Research Article

Combined effects of Hall and ion-slip current on MHD free convection flow in a vertical micro-channel

Basant K. Jha[1] · Peter B. Malgwi[1]

Abstract
Theoretic analysis on buoyancy-driven flow of conducting fluid in a vertical micro-channel taking into consideration the effect of Hall and ion-slip current is investigated. The micro-channel walls are assumed to be heated asymmetrically and flow is induced by buoyancy forces. The governing couple equations are solved exactly using the method of undetermined coefficient. Solutions are presented in dimensionless form. Effect of rarefaction parameter, Hartmann number, Hall current parameter and ion-slip parameter on fluid velocity, volume flow rate and skin friction along primary along primary and secondary flow directions are discussed.

Keywords Hall current · Ion-slip current · Micro-channel · Velocity slip · Temperature jump · Free convection

List of symbols

H_0	Constant magnetic flux density (Tesla)
g	Acceleration due to gravity (m s^{-2})
b	Channel width (m)
C_p, C_v	Specific heat at constant pressure and constant volume, respectively (J kg^{-1} K^{-1})
f_t, f_v	Thermal and tangential momentum accommodation coefficient, respectively (dimensionless variables)
β_t	Temperature jump (dimensionless variables)
β_v	Velocity slip (dimensionless variables)
λ	Mean free path (m)
ln	Fluid–wall interaction parameter β_t/β_v (dimensionless variables)
Kn	Knudsen number λ/b (dimensionless variables)
δ	Dimensionless volume flow rate
M	Hartmann number (dimensionless)
u'	Dimensional velocity along the x'-direction (m s^{-1})
w'	Dimensional velocity along the z'-direction (m s^{-1})
U_0	Characteristic velocity (m s^{-1})
u	Velocity along the x-direction (dimensionless)
w	Velocity along the z-direction (dimensionless)
Q	Complex velocity (dimensionless)
θ	Temperature (dimensionless)
ω_e	Cyclotron frequency of electrons
ω_i	Cyclotron frequency of ions
τ_e	Electron collision time
τ_i	Ion collision time
β_e	Hall parameter ($\omega_e\tau_e$)
β_i	Ion-slip parameter ($\omega_i\tau_i$)
T	Temperature of the fluid (K)
T_1	Temperature of the right wall (K)
T_2	Temperature of the left wall (K)

Greek letters

β	Coefficient of thermal expansion (K^{-1})
γ_s	Ratio of specific heat C_p/C_v
σ	Electrical conductivity of the fluid
μ_e	Magnetic permeability (N A^{-2})
ξ	Wall ambient temperature (dimensionless)
ρ	Density (kg m^{-3})

✉ Peter B. Malgwi, bumalpeter@gmail.com; Basant K. Jha, basant777@yahoo.co.uk | [1]Department of Mathematics, Ahmadu Bello University, Zaria, Nigeria.

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1 Introduction

Studies on hydromagnetic free convection flow of viscous incompressible and electrically conducting fluids like ionized gases (plasmas), liquid metal (mercury or liquid sodium) [1], in micro-channels are of immense relevance owing to its many scientific and technological applications like microelectromechanical system (MEMS), nanoelectromechanical system (NEMS), micro-pumps, micro-heat sink, flow meters, drug delivery, material processing, cooling of microelectronic devices, liquid robotics and colloidal energetic systems [2, 3]. In view of these applications, a lot of research have been conducted to improve the efficiency and performance of these devices. Some articles on hydromagnetic free convection flow under different physical situations include: Jha et al. [4], Jha and Aina [5–7], Avci and Aydin [8, 9], Bounomo and Manca [10], Hossain and Mandal [11], Ibrahim et al. [12], Eldabe et al. [13] and Gorla and Chamkha [14].

It is well known that subjecting the flow of electrically conducting fluid to a strong externally applied magnetic field induces many complex electromagnetic phenomena on flow characteristics such as Hall current, ion slip and Joule heating as stated by Cramer and Pai [15]. The Hall current becomes noticeable when an ionized gas with low density experiences the influence of a strong magnetic field (Sato [16]). Such types of flows are likely to find relevance in underground energy storage systems, spacecraft propulsion technology and in sensor technologies. Datta and Jana [17] presented an exact solution for Hall effect on hydromagnetic free convection flow of conducting fluid in a channel. They observed that shear stress at the hotter wall could be reduced with the increase in Hall parameter. In a related work, Datta and Jana [18] analyzed the effect of Hall current on mixed convection hydromagnetic flow of viscous incompressible conducting fluid in a vertical channel. They observed that Hall current strengthens both primary fluid velocity and secondary fluid velocity, whereas it decreases the rate of heat transfer at the walls. Mazumder et al. [19] investigated the influence of Hall current on MHD Ekman layer flow in a permeable channel. Guchhait et al. [20] presented an exact solution for time-dependent hydromagnetic Couette flow of conducting fluid in a horizontal permeable channel in the presence of Hall current. They observed that at large time, Hall parameter increases the primary component of fluid velocity whereas it decreases along the secondary direction. Recently, Jha et al. [21] examined the effect of Hall current on hydromagnetic flow in a micro-channel in the presence of natural convection current. They recorded an increase in volume flow rate with the increase in Hall parameter. Other notable articles on Hall current having different geometric conditions include: Singh [22], Seth and Ansari [23], Seth and Singh [24], Makinde et al. [25], Krishna and Jyothi [26], Tani [27], etc.

