Research Article

Application of spectral analysis and wavelet transforms to full-scale dynamic drainages at minesites

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Abstract

This paper focusses on the application of spectral analysis and continuous wavelet transforms to water drainages from full-scale minesite components, such as open pits, waste-rock piles, and tailings impoundments. Three minesite-drainage databases included high-frequency monitoring (as frequently as every 15 min) and/or long-term monitoring (up to 31 years) of both flows and aqueous chemistries. These databases were cleaned only by deleting very obvious outliers and ignoring statistical significance, so that extreme events and fractal patterns could be detected. In all three full-scale minesite-drainage databases, 1-over-f fractal slopes were common in the spectral analyses, but other slopes mostly between zero and 2.0 were also found. Spectral analyses also produced anomalous spectral slopes. Simple simulations showed these could be explained by major unseen seasonal changes in water retention by upstream buried ponds or subsurface aquitards. Wavelet transforms for the three minesite-drainage databases provided important observations such as (1) the varying strengths of periodicity with time, (2) the differing periodicities between physical drainage flows and their aqueous chemistries, and (3) the effect of placing a fine-grained soil/till cover over a waste-rock pile. Based on all three minesite-drainage databases, the most common wavelengths for strong, persistent periodicities were 1 year and 1 week. Other wavelengths of strong periodicity for at least two minesites were 10 years, approximately 4 months, and half-monthly to monthly. The minesite with data as frequent as every 15 min also showed strong periodicities over 1 day and less.

1 Introduction

Minesites can be divided into physically and environmentally distinct components, such as waste-rock piles, tailings impoundments, and open pits [51, 52, 58]. Each full-scale component typically has water draining through and from it, originating as direct precipitation and from any adjacent inflow.

If the drainage from a minesite component is contaminated, it typically has to be collected, managed, and treated before release to the surrounding environment. If the drainage is not contaminated, the flow still often has to be re-directed and managed so that other areas of the minesite and the downstream environment are not adversely affected by additional water.

Therefore, in most cases, understanding the dynamic natures of full-scale drainage chemistries and flows is important in optimum water management and environmental protection at minesites. This paper shows that these dynamic natures can be underestimated and

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misunderstood, primarily due to the common low-frequency monitoring at minesite drainages.

For example, for a full-scale, active, uncovered waste-rock pile (Fig. 1), a 1-day storm with significant variations in hourly rainfall produced significant variations in drainage outflow over 15-min intervals [43, 44]. This contradicts typical waste-rock models that show that most infiltration moves through finer-grained layers rather than through coarser layers [e.g. 14, 15, 17, 19, 26, 28, 39, 57]. This inaccuracy is most often likely due to simulating the turbulent open-channel flow in full-scale coarse waste rock using Darcian-type models with capillarity and matrix suction [7, 10]. More realistic approaches are available [e.g. 7, 10, 29, 30, 64].

For this same example of full-scale waste rock, the daily measurements of a single instantaneous flow, or the daily sums of flows and precipitation (Fig. 2), fail to reveal the real, dynamic nature of the outflow and its relationship to short periods of high precipitation (compare with Fig. 1). This also applies to aqueous chemistry, as shown below in this paper for mining catchments and in Fig. 3 for non-mining catchments. These short-term peaks, albeit short-lived and rarely detected, can lead to adverse effects on downstream ecosystems due to effects like [42]:

- ecological damage per unit of time,
- accumulating ecological damage through time,
- temporally aligned or offset synergistic and antagonistic interactions, and
- slowly reversible or non-reversible uptake and binding of some metals and other contaminants.

Therefore, high-frequency monitoring of minesite drainages, which is rare, can reveal errors in conceptual and digital models that are based on, or calibrated to, low-frequency monitoring. This has been well documented for non-mining catchments [e.g. 2, 35] and for ecosystems in general [e.g. 1]. This is discussed further in Sect. 5.

As shown later in this paper, the dynamic natures of full-scale drainage chemistries and flows, when portrayed as time series, include periodicities and fractal patterns at many wavelengths and frequencies of time. In simplistic words, the chemistries and the flows "hum" or "buzz" at various frequencies and wavelengths through time. This understanding determines the techniques (Sect. 2) and limits the databases (Sect. 3), available to assess more thoroughly and realistically the dynamics of minesite drainages.

For emphasis, this paper focusses on full-scale minesite components, containing more than 10^6 metric tonnes (t)

Fig. 1 High-frequency drainage flow, measured every 15 min (mathematically converted to mm, black line), and hourly precipitation (in mm, red vertical bars), from active, uncovered, full-scale waste-rock piles, showing dynamic, relatable trends in both precipitation and flow [43]; compare with Fig. 2 from about a month earlier with lower-frequency data
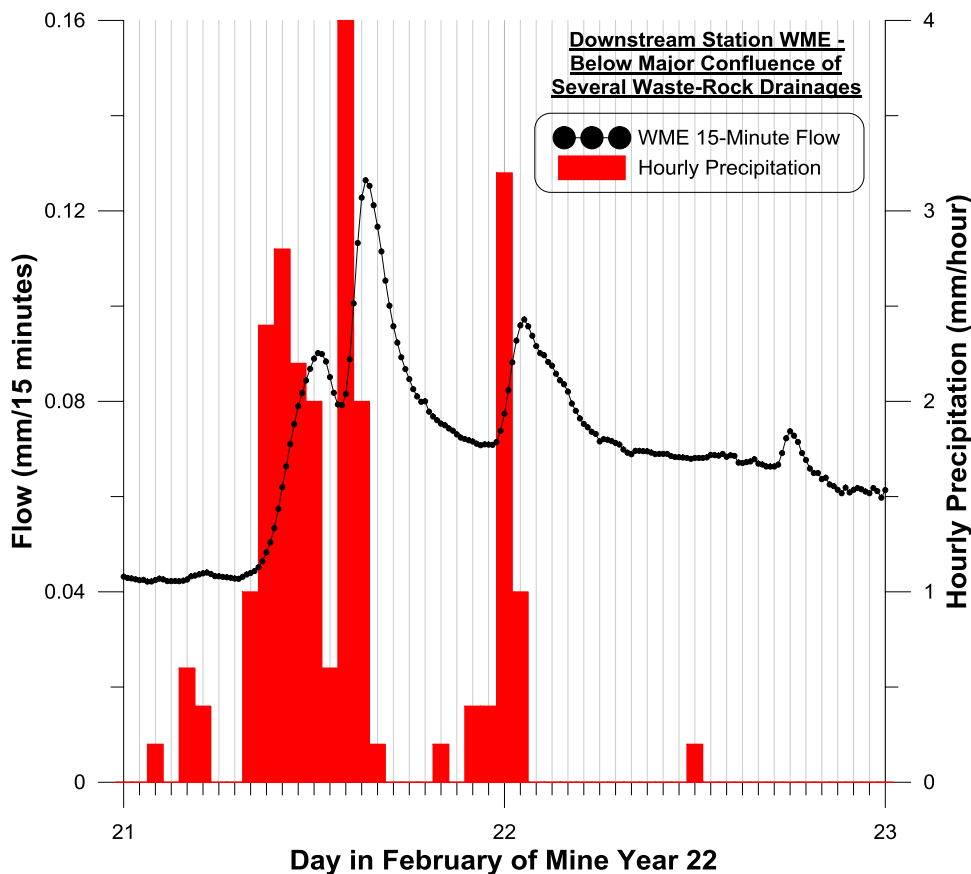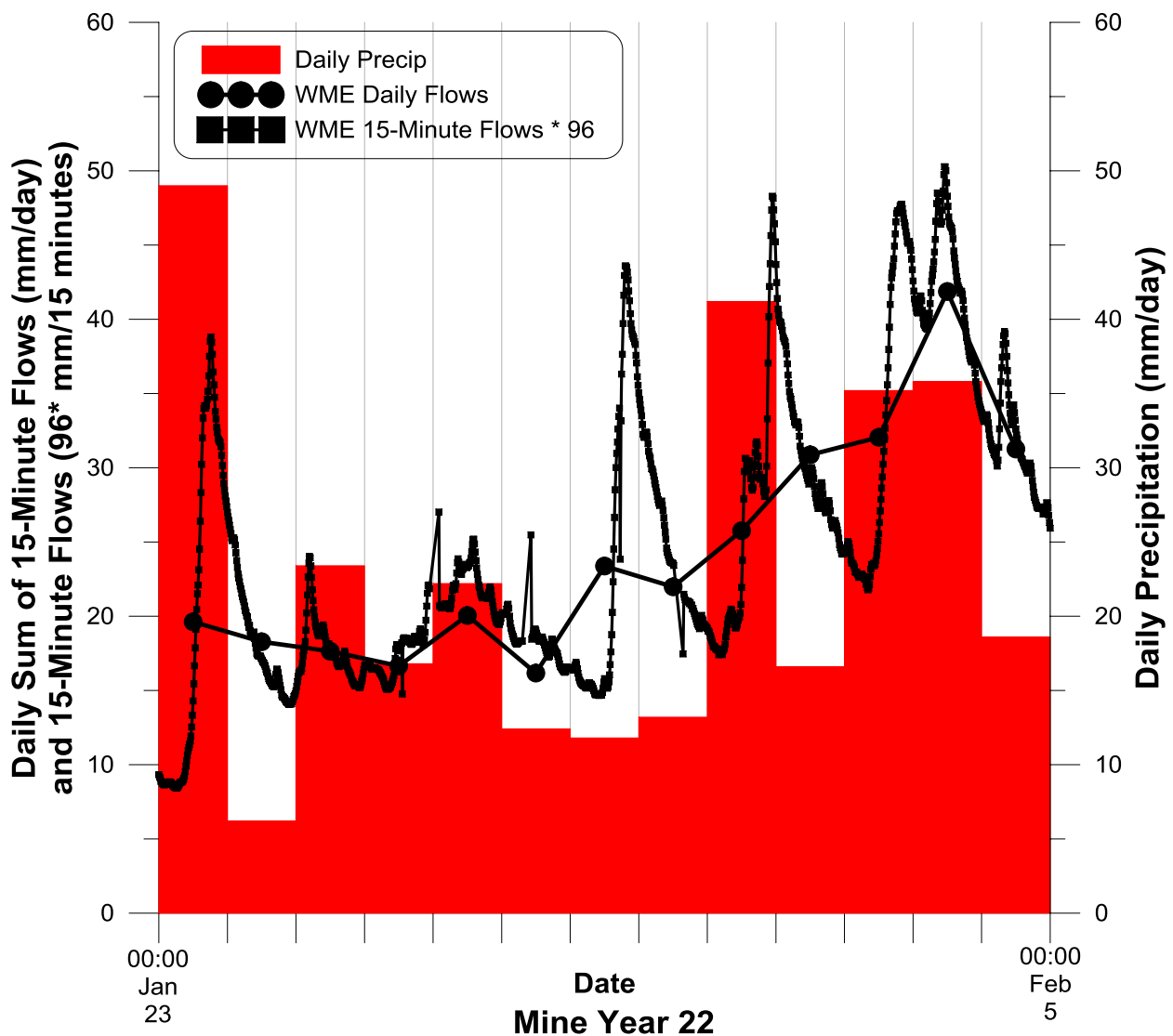
Fig. 2 High-frequency drainage flow measured every 15 min (mathematically converted to mm, black line marked with squares), daily sums of these short-term flows (in mm, black line marked with circles), and daily precipitation (in mm, red vertical bars), from active, uncovered, full-scale waste-rock piles [44]; compare with Fig. 1 from about a month later and note here the significant reduction in variability and in correlation with the lower-frequency daily data

of geological materials, constructed in the manner locally selected by the mining company. Laboratory-scale testing such as those containing kilograms, and pre-designed on-site testing such as "field-scale" piles, cannot provide reliable full-scale information [46, 50]. This is due to major changes with increasing scale, such as the emergence of certain critical properties and processes as the scale increases (discussed further in Sect. 4.5). Common expressions for this emergence with increasing scale are "the whole is greater than the sum of its parts" and "more is different".

Therefore, understanding the dynamics of full-scale minesite drainage requires full-scale monitoring data. Because full-scale monitoring cannot be neatly organized, planned, and executed like laboratory testing, full-scale data can be "messy" such as with periodic data gaps (Sect. 3). Nevertheless, these imperfect full-scale databases can still be used as explained below.

2 Mathematical approaches

This paper examines selected databases of full-scale mine-site drainage (Sect. 3) primarily through the mathematical and statistical techniques of spectral analysis (Sect. 2.1) and wavelet transforms (Sect. 2.2). Simple plotting of time series is also included here. The primary focus is on wavelengths and time periods with relatively significant

Fig. 3 Comparison of high-frequency and low-frequency monitoring in a full-scale non-mining catchment of aqueous chloride (top) and electrical conductivity (bottom) adapted from [35], showing the significant loss of dynamic variation and reality in less frequent analyses
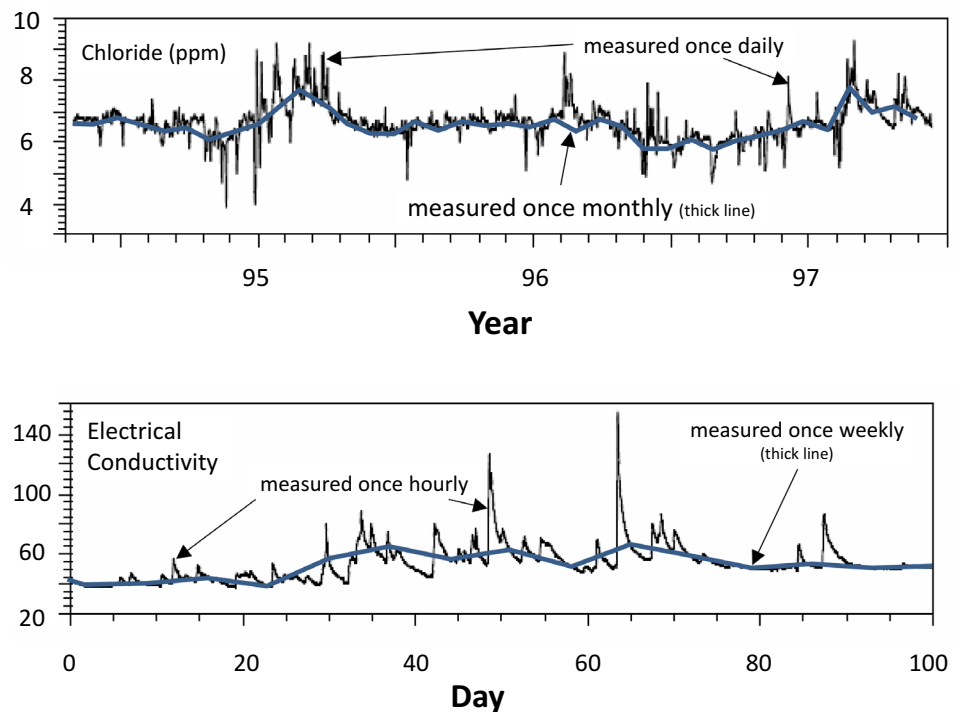
spectral or wavelet power. Relatively strong power signifies significant periodicity at that wavelength or during that time period.

