Natural nonlinear convection of dusty fluid in a suspension of multi-wall carbon nanotubes nanoparticles with different temperature of water and suction

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Abstract

This study addresses the nonlinear free convective flow of a conducting dusty nanofluid (multi wall carbon nanotubes + water + micrometer sized dust particles) over a stretched cylinder at different temperatures (10 °C and 50 °C) in the presence of Cattaneo–Christov heat flux, heat source. The mainly arising equations of motion such as continuity, momentum and energy are simplified with the suitable dimensionless variables and solved numerically using the Runge–Kutta method. A numerical simulation is executed with inclusion of multi-wall carbon nanoparticles and dust particles into different temperatures (10 °C and 50 °C) of base fluid water. Calculations for the wall friction coefficient and Nusselt number are carried out and demonstrated in tables. To authenticate the attained results we have offered the comparison with the already published results. A few graphical pictures are plotted to scrutinize the physical insight of the numerically modeled problem for the several flow parameters and found that the multi-walled carbon nanotubes + water at 10 °C dusty nanofluid shows better heat and mass transfer rate compared to temperature at 50 °C. Due to nonlinear convection the friction factor is reduced in the presence of the magnetic field. The Biot number encourages the wall friction, heat and mass transfer rates of both cases.

Keywords Carbon nanotubes · Suction · Nonlinear convection · Dust particles · Heat source · Cattaneo–Christov heat flux · Cylinder

List of symbols

$\bar{r}$	Coordinate measured in the radial direction	P	Pressure
$\bar{u}, \bar{v}$	Velocity components of fluid phase in $\bar{x}, \bar{r}$ directions (m/s ²)	N	Number density of particles
$\bar{u}_p, \bar{v}_p$	Velocity components of particle phase in $\bar{x}, \bar{r}$ directions (m/s ²)	α	Mass concentration of dust particles
g	Acceleration due to gravity (m/s ²)	l	Characteristic length
ν	Kinematic viscosity (m/s ²)	U_0	Constant
f, f'	Dimensionless velocities of fluid phase	d	Radius of the dust particle
F, F'	Dimensionless velocities of particle phase	K	Stokes resistance
T	Temperature of the fluid (K)	ζ	Similarity variable
T_p	Temperature of the dust particle (K)	σ	Electrical conductivity (S)
k_f	Thermal conductivity (W/m K)	β_T	Fluid particle interaction parameter for temperature
		β_v	Fluid particle interaction parameter for velocity
		Q_0	Heat generation/absorption coefficient

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γ	Curvature parameter
Q_H	Heat generation/absorption parameter
$(\rho\beta)_{nf}$	Volumetric thermal expansion coefficient
λ_1	Buoyancy parameter
λ_2	Non-linear convection parameter due to temperature
θ	Dimensionless temperature (K)
θ_p	Dimensionless temperature for the particle (K)
ϕ	Nano particle volume fraction
ϕ_d	Dust particle volume fraction
ρ_{nf}	Density of the nanofluid (kg/m ³)
ρ_f	Density of the base fluid (kg/m ³)
ρ_p	Density of the particle phase (kg/m ³)
ρ_s	Density of the solid nanoparticles (kg/m ³)
μ_f	Dynamic viscosity of the base fluid (N s/m ²)
μ_{nf}	Dynamic viscosity of the nanofluid (N s/m ²)
c_{pf}	Specific heat capacity at constant pressure of the fluid (J/kg K)
c_{mf}	Specific heat capacity at constant pressure of the dust particles (J/kg K)
k_{nf}	Thermal conductivity of nanofluid (W/m K)
$(\rho c_p)_{nf}$	Effective heat capacity of nanofluid (kg/m ³ K)
$(\rho c_p)_p$	Effective heat capacity of the particle medium (kg/m ³ K)
τ_v	Relaxation time of the dust particles
α_{nf}	Diffusion coefficient (m ² /s)
ν_{nf}	Kinematic viscosity (m ² /s)
Pr	Prandtl number
Ec	Eckert number
Λ	Thermal relaxation time
δ	Specific heat ratio
M	Magnetic parameter
C_f	Skin friction coefficient
Nu_x	Local Nusselt number
Re_x	Local Reynolds number