In all the aforementioned studies on hydromagnetic flow of conducting fluid with Hall effects, the influence of ion-slip current is neglected in Ohm's law. In general, the diffusion velocity of electron is very large in comparison with diffusion velocity of ions. However, inclusion of combined effect of Hall current and ion-slip current becomes essential when the diffusion velocity of electrons and ions is of same order of magnitude. Mittal et al. [28] analyzed the effect of Hall current and ion-slip current on hydromagnetic flow in a rectangular channel. Ram et al. [29] examined the influence of Hall and ion-slip current on buoyancy-driven flow of conducting fluid in a rotating system. Jha and Apere [30] investigated the effect of Hall and ion-slip currents on hydromagnetic flow of viscous incompressible fluid in a region between two permeable parallel plates in a rotating system due to impulsive motion of one of the porous plates. Naroua et al. [31] presented an analysis on time-dependent flow of partially ionized gas in the presence of Hall and ion-slip current in a rotating channel with heat source. Singh et al. [32] studied combined effect of Hall and ion-slip current on unsteady hydromagnetic Couette–Poiseuille flow in the presence of a moving magnetic field. They found out that both primary and secondary component of fluid velocity decreases with the increase in ion-slip parameter. Srinivasacharya and Shafeeurrahman [33] studied the combined effect of Hall and ion-slip current on hydromagnetic nano-fluid flow of viscous incompressible and electrically conducting fluid in an annulus in the presence of mixed convection current. They recorded a reduction in fluid temperature due to the presence of Hall and ion-slip current, while it increases fluid velocity and nanoparticle volume fraction. Some interesting works on the above subject include: Attia and Aboul-Hassan [34], Singh et al. [35], Makinde et al. [36], Ghosh et al. [37], Mollah et al. [38], etc.

The objective of the present work is to discuss the impact of Hall and ion-slip current on hydromagnetic natural convection flow of viscous incompressible and electrically conducting fluid in a vertical micro-channel. The micro-channel walls are assumed to be heated asymmetrically. The governing equations responsible for the present mathematical are solved exactly using the method of undetermined coefficient under relevant boundary conditions and presented in dimensionless form. Effect of pertinent flow parameters is portrayed using line graphs and tables. In fact the present work is the generalization of Jha et al. [21] by incorporating ion-slip current.

2 Mathematical analysis

Flow description of the present problem is shown in Fig. 1. Consider the steady, fully developed flow of viscous incompressible and electrically conducting fluid in a region between two vertical micro-channel walls situated at $y' = 0$ and $y' = b$, respectively. The two walls are assumed to be heated asymmetrically with temperature T_1 at the right micro-channel wall and T_2 at the left wall. It is assumed that the temperature at the left wall $y' = 0$ is smaller than the temperature at the right wall $y' = b$, i.e., $T_1 > T_2$ giving rise to free convection current. A constant magnetic field is imposed perpendicular to the conducting fluid flow, along the y'-axis. By assuming a negligibly small value for magnetic Reynolds number ($R_m = \sigma\mu_e U_0 b$), the induced magnetic field is neglected. The effects of the applied magnetic field are assumed to be strong enough to induce Hall and ion-slip current in a direction perpendicular to both the electric and magnetic field. Hence, the velocity field and magnetic field vectors are of the form $\vec{V} = (u', 0, w')$ and $\vec{H} = (0, H_0, 0)$. Recently, Jha et al. [21] discussed Hall effect on MHD natural convection flow in a vertical micro-channel. Following, Jha et al. [21] and incorporating ion-slip current, the dimensional governing equation describing the above physical situation can be written as:

$$\frac{d^2u'}{dy'^2} - \frac{\sigma H_0^2 \mu_e^2}{(\alpha_e^2 + \beta_e^2)\rho}(\alpha_e u' + \beta_e w') + g\beta(T' - T_0) = 0 \quad (1)$$

$$\frac{d^2w'}{dy'^2} + \frac{\sigma H_0^2 \mu_e^2}{(\alpha_e^2 + \beta_e^2)\rho}(\beta_e u' - \alpha_e w') = 0 \quad (2)$$

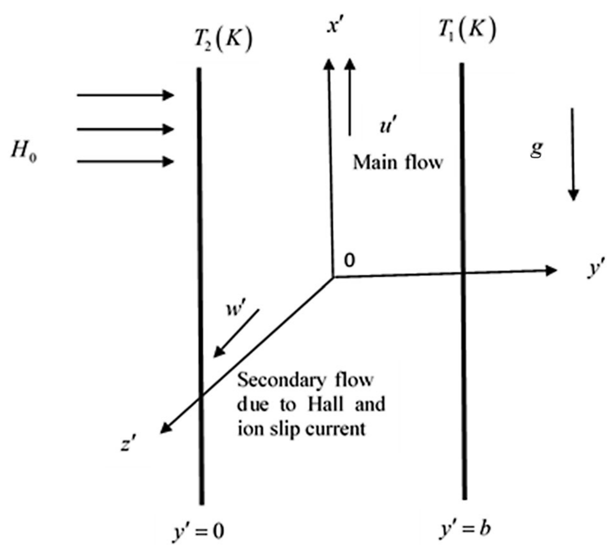

Fig. 1 Flow description

$$\frac{d^2T'}{dy'^2} = 0 \quad (3)$$

With dimensional momentum and thermal boundary conditions:

$$u'(y'=0) = \frac{2-f_v}{f_v}\lambda\frac{du'}{dy'}\Big|_{y'=0},\ w'(y'=0) = \frac{2-f_v}{f_v}\lambda\frac{dw'}{dy'}\Big|_{y'=0},$$
$$u'(y'=b) = -\frac{2-f_v}{f_v}\lambda\frac{du'}{dy'}\Big|_{y'=b},\ w'(y'=b) = -\frac{2-f_v}{f_v}\lambda\frac{dw'}{dy'}\Big|_{y'=b}, \quad (4)$$

$$T'(y'=0) = T_2 + \frac{2-f_t}{f_t}\frac{2\gamma_s}{\gamma_s+1}\frac{\lambda}{\Pr}\frac{dT'}{dy'}\Big|_{y'=0},$$
$$T'(y'=b) = T_1 - \frac{2-f_t}{f_t}\frac{2\gamma_s}{\gamma_s+1}\frac{\lambda}{\Pr}\frac{dT'}{dy'}\Big|_{y'=b}.$$

Employing the dimensionless variables and parameters:

$$y = \frac{y'}{b},\ M^2 = \frac{\sigma\mu_e^2 H_0^2 b^2}{\rho\upsilon},\ (u,w) = \frac{(u',w')}{U_0},\ U_0 = \frac{g\beta(T_1 - T_0)b^2}{\nu},$$
$$\Pr = \frac{\upsilon}{\alpha},\ \theta = \frac{T' - T_0}{T_1 - T_0} \quad (5)$$

