Analysis of buoyancy driven flow of a reactive heat generating third grade fluid in a parallel channel having convective boundary conditions

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Abstract

The study investigated the impacts of buoyancy force and internal heat source on a reactive third grade fluid flow within parallel channel. The results obtained for the coupled equations regulating the flow of fluid which are strongly nonlinear are secured by applying Adomian decomposition method (modified). The significant effects of buoyancy force and heat source are noticed to increase the speed transfer of heat within the flow regime. Also, the prevalent outcome of thermal energy at the lower and upper plates reduces with rising values of buoyancy force and reduces with heat source. The present result is compared with the earlier published article where the influence of buoyancy force and heat source were not accounted for.

Keywords Buoyancy force · Third-grade fluid · Heat source · Convective cooling · Adomian decomposition method (modified)

1 Introduction

Studies involving non-Newtonian fluids recently have been on tremendous increase due to its numerous and significant applications in industries, geology, petro-chemical engineering, technology, extraction of crude oil, to mention a few. Example of such fluids as illustrated in [1] are molten plastics, polymers, pulps, foods, mud and most biological fluids with higher molecular weight components. As yet, tremendous efforts are concluded on both Newtonian and non-Newtonian fluid models. One can assess the recent developments in this direction in [2–15]. These types of fluids are categorized as third-grade that belongs to a relevant category of non-Newtonian fluid alongside the complex behaviour and conditions that cannot easily be described, hence, so many considerable researches were carried out to formulate the fluid behaviour. However, according to the studies investigated in [16, 17], the third-grade design is simplified and efficient of halsening

and demonstrating the regular stress effect of shear thinning/thickening. In addition to that, of recent, [18, 19] studied the Magnetohydrodynamic (MHD) mixed convection flow of third grade fluid aimed at an exponentially stretching sheet where the broad, flat layer of the material on a surface is convectively heated and at the same time examined Magnetohydrodynamic (MHD) stretched flow of third grade Nano liquid with convective surface condition. Among relevant information on third grade fluid design with different physical aspects from researchers are extensively presented in [20–25]

Meanwhile, investigation in [20] revealed the significant effect of buoyancy on fluid motion that is subjected to gravitational forces and variations of measure in the mass of matter contained by specified volume which can occur when there is change in fluid temperature. In order to show the appreciable effect of buoyancy force [20], further explored the compound influence of buoyancy force and asymmetric convective cooling on unsteady MHD flow

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within the channel and heat transfer in a reactive third grade fluid. Additionally, [26] studied the impacts of thermal buoyancy on the boundary layer flow over an upright plate with convective surface boundary conditions. Moreover, [27] presented the analysis of first and second law of thermodynamics on fluid flow and heat transfer inside a vertical channel with respect to the compound actions of buoyancy force, constant pressure gradient and magnetic field strength. Other related surveys showing the impact of buoyancy force on fluid motion with other physical parameters can be found in existing literature, to mention a few [28–31].

However, from engineering and industrial applications, the effect of heat transfer on a fluid flow within parallel plates cannot be totally neglected as recently discussed in [32]. In support of that, the estimation of heat transfer depends on temperature which raises the interaction of moving fluid and thus the conduct of the internal energy of the flow system is presented in [33]. Other studies showing the effects of internal heat on the fluid flow include [34] where investigation of free convective flow of fluid with heat source/sink between vertical parallel porous plates. Recently, [35] proposed the general understanding of thermal transfer of magneto hydrodynamic non-Newtonian fluid flow over a dispersing sheet occurring together with exponential heat source. Also, in addition to that, [36] put forward the influence of heat flux design initiated by Cattaneo–Christov on the flow across a wedge and a cone of which the effect of non-uniform wall temperatures are also examined. Other investigations on the impact of heat source are extensively discussed in [37–39].