Subscripts

f	Fluid
w	Condition at the wall
∞	Condition at the free stream
nf	Nanofluid

1 Introduction

Mixed convection plays significant role in engineering improvements. The amalgamation of free and forced convection ascend in numerous transference approaches for example solar collectors, nuclear reactors, electronic equipment, biological flow modeling etc. The mixed convection is noteworthy after the buoyancy force unpaid to the temperature and concentration difference among the free stream and the solid surface turn into enormously,

which in order noticeably interrupt the boundary layer. In the overhead pondered works, a forced/free or mixed convection control was considered. Once there is thermal and concentration variation through the boundary and ambient fluid is extensively great the nonlinear density temperature and concentration dissimilarity in the buoyancy force term can't be ignored as the circulation of the flow, mass and heat transmission is momentarily exaggerated by it. In view of this uses, of late the authors [1–6] are studied the by considering nonlinear convection in the flow with various physical flow configurations. Lately, the investigators are attracted in usage of Cattaneo–Christov heat flux model in heat transportation mechanism ever since the Fourier's [7] heat conduction model create parabolic energy equation in the temperature gradient. Later on, Cattaneo [8] improved the Fourier's heat conduction classical model by including the relaxation time for heat flux. Further, Christov [9] reformed via Oldroyd's upper-convected derivative. In several literatures we found [10–12] by including the Cattaneo–Christov for analyzing the heat transfer mechanism. With that they found out-come as, the non-Fourier's heat flux model improves the immediate heating.

Depletion of energy has radically amplified throughout the modern era due to the growth in global world residents, the enormous industrial progresses, and exploitation of the transportation and expanding usage of energy in daily human activities. Utmost the energy spent in the world is created as a heat. Hence, lot of struggles have been commenced to improvise the thermal enactment of heat transportation developments as well as services consequently diminishing the expended energy. In the last period optimization of heat exchangers were done by adapting the metal type, interchange surface or isolation, among other parameters. In the present-day period, exertions are focused on the enrichment of the thermal possessions of occupied fluids. Nanofluids are dispersal of solid or metal or non-metal nanometer sized particles like graphene nanoparticles, copper (Cu), silver (Ag), alloy nanoparticles, aluminium oxide (Al₂O₃), carbon nanotubes (CNTs) and copper oxide (CuO) in a convectional occupied liquid such as lubricant oil, water and ethylene glycol etc. The area of nanofluid attained greatly consideration owing to its great presentation in industrial, manufacturing and sciences like solar energy, biomedicine, chilling of electronic equipment, heat exchangers, shrinking and evaporating systems, innumerable chemical developments etc. Allured by these uses researchers [13–20] inspected the domain of nanofluids experimentally and numerically. Later on, Nagendramma et al. [21] studied the magnetohydrodynamic double stratified hyperbolic fluid past a stretched permeable cylinder. Blasius and Sakiadis radiative flow characteristics in a suspension of carbon

nanotubes nanoparticle with CCHF (Cattaneo–Christov heat flux) was investigated by Babu et al. [22] and concluded that the thermal relaxation controls the thermal boundary. Nayak et al. [23] examined the partial slip and internal heating on dissipative radiated tangent nanofluid over a permeable Riga plate using Bungiorno nanomodel. The unsteady two phase magnetohydrodynamic nanofluid over porous stretched wedge with inclusion of tangent non-Newtonian fluid is considered by Mahdy and Chamkha [24]. Later on, the authors [25–32] analyzed the heat transfer characteristics of various flow configurations such as plate, shrinking surface and squeezing plates in a suspension of nanoparticles like copper, titanium, silver nanoparticle and combination of fluids (Fe_2O_3 , Al_2O_3 and graphene) and highlighted that the nanoparticles improves the conducting nature of the flow. A few investigators have deliberated the impact of magnetic field on the heat and mass transport enactment owing to its importance in a number of technological, industrialized and the energy generation viewpoint. Magnetohydrodynamic pumps are previously in usage in the chemical, energy technology in the progression of propelling electrically conducting fluids, power generation etc. The magnetization improves the rate of refrigeration in metallurgical processes. Due to this, recently the academicians [33–40] published their results with the application of magnetohydrodynamic.

Motivated from above mentioned literature, the present study is the extension of Mamatha et al. [10] and we have incorporated the temperature and concentrations in nonlinear form, convective conditions and suction effect with carbon nanotubes. The arising is governing system is solved with numerically and verified by validating present study with already existing study.