Equations (1)–(3) can be rewritten in dimensionless form as:

$$\frac{d^2u}{dy^2} - \frac{M^2}{(\alpha_e^2 + \beta_e^2)}(\alpha_e u + \beta_e w) + \theta = 0 \quad (6)$$

$$\frac{d^2w}{dy^2} + \frac{M^2}{(\alpha_e^2 + \beta_e^2)}(\beta_e u - \alpha_e w) = 0 \quad (7)$$

$$\frac{d^2\theta}{dy^2} = 0 \quad (8)$$

With dimensionless momentum and thermal boundary conditions:

$$u(y=0) = \beta_v Kn\frac{du}{dy}\Big|_{y=0},\ w(y=0) = \beta_v Kn\frac{dw}{dy}\Big|_{y=0},$$
$$u(y=1) = -\beta_v Kn\frac{du}{dy}\Big|_{y=1},\ w(y=1) = -\beta_v Kn\frac{dw}{dy}\Big|_{y=1}.$$
$$\theta(y=0) = \xi + \beta_v Kn\ln\frac{d\theta}{dy}\Big|_{y=0},\ \theta(y=1) = 1 - \beta_v Kn\ln\frac{d\theta}{dy}\Big|_{y=1}. \quad (9)$$

where

$$\beta_v = \frac{2-f_v}{f_v},\ \beta_t = \frac{2-f_t}{f_t}\frac{2\gamma_s}{\gamma_s+1}\frac{1}{\Pr},\ Kn = \frac{\lambda}{b},\ \ln = \frac{\beta_t}{\beta_v},\ \xi = \frac{T_2 - T_0}{T_1 - T_0}. \quad (10)$$

are, respectively, the velocity slip, temperature jump, Knudsen number, fluid wall interaction parameter and wall ambient temperature difference ratio as used in most microfluidic system. As defined in Eckert and Drake [39] and Goniak and Duffa [40], values for velocity slip and temperature jump at the micro-surfaces are $\beta_v = 1.000$ and $\beta_t = 1.667$ corresponding to $\Pr = 0.71$, $\gamma_s = 1.4$, $f_t = f_v = 1$.

By defining $F = u + iw$, Eqs. (6)–(7) and boundary conditions (9) can be combined as:

$$\frac{d^2F}{dy^2} - M_1^2 F + \theta = 0 \tag{11}$$

where $M_1^2 = \frac{M^2}{\alpha_e + i\beta_e}$ *and* $\alpha_e = 1 + \beta_e \beta_i$

$$\begin{aligned} F(0) &= \beta_v Kn \frac{dF}{dy}\Big|_{y=0} \qquad F(1) = -\beta_v Kn \frac{dF}{dy}\Big|_{y=1} \\ \theta(0) &= \xi + \beta_v Kn \ln \frac{d\theta}{dy}\Big|_{y=0} \quad \theta(1) = 1 - \beta_v Kn \ln \frac{d\theta}{dy}\Big|_{y=1} \end{aligned} \tag{12}$$

Using the method of undetermined coefficient, solution of Eqs. (8) and (11) subject to the boundary conditions (12) gives:

$$\theta(y) = C_1 + C_2 y \tag{13}$$

$$F(y) = C_3 \cosh(M_1 y) + C_4 \sinh(M_1 y) + \frac{1}{M_1^2}(C_1 + C_2 y) \tag{14}$$

where

$$C_1 = \xi + \frac{\beta_v Kn(1-\xi)}{1+2\beta_v Kn \ln}, \; C_2 = \frac{1-\xi}{1+2\beta_v Kn \ln}$$

$$C_3 = \frac{k_1k_5 + k_2k_4}{k_4 + k_1k_3}, \qquad C_4 = \frac{k_5 - k_2k_3}{k_4 + k_1k_3}$$

$k_1 \ldots k_5$ are constants defined in "Appendix."

2.1 Special cases

In the absence of ion-slip current, the present model reduces to the problem investigated by Jha et al. [21]. Also, in the absence of transversely applied magnetic field, the present work reduces to Chen and Weng [41].

Two important micro-channel flow characteristics are the volume flow rate and skin friction on the micro-channel surfaces. To obtain these features, we proceed as:

2.2 Volume flow rate and skin friction

Due to the combines effects of the Hall currents and ion slip, the present analysis considers the dimensionless volume flow rate in both the primary and secondary flow directions, given by;

$$\delta = Q_x + iQ_z = \int_0^1 F(y)dy \tag{15}$$

where Q_x and Q_z represents the volume flow rate along the primary and secondary flow directions.

Substituting Eq. (14) into Eq. (15) gives:

$$\delta = \frac{C_3}{M_1}\left(\sinh(M_1)\right) + \frac{C_4}{M_1}\left(\cosh(M_1) - 1\right) + \frac{1}{M_1^2}\left(C_1 + \frac{C_2}{2}\right) \tag{16}$$

Similarly, the skin friction at the micro-channel walls for both primary and secondary flows situated on the walls $y = 0$ and $y = 1$ is obtained as:

$$\tau_0 = \tau_{x0} + i\tau_{z0} = \frac{dF}{dy}\Big|_{y=0} = M_1 C_4 + \frac{C_2}{M_1^2} \tag{17}$$

$$\tau_1 = \tau_{x1} + i\tau_{z1} = \frac{dF}{dy}\Big|_{y=1} = \frac{C_2}{M_1^2} - M_1\left(C_3 \sinh(M_1) + C_4 \cosh(M_1)\right) \tag{18}$$

where τ_{x0}, τ_{z0} and τ_{x1}, τ_{z1} represent the skin friction along the primary and secondary flow directions at the micro-channel walls $y = 0$ and $y = 1$, respectively.