There are many books, conference proceedings, and papers published on spectral analysis and wavelet transforms. Thus, it is the objective of this paper not to discuss the underlying mathematics, but to focus on the application of these techniques to full-scale minesite drainage. Important concepts for proper full-scale application are summarized in Sect. 4.

A major issue with most mathematical and statistical algorithms for spectral analysis and wavelet transforms is the requirement for consistent, evenly spaced data in every time series. Full-scale environmental data rarely meet this requirement, due to events like equipment failure, equipment destruction during active mining, and long-term physical settlement of mine wastes with time. As a result, this study with real, unevenly spaced, full-scale data was greatly limited in the available applicable algorithms, leading to one for spectral analysis (Lomb–Scargle, Sect. 2.1) and one for wavelets (Morlet, Sect. 2.2). While a comparison of results using several algorithms would have been informative, it was not possible, but may be possible in the future. (Section 8 lists potential future work.)

While unevenly spaced data can be interpolated, imputed, and mapped into evenly spaced data [8], this requires some pre-selection of an assumed distribution or trend. For the databases examined here, such interpolation can mask or destroy fractal patterns and is thus not

recommended as the first step for such full-scale environmental databases.

2.1 Spectral analysis

The minesite-drainage databases in this paper (Sect. 3), plotted as time series, were evaluated using least-squares spectral analysis to obtain a periodogram with a frequency spectrum. This involved statistical least-squares fitting of multiple sine waves, with differing wavelengths (frequencies) and amplitudes, to the databases. Such fitting of sine waves, although sine waves are rounded and smooth, can still reliably address even non-curved or abrupt repeating patterns such as step (boxcar) and sawtooth when needed.

The least-squares technique used here is the Lomb–Scargle method [11, 12, 38, 61, 62, 66]. The Lomb–Scargle method is particularly suited for incomplete datasets with unequal spacing and data gaps, which are common in environmental monitoring due to unforeseen circumstances like equipment failure.

Statistical significance was dismissed so that log–log spectral slopes, like fractal 1-over-f slopes, could be evaluated. This is discussed further in Sect. 4.6.

The specific Lomb–Scargle algorithm used here is provided, maintained, and operated by the California Institute of Technology on the NASA Exoplanet Archive website [5]. Results of the spectral analyses were downloaded from that website for further compilation and interpretation.

Simplified explanations and examples of least-squares spectral analysis can be found in [41, 47], based on individual and composited sine waves. A brief example is included here in Sect. 4.2.

2.2 Wavelet transforms

This paper used a continuous, complex-valued Morlet wavelet, with a method [27] for handling the unevenly spaced and irregular minesite-monitoring data that invalidate most other wavelet software. This limitation to a continuous, complex-valued Morlet wavelet is not considered a disadvantage here for three primary reasons.

First, the full-scale time series shown in this paper resemble composited sine curves, as illustrated below, with radio waves and ocean waves as simple analogues. The Morlet wavelet is a sine curve encased in a Gaussian window. Thus, it is a suitable choice for these types of time series.

Second, this Morlet wavelet is continuous. The other basic category of wavelet is "discrete", which is less intensive computationally and results in a smaller data file. This is why discrete wavelets are more popular and are used for image compression, like JPEG2000. However, the resulting discrete-wavelet diagrams can be heavily "pixelated" or "blocky". As shown in Sect. 7 of this paper, it was important to use a continuous wavelet here to see the relatively "thin" wavelengths and time periods with significant wavelet power, which would be obscured by "pixelated" discrete wavelets.

Third, this complex-valued wavelet provided information on periodicity because it provided both real and imaginary parts of the complex-valued coefficients. The real parts were used as an indication of periodicity in this study.

For each wavelet diagram in this study, many tens of thousands of complex-valued wavelet coefficients were obtained using Package "mvcwt" in the R Language version 3.4.4 [59], with the Rstudio 1.1.447 user interface [60]. Results were plotted on 5000-by-5000 grids, using a base-10 logarithm for wavelet scale. Wavelet scale is generally equivalent to wavelength in this case, due to the selected Morlet wavelet. Edge effects, scale leakage, and "detection limits" were included in the interpretations. However, statistical significance was dismissed at this time so that fractal patterns could be evaluated (Sect. 4.6).

The basic difference between spectral analysis and wavelet transforms is that spectral analysis looks at the entire monitoring interval, such as 10 years, as one unit. On the other hand, wavelet transforms look at consecutive shorter portions of the monitoring interval, albeit with significantly fewer datapoints in the shorter portions. The mathematical reality is more complex than this simplified explanation [41].

Simplified explanations and examples of wavelet transforms can be found in [41], based on individual and composited sine waves. A brief example is included here in Sect. 4.2.

Future work (Sect. 8) on these databases will use alternative wavelet techniques for confirmation and refinement of these initial interpretations. These techniques will include least-squares wavelet analyses that can accommodate unevenly spaced data [20–23].

3 Minesite-drainage databases

In order to reliably evaluate long-wavelength and short-wavelength dynamics in full-scale minesite chemistries and flows, databases must contain high-frequency measurements, such as hourly or daily, continuing for years to decades. Such databases are rare.

Table 1 Summary of the three minesite-drainage databases

Minesite and deposit type	Number of primary monitoring stations	Maximum duration (years)[a]	Approximate number of datapoints at each station[b]
#1—Island Copper *Porphyry Cu ± Mo ± Au*	9	15	Hundreds to tens of thousands
#2—Equity Silver *Subvolcanic Cu–Ag–Au (As–Sb)*	8	23	Hundreds to thousands
#3—Bell and Granisle[c] *Porphyry Cu ± Mo ± Au*	15	31	Many dozens to hundreds

References for these databases are given in the text

[a]Duration of monitoring varies among the stations; the longest duration is given here

[b]The numbers of datapoints (measurements of aqueous concentrations and flows) vary widely with station and chemical element

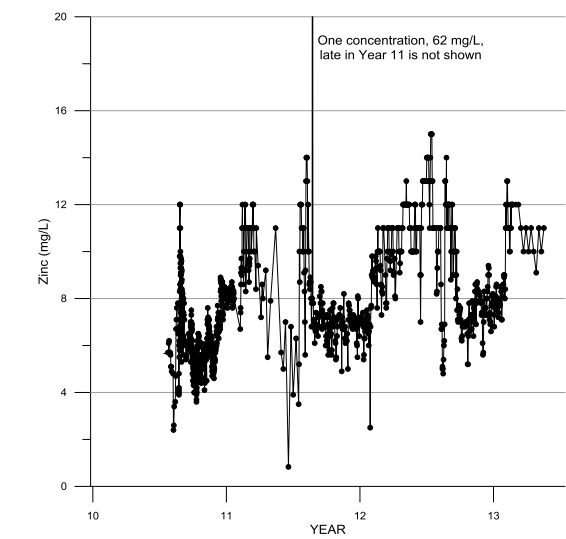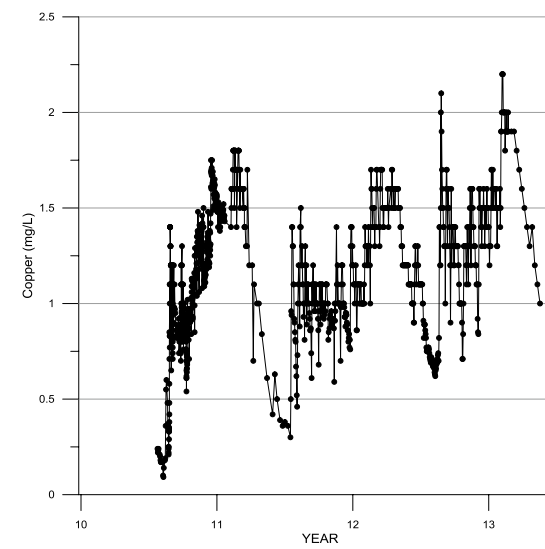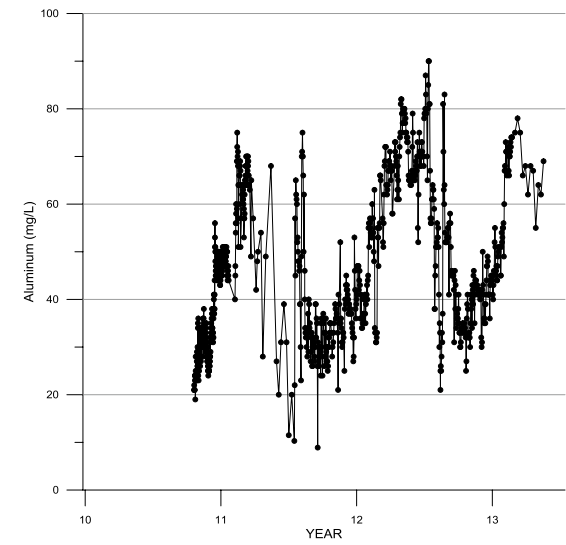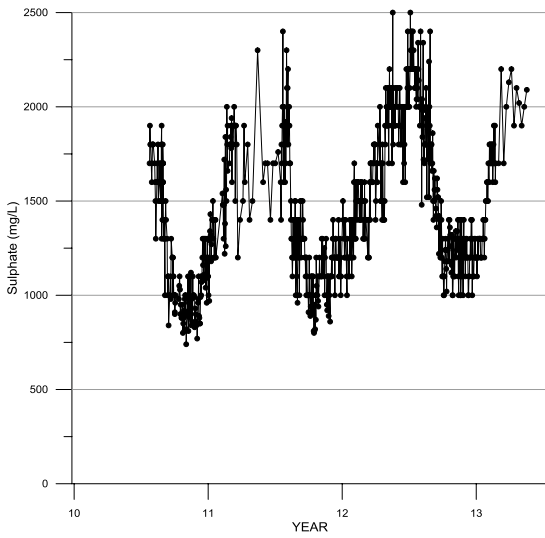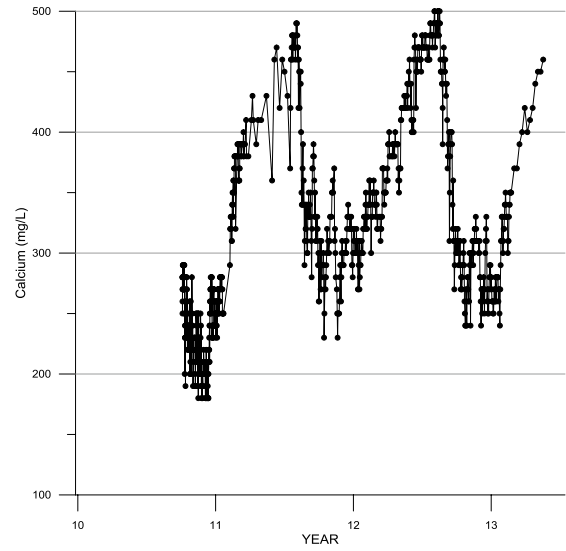[c]This database consists of two separate minesites located about 5 km apart
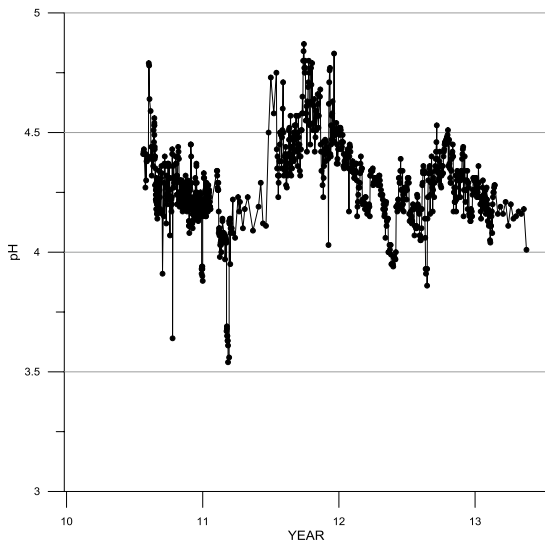
◄**Fig. 4** Examples of temporal trends in full-scale acidic minesite-drainage chemistry at Minesite 1, spanning a few years, based on near-instantaneous samples from one monitoring location, collected as frequently as every four hours. *Note* y-axis is arithmetic, not logarithmic like Fig. 5

Instead, minesite-drainage databases are typically based on:

- annual monitoring programs for regulatory or governmental purposes with weekly, monthly, or quarterly sampling,
- occasional synoptic sampling, each providing a single "snapshot" in time of drainage chemistry around and downstream of a minesite or one of its components, and
- intensive high-frequency (e.g. hourly or daily) sampling at one location for only a few days or weeks.

Such monitoring databases are not sufficient here.

For this paper, three databases were chosen that contained (1) high-frequency measurements as frequently as every 15 min continuing for years (inevitably with some gaps) and/or (2) less frequent measurements continuing for decades (inevitably with some gaps). These three databases have been well documented in the past [see references and data in 8, 9, 41, 46, 53]. They are summarized in Table 1 with selected time series shown in Figs. 4 and 5.

As part of their past documentation, these three databases have been checked for erroneous outliers that were corrected or deleted, as explained in those past references. Nevertheless, it is worth repeating here, because the identification and the deletion of outliers in each database can strongly determine the subsequent interpretations.

For example, if a database of acid rock drainage (ARD) contains pH values mostly around 3.0, with one value of 0.5, then the 0.5 is a geochemically unreasonable value and can be deleted. However, if pH values are around 3.0, with one value of 2.2, should that outlier be deleted?

The argument here is "no", because it may not truly be an erroneous outlier. Instead, it could be a rare and extreme value [e.g. 6] that is an important part of a real trend like a fractal or chaos (Sect. 4.3). If that value is later found to be part of a fractal pattern, then it is likely not an erroneous outlier. If such a value were initially deleted as an erroneous outlier before interpretations, then a fractal pattern might not be identified. As a result, only very apparent errors were deleted from these databases as outliers, and the resulting interpretations in this paper (Sects. 6, 7) show this careful approach was justified.

4 Conceptual development

As explained in Sect. 2, there are books written on spectral analysis and wavelet transforms, so a comprehensive summary is not possible here. Instead, the focus here is on the concepts and issues particularly applicable to time series of full-scale minesite drainage. The most important concepts are signal filtering and generation (Sect. 4.1), periodicity (Sect. 4.2), time-series fractals (Sect. 4.3), "1-over-f" (1/f) fractal slopes from spectral analysis (Sect. 4.4), emergence (Sect. 4.5), and statistical significance (Sect. 4.6).

4.1 Signal filtering and generation

Minesite components, such as open pits, waste-rock piles, and tailings impoundments, can be envisioned as signal filters and generators. For an "input signal" such as water infiltration (left side of Fig. 6), a minesite component can filter this input signal and create a different "output signal" in the outflowing drainage water. When no input signal exists, such as undetectable aqueous concentrations of metals in the infiltration (right side of Fig. 6), then a component can create a detectable output signal.