2 Mathematical formulation

In this study we considered, incompressible nonlinear thermal and diffusion convection of dusty nanofluid [water at 10°C + single-wall carbon nanotubes and water at 50°C + dust particles (micrometer in size, spherical in shape, uniform in size)] flows over a stretched cylinder of radius $\bar{R}$ with suction, heat source or sink and Cattaneo–Christov heat flux and magnetic field (see Fig. 1). The

Fig. 1 Physical model of the flow configuration (Ref. [10])

$\bar{x}$ -axis is considered along the axial direction of the cylinder whereas $\bar{r}$ -axis is normal to the axis of the cylinder. Velocity of the stretching cylinder is $\bar{u}_w(\bar{x}) = \frac{\bar{x}U_0}{l}$ where U_0 —is constant and l —characteristic length. T_w , C_w and T_∞ , C_∞ represents surface and ambient temperature and concentration respectively. The induced magnetic field is neglected in this study due to less Reynolds's number assumption.

The flow equations corresponding to this assumed system are given by (Refs. [2, 10])

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\bar{v}}{\bar{r}} \frac{\partial \bar{r}}{\partial \bar{r}} = 0 \quad (1)$$

$$\frac{\partial \bar{u}_p}{\partial \bar{x}} + \frac{\partial \bar{v}_p}{\partial \bar{r}} + \frac{\bar{v}_p}{\bar{r}} \frac{\partial \bar{r}}{\partial \bar{r}} = 0 \quad (2)$$

$$\begin{aligned} \rho_{nf}(1 - \phi_d) \left(\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{r}} \right) &= (1 - \phi_d) \bar{\mu}_{nf} \left(\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}}{\partial \bar{r}} \right) \\ &- \sigma_f B_0^2 \bar{u} + KN(\bar{u}_p - \bar{u}) \\ &+ (g[(\rho\beta_0)_{nf}(T - T_\infty) + (\rho\beta_2)_{nf}(C - C_\infty) \\ &+ ((\rho\beta_1)_{nf}(T - T_\infty)^2 + (\rho\beta_3)_{nf}(C - C_\infty)^2)]) \end{aligned} \quad (3)$$

$$\left(\bar{u}_p \frac{\partial \bar{u}_p}{\partial \bar{x}} + \bar{v}_p \frac{\partial \bar{u}_p}{\partial \bar{r}} \right) = \frac{K}{m} (\bar{u} - \bar{u}_p) \quad (4)$$

$$\begin{aligned} (\rho c_p)_{nf} \left(\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{r}} \right) &+ \lambda \left(\bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} \frac{\partial T}{\partial \bar{r}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{r}} \frac{\partial T}{\partial \bar{r}} + \bar{u}^2 \frac{\partial^2 T}{\partial \bar{x}^2} + \bar{v}^2 \frac{\partial^2 T}{\partial \bar{r}^2} \right. \\ &\left. + \bar{u} \frac{\partial \bar{u}}{\partial \bar{r}} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{x}} \frac{\partial T}{\partial \bar{r}} + 2\bar{v} \bar{u} \frac{\partial^2 T}{\partial \bar{r} \partial \bar{x}} \right) = k_{nf} \left(\frac{\partial^2 T}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial T}{\partial \bar{r}} \right) \\ &+ Q_0(T - T_\infty) - \frac{Nc_{pf}}{\tau_T} (T - T_p) + \frac{N}{\tau_v} (\bar{u}_p - \bar{u})^2 \end{aligned} \quad (5)$$

$$\begin{aligned}
 \left(\bar{u}_p \frac{\partial T_p}{\partial \bar{x}} + \bar{v}_p \frac{\partial T_p}{\partial \bar{r}} \right) &= \frac{c_{pf}}{c_{mf} \tau_T} (T - T_p) & (6) \\
 u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial r} &= D_B \frac{\partial^2 C}{\partial r^2} + \frac{\rho_p}{\rho_c} (C_p - C) & (7) \\
 u_p \frac{\partial C_p}{\partial x} + v_p \frac{\partial C_p}{\partial r} &= \frac{1}{\tau_c} (C - C_p) & (8)
 \end{aligned}$$

With boundary conditions:

$$\text{at } \bar{r} = \bar{R}, \bar{u} = \bar{u}_w, \bar{v} = \bar{v}_w, k_f \frac{\partial T}{\partial \bar{r}} = h_f (T_w - T), C = C_w$$

$$\text{as } \bar{r} \rightarrow \infty, \bar{u} \rightarrow 0, \bar{u}_p \rightarrow 0, \bar{v}_p \rightarrow \bar{v}, T \rightarrow T_\infty, C \rightarrow C_\infty, C_p \rightarrow C_\infty$$