Hence, the present study is to extend the investigation in [40] by examining the impact of buoyancy force on a reactive fluid of third-grade flowing steadily in-between upper and lower plates, subject to the effect of internal heat generation with symmetrical convective cooling the walls which was not accounted for in their study. The significant effect of heat source in the fluid motion and heat transfer cannot be overlooked as discussed in [41], hence; the study investigates the influence of heat generation on the fluid motion and thermal energy in the flow regime. The problem is strongly nonlinear with paired differential equations regulating the momentum and energy distributions obtained by means of employing the use of a rapidly convergent modified Adomian decomposition method. The intended modification method is seen to be more reliable as compared with the standard Adomian decomposition method. The major studies on this method are presented extensively in [42, 43] as the series converge with less iteration. Finally, the effects of the Grashof number (G_r) which is a function of buoyancy force and the internal heat generation that is a linear relation of temperature are thereby presented.

2 Mathematical formulation

Taking into account the stable flow of a reactive incompressible third-grade fluid driven by buoyancy force running through a channel within two parallel plates fixed at $y = -a$ and $y = a$ as shown in Fig. 1. Both plates are subjected to the impact of heat source which is a linear relation of temperature. Disregarding the consumption of the reacting viscous fluid, the equations controlling the fluid momentum and energy in non-dimensional form using [20, 27, 33, 40] may be written as:

$$-\frac{dp}{dx} + \mu \frac{d^2 u}{dy^2} + 6\beta_3 \frac{d^2 u}{dy^2} \left(\frac{du}{dy} \right)^2 + \rho g \beta (T - T_0) = 0 \quad (1)$$

$$k \frac{d^2 T}{dy^2} + \left(\frac{du}{dy} \right)^2 \left(\mu + 2\beta_3 \left(\frac{du}{dy} \right)^2 \right) + QC_0 A e^{-\frac{E}{RT}} + Q_0 (T - T_0) = 0 \quad (2)$$

The no slip state of fluid motion velocity obeying Newton's law of cooling is considered as

$$u = 0, \quad k \frac{dT}{dy} = -h(T - T_0) \quad \text{at} \quad y = \pm a \quad (3)$$

and the flow is symmetric with boundary requirement along the channel centreline as:

$$\frac{du}{dy} = \frac{dT}{dy} = 0, \quad \text{at} \quad y = 0 \quad (4)$$

However, the expression for the rate of entropy generation per unit volume together with appreciable buoyancy force and heat source with regards to [13, 14, 44, 45] is given as:

$$E_G = \frac{k}{T_0^2} \left(\frac{dT}{dy} \right)^2 + \frac{1}{T_0} \left[\left(\frac{du}{dy} \right)^2 \left(\mu + 2\beta_3 \left(\frac{du}{dy} \right)^2 \right) \right] \quad (5)$$

where p stands for the pressure, U represents the reference velocity, u is the dimensional fluid velocity, μ is the viscosity

Fig. 1 Geometry of the problem

of the fluid, β_3 the material coefficient, ρ is the density evaluated at the mean temperature and g is the gravitational constant. In addition to that, β represents the coefficient of thermal expansion, k stands for the thermal conductivity coefficient, T is the dimensional fluid temperature, Q is the heat of the reaction term, C_0 is the initial concentration of the reactant species, A is the reaction rate constant, E represents activation energy. Also, R stands for the universal gas constant, Q_0 represents the dimensional heat generation coefficient, h is the heat transfer coefficient, T_0 is the wall temperature and E_G is the rate of entropy generation in non-dimensionless form. It is worthy to note that the last term in the momentum equation is the additional term to extend the study in [40] by examining the impact of buoyancy force as in [27] and the final term in energy equation is resulting from heat source [33, 34, 39].

With the introduction of the under-listed non-dimensional quantities:

$$\begin{aligned}\theta &= \frac{E(T - T_0)}{RT_0^2}, \quad \bar{y} = \frac{y}{a}, \quad \lambda = \frac{QE A a^2 C_0}{k R T_0^2} e^{-\frac{E}{RT_0}}, \\ W &= \frac{u}{UG}, \quad Bi = \frac{ha}{k}, \quad m = \frac{\mu G^2 U^2}{Q A a^2 C_0} e^{\frac{E}{RT_0}}, \quad \epsilon = \frac{RT_0}{E} \\ G &= -\frac{a^2}{\mu U}, \quad \frac{dp}{dx}, \quad \gamma = \frac{\beta_3 U^2 G^2}{a^2 \mu}, \\ \alpha &= \frac{Q_0 R T_0^2}{Q C_0 A} e^{\frac{E}{RT_0}}, \quad Br = \frac{E \mu U^2 G^2}{k R T_0^2} \quad \text{and} \\ G_r &= \frac{\rho g \beta a^2}{\mu U G} \left(\frac{RT_0}{E} \right)\end{aligned}\quad (6)$$

