum at (p_{ci}^*, r_c^*).

Here $\Delta_1 = \partial^2 E(v_c)/\partial p_1^2 = -b_1(2-\theta) < 0$, $\Delta_2 = \begin{vmatrix} -b_1(2-\theta) & 0 \\ 0 & -b_2(2-\theta) \end{vmatrix} = b_1 b_2 (2-\theta)^2 > 0$ and $\Delta_3 = \begin{vmatrix} -b_1(2-\theta) & 0 & 0 \\ 0 & -b_1(2-\theta) & 0 \\ 0 & 0 & -b_3(2-\theta) \end{vmatrix} = -b_1 b_2 b_3 (2-\theta)^3 < 0$. Clearly, $\Delta_n = (-1)^n b_1 b_2 ... b_n (2-\theta)^n$. $\Delta_n > 0$ if n is even and $\Delta_n < 0$ if n is odd . Now,

$$\Delta_{n+1} = |H_c| = \begin{vmatrix} -b_1(2-\theta) & 0 & 0 & - & - & b_1 T \\ 0 & -b_2(2-\theta) & 0 & ... & ... & b_2 T \\ 0 & 0 & -b_3(2-\theta) & ... & ... & b_3 T \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ b_1 T & b_2 T & b_3 T & ... & ... & -2\beta \end{vmatrix}$$

On simplification we have, $\Delta_{n+1} = (-1)^n [2b_1 b_2 \ldots b_n \beta (2-\theta)^n + b_1 b_2 \ldots b_n T^2 (2-\theta)^{(n-1)} (\sum_{i=1}^n b_i)]$. Hence, $E(v_c)$ will be concave if $\Delta_{n+1} > 0$ when n is odd and $\Delta_{n+1} < 0$ when n is even. On other words, $E(v_c)$ will be concave if the Hessian matrix, $E(v_c)$ is negative definite, i.e., the eigenvalues of the matrix are all negative real numbers. From the above discussion we have the following proposition.

Appendix 2

Differentiating optimal whlesale price and selling prices of the decentralized channel with respect to θ we get,

$$\frac{dp_i^*}{d\theta} = -\frac{\left(\sum_{i=1}^n a_i - \sum_{i=1}^n b_i c_m\right)\beta^2}{(\beta(4-\theta) - (c_4 - c_1)E(x)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3))\sum_{i=1}^n b_i)^2} < 0$$

$$\frac{dw^*}{d\theta} = -\frac{2\left(\sum_{i=1}^n a_i - \sum_{i=1}^n b_i c_m\right)\beta^2}{(\beta(4-\theta) - (c_4 - c_1)E(x)((c_1 + c_3)E(x) - (c_0 + c_2 + c_3))\sum_{i=1}^n b_i)^2} < 0$$

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