The nanofluid thermo physical properties are (Babu et al. [22])

$$\left. \begin{aligned}
 \rho_{nf} - \phi \rho_s &= (1 - \phi) \rho_f, \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, (\rho c_p)_{nf} - \phi (\rho c_p)_s = (1 - \phi) (\rho c_p)_f, \\
 (\rho \beta)_{nf} &= (1 - \phi) (\rho \beta)_f + \phi (\rho \beta)_s, \frac{k_{nf}}{k_f} = \frac{k_s + (n - 1) k_f - \phi (n - 1) (k_f - k_s)}{k_s + (n - 1) k_f + \phi (k_f - k_s)},
 \end{aligned} \right\} \quad (10)$$

Similarity transformations for the assumed flow problem are

$$\left. \begin{aligned}
 \bar{u} &= \frac{U_0 \bar{x}}{l} f'(\zeta), \bar{v} = -\frac{1}{\bar{r}} \left(\frac{v_f U_0}{l} \right)^{0.5} \bar{R} f(\zeta), \bar{v}_p = -\frac{1}{\bar{r}} \left(\frac{v_f U_0}{l} \right)^{0.5} \bar{R} F(\zeta), \frac{T - T_\infty}{(T_w - T_\infty)} = \theta(\zeta), \\
 \zeta &= \left(\frac{\bar{r}^2 - \bar{R}^2}{2 \bar{R}} \right) \left(\frac{U_0}{v_f l} \right)^{0.5}, \frac{T_p - T_\infty}{(T_w - T_\infty)} = \theta_p(\zeta), \frac{C - C_\infty}{(C_w - C_\infty)} = G(\zeta), \frac{C_p - C_\infty}{(C_w - C_\infty)} = G_p(\zeta), \bar{u}_p = \frac{U_0 \bar{x}}{l} F'(\zeta),
 \end{aligned} \right\} \quad (11)$$

Equation (11) automatically satisfies the continuity Eqs. (1) and (2). The Eqs. (3)–(9) are transformed as

$$\begin{aligned}
 &\frac{(1 - \phi_d)}{(1 - \phi)^{2.5}} \left[\frac{d^3 f}{d\zeta^3} (2\zeta\gamma + 1) + 2\gamma \frac{d^2 f}{d\zeta^2} \right] \\
 &+ \left(1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right) \right) (1 - \phi_d) \left(f(\zeta) \frac{d^2 f}{d\zeta^2} - \left(\frac{df}{d\zeta} \right)^2 \right) - M \frac{df}{d\zeta} \\
 &+ \lambda_1 \left((1 - \phi) + \left(\frac{\phi (\rho \beta)_s}{(\rho \beta)_f} \right) \right) (\theta(\zeta) + \Lambda_1 \theta(\zeta)^2 + \Delta (G(\zeta) + \Lambda_2 G(\zeta)^2)) \\
 &+ \left[\frac{dF}{d\zeta} - \frac{df}{d\zeta} \right] (\alpha \beta_v) = 0
 \end{aligned} \quad (12)$$

$$\frac{d^2 F}{d\zeta^2} (F) - \left(\frac{dF}{d\zeta} \right)^2 + (\beta_v) \left(\frac{df}{d\zeta} - \frac{dF}{d\zeta} \right) = 0 \quad (13) \quad \left(\frac{dG_p}{d\zeta} \right) F(\zeta) - \left(\frac{dF}{d\zeta} \right) G_p(\zeta) + \beta_c [G(\zeta) - G_p(\zeta)] = 0 \quad (17)$$

The transformed boundary conditions:

$$\left. \begin{aligned} \frac{df}{d\zeta}(0) = 1, f(0) = S \theta'(0) = -Bi(1 - \theta(0)), \theta(\infty) = 0, \theta_p(\infty) = 0, G(0) = 0, \\ F(\infty) = -f(\infty), \frac{dF}{d\zeta}(\infty) = 0, \frac{df}{d\zeta}(\infty) = 0, G(\infty) = 0, G_p(\infty) = 0 \end{aligned} \right\} \quad (18)$$

The arising set of governing equations are depending on the following parameters. In above equations Δ is buoyancy ratio, Λ_1 is the non-linear thermal convection, Λ_2 is the non-linear concentration convection, Bi is Biot number and S is suction parameter.

