Study of entropy generation impacts on magneto-hydrodynamic flow and heat transmission over a contracting surface

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Abstract

The present article aims to investigate the entropy impacts on boundary layer flow and heat transmission of a hydro-magnetic viscous fluid past a contracting sheet. Similarity transmutations are utilized to transform the momentum and energy equations into ordinary differential equations. The expressions of velocity (V) and temperature (T) for different parameters are obtained to compute local entropy generation number (N_g) and Bejan number (Be). A profound study is performed to attain the exact solution for relevant parameters, and the outcomes are elucidated graphically. The solution is presented in the form of an incomplete Gamma function. Heat transfer entropy effects are dominant at contracting sheet surface while entropy effects due to fluid friction and magnetic field are significant in boundary layer region. The outcomes of current study illustrate that the optimal design and the efficient performance of a thermally designed system can be improved by selecting the appropriate values of the flow parameters. It will enable scientists and engineers to minimize the impacts of entropy generation and to construct an efficient and productive thermodynamical system.

Keywords Contracting surface · Magneto-hydrodynamics (MHD) · Entropy generation · Heat transmission

1 Introduction

Boundary layer flows and heat transmission over a contracting sheet has attracted numerous engineers and scientists due to its fascinating properties. Shrinking sheet phenomena can be observed in industrial processes such as packaging, extrusion of metals and plastic, crystal growing, glass, and paper production and wrapping of bulk products by shrinking films. Such kind of wrapping can be removed by providing adequate heat. Moreover, this phenomenon can be observed physically in case of rising and shrinking balloon.

Miklavčič and Wang [1] explored the flow past a surface contracting to a fixed point, thus presented the exact solution of the problem. They investigated that mass suction across the surface is essential in order to retain the flow.

Wang [2] analyzed the stagnation point flow over a contracting sheet. According to him, the solution does not exist for still fluid for a shrinking sheet, and it is necessary to consider stagnation flow so that similarity solutions may exist. Fang [3] examined the flow near boundary and mass transmission past a sheet contracting continuously with surface velocity. Fang and Zhang [4] presented the closed-form solution of magneto-hydrodynamic viscous-flow over a contracting sheet. Since then, interesting investigations have been made relating flow and transmission of heat over a contracting surface [5–7].

Entropy generation is endured in various applications linked with energy, like the cooling of latest electronic and geothermal energy systems. In literature, Entropy generation is utilized to treat the internal irreversibility. The groundbreaking piece of work by Bejan [8–10]

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elucidates how the idea of entropy generation minimization (EGM) can be utilized in order to intensify heat transmission for various industrial applications. Poulikakos and Bejan [11] used EGM to deduce the theoretical framework for an optimal fin geometry in forced convection. Giangaspero and Sciubba [12, 13] employed the entropy generation to explore thermal management for various electric machines. Moreover, EGM is utilized to optimize thermodynamic processes, such as the analyses of heat exchangers and the optimization of power plants, the Rankine cycle and the absorption chillers. Numerous researchers [14–17] have employed EGM to improve heat sink geometry.

Efficient and Productive use of energy is the fundamental aspiration in the construction of any thermodynamical system which can be attained by minimizing the values of entropy in procedures. These phenomena are related to viscous dissipation, heat and mass transmission etc. [18–20]. All flow and heat transfer phenomena in real life are irrevocable. A system in which changes due to flow and heat transmission occur can't be reverted to its original position. This causes disorders in the system and it can be quantified in terms of entropy. Bejan [21, 22] proposed that the irreversible changes are taking place in flow and heat transmission phenomenon that can be studied in terms of SG. He also mentioned that entropy generation effects can be minimized by identifying and controlling the factors that cause disorders. Later on, researchers analyzed different flow and heat transmission problems to examine the entropy impacts and to seek methods to minimize them [23–27]. They found that entropy generation in a system can be minimized by selecting suitable flow parameters. Features of heat, entropy generation, and MHD can be seen in renowned studies given in Refs. [28–39].

Up to authors' best knowledge, no efforts are yet made to explore the effects of entropy generation in MHD flows and heat transmission over a contracting surface. Therefore to full fill this gap, the goal of the current article is to inspect the impacts of entropy generation in fluid flow and heat transmission over a sheet contracting with a linear velocity. Exact solutions are attained for the considered problem and results for different parameters are elucidated through graphs. The dominance of entropy impacts due to heat transmission is noted at the surface of the contracting sheet while entropy effects due to fluid friction and magnetic field are prominent in the boundary layer region. The results attained through current study illustrate that the optimal design and the efficient performance of a thermally designed system can be improved by selecting the appropriate values of the physical parameters. This will enable us to minimize the impacts of entropy generation within the system (Fig. 1).