To be clear, this paper remarkably shows that full-scale minesite components act as reliable and persistent signal filters and generators. This happens despite these components containing more than 10^6 tonnes of large, relatively young, highly disturbed, human-constructed masses of blasted and broken rock. This forms the basis of the following subsections.

4.2 Periodicity

A fundamental concept here is that, through time, minesite-drainage flows and chemistries created by the signal filters and generators (Sect. 4.1) can display periodicity, that is, they generally repeat themselves in patterns. The most obvious pattern is seasonal values that approximately repeat every year, resulting in time-series periodicity with a wavelength of 1 year. Such annual periodicities, or oscillations, often display large seasonal variations (amplitudes) in values.

Minesite drainage can also display short-wavelength periodicities such as 1 day [e.g. 13, 54–56, 63], or even shorter and longer wavelengths. Daily amplitudes (variabilities) are typically much less than annual amplitudes, as illustrated below.

The complexity here is that time series of minesite drainages can be composites of many periodicities of varying wavelengths and amplitudes. Approximately 200 years ago, Fourier analysis was first developed to identify and separate each significant wavelength and its amplitude
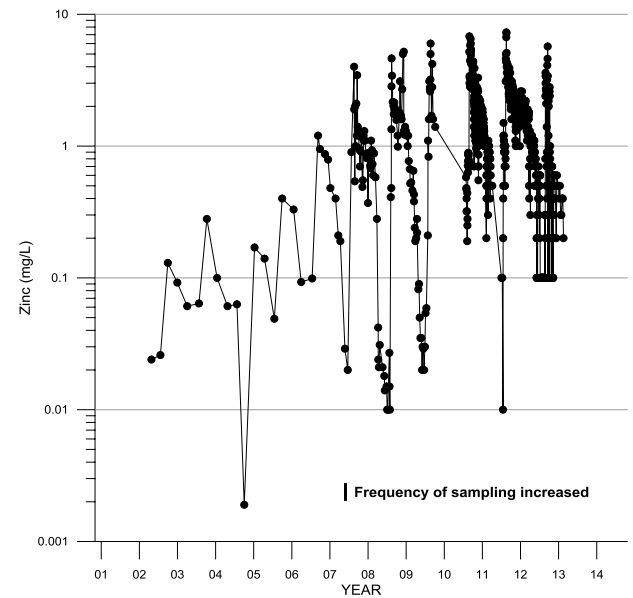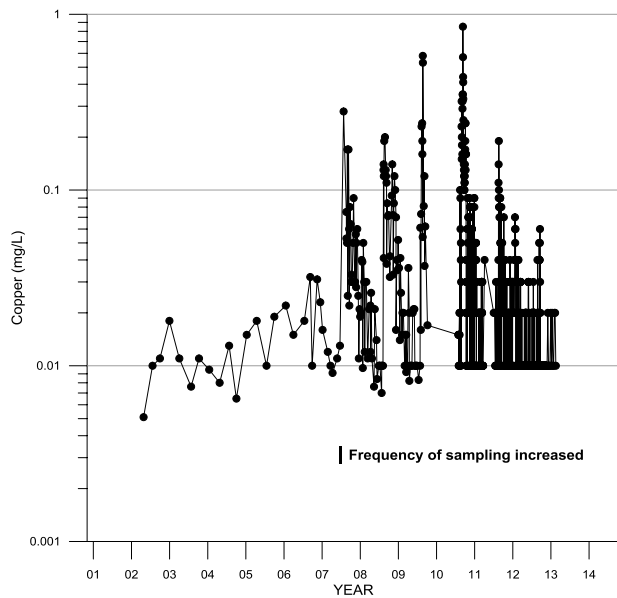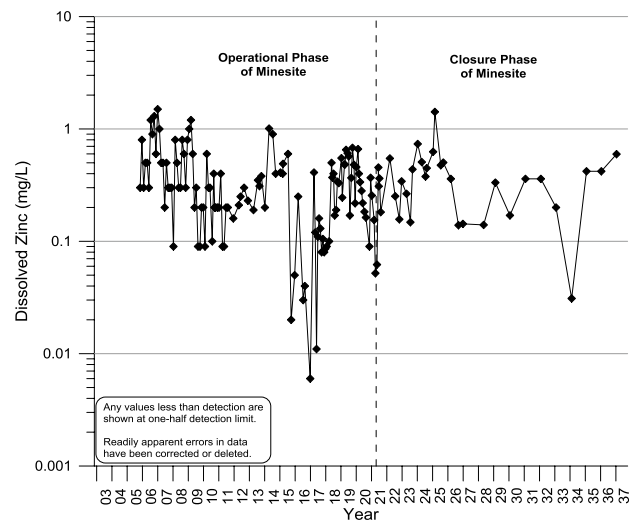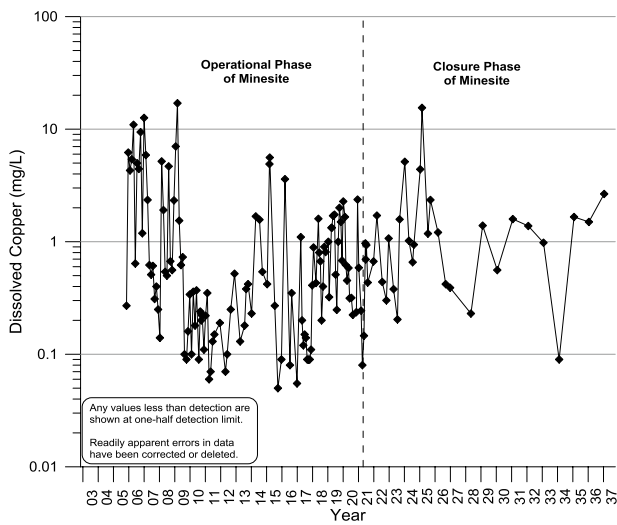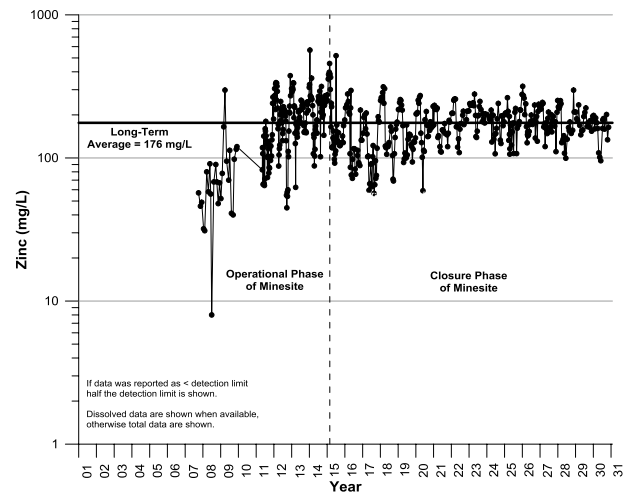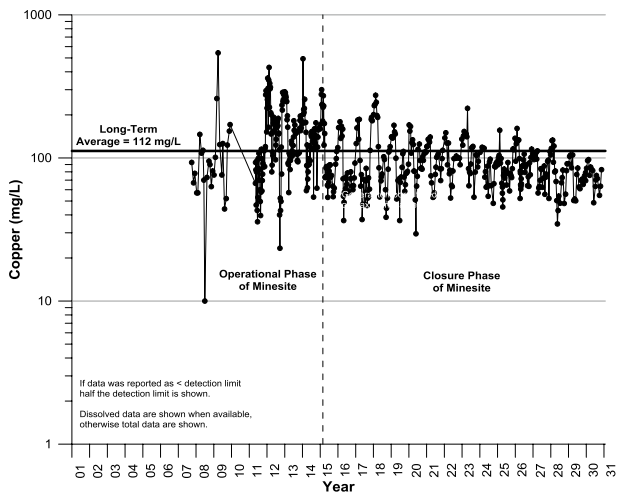
◀**Fig. 5** Examples of temporal trends in full-scale minesite-drainage chemistry, spanning at least one decade, based on near-instantaneous samples. Upper row (Minesite 1): acidic drainage; middle (Minesite 2) and lower (Minesite 3) rows: near-neutral drainage. Left column: copper; right column: zinc. *Note* y-axis is log10 (concentration)

from the others. Spectral analysis, used here (Sect. 2.1), is a generalization of Fourier analysis. Additionally, unlike spectral analysis, wavelet transforms used here (Sect. 2.2) look at discrete portions of the time series.

As simplified examples, the upper left side of Fig. 7 shows a time series of sine waves with composited wavelengths of 1 year, 1 month, and 1 week. The amplitudes (ranges of highs to lows) decrease with decreasing wavelength, and these amplitudes were carefully selected here to create a certain spectral slope across the major peaks during spectral analysis. As expected, spectral analysis produced a log–log periodogram with three major peaks at these wavelengths (right side of Fig. 7). The spectral slope connecting the three major peaks has a value of 1.0 (also called a 1-over-f slope, see Sect. 4.4). The smaller peaks between the major ones are mathematical artefacts caused by the input data at discrete times appearing as a type of Dirac comb to the algorithm.

Also, as expected, the corresponding wavelet transform created three horizontal bands of relatively strong wavelet power (darker colour) at the three wavelengths

Signal Filter and Signal Generator
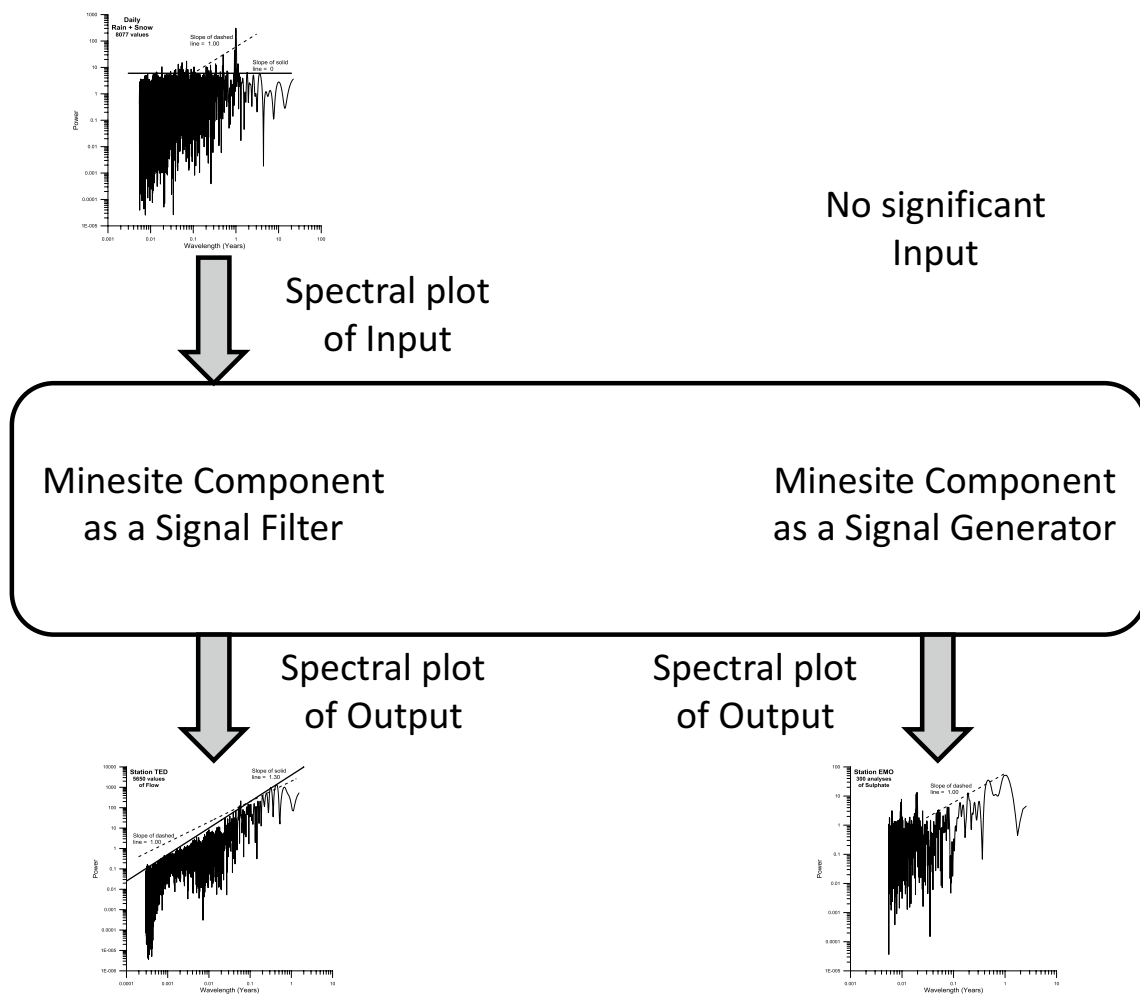
Fig. 6 For minesite drainage, signal filtering by a minesite component causes the spectral slope of the input signal (shown on the upper left as primarily random precipitation and infiltration with a spectral slope of zero) to change in the output signal (shown on the lower left as outflowing drainage water with a spectral slope of 1.3); signal generation creates a spectral slope in the output signal (shown on the lower right as aqueous concentrations in outflowing drainage with a spectral slope of 1.0) where input concentrations were undetectable