$$\begin{aligned} \beta_v = \frac{1}{a\tau_v}, \gamma = \left(\frac{v_f l}{aR^2} \right)^{\frac{1}{2}}, Bi = \frac{h_f r}{k_f R} \left(\frac{v_f l}{c} \right), \alpha = \frac{\rho_p}{\rho_f}, Pr = \frac{(\mu c_p)_f}{k_f}, M = \frac{\sigma B_0^2}{a\rho_f}, K = 6\pi\mu_f d, \\ \tau_v = \frac{m}{K}, Ec = \frac{\bar{U}_w^2}{(T_w - T_\infty)(c_p)_f}, \rho_p = Nm, \Lambda = \frac{\lambda a}{l}, \delta = \frac{c_{pf}}{c_{mf}}, Q_H = \frac{lQ_0}{a(\rho c_p)_f}, \\ \lambda_1 = \frac{Gr_x}{Re_x^2}, Gr_x = \frac{g\beta_f l^2 (T_w - T_\infty)}{U_0^2 x}, S = -\frac{v_w r}{R(v_f c/l)}, \beta_c = \frac{1}{b\tau_c}, Le = \frac{v_f}{D_B}, \\ \beta_t = \frac{1}{a\tau_T}, \Lambda_1 = \frac{\beta_1}{\beta_0} (T_w - T_\infty), \Lambda_2 = \frac{\beta_3}{\beta_2} (C_w - C_\infty), \Delta = \frac{\beta_2}{\beta_0} \left(\frac{C_0 - C_\infty}{T_0 - T_\infty} \right), \end{aligned} \quad (19)$$

Shear stress coefficient C_f , heat transfer and mass transfer coefficient Nu_x and Sh_x are given by

$$\begin{aligned} C_f = \left(\frac{\mu_{nf}}{U_w^2 \rho_f} \right) \left(\frac{\partial \bar{u}}{\partial \bar{r}} \right)_{\bar{r}=\bar{R}}, Nu_x = \left(\frac{k_{nf}}{k_f (T_w - T_\infty)} \right) \left(\frac{\partial T}{\partial \bar{r}} \right)_{\bar{r}=\bar{R}}, Sh = \frac{xj_w}{D_B (C_w - C_\infty)}, j_w = -D_B \left(\frac{\partial G}{\partial \bar{r}} \right)_{\bar{r}=\bar{R}}, \\ C_f (Re_x)^{0.5} = (1 - \phi)^{-2.5} \frac{d^2 f(0)}{d\zeta^2}, Nu_x (Re_x)^{-0.5} = -\left(\frac{k_{nf}}{k_f} \right) \left(\frac{\partial \theta(0)}{\partial \zeta} \right), Sh_x (Re_x)^{-0.5} = -\frac{\partial G(0)}{\partial \zeta} \end{aligned} \quad (20)$$

where $Re_x = \left(\frac{\bar{U}_w \bar{x}}{\nu_f} \right)$.

3 Method of solution

The nonlinear differential equations (ODEs) (12)–(17) with the boundary constraints (18) are solved numerically R-K with shooting method. Initially, the set of nonlinear ODEs

converted to 1st order differential equations, by using the following process:

$$\begin{aligned} f = y_1, f' = y_2, f'' = y_3, F = y_4, F' = y_5, \theta = y_6, \theta' = y_7, \\ \theta_p = y_8, G = y_9, G' = y_{10}, G_p = y_{11} \end{aligned} \quad (21)$$

$$y_3' = \left(\frac{(1 - \phi)^{2.5}}{(1 - \phi_d)(2\zeta\gamma + 1)} \right) \left(\left(1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right) \right) (y_2^2 - y_1 y_3) - \frac{(1 - \phi)^{2.5}}{(1 - \phi_d)} 2\gamma y_3 + M y_2 \right) \\ - \alpha \beta_v (y_5 - y_2) - \left((1 - \phi) + \left(\frac{\phi(\rho\beta)_s}{(\rho\beta)_f} \right) \right) \lambda_1 (y_6 + \lambda_2 y_8) \quad (22)$$

Table 1 Validation of the local Nusselt number $[-\theta'(0)]$ for the case of $\Lambda = \Lambda_1 = \Lambda_2 = M = A = \phi = \phi_d = Q_H = \beta_c = Bi = 0, l = 1$

Pr	Mamatha et al. [10]	Present studies
10.0	4.7968	4.796
0.72	1.0885	1.088
1.0	1.3333	1.333

Table 2 Thermo physical properties of base fluids (water 10 °C and 50 °C) and SWCNT nanoparticles

Thermo physical properties	SWCNT	Water	
		10 °C	50 °C
ρ (kg m ⁻³)	2600	999.6	987.7
c_p (J kg ⁻¹ K ⁻¹)	425	4090	4066
k (W m ⁻¹ K ⁻¹)	6600	0.5884	0.6440
Prandtl number	–	8.80	3.35