Therefore, the dimensionless coupled differential equations governing the momentum and energy of the fluid flow with appropriate boundary conditions are written as follows:

$$\frac{d^2 W}{dy^2} + 6\gamma \frac{d^2 W}{dy^2} \left(\frac{dW}{dy} \right)^2 + G_r \theta + 1 = 0 \quad (7)$$

$$\frac{d^2 \theta}{dy^2} + \lambda \left[e^{\frac{\theta}{1+\epsilon\theta}} + m \left(\left(\frac{dW}{dy} \right)^2 \left(1 + 2\gamma \left(\frac{dW}{dy} \right)^2 \right) \right) + \alpha \theta \right] = 0 \quad (8)$$

subject to the following conditions

$$\begin{aligned}W &= 0, \quad \frac{d\theta}{dy} = -Bi\theta \quad \text{on } y = \pm 1 \quad \text{and} \\ \frac{dW}{dy} &= \frac{d\theta}{dy} = 0 \quad \text{on } y = 0\end{aligned}\quad (9)$$

And, the expression for the rate of entropy generation in dimensionless form applying the existing dimensionless variables and parameters are given as:

$$N_s = \left(\frac{d\theta}{dy} \right)^2 + \frac{Br}{\Omega} \left(\frac{dW}{dy} \right)^2 \left[1 + 2\gamma \left(\frac{dW}{dy} \right)^2 \right] \quad (10)$$

where W and θ respectively represent the velocity and temperature of the fluid. Also, γ , λ , ϵ , m , α and Ω are respectively parameters for the dimensionless non-Newtonian, Frank–Kamenetski, activation energy, viscous heating, heat source and wall temperature. Others are numbers for Grashof (G_r), Biot (Bi) and Brinkman (Br). Finally, N_s is the rate of entropy generation in dimensionless form.

3 Method of solution

The fluid velocity and energy equations with the boundary conditions in (7–9) are couple equations that need to be solved simultaneously using modified Adomian decomposition method. To begin, we integrate (7) and (8) respectively to obtain:

$$\begin{aligned}W(y) &= a_0 - \frac{y^2}{2} - 6\gamma \int_0^y \int_0^y \frac{d^2 W}{dy^2} \left(\frac{dW}{dy} \right)^2 dY dY \\ &\quad - G_r \int_0^y \int_0^y \theta(y) dY dY\end{aligned}\quad (11)$$

$$\begin{aligned}\theta(y) &= b_0 - \lambda \int_0^y \int_0^y \left(e^{\frac{\theta}{1+\epsilon\theta}} + m \left(\frac{dW}{dy} \right)^2 \right. \\ &\quad \left. + 2m\gamma \left(\frac{dW}{dy} \right)^4 + \alpha \theta \right) dY dY\end{aligned}\quad (12)$$

where a_0 and b_0 are respectively equal to $W(0)$ and $\theta(0)$ to be determined by other boundary conditions stated in (9). Nevertheless, in order to find solutions to the paired Eqs. (7) and (8), we introduce an unlimited series solutions in the formation of

$$W(y) = \sum_{n=0}^{\infty} W_n(y) \quad \text{and} \quad \theta(y) = \sum_{n=0}^{\infty} \theta_n(y) \quad (13)$$

to the extent that when (13) is substituted into (11) and (12), then we have,

$$\begin{aligned}W(y) &= a_0 - \frac{y^2}{2} - 6\gamma \int_0^y \int_0^y \frac{d^2 (\sum_{n=0}^{\infty} W_n(y))}{dy^2} \\ &\quad \left(\frac{d(\sum_{n=0}^{\infty} W_n(y))}{dy} \right)^2 dY dY \\ &\quad - G_r \int_0^y \int_0^y \left(\sum_{n=0}^{\infty} \theta_n(y) \right) dY dY\end{aligned}\quad (14)$$