Fig. 1 Physical model and coordinate system

2 Mathematical formulation

Let us take into account, a two dimensional (2D) steady boundary-layer laminar flow past a contracting surface immersed in a viscous fluid in which the horizontal axis coincides with the contracting surface and runs along the converse direction of the contracting sheet. The vertical axis and contracting sheet are perpendicular to each other. A constant magnetic field strength (B_0) at a right angle to the contracting surface is implemented. Magnetic field induction is presumed negligibly small and there is no electric field present. Then the equations governing the MHD viscous flow and heat transmission are given as:

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2}{\rho} u, \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad (4)$$

The associated boundary conditions are

$$\begin{aligned} u &= U_w(x) = -ax, \quad v = v_w \quad \text{at} \quad y = 0, \\ u &\rightarrow 0 \quad \text{as} \quad y \rightarrow \infty. \end{aligned} \quad (5)$$

and

$$\begin{aligned} T &= T_w \quad \text{at} \quad y = 0, \\ T &= T_\infty \quad \text{as} \quad y \rightarrow \infty. \end{aligned} \quad (6)$$

Velocity components along horizontal and vertical directions are denoted by u and v respectively, ν is the kinematic viscosity, ρ is the density, σ is the electrical conductivity, p is the pressure, $a(> 0)$ is the shrinking constant, v_w is the suction velocity and α is the thermal diffusivity, T is fluid temperature near the boundary, T_w represents the uniform temperature on the contracting sheet and the temperature of the ambient fluid is represented by T_∞ .

By utilizing the given similarity transmutations:

$$u = axf'(\eta), \quad v = -\sqrt{av}f(\eta), \quad \eta = \sqrt{\frac{a}{\nu}}y, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}. \quad (7)$$

Then the velocity of mass transmission becomes $v_w(x) = -\sqrt{av}f(0)$ and by substituting (7) in Eqs. (2–4), the following nonlinear ODE's are obtained:

$$f''' + ff'' - f'^2 - M^2 f' = 0, \quad (8)$$

$$\theta'' + \text{Pr} f \theta' = 0, \quad (9)$$

The non-dimensional conditions for boundary are:

$$f(0) = s, \quad f'(0) = -1, \quad \text{and} \quad f'(\infty) = 0, \quad (10)$$

$$\theta(0) = 1 \quad \text{and} \quad \theta(\infty) = 0, \quad (11)$$

where $\text{Pr} = \frac{\nu}{\alpha}$, $M^2 = \frac{\sigma B_0^2}{\rho a}$ and $s = -\frac{v_w}{\sqrt{av}}$ is the mass transmission parameter with $s > 0$ for suction, $s < 0$ for injection and $s = 0$ for an impermeable surface. By integrating Eq. (3), the pressure can be obtained as

$$\frac{p}{\rho} = \nu \frac{\partial v}{\partial y} - \frac{v^2}{2} + \text{constant}. \quad (12)$$

3 Entropy generation

All thermo-dynamical systems encounter entropy generation. It is directly related to thermodynamic irreversibility. It is essential to figure out the rate of entropy generation to maximize the energy for the operational efficiency of the system. The volumetric rate of entropy generation for the flow of viscous-fluid in the existence of a magnetic field is denoted and described as [40, 41]

$$S_G = \frac{k}{T_\infty^2} \left(\frac{\partial T}{\partial y} \right)^2 + \frac{\mu}{T_\infty} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{\sigma B_0^2}{T_\infty} u^2. \quad (13)$$

In Eq. (13), the entropy generation is described in three parts.

1. Entropy generation due to heat transfer.
2. Entropy generation due to fluid friction.
3. Entropy generation due to an applied magnetic field.

By introducing the similarity variables, the entropy generation in dimensionless form takes the form

$$Ns = \frac{S_G}{S_o} = \text{Re}_L \theta'^2 + \text{Re}_L \frac{Br}{\Omega} f''^2 + \text{Re}_L \frac{Br}{\Omega} M f'^2, \quad (14)$$

$$\text{where } S_o = \frac{k(T_w - T_\infty)^2}{T_\infty^2 L^2}, \quad \Omega = \frac{T_w}{T_w - T_\infty} \quad \text{and} \quad Br = \frac{\mu U_w^2}{k(T_w - T_\infty)}$$

To check the dominance of entropy generation on heat transmission, we can introduce the irreversibility parameter Be as

$$Be = \frac{\text{Entropy generation due to heat transfer}}{\text{Total entropy generation}}. \quad (15)$$

By seeing (15), it is evident that the values of Be lie from 0 to 1. The irreversibility dominates due to heat transmission in the case when $Be > 0.5$ while $Be < 0.5$ indicates that dominance of irreversibility due to fluid friction and magnetic field. The entropy generation contributes equally to both cases when $Be = 0.5$.