3 Results and discussion

Using expression for fluid velocity, volume flow rate and skin friction obtained in the previous section, a MATLAB code is prepared to investigate effect of pertinent governing parameters on flow formation within the micro-channel. Unless otherwise indicated, range of values used include: $5 \le M \le 9$, $0.01 \le \beta_e \le 0.10$, $1 \le \beta_i \le 9$ and $0.01 \le \beta_v Kn \le 0.10$ with fixed values $M = 5$, $\beta_i = 3$, $\beta_e = 0.10$, $\ln = 1.667$ and $\beta_v Kn = 0.05$ all arbitrarily chosen to investigate their effect on flow field as used in Jha et al. [21], Mollah et al. [38] and Chen and Weng [41]. The present analysis is carried out for three important cases of the wall ambient temperature difference ratio ξ: $\xi = 1$ (symmetric heating) means the physical condition when the two micro-channel walls are heated symmetrically at the same constant temperature, $\xi = 0$ (asymmetric heating) means the condition when one of the micro-channel wall is heated and the other not heated, $\xi = -1$ (asymmetric heating) means the physical condition when one of the walls is heated and the other is cooled at different ambient temperatures as defined in Weng and Chen [42].

Expression for energy equation in the micro-channel as given in Eq. (8) and solution in Eq. (13) is the same with those of Chen and Weng [40]. Detailed analysis on effect of rarefaction parameter ($\beta_v Kn$), fluid wall interaction parameter (ln) and wall ambient temperature difference ratio (ξ) on fluid temperature and rate of heat transfer is discussed in their work. The main focus of this paper is to analyze the hydrodynamics of the flow in presence of Hall and ion-slip currents.

Figure 2a, b illustrates the effect of rarefaction parameter ($\beta_v Kn$) on fluid velocity profile along primary and secondary flow direction in symmetrically ($\xi = 1$) or asymmetrically ($\xi = 0, -1$) heated micro-channel in the presence of Hall and ion-slip current. As expected, both primary and secondary fluid velocities are observed to be directly proportional to increase in $\beta_v Kn$ for $\xi = 1$ and $\xi = 0$ heating conditions yielding an increase in velocity profiles. This is due to the fact that the retarding effect of the boundary walls reduces with an increase in $\beta_v Kn$. Figure 3 (a, b) displays how the Hall current parameter (β_e) influences fluid velocity distribution in the presence of ion-slip current parameter for different micro-channel heating conditions (ξ). From these figures, we conclude that the impact of Hall parameter on primary velocity is almost negligible while it increases the secondary velocity for considered range of numerical values.

The effect of ion-slip (β_i) parameter as well as wall ambient temperature difference ratio (ξ) on fluid velocity is displayed in Fig. 4a, b for fixed values of Hall current parameter, Hartmann number, and rarefaction parameter. From the figure, it is interesting to observe that when the micro-channel walls are heated either symmetrically ($\xi = 1$) or asymmetrically ($\xi = 0$), ion-slip parameter augment flow formation along the primary direction causes an increase

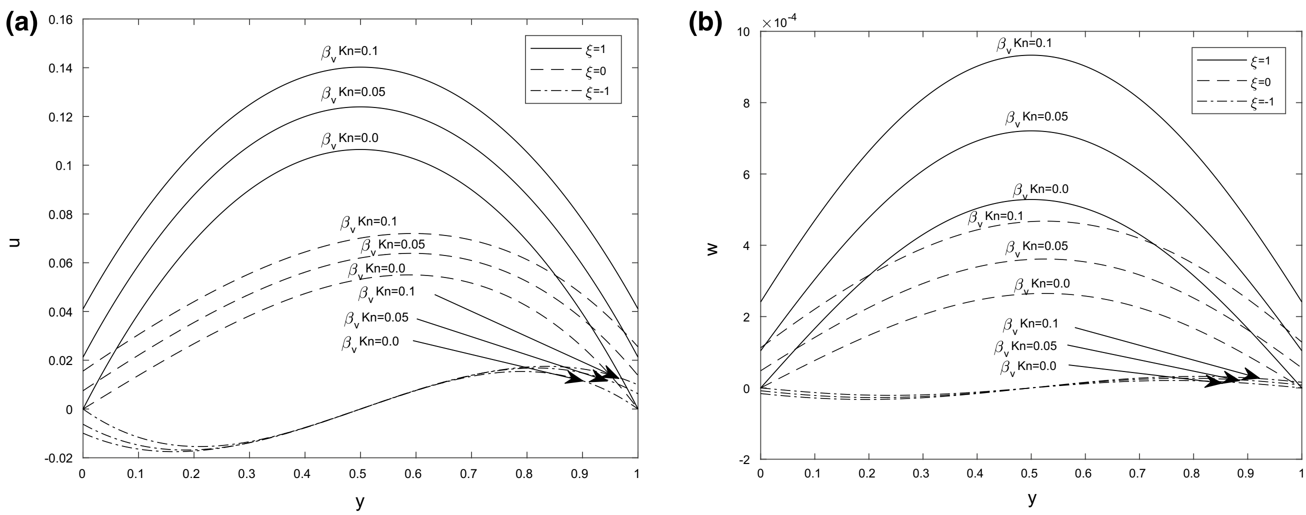

Fig. 2 Effect of rarefaction parameter ($\beta_v Kn$) on primary and secondary velocity profiles for $\beta_e = 0.10$, $\beta_i = 3.0$, $\ln = 1.667$ and $M = 5.0$