$$\begin{aligned} \theta(y) = b_0 - \lambda \int_0^y \int_0^y & \left(e^{\frac{\sum_{n=0}^{\infty} \theta_n(y)}{1+\epsilon(\sum_{n=0}^{\infty} \theta_n(y))}} \right. \\ & + m \left(\frac{d(\sum_{n=0}^{\infty} W_n(y))}{dy} \right)^2 \Bigg) dY dY \\ & - 2m\gamma \lambda \int_0^y \int_0^y \left(\left(\frac{d(\sum_{n=0}^{\infty} W_n(y))}{dy} \right)^4 \right. \\ & \left. + \alpha \left(\sum_{n=0}^{\infty} \theta_n(y) \right) \right) dY dY \end{aligned} \quad (15)$$

At this point, the non-linear terms in (14) and (15) amount to the following in order to use ADM as:

$$\begin{aligned} \sum_{n=0}^{\infty} A_n(y) &= e^{\frac{\sum_{n=0}^{\infty} \theta_n(y)}{1+\epsilon(\sum_{n=0}^{\infty} \theta_n(y))}}, \\ \sum_{n=0}^{\infty} B_n(y) &= \left(\frac{d(\sum_{n=0}^{\infty} W_n(y))}{dy} \right)^2 \\ \sum_{n=0}^{\infty} C_n(y) &= \left(\frac{d(\sum_{n=0}^{\infty} W_n(y))}{dy} \right)^4 \quad \text{and} \\ \sum_{n=0}^{\infty} D_n(y) &= \frac{d^2(\sum_{n=0}^{\infty} W_n(y))}{dy^2} \left(\frac{d(\sum_{n=0}^{\infty} W_n(y))}{dy} \right)^2. \end{aligned} \quad (16)$$

where the respective components $A_0, A_1, A_2, \dots, B_0, B_1, B_2, \dots, C_0, C_1, C_2, \dots$ and $D_0, D_1, D_2, \dots$ are termed Adomian polynomials. Then (16) is consequently expanded in this manner as:

$$\begin{aligned} A_0 &= e^{\frac{\theta_0(y)}{\epsilon\theta_0(y)+1}}, \quad A_1 = \frac{\theta_1(y)e^{\frac{\theta_0(y)}{\epsilon\theta_0(y)+1}}}{(\epsilon\theta_0(y)+1)^2}, \\ A_2 &= \frac{\lambda^2 e^{\frac{\theta_0(y)}{\epsilon\theta_0(y)+1}} (\theta_1(y)^2 (-2\epsilon^2\theta_0(y) - 2\epsilon + 1) + 2\theta_2(y)(\epsilon\theta_0(y) + 1)^2)}{2(\epsilon\theta_0(y) + 1)^4}, \dots, \\ B_0 &= W_0'(y)^2, \quad B_1 = 2W_0'(y)W_1'(y), \quad B_2 = W_1'(y)^2 + 2W_0'(y)W_2'(y), \dots, \\ C_0 &= W_0'(y)^4, \quad C_1 = 4W_0'(y)^3W_1'(y), \quad C_2 = 2W_0'(y)^2(3W_1'(y)^2 + 2W_0'(y)W_2'(y)), \dots, \\ D_0 &= W_0'(y)^2W_0''(y), \quad D_1 = W_0'(y)(2W_1'(y)W_0''(y) + W_0'(y)W_1''(y)), \\ D_2 &= W_0'(y)^2W_2''(y) + 2W_1'(y)W_0'(y)W_1''(y) + (W_1'(y)^2 + 2W_0'(y)W_2'(y))W_0''(y) \dots, \end{aligned} \quad (17)$$