Fig. 2 Effect of M on temperature profiles of fluid and dust phase

$$y'_5 = (y_5^2 - \beta_v(y_2 - y_5)) \left(\frac{1}{y_4} \right) \quad (23)$$

$$y'_7 = \frac{1}{\left(\frac{k_{nf}}{k_f} \frac{1}{Pr} (1 + 2\zeta\gamma) + \Lambda y_1^2 \right)} \left(\begin{array}{l} \Lambda y_7 y_1 y_2 - Q_H y_6 - y_1 y_7 \left(1 - \phi + \phi \frac{(\rho c_p)_s}{(\rho c_p)_f} \right) \\ -\alpha \beta_t (y_8 - y_6) - \alpha Ec \beta_v (y_5 - y_2)^2 - \frac{k_{nf}}{k_f} \frac{1}{Pr} 2\gamma y_7 \end{array} \right) \quad (24)$$

$$y'_8 = (\gamma \beta_t (y_8 - y_6)) (1/y_4) \quad (25)$$

$$y'_{10} = -Le Pr (y_{10} y_1 + l \beta_c (y_{11} - y_9)) \quad (26)$$

Fig. 3 Effect of M on velocity profiles of dust and fluid phase **Fig. 4** Effect of M on concentration profiles of fluid and dust phase

$$y'_{11} = (y_{11} y_5 - \beta_c (y_9 - y_{11})) (1/y_4) \quad (27)$$

With boundary conditions as

$$\begin{array}{l} y_1 = S, y_2 = 1, y_7 = -Bi(1 - y_6), y_9 = 1, \text{ at } \zeta \rightarrow 0 \\ y_2 = 0, y_5 = 0, y_4 = -y_1, y_6 = 0, y_8 = 0, y_9 = 0, y_{11} = 0, \text{ at } \zeta \rightarrow \infty \end{array} \quad (28)$$

Fig. 5 Effect of ϕ on velocity profiles of dust and fluid phase

Fig. 7 Effect of ϕ_d on temperature profiles of dust and fluid phase

Fig. 6 Effect of ϕ on temperature profiles of fluid and dust phase

Fig. 8 Effect of ϕ_d on velocity profiles of fluid and dust phase

We guess the values of $y_3(0), y_5(0), y_7(0), y_8(0), y_{10}(0)$ which are not given at the initial conditions. The Eqs. (21)–(28) are integrated by taking the help of Runge–Kutta method with the successive iterative step length is 0.01. The correctness of the hypothetical values is tested by equating the considered values y_2, y_6, y_8 at $\zeta = \zeta_{\max}$ with their assumed values at $\zeta = \zeta_{\max}$. If there is any variance occur the method is sustained up to the vital worthy values. Alternatively, we are used the shooting technique to acquire the precisely found the initial values of $y_3(0), y_5(0), y_7(0), y_8(0), y_{10}(0)$. This method is repeated till the difference among the considered numerical value and the constraint is specified at $\zeta = \zeta_{\max}$ is within the quantified degree of precision 10^{-5} .

4 Interpretation of the results

The non-linear coupled differential Eqs. (12)–(17) along with the constraints (18) are computed numerically for solutions using RK method. In this analysis the values of the material parameters are considered as $\alpha = Q_H = \beta_v = \beta_c = \beta_t = 0.5, \phi = \phi_d = 0.05, \Lambda = 0.3, Ec = 0.1, M = 1, \gamma = 0.2, \Delta = 0.5, \delta = 1, \lambda_1 = 0.5, \Lambda_1 = 0.3, \Lambda_2 = 0.3, S = 0.1, Bi = 0.2, Le = 0.2, l = 0.2$. These numeric values were common in entire study apart from the assorted values as portrayed in relevant figures and tables. To assess the accuracy of the obtained data, comparative study is presented in Table 1. The thermo physical properties of nano particle and the base fluid water are presented in Table 2. The ultimate aim

Fig. 9 Effect of Λ on temperature profiles of dust and fluid profiles

Fig. 11 Effect of S on velocity profiles of dust and fluid phase

Fig. 10 Effect of Bi on temperature profiles of fluid and dust phase

Fig. 12 Effect of S on temperature profiles of fluid and dust phase

of this section is to survey on the physical behavior of fluid and dust phase velocity ($f'(\zeta)$, $(F(\zeta))$, concentration ($G(\zeta)$, $(G_p(\zeta))$ and temperature ($\theta(\zeta)$, $(\theta_p(\zeta))$ for the magnetic parameter (M), volume fraction of nanoparticle, volume fraction of dust particles, thermal relaxation (Λ), Biot number, Suction parameter, particle interaction parameters (β_v , β_t and β_c), heat source or absorption (Q_H) and non-linear convection parameter due to concentration (Λ_2). For those trends are plotted at the end from Figs. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21 and 22. Numerical values of wall friction coefficient ($C_f(Re_x)^{0.5}$), Sherwood $Sh_x(Re_x)^{-0.5}$ and Nusselt ($Nu_x(Re_x)^{-0.5}$) numbers ($-\theta(0)$) is presented in Table 3 and 4.