4 Solution of the problem

To find the closed form solution of Eq. (8), we assume:

$$f(\eta) = A + Be^{-\beta\eta}. \quad (16)$$

Putting this relation to Eq. (8), we get the values of A and B as:

$$A = \frac{1}{\beta}, \quad B = s - \frac{1}{\beta}, \quad (17)$$

where

$$\beta = \frac{s \pm \sqrt{s^2 - 4(1 - M^2)}}{2}. \quad (18)$$

Then the solution for flow equation Eq. (8) takes the form

$$f(\eta) = s - \frac{(1 - e^{-\beta\eta})}{\beta}. \quad (19)$$

For temperature Eq. (9), a new variable $\xi = \text{Pr} \frac{e^{-\beta\eta}}{\beta^2}$ is proposed. By substitution of this variable in Eq. (9), the equation becomes

$$\xi \frac{d^2 \theta}{d\xi^2} + (1 - \gamma - \xi) \frac{d\theta}{d\xi} = 0, \quad (20)$$

with boundary conditions

$$\theta \left(\frac{\text{Pr}}{\beta^2} \right) = 1, \quad \theta(0) = 0. \quad (21)$$

Here $\gamma = \frac{\text{Pr}(\beta s - 1)}{\beta^2}$. The solution of Eq. (20) yields

$$\theta(\xi) = c_1 + c_2 \Gamma(\gamma, -\xi), \quad (22)$$

where $\Gamma(a, x)$ is the incomplete Gamma function.

On applying the boundary conditions (21) on Eq. (22), we have

$$\theta(\xi) = \frac{\Gamma(\gamma, 0) - \Gamma(\gamma, -\xi)}{\Gamma(\gamma, 0) - \Gamma\left(\gamma, -\frac{\text{Pr}}{\beta^2}\right)}. \quad (23)$$

In term of variable η , Eq. (23) has the form

$$\theta(\eta) = \frac{\Gamma(\gamma, 0) - \Gamma\left(\gamma, -\frac{\text{Pr}}{\beta^2} e^{-\beta\eta}\right)}{\Gamma(\gamma, 0) - \Gamma\left(\gamma, -\frac{\text{Pr}}{\beta^2}\right)}. \quad (24)$$

By taking derivatives of Eqs. (19) and (24) and using them in (14–15), the expressions for N_s and B_e are obtained.

5 Results and discussion

Analysis of entropy generation (EG) impacts on boundary layer flow and heat transfer of hydro-magnetic viscous fluid past a contracting sheet is presented. Exact solutions are obtained for non-dimensional momentum, and heat transfer equations in the considered problem. A comparison of numerically obtained results is made with the existing literature. Table 1 shows the comparison of different values of $f''(0)$ of present study with those of Raftari [42] when $M=2$ and $s=1$. They presented an approximate analytical solution of MHD viscous flow over a shrinking sheet by using a two-parameter homotopy Perturbation method (HPM). Comparison of numerical values of $f''(0)$ for various values of s when $M^2 = 0.25$ for our results and Ali et al. [43] is given in Table 2. They investigated MHD boundary layer flow and heat transmission due over a shrinking sheet with prescribed surface heat flux in the presence of applied magnetic field. However, entropy effects are not discussed in the above mentioned studies. We found that our results are in good agreement with the above mentioned studies which authenticate the validity and precision of current study.

By observing the momentum solution (19), it is noticed that physical solution exists only for positive values of β , i.e., $\beta > 0$. Moreover, it can be observed from (18) that two solution branches occur for the values of M between 0

Table 2 Comparison of numerical values of $f''(0)$ for various values of s when $M^2 = 0.25$

s	Ali et al. [43]	Present
1.75	1.0000	1.0000
1.80	1.1449	1.1449
1.85	1.2500	1.2500
1.90	1.3405	1.3405

and 1 while only one solution $\beta = s$, ($s > 0$), is possible for $M = 1$. Moreover, there is one solution associated to '+' sign branch for $M > 1$. Since we are interested in examining the entropy generation at higher values of Hartmann number, we have taken $M \geq 1$. This is because entropy generation impacts are more remarkable for greater values of M .

Figure 2 illustrates the impacts of mass transmission parameter s on $f'(\eta)$. It is noticeable that the velocity increases with the rise in the value of s . However, a decline in boundary layer thickness is observed as mass transfer parameter becomes large. The impacts of M on velocity profile $f'(\eta)$ are elucidated in Fig. 3. An increase in $f'(\eta)$ is observed by increasing the value of Hartmann number which is mainly due to the Lorentz force arising during the flow of an electrically conducting fluid. In Figs. 4, 5 and 6, the impacts of s , M and Pr on $\theta(\eta)$ are presented. In all cases mentioned above-obtained results show a decline in thermal boundary-layer. The effects of all pertinent parameters in the considered problem N_s are shown in Figs. 7, 8, 9 and 10. By observing Figs. 7 and 8, a rise in N_s with rising in s and M is observed near the shrinking surface. However, the situation is reversed near the boundary in both cases. In the region far from the boundary, the SG rate is very low.