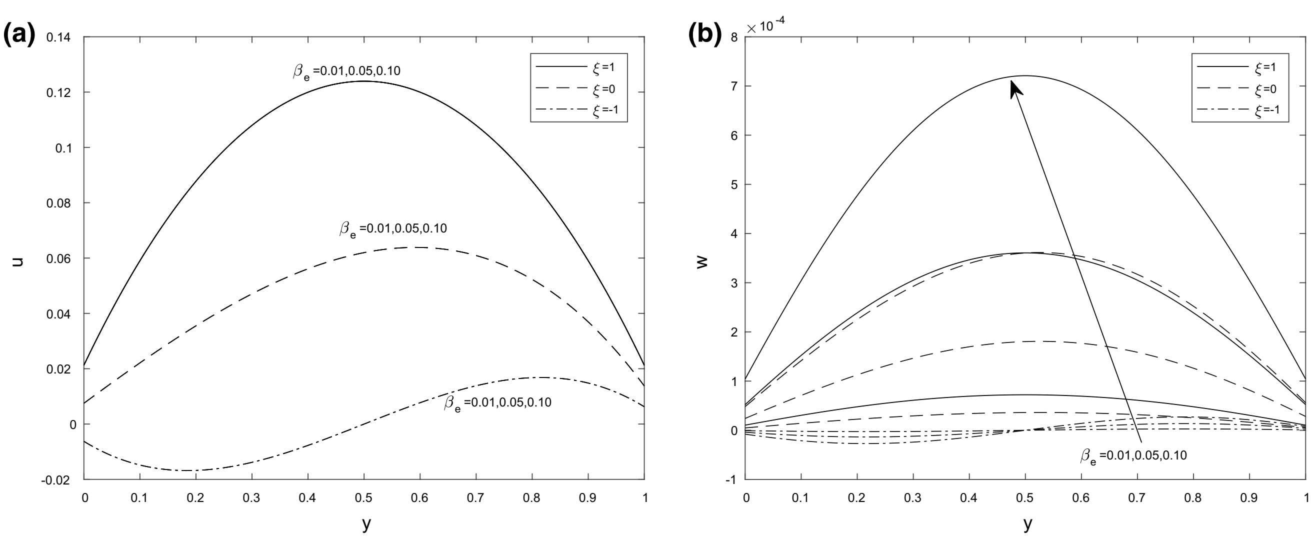

Fig. 3 Effect of Hall current parameter (β_e) on primary and secondary velocity profiles for $\beta_v Kn = 0.05$, $\beta_i = 3.0$, $\ln = 1.667$ and $M = 5.0$

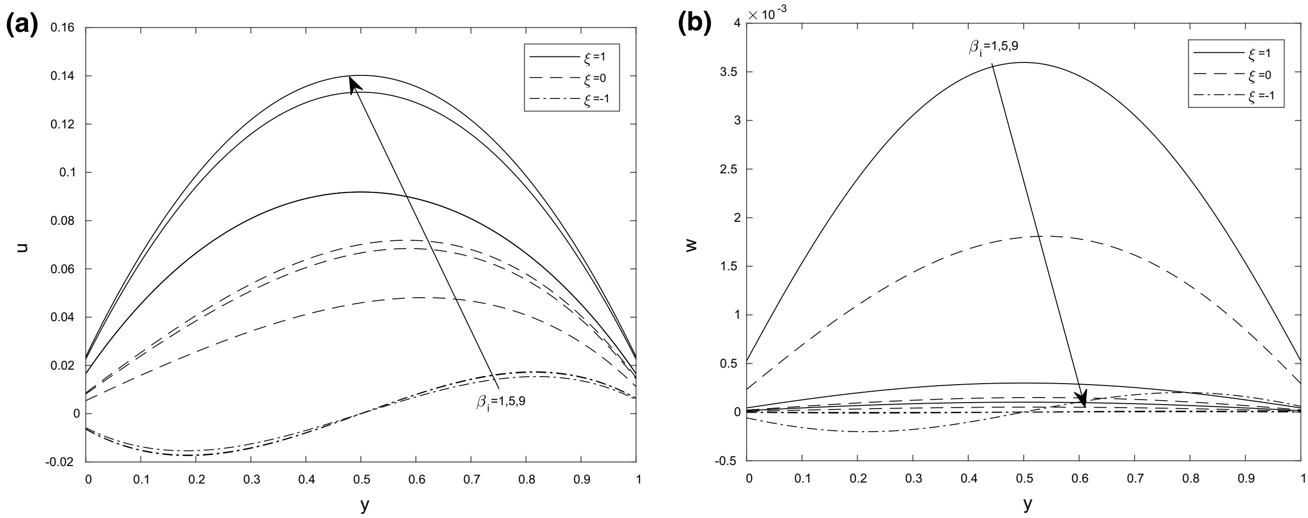

Fig. 4 Effect of ion-slip parameter (β_i) on primary and secondary velocity profiles for $\beta_e = 0.10,\ \beta_v Kn = 0.05,\ \mathrm{In} = 1.667$ and $M = 5.0$

in fluid velocity profile whereas it displays a dual character for asymmetric heating ($\xi = -1$). The reverse of this phenomenon is evident along the secondary flow direction (Fig. 4b). Figure 5 illustrates the effect of Hartmann number (M) in the presence of Hall (β_e) and ion-slip current (β_i) on fluid velocity for different values of ξ. The result indicates that for symmetric ($\xi = 1$) and asymmetric heating ($\xi = 0, -1$), the primary fluid velocity is lowered with the increase in Hartmann number all through the flow domain. This result is just contrast for secondary velocity.

Figures 6, 7 and 8 examine the effect of Hartmann number (M), Hall parameter (β_e), and ion-slip parameter (β_i) on volume flow rate along primary and secondary flow directions (Q_x, Q_z) for different wall heating conditions (ξ). The combined effect of Hall current parameter (β_e) and rarefaction parameter ($\beta_v Kn$) on volume flow rate is portrayed in Fig. 6 for both symmetric ($\xi = 1$) and asymmetric ($\xi = 0, -1$) wall heating conditions. The figure reveals how the wall ambient temperature affects the influence of the Hartmann number on volume flow rate in the presence of Hall (β_e) and ion-slip current (β_i). It is evident from Fig. 6a that for $\xi = 1$ and $\xi = 0$, volume flow decreases along the primary flow direction with the increase in Hartmann number whereas it increases with the increase in rarefaction parameter ($\beta_v Kn$). Interestingly, it is evident that for $\xi = 0$, simultaneously increasing the Hartmann number (M) and rarefaction parameter ($\beta_v Kn$) causes an enhancement in volume flow rate along secondary flow direction (Q_z). In both flow directions, however, it is observed that