Figures 2, 3, and 4 elucidate the influence of magnetic field on the temperature, velocity and concentration fields ($\theta(\zeta)$, $(\theta_p(\zeta))$, $(f'(\zeta))$, $(F(\zeta))$ and $(G(\zeta))$, $(G_p(\zeta))$. As anticipated fluid phase and dust phase velocity declines and concentration profiles are remarkably increases for higher values of M . But qualitatively mixed performance is acknowledged for the temperature profiles. Whereas in dust phase temperature profiles improved due to conducting nature of dust particles. Quite interesting to note that there is a noteworthy difference in the concentration, velocity & temperature profiles in both the fluid and dust phase when the temperature is increased from 10 to 50 °C. Whereas the concentration and velocity profiles are higher in 50 °C compared to 10 °C. The velocity and temperature of both fluid and dust phase profiles are improved with

Fig. 13 Effect of S on concentration profiles of dust and fluid phase

Fig. 15 Effect of β_t on temperature profiles of dust and fluid phase

Fig. 14 Effect of β_t on velocity profiles of fluid and dust phase

Fig. 16 Effect of β_c on concentration profiles of fluid and dust phase

improving values of nanoparticle volume fraction, these are plotted in Figs. 5 and 6. The influence of dust particle volume fraction is plotted in Figs. 7 and 8. It displays the reduction in velocity profiles and mixed sense in thermal boundary layer. As expected dust particles volume fraction encourages the conductivity nature due to this we saw enhancement in dust phase temperature field. Impact of increasing in thermal relaxation Λ on temperature fields $(\theta(\zeta))$, $(\theta_p(\zeta))$ is depicted in Fig. 9. The dust phase temperature profiles $(\theta(\zeta))$, $(\theta_p(\zeta))$ improved with Λ and fluid phase shows mixed performance in temperature from 10 to 50 °C. But it is observed that when the temperature is raised to 10 °C $\theta(\zeta)$ profiles shows lower boundary layer thickness where as in dust phase temperature field $\theta_p(\zeta)$ is increased the boundary layer thickness. Therefore, we

conclude that the lower temperatures improve the thermal boundary layer in case of dust phase temperature. Improvement in Biot number Bi shows higher temperature profiles in fluid phase, but mixed performance in dust phase which is witnessed in Fig. 10. Initially, $\theta_p(\zeta)$ profiles show decrement near the boundary but as $\zeta \geq 1.2$ thermal boundary thickness improved. Figures 11, 12, and 13 show the effect of suction on velocity, temperature and concentration fields for both the fluid and dust phases. It noticed that the velocity profiles are improved and temperature shows mixed performance, concentration display the reduction profiles. As expected rising values of suction boost the pressure force on the flow due to this improves the velocity, whereas due to nonlinear convection depreciation in concentration profiles. Due to ascendancy of

Fig. 17 Effect of β_v on velocity profiles of dust and fluid phase

Fig. 19 Effect of Q_H on temperature profiles of dust and fluid phase

Fig. 18 Effect of β_v on temperature profiles of fluid and dust phase

Fig. 20 Effect of Λ_2 on temperature profiles of fluid and dust phase

thermal dust particles interaction parameter reduces the velocity and shows dual behavior in temperature (fluid phase is improved and dust phase is decreased), these are displayed in Figs. 14 and 15. The effect of concentration particle interaction on concentration field ($G(\zeta)$), ($G_p(\zeta)$) is shown in Fig. 16. Due to dust phase concentration is increased and fluid phase concentration is reduced with β_c . Figs 17 and 18 illustrated the effect of β_v on ($f'(\zeta)$), ($F(\zeta)$), ($\theta(\zeta)$), ($\theta_p(\zeta)$) profiles. The velocity particle interaction shows mixed sense in both the ($f'(\zeta)$), ($F(\zeta)$), ($\theta(\zeta)$), ($\theta_p(\zeta)$) fields, this happen may be the nonlinear convection domination in the flow. Fig 19 displays the influence of Q_H on