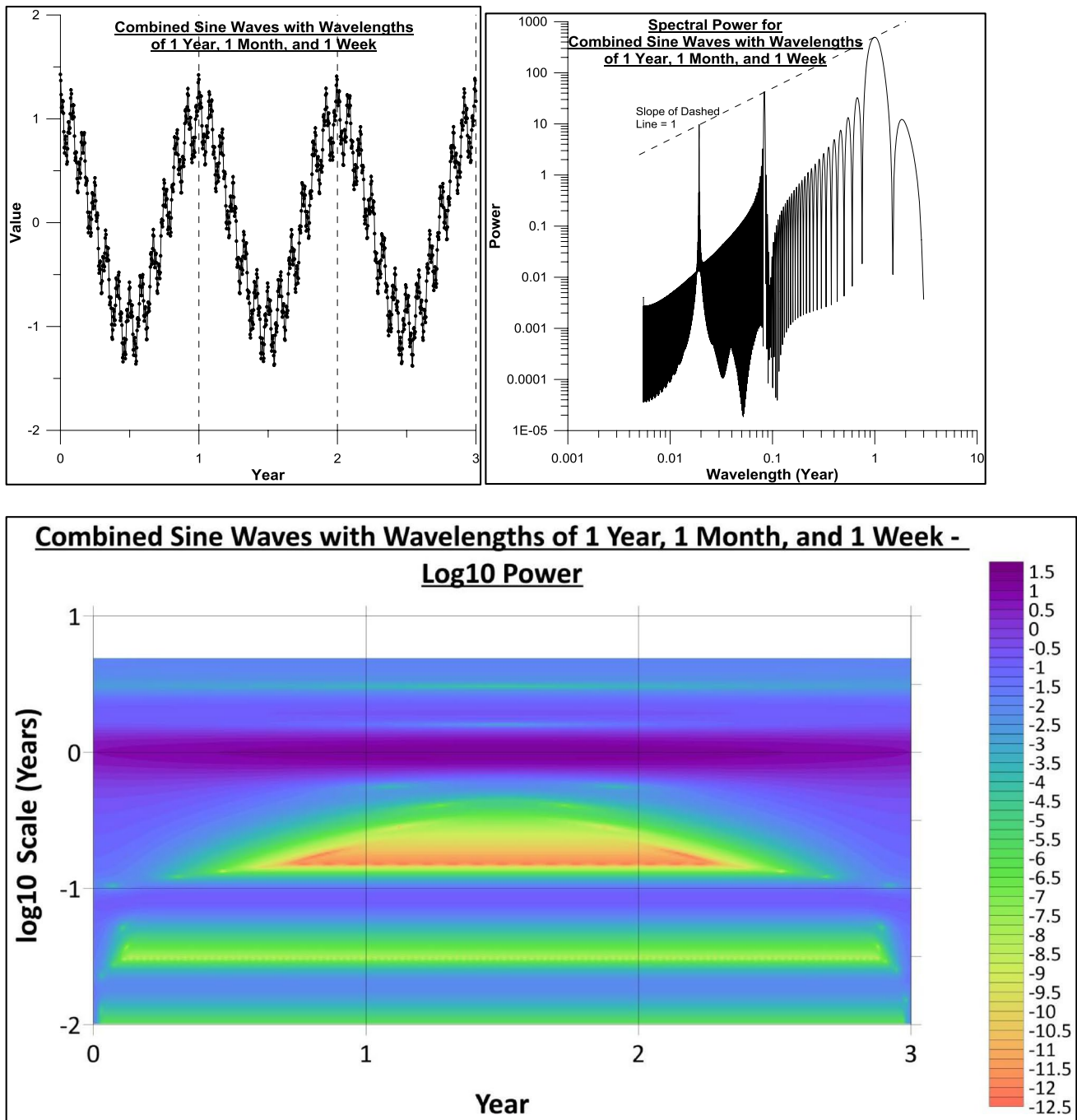**Fig. 7** (Upper left) three-year time series of combined sine waves with wavelengths of 1 year, 1 month, and 1 week, and with amplitude decreasing with decreasing wavelength; (upper right) log–log periodogram from spectral analyses of this time series showing major peaks at 1 year, 1 month (0.083 years), and 1 week (0.02 years) with a spectral slope of 1.0 connecting the peaks; (bot-tom) wavelet transform of this time series showing dark-coloured horizontal bands of strong wavelet power persisting for all three years at scales (wavelengths) of 1 year (log10 0.0), 1 month (log10 −1.1 years), and 1 week (log10 −1.7 years), and power decreasing (lighter colour) with decreasing wavelength of the three bands

or scales (bottom of Fig. 7): at 1 year (log10 0.0), 1 month (log10 − 1.1 years), and 1 week (log10 − 1.7 years). Because the time series was consistent for all three years in this example (upper left of Fig. 7), the horizontal wavelet bands persist for all three years. However, if one wavelength was missing in Year 3 in the time series, for example, then the horizontal band for that wavelength would also be missing in Year 3 from the wavelet transform.

Fig. 8 The one-dimensional fractal Weierstrass equation adjusted to a dominant one-year wavelength, showing the composite of the first five terms of the infinite series; compare with Figs. 4 and 5
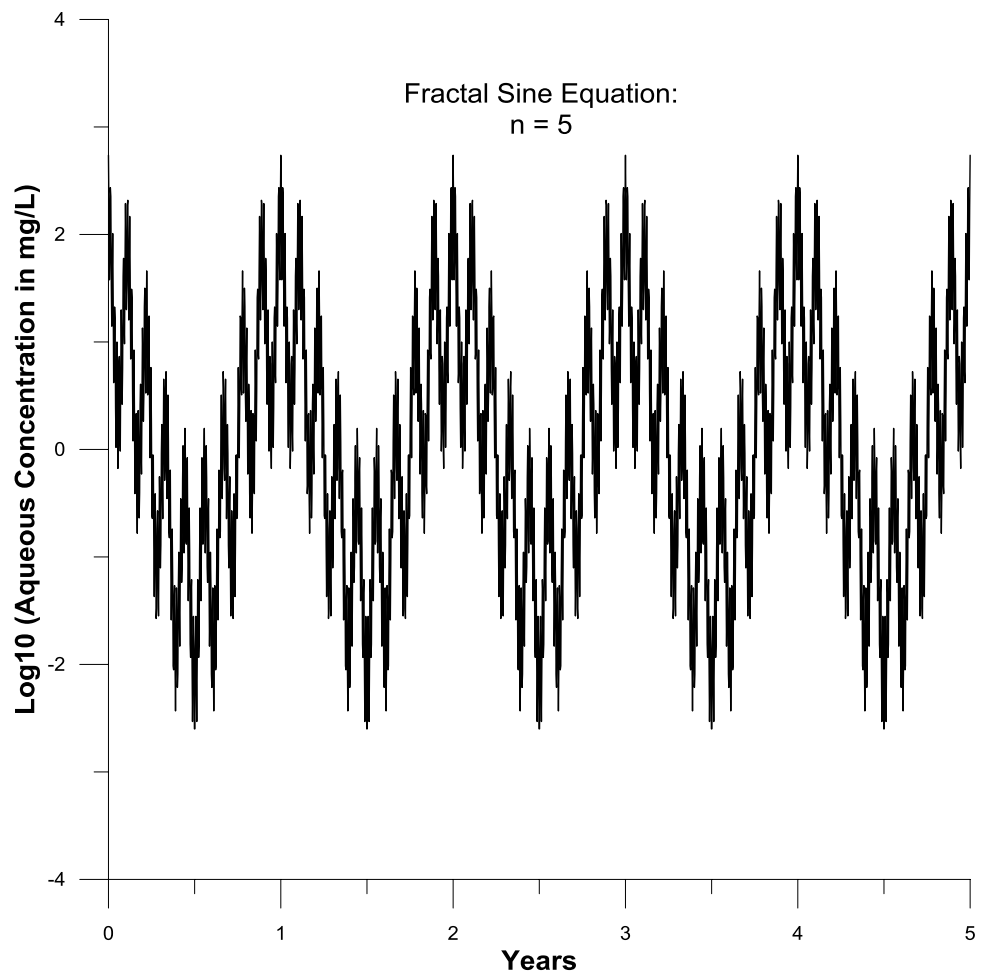
4.3 Time-series fractals

The identification and separation of composite periodicities and amplitudes can reveal patterns, which leads to the next important concept: fractals. In its basic form, a fractal is a repeating pattern that looks the same as one zooms in closer and closer to the object. Terms associated with fractals include "self-similar at various scales" and "scale independent".

Fractals in two and three spatial dimensions are well documented and beautiful in appearance. An Internet search will show many, such as the Mandelbrot set.

Less well documented (and still beautiful in this author's opinion) are "one-dimensional" line fractals, particularly for time series. This is an important distinction. Time-series fractals cannot reverse like spatial fractals, but can only move forward unidirectionally in time due to the Arrow of Time.

In reality, fractals are fractionally dimensional. As a result, a "one-dimensional" fractal drawn as a line actually has a fractal dimension between 1.0 and 2.0, but this complexity is ignored in the simplistic terminology used here.

Around the mid-1800s, the equation for these "one-dimensional" line fractals (long before they were called "fractals") was published. It is now known as the Weierstrass equation or function and is a summation of an infinite series of cosine or sine waves of varying wavelengths and amplitudes. A plot of the first five terms of this series is shown in Fig. 8. This plot resembles Figs. 4 and 5, suggesting they are also fractal, which turns out to be the case (Sect. 6).

Spectral analysis of time series is a primary method for identifying a fractal time series, where a line connecting the major spectral peaks plot as a straight line on a log–log plot of wavelength (or its inverse, frequency) and spectral power (proportional to the square of the amplitude). This is also known as "power laws".

For minesite drainage (Sect. 6) and non-mining-related drainages (Sect. 5), the dominate range of spectral slope is from zero (with zero slope signifying random or "white noise") to two (signifying random walk or "red noise"). One important slope lies at 1.0 (upper right of Fig. 7), also called "1-over-f" slope and "pink noise"

Fig. 9 Overlap of individual process having partial spectral slopes of zero and two, resulting in a single 1-over-f slope as one potential explanation (adapted from [47])
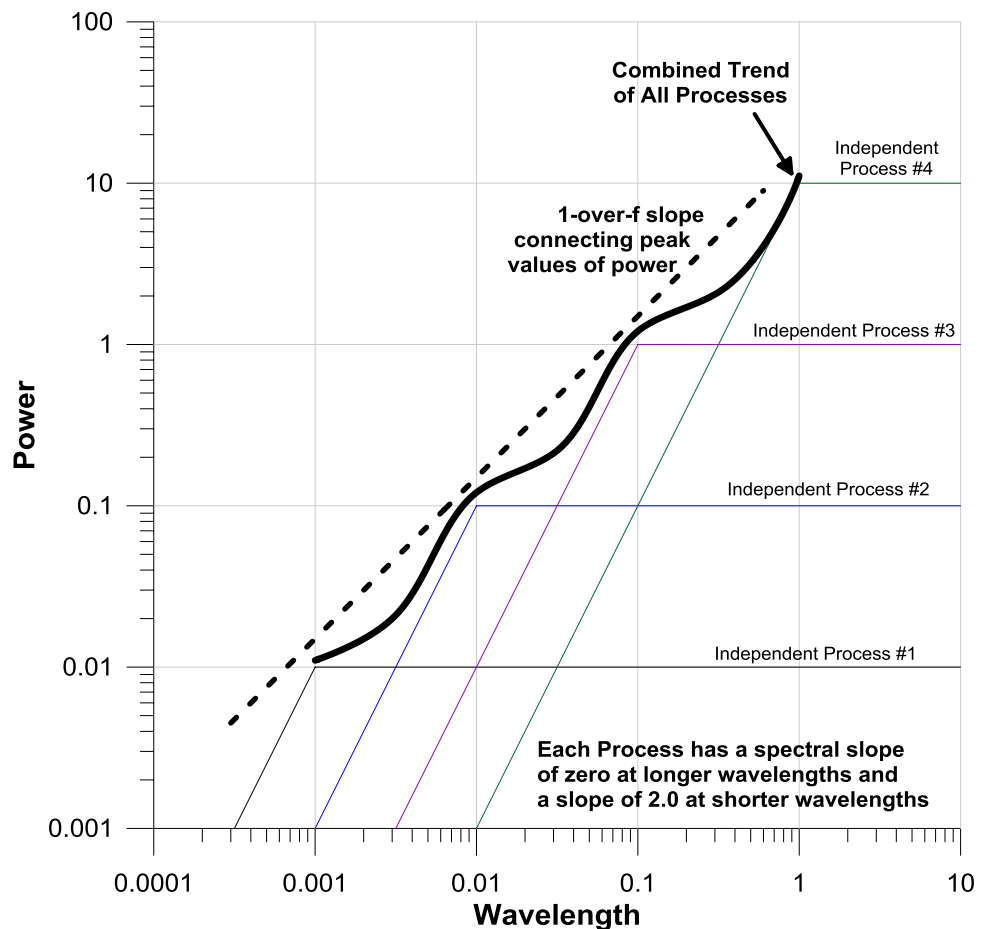
(Sect. 4.4). Other slopes can be seen at the bottom of Fig. 6 and in Sect. 6.

4.4 1-over-f spectral slopes

As explained in Sect. 4.3, spectral slopes in mining- and non-mining full-scale catchments often lie between zero (random) and two (random walk). A slope of one is so special that it is often referred to as "1-over-f" (1/f) or "pink noise".

This 1-over-f slope represents a dividing line between various mathematical and statistical features such as fractal models, statistical stationarity, and change in statistical variance as scale and wavelength change [25]. Thus, a 1-over-f slope, as a dividing line, is theoretically not stable and should not persist.

Remarkably, 1-over-f slopes have been documented in many sciences and arts. Here is a partial list of their occurrence [compiled in 47]: earthquakes described through the Gutenberg–Richter law, climatic temperature and precipitation, highway traffic flow, river flow, tides, heart beats, neural activity, biologic evolution, solar flares,

psychological models of mental states, light from quasars, electrical current in solid-state devices, epidemics, variations in musical styles, insulin uptake by diabetics, economic trends, forest fires, application of automotive paint, and cavitation in pumps.

This glaring discrepancy has led researchers to say:

> We became obsessed with the origin of the mysterious phenomenon of 1/f noise, or more appropriately, the 1/f 'signal' that is emitted by numerous sources on earth and elsewhere in the universe. [6]
> ... the ubiquity of 1/f noise is one of the oldest puzzles of contemporary physics and of science in general. [67]

Explanations for 1-over-f slopes range from coincidence, to overlap of individual processes and filters (e.g. Fig. 9), to self-organized criticality (SOC). SOC was summarized by [6]:

> ... complex behavior in nature reflects the tendency of large systems with many components to evolve into a poised, 'critical', state, way out of balance, where minor disturbances may lead to events,
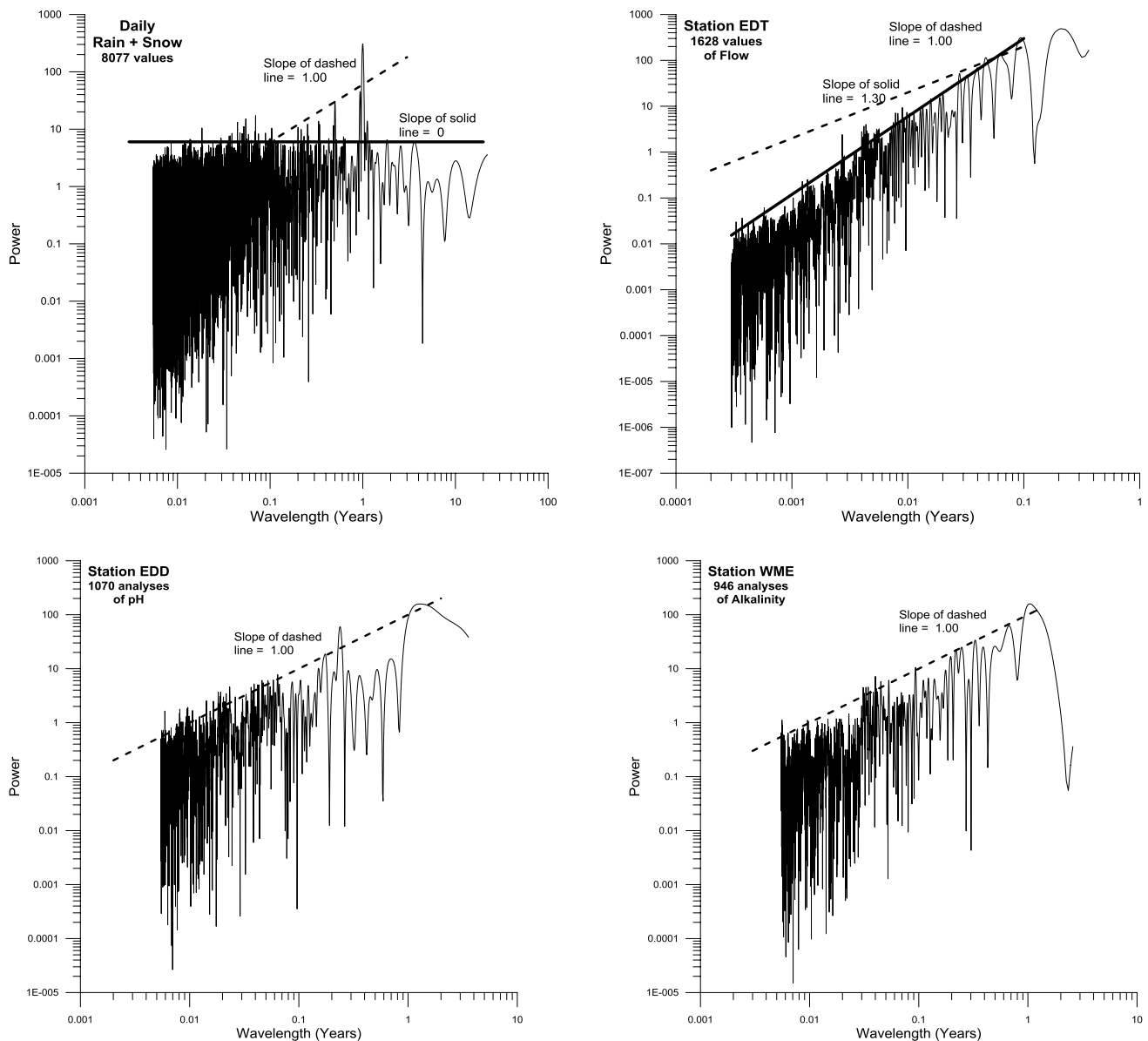
Fig. 10 Examples of spectral slopes from Minesite 1

called avalanches, of all sizes. Most of the changes take place through catastrophic events rather than by following a smooth gradual path. The evolution of this very delicate state occurs without design from any outside agent.

As explained in Sect. 3, the identification and deletion of outliers in the mining databases used here were done carefully with the intent of not deleting the critical "catastrophic events". This is also why statistical significance was not included here in spectral analysis and wavelet transforms (Sect. 4.6).

4.5 Emergence

As emphasized several times, this paper focusses on drainages from full-scale minesite components to understand the real environmental dynamics and the potential effects on downstream ecosystems. This is due to a concern over emergence. Smaller-scale laboratory-based and on-site test work might be more thorough with less data gaps than full-scale databases (Sect. 3), but the smaller-scale test work would not likely characterize the full scale reliably.

Emergence can generally be defined as the appearance of distinct patterns or properties as scale increases, often attributed to self-organization in complex systems.
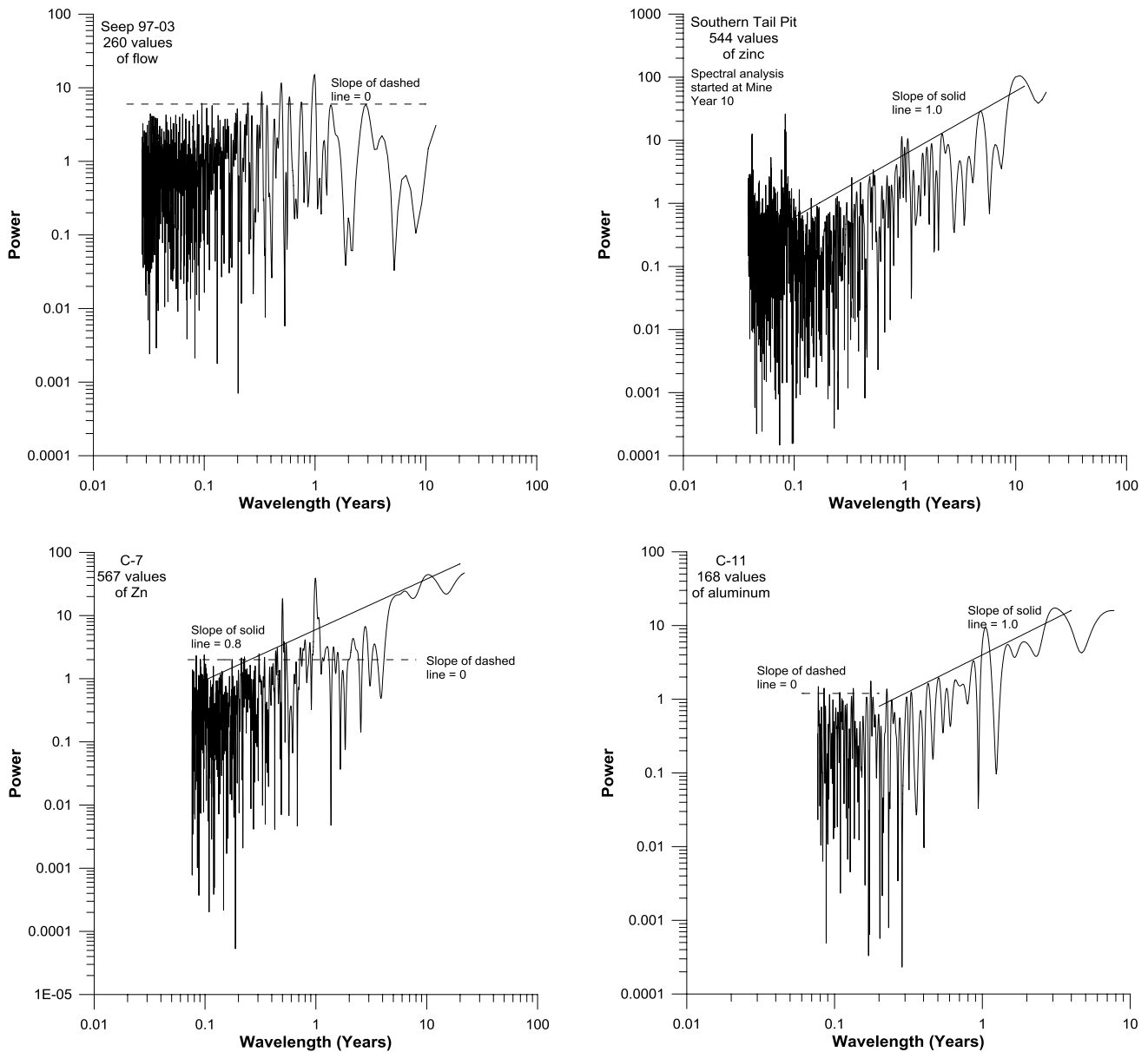
Fig. 11 Examples of spectral slopes from Minesite 2

For example, "the whole becomes not merely more, but very different from the sum of its parts" [3]. Also, "Meaning is found in the effective theories or models developed to explain a particular phenomenon at its appropriate level [scale] of description." [16]

In fact, for minesite drainage, small-scale laboratory results were found to be complex even at the scale of 1 kg [46]. However, this small-scale complexity was notably different than full-scale complexity. Thus, this paper focusses on drainage from full-scale minesite components.

4.6 Statistical significance

Common statistical significance, for example based on *t* tests, *Z* tests, and *F* test, plays important roles in statistical analysis, but should not always be applied rigorously. Statistical significance can be assessed for spectral analysis and wavelet transforms in more complex ways [e.g. 65]. However, others argue successfully why statistical significance should sometimes be discounted or ignored [e.g. 68–70].

This study ignores statistical significance for a simple reason. Fractal distributions are common, even universal,
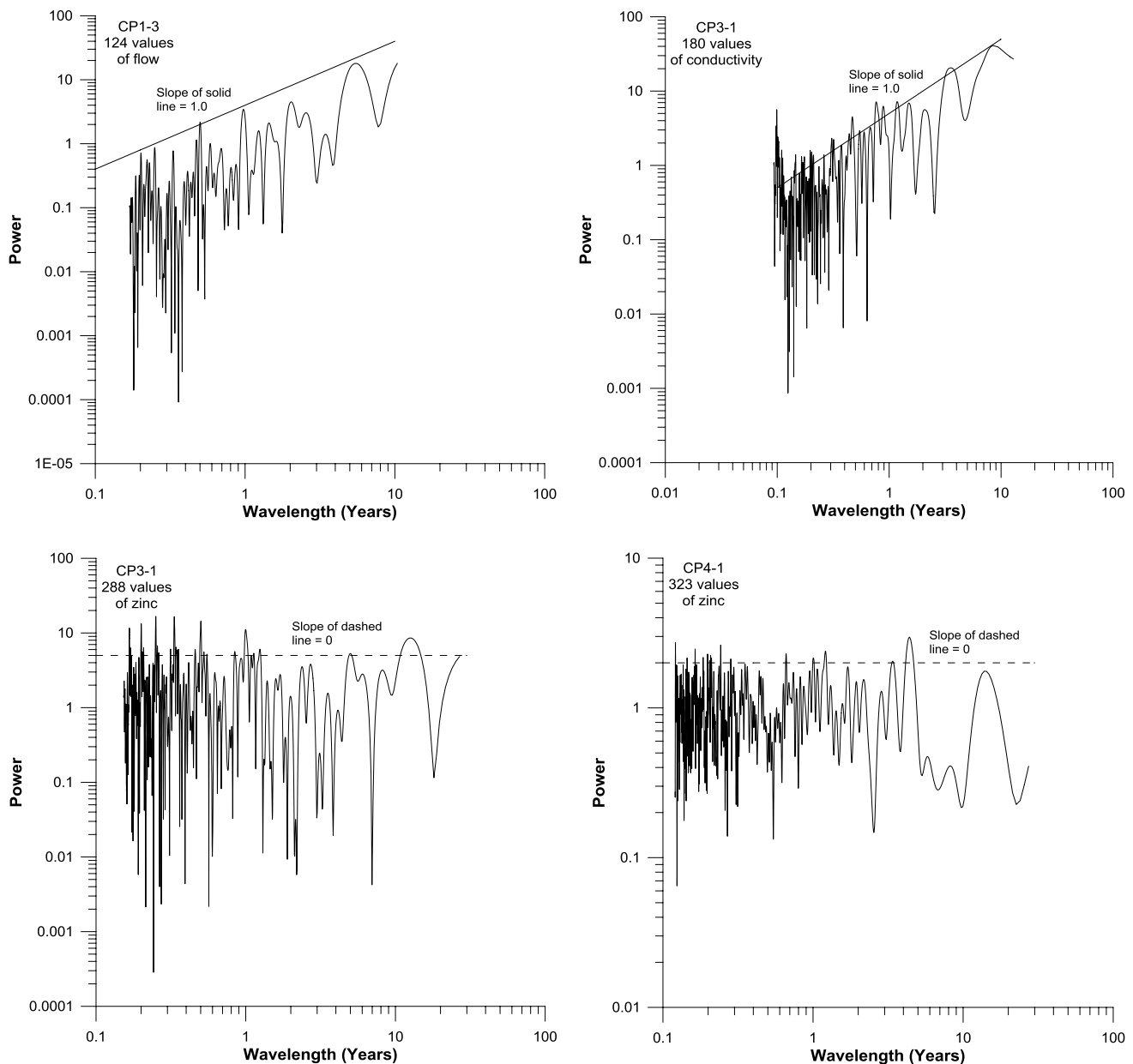
Fig. 12 Examples of spectral slopes from Minesite 3

in drainages from mining-related and non-mining-related catchments (discussed further in Sects. 5, 6, 7). This means that very weak power peaks, which would not be statistically significant by standard tests, are important parts of fractal interpretations. For example, the dismissal of minor peaks starting in Fig. 7 (top right) and subsequent figures, that are only 1% to 10% of the maximum peak and thus not likely statistically significant in a general sense, would preclude the assessment of 1-over-f slopes.

Statistical significance has its place. However, that does not necessarily include fractal time series of drainages from full-scale minesite components unless performed

with careful knowledge of the fractal pattern. This paper is the first step in delineating those fractal patterns in minesite drainage, so statistical significance may become important later (Sect. 8).

5 Previous work

For minesite drainage exclusively, years of searching paper documents and the Internet have failed to yield any previous full-scale studies focussed on spectral analysis and
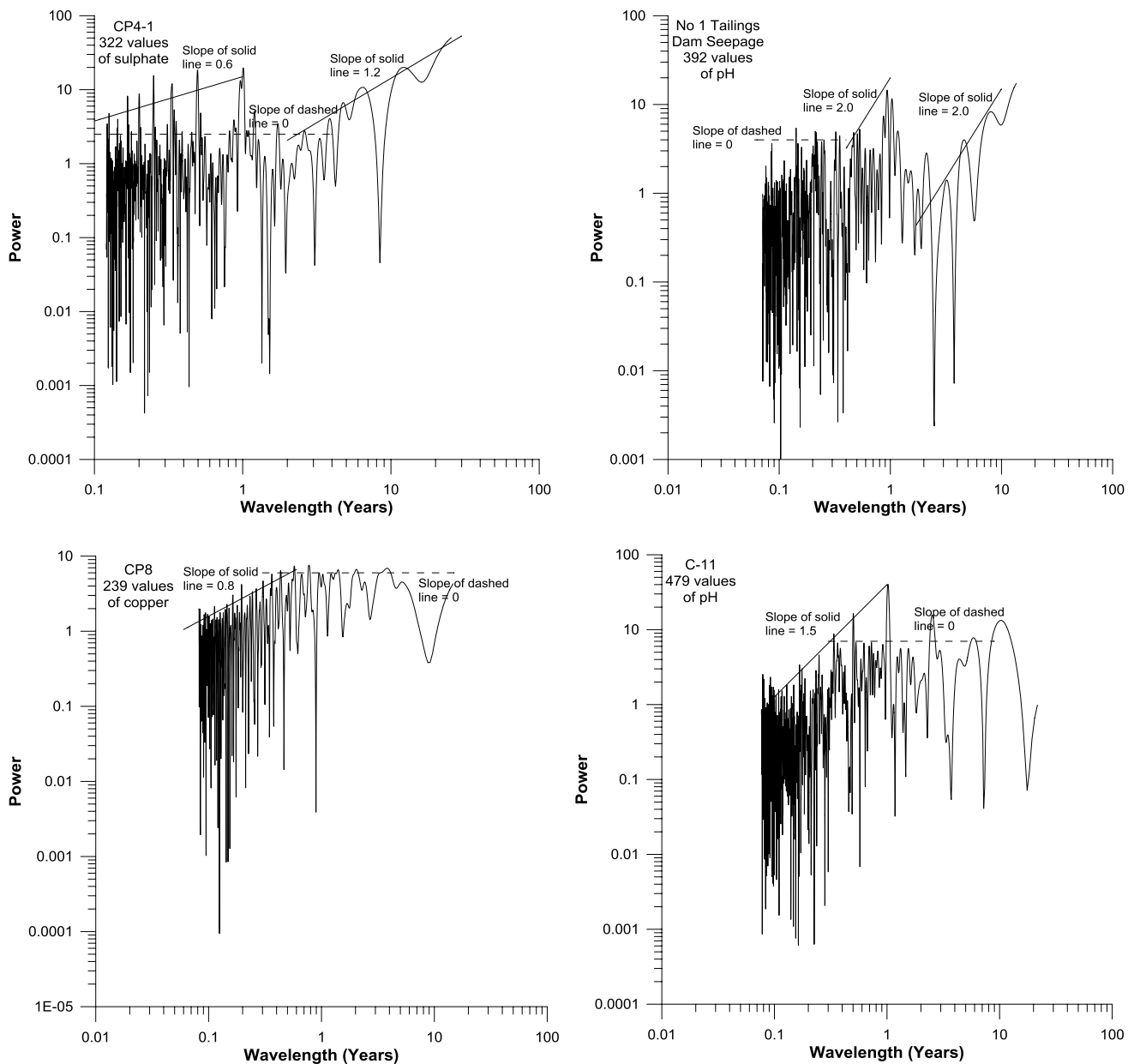
Fig. 13 Examples of spectral slope anomalies from the three databases, showing discontinuities, ranges of wavelengths with two slopes of spectral power, and changes in spectral slope at specific wavelengths. This can be explained with simple simulations, as shown in Fig. 14, and thus reflect real full-scale processes active in minesite components

wavelet transforms, other than those related to this paper [41, 45–49]. Therefore, this is a novel application of both spectral analysis and wavelet transforms.