With (16), the energy and momentum equations respectively scaled down to:

$$\begin{aligned} \theta(y) = b_0 - \lambda \int_0^y \int_0^y & \left(\left(\sum_{n=0}^{\infty} A_n(y) \right) + m \left(\sum_{n=0}^{\infty} B_n(y) \right) \right. \\ & \left. + 2m\gamma \left(\sum_{n=0}^{\infty} C_n(y) \right) + \alpha \left(\sum_{n=0}^{\infty} \theta_n(y) \right) \right) dY dY \end{aligned} \quad (18)$$

$$\begin{aligned} W(y) = a_0 - \frac{y^2}{2} - 6\gamma \int_0^y \int_0^y & \left(\sum_{n=0}^{\infty} D_n(y) \right) dY dY \\ & - G_r \int_0^y \int_0^y \left(\sum_{n=0}^{\infty} \theta_n(y) \right) dY dY \end{aligned} \quad (19)$$

Subsequently, the iterative function of the zeroth subdivision as mentioned in [42, 43] are obtained as follows:

$$\theta_0(y) = 0, \quad W_0(y) = a_0 - \frac{y^2}{2}, \quad (20)$$

$$\begin{aligned} \theta_1(y) &= b_0 - \lambda \int_0^y \int_0^y (A_0(y) + m(B_0(y)) \\ & + 2m\gamma(C_0(y)) + \alpha(\theta_0(y)) \quad dY dY \\ W_1(y) &= -6\gamma \int_0^y \int_0^y (D_0(y)) \quad dY dY \\ & - G_r \int_0^y \int_0^y (\theta_0(y)) \quad dY dY \end{aligned} \quad (21)$$

$$\begin{aligned}\theta_{n+1}(y) &= \lambda \int_0^y \int_0^y (A_n(y)) + m(B_n(y)) \\ &\quad + 2m\gamma(C_n(y)) + \alpha(\theta_n(y)) \, dY \, dY, \\ W_{n+1}(y) &= -6\gamma \int_0^y \int_0^y (D_n(y)) \, dY \, dY \\ &\quad - G_r \int_0^y \int_0^y (\theta_n(y)) \, dY \, dY, \quad n \geq 1\end{aligned}\quad (22)$$

Equations (20)–(22) are thereby programmed in computer software to secure the estimated solutions used and discussed in the subsequent section as

$$\theta(y) = \sum_{n=0}^3 \theta_n(y) \quad \text{and} \quad W(y) = \sum_{n=0}^3 W_n(y) \quad (23)$$

Moreover, to determine the rate of entropy production across the flow channels which are endless owing to transfer of heat and fluid flow. For easy computation we split-up N_s in (10) as follows:

$$\begin{aligned}N_1 &= \left(\frac{d\theta}{dy} \right)^2 \\ N_2 &= \frac{Br}{\Omega} \left(\frac{dW}{dy} \right)^2 \left[1 + 2\gamma \left(\frac{dW}{dy} \right)^2 \right]\end{aligned}\quad (24)$$

where N_1 indicates the irreversibility due to thermal energy and N_2 represents local rate of entropy production because of the effect of the viscous diffusion of the flow system.

Alternatively, the irreversibility disposition is defined as (ϕ) and is given as :

$$\phi = \frac{N_1}{N_2}, \quad (25)$$

which signifies that heat transfer exercises control over during the time when $0 \leq \phi < 1$ and fluid friction does

Fig. 2 Velocity profile with change in G_r

the same when $\phi > 1$. This is also important to control the significant addition of heat transfer in many manufacturing industries. In order to complement the irreversibility dispersion variable, the Bejan number (Be) is marked to be

$$Be = \frac{N_1}{N_s} = \frac{1}{1 + \phi} \quad (26)$$

where Be lies between 0 and 1.

4 Results and discussion

This portion extensively revealed the impact of buoyancy on a reactive third grade fluid in-between parallel plates with convective cooling the walls under the influence of internal heat source combined with other essential flow properties are thereby presented and discussed. It is worthy to note that our result shall be equivalent to that of [40] when the heat source parameter (α) and buoyancy effect parameter known as Grashof number (G_r) are both equal to 0.