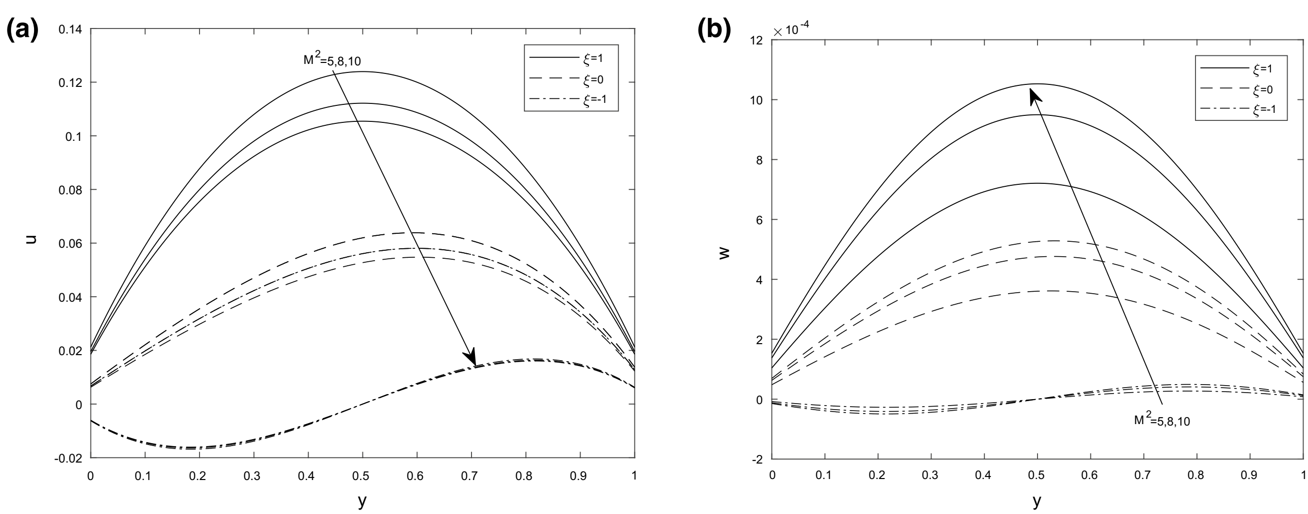

Fig. 5 Effect of Hartmann number (M) on primary and secondary velocity profiles for $\beta_e = 0.10,\ \beta_i = 3.0,\ \mathrm{In} = 1.667$ and $\beta_v Kn = 0.05$

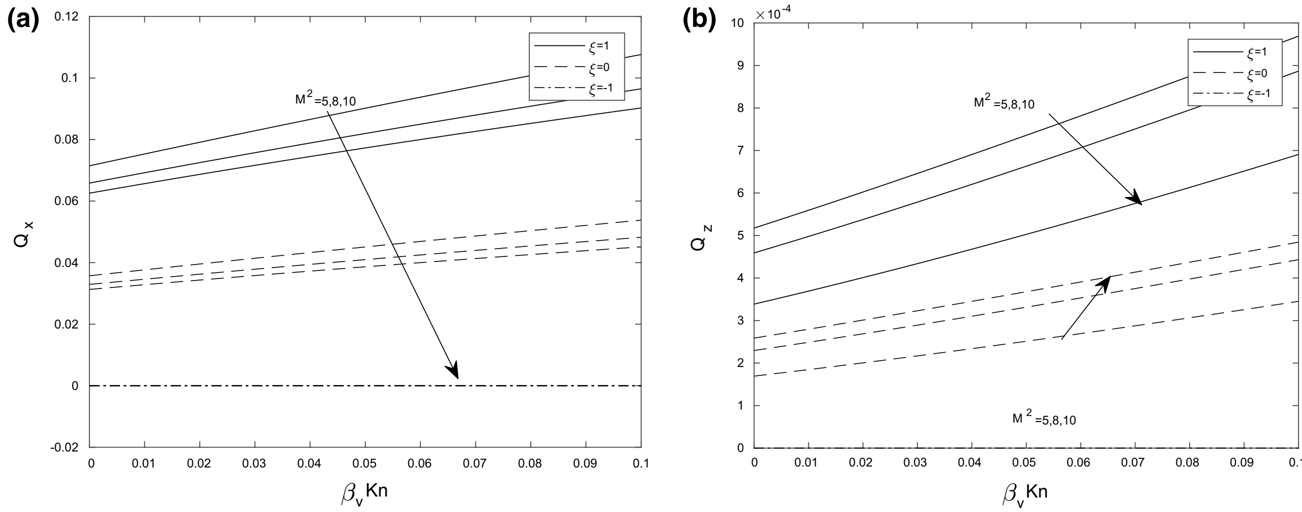

Fig. 6 Effect of Hartmann number (M) on primary and secondary volume flow rate for $\beta_e = 0.10$, $\beta_i = 3.0$ and $\ln = 1.667$

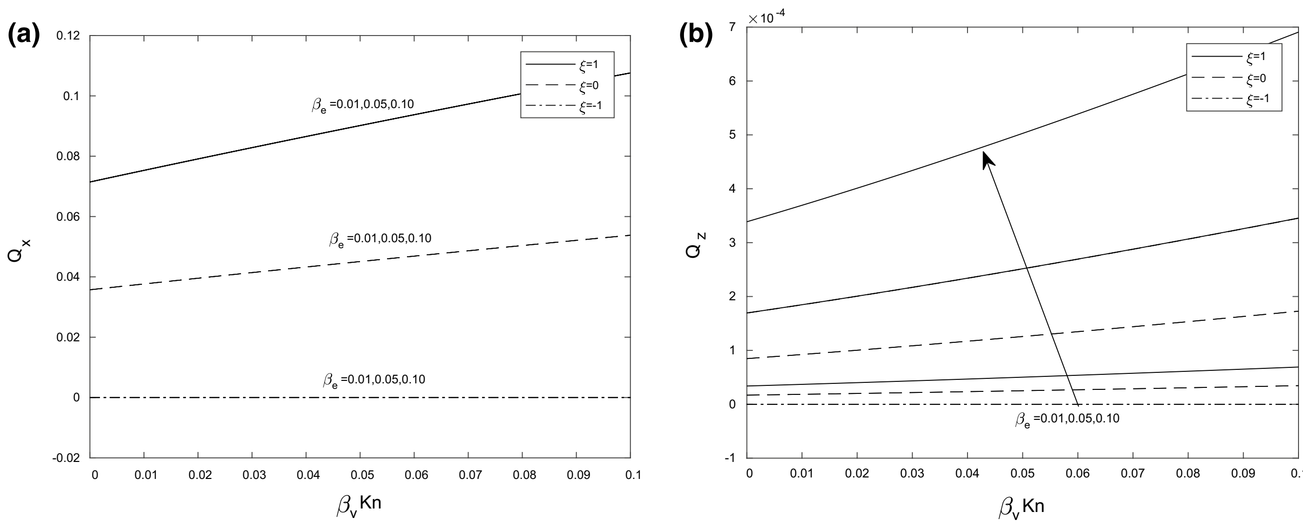

Fig. 7 Effect of Hall current parameter (β_e) on primary and secondary volume flow rate for $M = 5.0$, $\beta_i = 3.0$ and $\ln = 1.667$

for asymmetric heating ($\xi = -1$) net volume flow is almost zero for all considered values of M and $\beta_v Kn$.