However, for full-scale non-mining catchments, excellent work with spectral analysis was conducted, primarily between the years of 2000 and 2013, by Kirchner and colleagues [e.g. 4, 18, 24, 31–37]. For these non-mining catchments, spectral analysis of high-frequency analyses, including up to dozens of chemical elements and parameters, showed that fractal 1-over-f slopes were so common as to sometimes be called "universal". However, these past papers did not include wavelet transforms which provide additional information on shorter intervals (e.g. Fig. 7).

Therefore, 100% of every non-mining catchment with sufficient high-frequency monitoring over a long period of time showed at least some fractal 1-over-f slopes. As presented in this paper (Sects. 6, 7), this 100% occurrence in high-frequency monitoring also applies to the few detailed databases for full-scale minesite drainage (Sect. 3).

Where this long-term high-frequency monitoring is lacking in other studies, the data are treated stochastically or deterministically, yet both could be incorrect. There
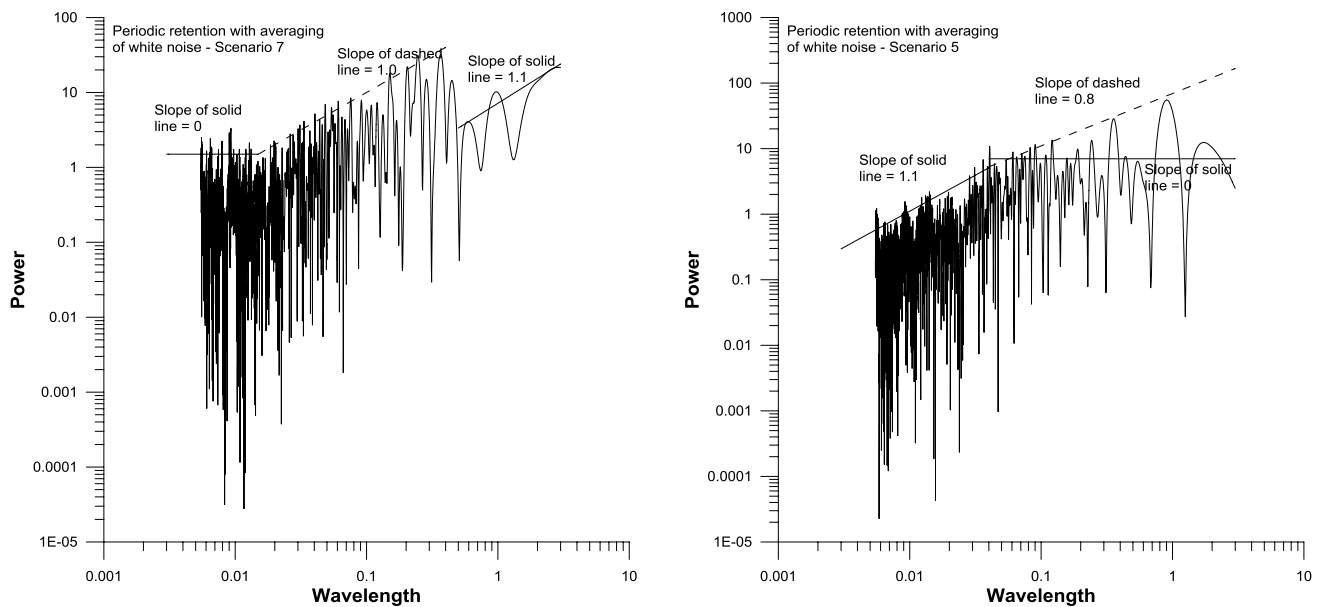
Fig. 14 Spectral analyses of simple spreadsheet models with major variations in seasonal upstream water retention by ponds or aquitards, explaining anomalies observed for full-scale drainage in Fig. 13 (adapted from [49])

would be no way to be certain without high-frequency monitoring.

6 Results of spectral analyses

Most spectral slopes for all three databases (Table 1) generally followed straight lines on the log–log plots over at least two orders of magnitude of wavelength. This indicated their practical fractal nature. Some of these spectral slopes are shown in Figs. 10, 11, and 12, and hundreds more can be found in the appendices of [47].

Spectral analyses of drainages from non-mining catchments found 1-over-f fractal spectral slopes so common as to be "universal" in some catchments (Sect. 5). Spectral analyses of the three full-scale drainage databases examined here (Sect. 3) showed that 1-over-f slopes were indeed common. However, other slopes between zero and approximately 2.5 were also observed for drainage flows and aqueous elements.

The 1-over-f slopes occurred for parameters as diverse as pH, alkalinity, aluminium, zinc, electrical conductivity, and flow (Figs. 10, 11, 12). Nevertheless, 1-over-f slopes could exist for parameters like zinc at some minesites (Fig. 11), but be virtually zero (random) for zinc at others (Fig. 12).

In general, most spectral slopes of aqueous elements at the minesites did not match the corresponding slopes of flow. This further supports the widely observed general independence of full-scale flow and full-scale chemistry from minesite components [e.g. 47, 51, 52].

Electrical conductivity reflects the ionic content of water and is easily and frequently measured in-field with a probe, metre, and data logger. However, at many monitoring stations, conductivity displayed significant spectral differences from those of individual elements. As a result, the high-frequency measurement of this easily measured in-field parameter would not have been a reliable surrogate for laboratory-based chemical analyses, which was also observed in non-mining watersheds (Sect. 5).

At Minesite 2 (Fig. 11), drainage from the waste-rock pile originates as a poorly constrained combination of (1) infiltration through the fine-grained soil cover placed at closure and (2) upwelling groundwater from the upgradient, undisturbed catchment. Because these two sources mix together before exiting the waste rock, they cannot be readily differentiated. However, based on upgradient monitor wells and on variations in infiltration through the cover, the two sources could be distinguished based on the relative spectral power of the two peaks at 0.5 and 1.0 years (e.g. bottom left and bottom right of Fig. 11). In effect, the spectral peaks can sometimes act as a "tracer" of the original water sources.

Of note, spectral analysis assumes the entire record of data repeats consistently. If this is not the case, spectral analysis can yield unusual results that could be considered "errors". However, if such errors are caused by real processes and events, then the erroneous results lead

Minesite closed, and
fine-grained soil/till
cover completed in
Mine Year 14

Scale:
log10 (1 year) = 0
log10 (1 month) = -1.1
log10 (1 week) = -1.7
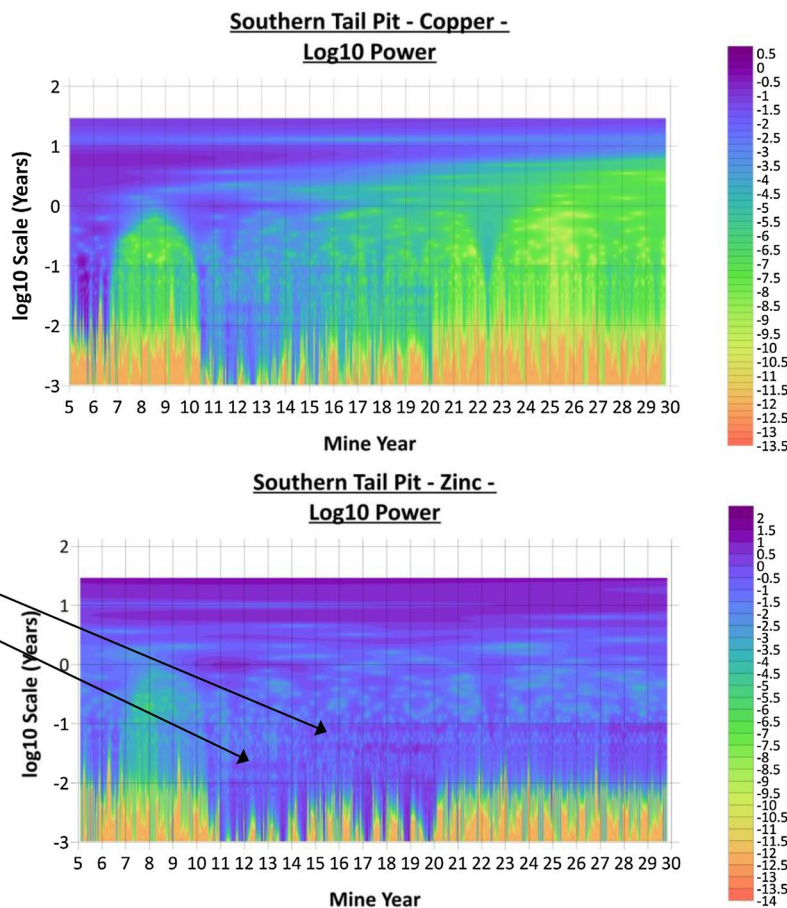log10 (1 day) = -2.6

Fig. 15 Wavelet transforms for aqueous copper (top) and zinc (bottom) at a monitoring station at Minesite 2, showing the periodicity of copper decreasing (lighter colour) across many wavelengths (scales) through the years and the periodicity of zinc increasing (darker colour); orange-coloured spikes rising from the bottom represent missing data at those wavelengths (frequencies)

to important observations about the full-scale minesite component.

For example, Fig. 13 contains anomalous full-scale spectral slopes including (1) discontinuities in spectral power, (2) ranges of wavelengths with two slopes of spectral power, and (3) changes in spectral slope at specific wavelengths. Relatively simple hypothetical simulations (Fig. 14) with spreadsheets show that these anomalies can be explained by major seasonal changes in water retention by upstream ponds or subsurface aquitards [49]. Such upstream retention was not visibly apparent in some full-scale minesite components, yet apparently affected the periodicities of the flows and chemistries of the drainage waters passing those monitoring points.

7 Results of wavelet transforms

With wavelet transforms, the primary interest here is (1) the length of time that periodicity at a particular wavelength persists (horizontal bands of darker colour, bottom of Fig. 7) and (2) a short-lived increase in periodicity across many wavelengths (vertical bands of darker colour). Wavelet transforms for the three full-scale minesite databases (Table 1) are shown in Figs. 15, 16, 17, 18, and 19, and dozens more can be found in the appendices of [41].

Figure 15 contains wavelet transforms for a monitoring station at Minesite 2, with aqueous copper at the top and aqueous zinc at the bottom. The overall temporal trend in copper over the decades was towards lower concentrations, and the top of Fig. 15 shows this was accompanied by a trend towards relatively weaker periodicity (lighter colours) at many wavelengths ("scales"). The orange-coloured spikes at the bottom represent missing data at those wavelengths.
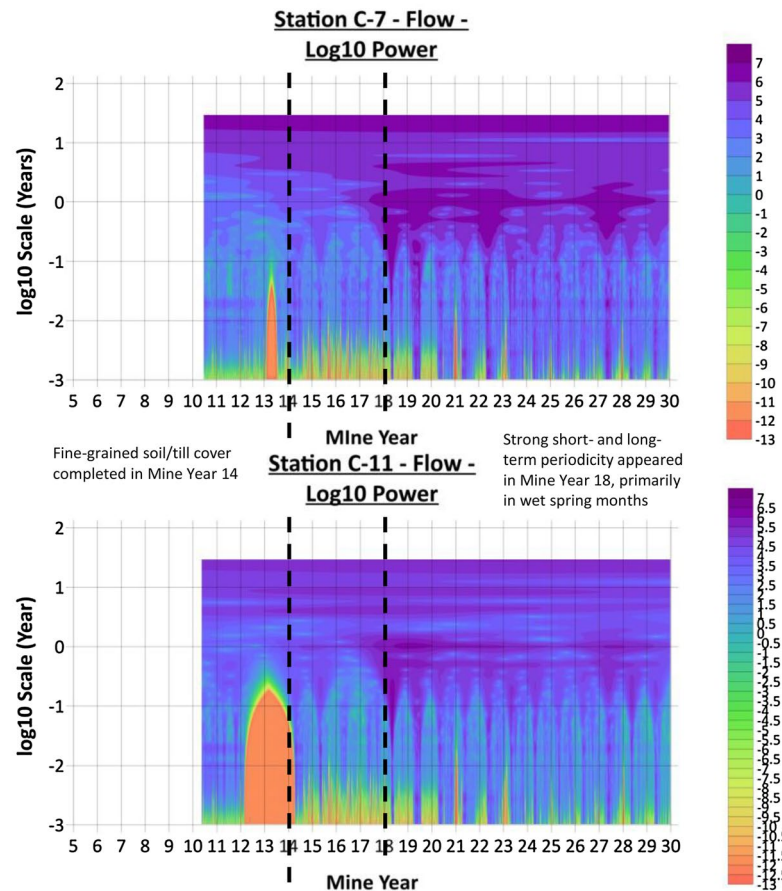
Fig. 16 Wavelet transforms for flow at two monitoring stations at Minesite 2, showing some increase in spring-time periodicity (vertical bands of darker colour) after the minesite closed and a fine-grained soil/till cover was completed in Mine Year 14; significant increases in spring periodicity at many wavelengths at Mine Year 18 suggest high spring infiltration could not be as reduced and as smoothed as before; orange-coloured spikes rising from the bottom represent missing data at those wavelengths (frequencies)

In contrast, zinc generally increased in concentrations through the decades (bottom of Fig. 15). This was accompanied by relatively increasing periodicity at wavelengths such monthly (log10 − 1.1) and half-monthly (log10 − 1.4). Periodicity particularly strengthened around Mine Year 14, when the minesite closed and a fine-grained soil/till cover was completed to reduce infiltration into the waste-rock pile.