Table 1 displayed the collation of arithmetical solutions of velocity profiles between the formerly existed results

Table 1 Comparison of numerical results of the velocity distribution

$\epsilon = \gamma = 0.1, \lambda = 0.5, Bi = 10, m = 1, \alpha = G_r = 0$			
y	$W(y)_{PM} [40]$	$W(y)_{mADM}$	Absolute error
-1	0	$4.510281037534 \times 10^{-17}$	$4.510281037534 \times 10^{-17}$
-0.75	0.19851653823852540	0.19851653823852541	2.77556×10^{-17}
-0.50	0.34465078125000004	0.34465078125000004	0
-0.25	0.43574060440063480	0.43574060440063480	0
0	0.46680000000000000	0.46680000000000005	5.55112×10^{-17}
0.25	0.43574060440063480	0.43574060440063480	0
0.50	0.34465078125000004	0.34465078125000004	0
0.75	0.19851653823852540	0.19851653823852541	2.77556×10^{-17}
1	0	$4.510281037534 \times 10^{-17}$	$4.510281037534 \times 10^{-17}$

Fig. 3 Velocity profile with change in α

Fig. 5 Effects of α on $\theta(y)$

Fig. 4 Effects of G_r on $\theta(y)$

Fig. 6 Effects of G_r on N_s

in [40], whereas perturbation technique (PT) was used and the newly obtained result from the modification of Adomian decomposition method. The impact of buoyancy force and internal heat source are not accounted for in the previously obtained results and these parameters are both zero to show the significant effects. Therefore, the validity of present result is shown in Table 1 with absolute error of average order of 10^{-17} obtained with size-able number of iterations done.

Figures 2 and 3 respectively displayed the effects of buoyancy force and heat source on fluid motion. It is detected that the maximum fluid motion occur at the greatest values of (G_r) and (α). Meanwhile, the effect on fluid motion is more pronounced in the presence of buoyancy force as shown in Fig. 2 and slightly noticed with the heat source in Fig. 3.

Fig. 7 Effects of α on N_s

Fig. 8 Effects of G_r on Be

Fig. 9 Effects of α on Be

The temperature profiles for variations in the Grashof number (G_r) and heat source term (α) are respectively shown in Figs. 4 and 5. The utmost temperature is perceived at the centreline with rising values of (G_r) and (α). This is so due to the presence of internal energy produced during fluid interactions, that is stored within the flow regime due to viscous heating thereby increases the fluid temperature.

Figures 6 and 7 depicted the rate of entropy generation versus the channel width for Grashof number (G_r) and heat source term (α). The slightest value of the rate of entropy generation occur around the core region and increases to a maximum around the plate surfaces. It is observed that a rise in (G_r) and heat source term (α) also bring about a rise in the rate of entropy generation.

Figures 8 and 9 show the effects of Bejan number (Be) with respect to buoyancy force and heat source. Generally, the heat transfer exercises control over at both lower and upper surfaces while the fluid friction irreversibility

exercises control over around the core region. The controlling effects of heat transfer irreversibility at the lower plates reduce with rising values of (G_r) in Fig. 8, while an increase is observed with the rising values of heat source (α) around the core region in Fig. 9.

5 Conclusion

The study investigated the impacts of buoyancy force and heat source on a reactive third grade fluid flow within parallel plates. The results obtained for the coupled equations controlling the fluid regime which are strongly nonlinear are secured using the modification of Adomian decomposition method. The present result is compared with the one in [40] where the influences of buoyancy force and heat source were not accounted for. Also, the fluid motion and heat transfer distributions are used to evaluate the rate of entropy generation together with the Bejan number in the flow regime. The investigation gives the following conclusions:

- The fluid motion increases with rising values of (G_r) and (α).
- The maximum temperature is obtained at the centreline with increasing rates of (G_r) and (α).
- The least rate of entropy generation occur around the significance zone and increases to the uttermost around the plate surfaces with respect to (G_r) and (α).
- The controlled impact of thermal energy irreversibility at the lower and upper plates reduces the impact of (G_r) and (α)

Hence, the significant effects of buoyancy force and heat source are noticed to increase the fluid motion and heat transfer within the flow regime.

Compliance with ethical standards

Conflict of interest The authors hereby declare that there is no conflict of interests as regards the publication of this investigation.

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