Fig. 2 Mass transfer parameter (s) effecting velocity profile [$M=2$]

Table 1 Comparison of the values of $f''(0)$ present study with other applied methods when $M=2$ and $s=1$

	Raftari and Yildirim [42] $f''(0)$	Present $f''(0)$
$M=2, s=1$	2.30277	2.30277

Fig. 3 Hartmann number (M) effecting velocity profile [$s=1$]

Fig. 5 Hartmann number (M) effecting temp. Profile (T) [$s=1$, $Pr=1$]

Fig. 4 Impacts of mass transfer parameter (s) on temp. Profile [$M=2$, $Pr=1$]

Fig. 6 Prandtl number (Pr) effecting temp. Profile [$s=1$, $M=2$]

In Fig. 9, it is elucidated that for large Prandtl number, the entropy generation rate is more significant in comparison with low Prandtl number. Figure 10 depicts the effects of Br/Ω on entropy generation. Near the contracting surface, N_s is high for higher values of Br/Ω as compared to low values. This is due to the reason that viscous effects are more prominent for large group parameter values.

In order to determine whether entropy generation due to heat transmission dominates over entropy generation due to fluid friction and magnetic field or vice versa, Figs. 11, 12, 13 and 14 are drawn. Figure 10

elucidates that entropy influences because of these parameters become slightly less with a rise in mass transfer parameter s at the shrinking sheet surface. However, these impacts become fully dominant just a little distance far from the sheet and remain dominant in the region of boundary-layer. In Fig. 12, it is evident entropy impacts of these parameters are dominant and this dominance increase with the rise in the value of M . On the other hand, Fig. 13 shows that with a rise in values of Pr , heat transfer entropy effects become more dominant at the surface of the contracting

Fig. 7 Mass transfer parameter (s) on Ns [$M=2$, $Pr=1$, $Re_L=3$, $Br/\Omega=1$]

Fig. 9 Impact of Prandtl number (Pr) on Ns [$s=1$, $M=2$, $Re_L=3$, $Br/\Omega=1$]

Fig. 8 Impact of Hartmann number (M) on Ns [$s=1$, $Pr=1$, $Re_L=3$, $Br/\Omega=1$]

Fig. 10 Impact of group parameter (Br/Ω) on Ns [$s=1$, $M=2$, $Re_L=3$, $Pr=1$]

sheet, while in the boundary-layer region, the dominance of fluid friction and magnetic field entropy effects is observed. Moreover, it is noticed that for higher values of Prandtl number, entropy impacts because of fluid friction and magnetic field dominate earlier in the boundary-layer region as compared to low values. Figure 14 illustrates that entropy impacts of fluid friction and magnetic field dominate because of the rise in values of Br/Ω .

6 Concluding remarks

In the present article, entropy generation (EG) impacts on boundary layer flow and heat transfer of hydro-magnetic viscous fluid past a contracting sheet are presented. A comprehensive study is performed to compute the exact solution for relevant parameters and the outcomes are elucidated graphically. The major findings of our study are as under.

Fig. 11 Impact of mass transfer parameter (s) on Bejan number (Be) [$Br/\Omega = 1$, $M = 2$, $Re_L = 3$, $Pr = 1$]

Fig. 13 Impact of Prandtl number (Pr) on Bejan number (Be) [$Br/\Omega = 1$, $s = 1$, $Re_L = 3$, $M = 2$]

Fig. 12 Impact of Hartmann number (M) on Bejan number (Be) [$Br/\Omega = 1$, $s = 1$, $Re_L = 3$, $Pr = 1$]

Fig. 14 Impact of group parameter (Br/Ω) on Bejan number (Be) [$Br/\Omega = 1$, $M = 2$, $Re_L = 3$, $Pr = 1$]

- Thermal boundary layer thickness is decreased with the rise in values of Pr , M and s .
- The entropy generation number (N_s) increases with magnetic parameter M , and group parameter Br/Ω .
- The influence of entropy due to fluid friction and magnetic field fully dominates near the stretching surface in case of M and Br/Ω while for mass transfer parameter (s), it is less at the shrinking surface by increasing s . However, the trend is totally opposite in the boundary layer region.

- Entropy impacts due to heat transfer are prominent in the regime of contracting sheet.
- The local entropy generation number increases with group parameter Br/Ω , mass transfer parameter s and Hartmann number M .
- A boost in the entropy generation number is noted with the rise in Prandtl number (Pr).
- The analysis made through current study illustrates that the optimal design and the efficient performance of a thermally designed system can be improved by select-

ing the appropriate values of the physical parameters. This will enable us to minimize the impacts of entropy generation within the system.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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