At two nearby monitoring stations, the wavelet transform for drainage flow showed some increase in periodicity when the minesite closed in Mine Year 14 (Fig. 16), with slightly darker, vertical bands of strong periodicity typically occurring in very wet spring months (April–June) with high precipitation and snowmelt. Notably, these vertical bands of periodicity in spring increased significantly in strength starting in Mine Year 18. This is interpreted as weakening of the soil cover's ability to reduce and smooth high spring infiltration after only four years of closure. Although flow was affected this way, corresponding aqueous chemistry was not (see pH in Fig. 17), consistent with the common observation of general independence between full-scale flow and its corresponding aqueous chemistry.

For minesites with acid rock drainage (ARD) from sulphide mineral oxidation, near-neutral pH can only be maintained if neutralizing minerals such as calcite ($CaCO_3$) are sufficiently and aggressively reactive and long-lasting. Where this rapid neutralization is occurring, dynamic geochemical tension is high [41] and, like stretched and oscillating coiled springs, periodicities of the relevant aqueous parameters are high at many wavelengths. For example, several persistent horizontal bands of strong periodicity can be seen in wavelet transforms of alkalinity and calcium from a monitoring station at Minesite 1 with ongoing sulphide oxidation and acid generation (Fig. 18).

In full-scale waste-rock piles with more than 10^6 tonnes of broken, blasted, and unsorted rock, long-term ongoing breakage and settling of the rock is typical and expected but rarely documented. Like an "avalanche" [6] (see also Sect. 4.4), an extreme degree of such breaking and settling should occur rarely. From a geochemical
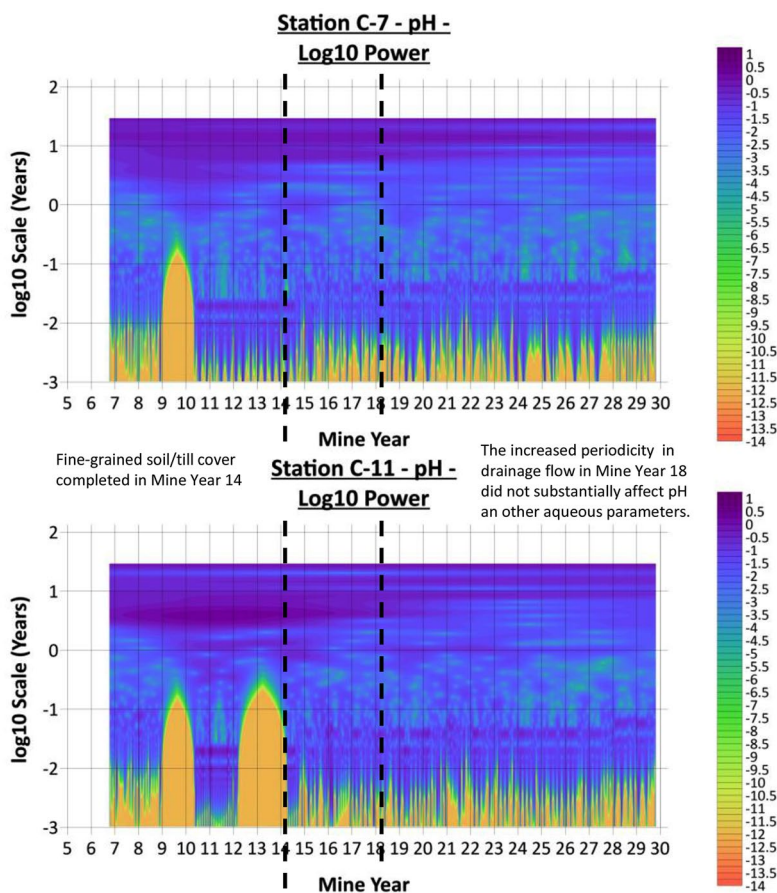
Fig. 17 Wavelet transforms for pH at the same monitoring stations as Fig. 16, but showing no increased spring-time periodicity at Mine Year 18 as shown by drainage flow; orange-coloured spikes rising from the bottom represent missing data at those wavelengths (frequencies)

perspective, this can appear as a short disruption in aqueous concentrations.

At Minesite 2 with strong ARD, such a disruption was detected around Mine Year 21–22 as short-term increases and decreases in concentrations of acidity (Fig. 19). The wavelet transform (right side of Fig. 19) showed this occurred as briefly increased periodicities across a large range of wavelengths (scales) as a thin vertical band of darker colour. Such brief peaks of aqueous concentrations can cause additional adverse impacts on downstream ecosystems (discussed in Sect. 1), but are rarely detected by common, low-frequency monitoring at minesites.

When wavelet transforms from all three databases (Table 1) are compiled, strong periodicities at certain wavelengths are common. The two most common wavelengths are:

- one year (annually), likely due to annual Canadian climate cycles, and,

- one week, which is not a commonly expected strong cycle but detected at all minesites, except at Granisle which did not have the higher-frequency monitoring to check for it.

The physical, geochemical, and/or biological processes accounting for the common weekly periodicity are not readily apparent, but are obviously common and important. This weekly periodicity persisted even through major changes in pH, showing the processes causing weekly oscillations were not strongly pH dependent. Moreover, the persistent periodicity through major changes in pH and aqueous chemistry showed there was no chaotic behaviour, in the mathematical or general sense of the word [40].

Other wavelengths, common for at least two of the minesites, were: equal to or greater than 10 years, approximately 4 months, and half-monthly to monthly. Because Minesite 1 was the only one to have data as frequent as every 15 min, it was the only one to produce strong periodicities at wavelengths of 1 day and less.
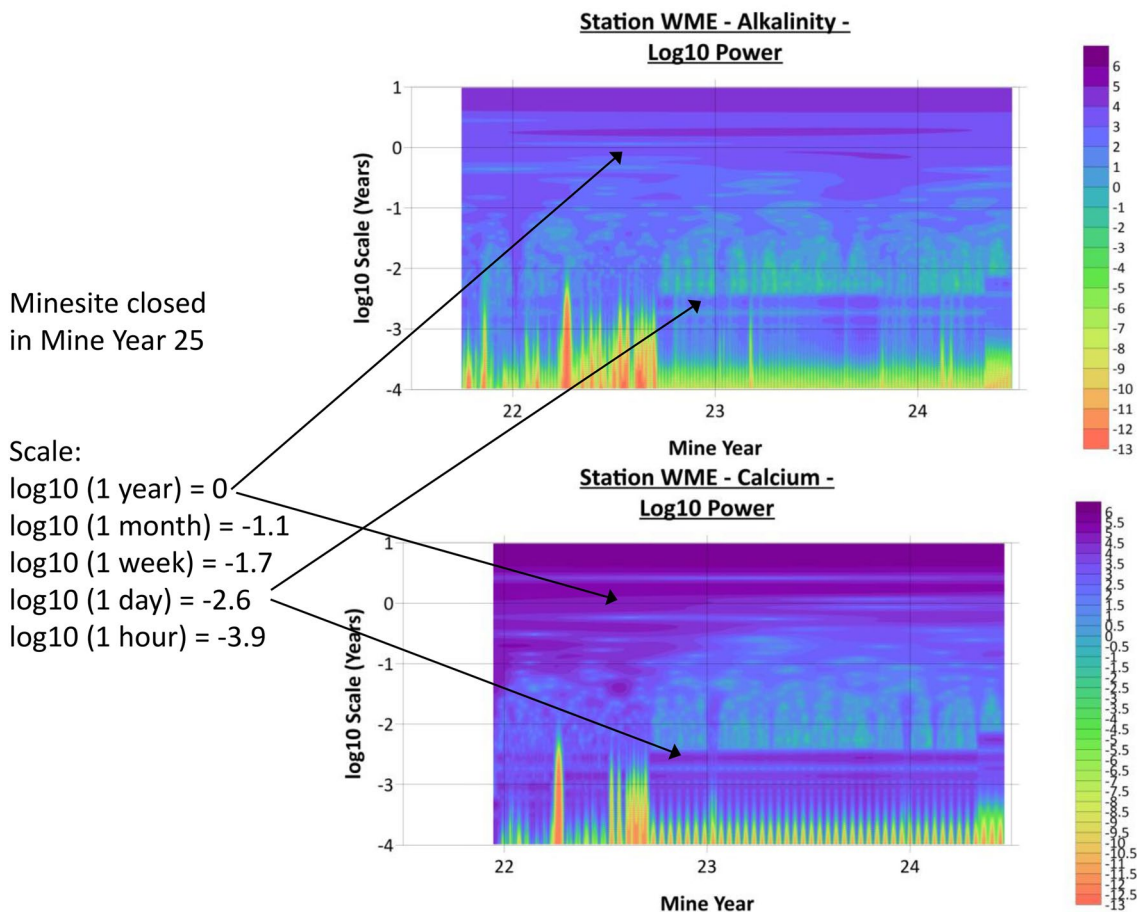
Fig. 18 Wavelet transforms for aqueous alkalinity and calcium at a monitoring station at Minesite 1, showing strong periodicities (darker horizontal bands) at many wavelengths as calcite ($CaCO_3$) rapidly neutralizes acidity generated by sulphide mineral oxidation; orange-coloured spikes rising from the bottom represent missing data at those wavelengths (frequencies)
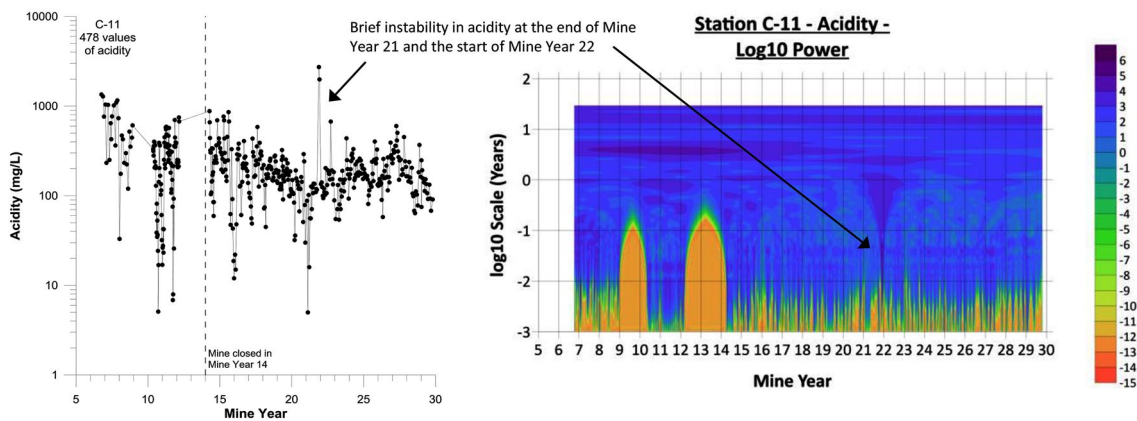
Fig. 19 Time series (left) and wavelet transform (right) for acidity at a monitoring station at Minesite 2, showing a brief instability with strong periodicities across wavelengths (vertical darker-coloured band) in Mine Years 21 and 22; orange-coloured spikes rising from the bottom of the wavelet transform represent missing data at those wavelengths (frequencies)

8 Future work

This study is the first step in the detailed characterization and explanation of oscillations in full-scale minesite drainage. The ultimate goal is to understand and predict more reliably (1) these dynamic physical flows and aqueous chemistries of the drainages and (2) their consequent impacts on downstream/downgradient ecosystems not often detected and explained by current regular monitoring (e.g. Fig. 3).

As the first step, several paths of future work and interpretation are available, although heavily limited by the unevenly spaced data in full-scale databases. These include:

- application of other wavelets, such as the Mexican Hat which can better characterize the infrequent discontinuities,
- least-squares wavelet analysis which accommodates unevenly spaced data [20–23],
- finely scaled discrete wavelets,
- other non-spectral time-series techniques for examining periodicity in unevenly spaced environmental data,
- closer examination of narrow wavelengths with significant spectral and wavelet power,
- implementation of statistical techniques like binning as well as appropriate statistical significance, and
- any newly developed algorithms that accommodate unevenly spaced environmental data.

9 Conclusion

This paper has applied both spectral analysis in the wavelength-frequency domain and continuous wavelet transforms to drainages from full-scale minesite components. These components were located at three minesites with (1) high-frequency measurements as frequently as every 15 min continuing for years (inevitably with some gaps) and/or (2) less frequent measurements continuing for decades (inevitably with some gaps). The monitoring databases were cleaned only by deleting very obvious outliers and ignoring statistical significance, so that extreme events and fractal patterns could be detected in this initial evaluation.

Important concepts in the application of spectral analysis and wavelet transforms included signal filtering and signal generation by minesite components, periodicity, time-series fractals, 1-over-f spectral slopes, emergence, and statistical significance. This is a novel application for minesite drainage, but spectral analysis without wavelet transforms has been applied previously to full-scale non-mining catchments.

The non-mining spectral analysis showed that fractal 1-over-f spectral slopes were so common as to be "universal" in some catchments for dozens of aqueous elements throughout the Periodic Table. Therefore, 100% of non-mining catchments with sufficient high-frequency monitoring over a long period of time showed fractal 1-over-f slopes.

Similarly, for all three minesite-drainage databases examined here, 1-over-f fractal slopes were common, but other slopes mostly between zero and 2.0 were also found. In particular, spectral slopes of flow often differed from the slopes of the aqueous elements it carried, supporting the common observation that full-scale flows are generally independent of the corresponding full-scale aqueous chemistries. Spectral analysis also indicated electrical conductivity, which is easily measured at high frequency, is not necessarily a reliable surrogate for chemical analyses of individual elements. At one minesite, the relative strengths of two spectral peaks could be used as a "tracer" to distinguish two sources of inflowing water.

Additionally, spectral analyses produced anomalous full-scale spectral slopes including (1) discontinuities in spectral power, (2) ranges of wavelengths with two slopes of spectral power, and (3) changes in spectral slope at specific wavelengths. While these can be considered "errors", relatively simple simulations showed that these anomalies can be explained by major seasonal changes in water retention by upstream buried ponds or subsurface aquitards. In other words, the errors pointed to real full-scale processes that may not otherwise be detected.

Whereas spectral analysis evaluates an entire time series as a unit, wavelet transforms evaluate shorter portions of a time series, albeit based on fewer datapoints in the shorter portions. Wavelet transforms for the three minesite-drainage databases provided important observations such as (1) the increasing or decreasing strengths of periodicity and oscillation with time, (2) the differing periodicities and strengths between physical drainage flows and their aqueous chemistries, (3) the effect of closing a minesite and placing a fine-grained soil/till cover over a waste-rock pile, (4) the strong periodicities caused by pH-neutralizing minerals forced to dissolve quickly in response to acid-generating sulphide oxidation, and (5) the extreme but short-lived events when periodicities in aqueous concentrations increase sharply across a large range of wavelength.