Volume flow rate variations with Hall current parameter (β_e) and rarefaction parameter ($\beta_v Kn$) are illustrated in Fig. 7 under different micro-channel heating conditions. It is be observed that as Hall parameter (β_e) increases, the volume flow along the primary flow direction remains unaffected for all considered values of ξ. This result is in excellent agreement with Fig. 3a as observed earlier. Increase in rarefaction parameter on the other hand yields an increase in volume flow for both symmetric ($\xi = 1$) and asymmetric ($\xi = 0$) heating conditions. For secondary volume flow rate, it is observed that simultaneously increasing $\beta_v Kn$ and β_e causes an enhancement in volume flow rate for $\xi = 1$ and $\xi = 0$.

Figure 8a shows that ion-slip parameter (β_i) supports flow formation for all considered values of $\beta_v Kn$ when the micro-channel walls are heated either symmetrically ($\xi = 1$) or asymmetrically ($\xi = 0$) yielding an increase in primary volume flow rate (Q_x), whereas the reverse is encountered for secondary volume flow rate (Q_z).

Table 1 demonstrates the effects of Hartmann number (M), Hall parameter (β_e) and ion-slip parameter (β_i) on skin friction in both primary and secondary flow directions (τ_x, τ_z) on micro-channel walls situated at $y = 0$ and $y = 1$, respectively, when $\beta_v Kn = 0.05$, $\ln = 1.667$ and $\xi = 0$.

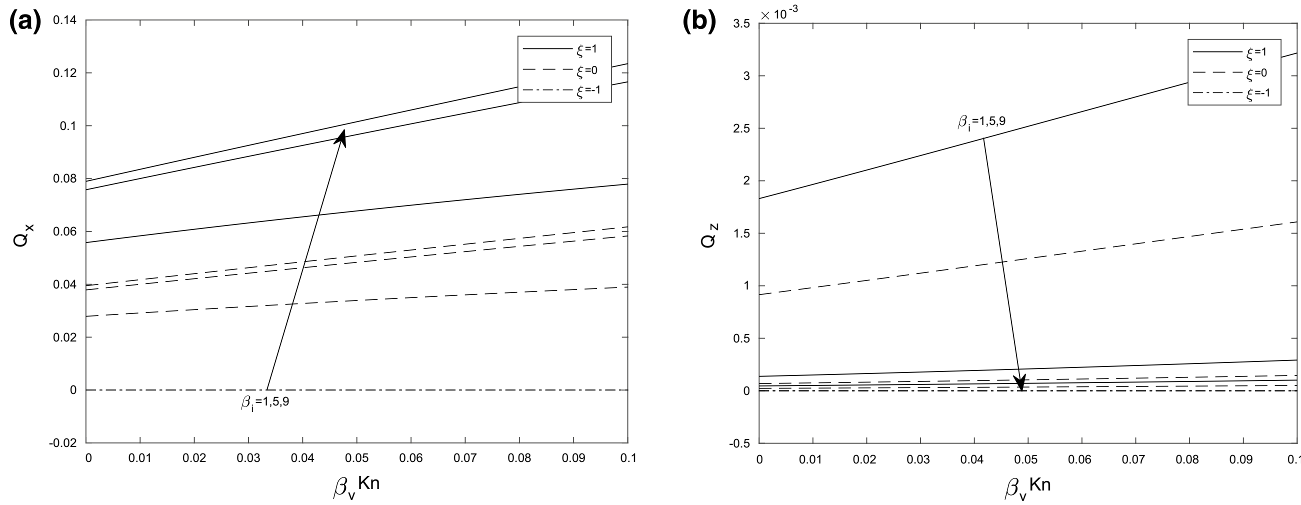

Fig. 8 Effect of ion-slip parameter (β_i) on primary and secondary velocity profiles for $\beta_e = 0.10,\ M = 5.0$ and $\ln = 1.667$

Table 1 Effects of Hartmann number (M), Hall parameter (β_e) and ion-slip parameter (β_i) on skin friction on micro-channel walls situated at $y = 0$ and $y = 1$. For $\beta_v Kn = 0.05,\ \ln = 1.667$ and $\xi = 0$

M	β_e	β_i	τ_{x0}	τ_{z0}	τ_{x1}	τ_{z1}
5	0.01	1	0.1074	4.6888e−04	−0.2236	−5.8902e−04
		5	0.1624	4.0136e−05	−2.2892	−4.6082e−05
		9	0.1718	1.3737e−05	−0.3000	−1.5619e−05
	0.05	1	0.1075	0.0023	−2.2237	−0.0029
		5	0.1624	2.0067e−04	−0.2892	−2.3039e−04
		9	0.1718	6.8682e−05	−0.3000	−7.8092e−05
	0.10	1	0.1077	0.0047	−0.2239	−0.0059
		5	0.1624	4.0124e−04	−0.2893	−4.6067e−04
		9	0.1718	1.3735e−04	−0.3000	−1.5617e−04
8	0.01	1	0.0848	4.8431e−04	−0.1942	−6.5069e−04
		5	0.1512	5.6265e−05	−0.2762	−6.5463e−05
		9	0.1647	2.0330e−05	−0.2918	−2.3285e−05
	0.05	1	0.0849	0.0024	−0.1943	−0.0033
		5	0.1512	2.8131e−04	−0.2762	−3.2729e−04
		9	0.1647	1.0165e−04	−0.2918	−1.1642e−04
	0.10	1	0.0850	0.0048	−0.1946	−0.0065
		5	0.1512	5.6250e−04	−0.2762	−6.5445e−04
		9	0.1647	2.0328e−04	−0.2918	−2.3282e−04
10	0.01	1	0.0741	4.7111e−04	−0.1796	−6.6117e−04
		5	0.1444	6.4670e−05	−0.2683	−7.5918e−05
		9	0.1602	2.4162e−05	−0.2867	−2.7809e−05
	0.05	1	0.0742	0.0024	−0.1796	−0.0033
		5	0.1444	3.2333e−04	−0.2683	−3.7956e−04
		9	0.1602	1.2080e−04	−0.2867	−1.3904e−04
	0.10	1	0.0743	0.0047	−0.1799	−0.0066
		5	0.1444	6.4654	−0.2683	−7.5898e−04
		9	0.1602	2.4159e−04	−0.2867	−2.7806e−04