Based on all three minesite-drainage databases, the most common wavelengths for strong, persistent periodicities were 1 year and 1 week. Whereas strong periodicity over 1 year is expected based on annual Canadian

climate cycles, plausible explanations for 1 week are not apparent. Other wavelengths of strong periodicity for at least two minesites were 10 years, approximately 4 months, and half-monthly to monthly. The minesite with data as frequent as every 15 min also showed strong periodicities over 1 day and less.

Compliance with ethical standards

Conflict of interest The author declares that he has no conflict of interest.

References

1. Adrian R, Gerten D, Huber V, Wagner C, Schmidt SR (2012) Windows of change: temporal scale of analysis is decisive to detect ecosystem responses to climate change. Mar Biol 159:2533–2542
2. Aguilera R, Livingstone DM, Marcé R, Jennings E, Piera J, Adrian R (2016) Using dynamic factor analysis to show how sampling resolution and data gaps affect the recognition of patterns in limnological time series. Inland Waters 6:284–294
3. Anderson PW (1972) More is different: broken symmetry and the nature of the hierarchical structure of science. Science 177:393–396
4. Aubert AH, Kirchner JW, Gascuel-Odoux C, Faucheux M, Gruau G, Mérot P (2013) Fractal water quality fluctuations spanning the periodic table in an intensively farmed watershed. Environ Sci Technol 48:930–937
5. Akeson RL, Chen X, Ciardi D, Crane M, Good J, Harbut M, Jackson E, Kane SR, Laity AC, Leifer S, Lynn M, McElroy DL, Papin M, Plavchan P, Ramírez SV, Rey R, von Braun K, Wittman M, Abajian M, Ali B, Beichman C, Beekley A, Berriman GB, Berukoff S, Bryden G, Chan B, Groom S, Lau C, Payne AN, Regelson M, Saucedo M, Schmitz M, Stauffer J, Wyatt P, Zhang A (2013) The NASA exoplanet archive: data and tools for exoplanet research. Publ Astron Soc Pac 125:989–999
6. Bak P (1996) How nature works: the science of self-organized criticality. Springer, New York. https://doi.org/10.1007/978-1-4757-5426-1. ISBN 978-0-387-98738-5
7. Bannerjee A, Pasupuleti S, Singh MK, Pradeep Kumar GN (2018) A study on the Wilkins and Forchheimer equations used in coarse granular media flow. Acta Geophys 66:81–91
8. Betrie GD, Sadiq R, Tesfamariam S, Morin KA (2016) On the issue of incomplete and missing water-quality data in mine site databases: comparing three imputation methods. Mine Water Environ 35:3–9
9. Betrie GD, Tesfamariam S, Morin KA, Sadiq R (2013) Predicting copper concentrations in acid mine drainage: a comparative analysis of five machine learning techniques. Environ Monit Assess 185:4171–4182
10. Beven K, Germann P (2013) Macropores and water flow in soils revisited. Water Resour Res 49:3071–3092
11. Bretthorst GL (2003) Frequency estimation and generalized Lomb-Scargle periodograms. In: Statistical challenges in astronomy. Springer, New York. https://doi.org/10.1007/0-387-21529-8_21
12. Bretthorst GL (2001) Generalizing the Lomb–Scargle periodogram—the nonsinusoidal case. In: Bayesian inference and maximum entropy methods in science and engineering, 20th international workshop, AIP conference proceedings, vol 568, pp 246–251. https://doi.org/10.1063/1.1381889
13. Cánovas CR, Olías A, Nieto JM, Galván L (2010) Wash-out processes of evaporitic sulfate salts in the Tinto river: hydrogeochemical evolution and environmental impact. Appl Geochem 25:288–301
14. Dawood I, Aubertin M (2012) Influence of internal layers on water flow inside a large waste rock pile. Report EPM-RT-2012-01, Department of Civil, Geologic, and Mining Engineering, École Polytechnique de Montréal
15. Dawood I, Aubertin M, Intissar R, Chouteau M (2011) A combined hydrogeological–geophysical approach to evaluate unsaturated flow in a large waste rock pile. In: 2011 Pan-Am CGS geotechnical conference
16. English LQ (2017) There is no theory of everything. Springer, New York. ISBN 978-3-319-59149-0
17. Fala O, Aubertin M, Molson J, Bussiere B, Wilson GW, Chapuis R, Martin V (2003) Numerical modelling of unsaturated flow in uniform and heterogeneous waste rock piles. In: Farrell T, Taylor G (eds) Proceedings from the sixth international conference on acid rock drainage, July 14–17, Cairns, Australia, pp 895–902
18. Feng X, Kirchner JW, Neal C (2004) Spectral analysis of chemical time series from long-term catchment monitoring studies: hydrochemical insights and data requirements. Water Air Soil Pollut 4:221–235
19. Franklin M., Fernandes H, Yeh G, de Azevedo JPS (2006) Preliminary results of the water flow modeling in an acid drainage generating waste rock pile located at the uranium mining site of Poços De Caldas—Brazil. In: Barnhisel RI (ed) Proceedings of the 7th international conference on acid rock drainage (ICARD), March 26–30, 2006, St. Louis, pp 611–622
20. Ghaderpour E, Pagiatakis SD (2019) LSWAVE: a MATLAB software for the least-squares wavelet and cross-wavelet analyses. GPS Solut 23(2):8
21. Ghaderpour E, Pagiatakis SD (2017) Least-squares wavelet analysis of unequally spaced and non-stationary time series and its applications. Math Geosci 49(7):819–844
22. Ghaderpour E, Ince ES, Pagiatakis SD (2018) Least-squares cross-wavelet analysis and its applications in geophysical time series. J Geodesy 92(10):1223–1236
23. Ghaderpour E, Liao W, Lamoureux MP (2018) Antileakage least-squares spectral analysis for seismic data regularization and random noise attenuation. Geophysics 8(3):V157–V170
24. Godsey SE, Aas W, Clair TA, de Wit HA, Fernandez IJ, Kahl JS, Malcolm IA, Neal C, Neal M, Nelson SJ, Norton SA, Palucis MC, Skjelvåle BL, Soulsby C, Tetzlaff D, Kirchner JW (2010) Generality of fractal 1/f scaling in catchment tracer time series, and its implications for catchment travel time distributions. Hydrol Process 24:1660–1671
25. Halley JM, Inchausti P (2004) The increasing importance of 1/f-noises as models of ecological variability. Fluct Noise Lett 4:R1–R26
26. Herasymuik GM (1996) Hydrogeology of a sulphide waste rock dump. M.Sc. Thesis, Department of Civil Engineering, University of Saskatchewan
27. Keitt TH (2008) Coherent ecological dynamics induced by large-scale disturbance. Nature 454:331–335
28. Kempton H (2012) A review of scale factors for estimating waste rock weathering from laboratory tests. In: Proceedings of the 2012 international conference on acid rock drainage, Ottawa, Canada, May 22–24

29. Kirchner JW (2016) Aggregation in environmental systems—part 1: seasonal tracer cycles quantify young water fractions, but not mean transit times, in spatially heterogeneous catchments. Hydrol Earth Syst Sci 20:279–297

30. Kirchner JW (2016) Aggregation in environmental systems—part 2: catchment mean transit times and young water fractions under hydrologic nonstationarity. Hydrol Earth Syst Sci 20:299–328

31. Kirchner JW (2009) Catchments as simple dynamical systems: catchment characterization, rainfall-runoff modeling, and doing hydrology backward. Water Resour Res 45(W02429):1–34

32. Kirchner JW (2005) Aliasing in 1/f noise spectra: origins, consequences, and remedies. Phys Rev E 71:066110-1–066110-16

33. Kirchner JW (2003) A double paradox in catchment hydrology and geochemistry. Hydrol Process 17:871–874

34. Kirchner JW, Neal C (2013) Universal fractal scaling in stream chemistry and its implications for solute transport and water quality trend detection. Proc Natl Acad Sci 110:12213–12218

35. Kirchner JW, Feng X, Neal C, Robson AJ (2004) The fine structure of water-quality dynamics: the (high-frequency) wave of the future. Hydrol Process 18:1353–1359

36. Kirchner JW, Feng X, Neal C (2001) Catchment-scale advection and dispersion as a mechanism for fractal scaling in stream tracer concentrations. J Hydrol 254:82–101

37. Kirchner JW, Feng X, Neal C (2000) Fractal stream chemistry and its implications for contaminant transport in catchments. Nature 403:524–527

38. Lomb NR (1976) Least-squares frequency analysis of unevenly spaced data. Astrophys Space Sci 39:447–462

39. Momeyer SA (2014) Hydrologic processes in unsaturated waste rock piles in the Canadian Subarctic. M.Sc. Thesis, University of British Columbia

40. Morin KA (2019) Geochemical transitions from near-neutral to acidic conditions in minesite drainage—chaotic or organized? MDAG Internet Case Study #60. www.mdag.com/case_studies/cs60.html. Accessed 30 June 2019

41. Morin KA (2018) Wavelet transforms of drainage from highly reactive geologic materials. MDAG Publishing, Surrey. ISBN 978-0-9952149-3-4

42. Morin KA (2017) Time-dependent toxicity related to short-term peaks of contaminant release. MDAG Internet Case Study #49. www.mdag.com/case_studies/cs49.html. Accessed 30 June 2019

43. Morin KA (2017) A case study applying first-passage percolation theory and reservoir modelling to full-scale waste-rock drainage after a one-day dynamic storm. MDAG Internet Case Study #52. www.mdag.com/case_studies/cs52.html. Accessed 30 June 2019

44. Morin KA (2017) A case study of rapid water flow through full-scale waste-rock piles. MDAG Internet Case Study #45. www.mdag.com/case_studies/cs45.html. Accessed 30 June 2019

45. Morin KA (2017) Spectral analysis for identifying interactions of physical, geochemical, and biological processes creating contaminated drainages at minesites. In: GeoOttawa 2017, the 70th Canadian geotechnical conference and the 12th Joint CGS/IAH-CNC groundwater conference, October 1–4, Ottawa, ON

46. Morin KA (2016) Geochemical dynamics and complexities of 1-kg humidity cells: potentials for geochemical signal generation, phase transitions, harmonic oscillations, and conceptual stacking for large-scale predictions. MDAG Internet Case Study #44. www.mdag.com/case_studies/cs44.html. Accessed 30 June 2019

47. Morin KA (2016) Spectral analysis of drainage from highly reactive geologic materials. MDAG Publishing, Surrey. ISBN 978-0-9952149-1-0

48. Morin KA (2016) Fractal 1/f temporal trends in minesite drainage from waste-rock dumps. In: 14th experimental chaos and complexity conference, May 16–19, Banff Center, Banff, Canada

49. Morin KA (2016) Effects of water-retention structures on temporal power spectra for drainage waters. MDAG Internet Case Study #43. www.mdag.com/case_studies/cs43.html. Accessed 30 June 2019

50. Morin KA (2014) Applicability of scaling factors to humidity-cell kinetic rates for larger-scale predictions. In: 21st annual BC/MEND metal leaching/acid rock drainage workshop, challenges and best practices in metal leaching and acid rock drainage December 3–4, 2014, Simon Fraser University Harbour Centre, Vancouver, BC, Canada

51. Morin KA, Hutt NM (2001) Environmental geochemistry of minesite drainage: practical theory and case studies, digital edition. MDAG Publishing, Surrey. ISBN 0-9682039-1-4

52. Morin KA, Hutt NM (1997) Environmental geochemistry of minesite drainage: practical theory and case studies. MDAG Publishing, Surrey. ISBN 0-9682039-0-6

53. Morin KA, Hutt NM, Aziz M (2012) Case studies of thousands of water analyses through decades of monitoring: selected observations from three minesites in British Columbia, Canada. In: Proceedings of the 2012 international conference on acid rock drainage, Ottawa, Canada, May 22–24

54. Nimick DA, Gammons CH, Parker SR (2010) Diel biogeochemical processes and their effect on the aqueous chemistry of streams: a review. Chem Geol 283:3–17

55. Nimick DA, McClesky BR, Gammons CH, Cleasby TE, Parker SR (2007) Diel mercury-concentration variations in streams affected by mining and geothermal discharge. Sci Total Environ 373:344–355

56. Nimick DA, Gammons CH, Cleasby TE, Madison JP, Skaar D, Brickand CM (2003) Diel cycles in dissolved metal concentrations in streams: occurrence and possible causes. Water Resour Res 39:1247, 2-1 - 2-17

57. Panda BB, Freiman TJ, Manepally C (1999) A comparison of simulated unsaturated flow through waste rocks using two common used computer models. In: 1999 national meeting of the American society for surface mining and reclamation, Scottsdale, Arizona, USA, August 13–19, pp 527–538

58. Price WA (2009) Prediction manual for drainage chemistry from sulphidic geologic materials. Canadian Mine Environment Neutral Drainage Report 1.20.1, Natural Resources Canada, dated December 2009

59. R Core Team (2016) R: a language and environment for statistical computing. R Foundation for Statistical Computing, Vienna

60. RStudio Team (2015) RStudio: integrated development for R. RStudio Inc, Boston

61. Scargle JD (1989) Studies in astronomical time series analysis. III. Fourier transforms, autocorrelation and cross-correlation functions of unevenly spaced data. Astrophys J 343:874–887

62. Scargle JD (1982) Studies in astronomical time series analysis II. Statistical aspects of spectral analysis of unevenly sampled data. Astrophys J 263:835–853

63. Shope CL, Xie Y, Gammons CH (2006) The influence of hydrous Mn–Zn oxides on diel cycling of Zn in an alkaline stream draining abandoned mine lands. Appl Geochem 21:476–491

64. Smith L, Lopez D, Beckie R, Morin K, Dawson R, Price W (1995) Hydrogeology of waste rock dumps. MEND Associate Project PA-1

65. Torrence C, Compo GP (1998) A practical guide to wavelet analysis. Bull Am Meteorol Soc 79:61–78. See also Wavelet Analysis & Monte Carlo at http://paos.colorado.edu/research/wavelets/

66. VanderPlas JT (2018) Understanding the Lomb–Scargle periodogram. Astrophys J Suppl Ser 236(16):1–28

67. Ward LM, Greenwood PE (2010) The mathematical genesis of the phenomenon called "1/f noise" (10frg132). Final report of the workshop. The Banff International Research Station for Mathematical Innovation and Discovery, June 6–12

68. Ziliak ST (2016) Statistical significance and scientific misconduct: improving the style of the published research paper. Rev Soc Econ 74:83–97

69. Ziliak ST, McCloskey DN (2009) The cult of statistical significance. The joint statistical meetings (JSM), Aug. 3rd, 2009, in Washington, DC, USA

70. Ziliak ST, McCloskey DN (2008) The cult of statistical significance. How the standard error costs us jobs, justice, and lives. University of Michigan Press, Michigan. ISBN 978-0-472-05007-9