Details from the table reveals that in the presence of Hall current (β_e) and ion slip (α_e), increase in Hartmann number (M) causes a reduction in τ_{x0}, whereas it increases τ_{x1}. Increase in ion slip (β_i), however, decreases τ_{x0} while it displays a dual character for τ_{x1}. For the secondary component, on the other hand (τ_z), evidence from the table shows that τ_{z0} increases with Hall parameter whereas it decreases τ_{z1}. Increase in Hartmann number causes an increase in τ_{z0} while it displays a dual character for τ_{z1}.

Numerical results for fluid velocity obtained in this work are compared with those of Jha et al. [21] in the absence of ion-slip current and presented in Table 2. Details from the table suggest that in the absence of ion-slip current ($\beta_i = 0.0$), numerical values of present work are in excellent agreement with those of Jha et al. [21], hence validating the solution obtained herein. Also as a special case, numerical values for fluid velocity obtained in the present work are validated with those of Chen and Weng [41] and presented in Table 3 in the absence of a transversely applied magnetic field, i.e., $M \to 0$.

4 Conclusion

Combined effect of Hall and ion-slip current on hydromagnetic free convection flow of viscous incompressible and electrically conducting fluid in a vertical micro-channel is investigated taking into consideration the asymmetric heating of the micro-channel walls. Detail from the analysis leads to the resulting conclusions:

1. In the presence of ion-slip current, primary fluid velocity and volume flow remains unaffected with variations in Hall current parameter for all micro-channel heating conditions.
2. Increase in Hartmann number decreases fluid velocity along the primary direction, whereas it enhances it along the secondary flow direction.
3. In the presence of ion-slip current, secondary volume flow rate could be enhanced by simultaneously increasing the rarefaction parameter and Hall current parameter.
4. For asymmetric heating ($\xi = 0$), skin friction for the primary flow direction at $y = 0$ could be reduced with the increase in Hartmann number.

Table 3 Numerical comparison of the values of the primary fluid velocity obtained in the present work (for $M \to 0$, i.e., in the absence of Hall and ion-slip current) with those of Chen and Weng [41]. When $\ln = 1.667$ and $\beta_v Kn = 0.05$

ξ	y	Chen and Weng [41]	Present work (When $M \to 0$)
1.0	0.0	0.0250	0.0250
	0.2	0.1050	0.1050
	0.4	0.1450	0.1450
	0.6	0.1450	0.1450
	0.8	0.1050	0.1050
	1.0	0.0250	0.0250
0.0	0.0	0.0157	0.0157
	0.2	0.0437	0.0437
	0.4	0.0684	0.0684
	0.6	0.0766	0.0766
	0.8	0.0613	0.0613
	1.0	0.0157	0.0157
−1.0	0.0	−0.0065	−0.0065
	0.2	−0.0176	−0.0176
	0.4	−0.0081	−0.0081
	0.6	0.0081	0.0081
	0.8	0.0176	0.0176
	1.0	0.0065	0.0065

Table 2 Numerical comparison for fluid velocity in the present work with those of Jha et al. [21] in the absence of ion-slip current ($\beta_i = 0.0$), when $\beta_v Kn = 0.05$, $\ln = 1.667$, $M = 5$ and $\beta_e = 0.5$

ξ	y	u Present work	w Present work	u Jha et al. [21]	w Jha et al. [21]
1.0	0.2	0.0679	0.0110	0.0679	0.0110
	0.4	0.0938	0.0158	0.0938	0.0158
	0.6	0.0938	0.0158	0.0938	0.0158
	0.8	0.0697	0.0110	0.0697	0.0110
0.0	0.2	0.0270	0.0051	0.0270	0.0051
	0.4	0.0433	0.0077	0.0433	0.0077
	0.6	0.0505	0.0081	0.0505	0.0081
	0.8	0.0427	0.0059	0.0427	0.0059
−1.0	0.2	−0.0156	−0.8508e−03	−0.0156	−0.8508e−03
	0.4	−0.0071	−0.4339e−03	−0.0071	−0.4339e−03
	0.6	0.0071	0.4339e−03	0.0071	0.4339e−03
	0.8	0.0159	0.8508e−03	0.0159	0.8508e−03

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Appendix

$$C_1 = \xi + \frac{\beta_v Kn(1-\xi)}{1 + 2\beta_v Kn \ln}$$
$$C_2 = \frac{1-\xi}{1 + 2\beta_v Kn \ln}$$

$$k_1 = \beta_v Kn M_1$$
$$k_2 = \frac{1}{M_1^2(A_1\beta_v Kn - A_0)}$$
$$k_3 = \cosh(M_1) + M_1\beta_v Kn \sinh(M_1)$$
$$k_4 = \sinh(M_1) + M_1\beta_v Kn \cosh(M_1)$$

$$k_5 = -\frac{\left((C_1 + C_2) + C_2\beta_v Kn\right)}{M_1^2}$$
$$C_3 = \frac{k_1k_5 + k_2k_4}{k_4 + k_1k_3}$$
$$C_4 = \frac{k_5 - k_2k_3}{k_4 + k_1k_3}$$

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