($\theta(\zeta)$), ($\theta_p(\zeta)$). It is witnessed that the elevating values of Q_H declines the temperature boundary layer thickness. $\theta(\zeta)$ Profiles are higher at 50 °C and $\theta_p(\zeta)$ higher at 10 °C. This may happen due to the dust particles in the flow. Thus dusty nanofluid facilitate enhancement the thermal boundary thickness once it reaches the critical temperature level. Figures 20, 21, and 22 illustrate the influence of Λ_2 on ($\theta(\zeta)$), ($\theta_p(\zeta)$), ($f'(\zeta)$), ($F(\zeta)$) and ($G(\zeta)$), ($G_p(\zeta)$) profiles. Higher values of Λ_2 gives upsurge to buoyancy force hence velocity and temperature field are amplified, but opposite phenomena is observed in concentration field. Quite interestingly we have observed the divergent case

Fig. 21 Effect of Λ_2 on velocity profiles of dust and fluid phase

Fig. 22 Effect of Λ_2 on concentration profiles of dust and fluid phase

phenomena after the range $\zeta = 2$. Since if we increase the range values we cannot able to control the interaction between the fluid (water different temperatures), dust and nanoparticles (carbon nanotubes) due to this nonlinear phenomena of dust and fluid phases we have seen asymptotic profiles in momentum boundary layer.

Tables 3 and 4 presented to examine on the variation of wall friction $C_f(Re_x)^{0.5}$, Nusselt $Nu_x(Re_x)^{-0.5}$ and Sherwood $Sh_x(Re_x)^{-0.5}$ numeral values of several physical parameters ($M, \phi, \phi_d, \Lambda, Bi, S, \beta_t, \beta_c, \beta_v, Q_H, \Lambda_2$) for multi-walled carbon nanotubes + water dusty nanofluid, multi-walled carbon nanotubes + water dusty nanofluid at 10–50 °C. The escalating values of ϕ_d, Bi, Λ_2 improves whereas $M, \phi, \Lambda, S, \beta_t, \beta_c, Q_H$ declines the wall skin friction $C_f(Re_x)^{0.5}$ performance. Development in ($M, \phi, \phi_d, \Lambda, Bi, \beta_c, \beta_v, Q_H$) boosts and S, β_t, Λ_2 diminishes $Nu_x(Re_x)^{-0.5}$ for multi-walled carbon nanotubes + water dusty nanofluid, multi-walled carbon nanotubes + water dusty nanofluid at 10–50 °C. In all parameters takes less time take for execution at multi-walled carbon nanotubes + temperature 50 °C whereas higher execution time for multi-walled carbon nanotubes + water nanofluid 10 °C. This happens due to the higher temperatures reduces the viscous nature of the flow, this helps to less execution time.

5 Conclusions

In this study, we analyzed the influence of non-linear convection, heat source, Cattaneo–Christov heat flux and uniform magnetic field on the dusty nanofluid (multi-walled carbon nanotubes + water) flow past a stretched cylinder at different temperatures 10 °C and 50 °C with thermal convective conditions are considered. The arising set of governing equations is solved with help of graphs and table. The major outcomes are:

- Multi-walled carbon nanotubes + water at 10 °C dusty nanofluid shows better heat and mass transfer rate compared to temperature 50 °C.
- Due to nonlinear convection the friction factor is reduced in the presence of magnetic field.
- The Biot number encourages the wall friction, heat and mass transfer rates of both the cases.
- The velocity and temperature profiles in fluid phase are lesser in multi-walled carbon nanotubes + water at 50 °C. But in dust phase we observed opposite phenomena in multi-walled carbon nanotubes + water at 10 °C shows higher profiles.

Table 3 The values of $C_f(Re_x)^{0.5}$, $Nu_x(Re_x)^{-0.5}$ and $Sh_x(Re_x)^{-0.5}$ for various values of $\beta_v, Q_H, \Lambda_2, M, \phi, \phi_d, \Lambda, Bi, S, \beta_t, \beta_c$, with SWCNT + water 10 °C mixture

M	ϕ	ϕ_d	Λ	Bi	S	β_t	β_c	β_v	Q_H	Λ_2	$C_f(Re_x)^{0.5}$	$Nu_x(Re_x)^{-0.5}$	$Sh_x(Re_x)^{-0.5}$	Time execution
0.5											− 1.001045	0.140445	1.078759	7.171770
1.5											− 1.092510	0.147080	1.068807	
2.5											− 1.179341	0.152126	1.059586	
	0.05										− 1.177295	0.160313	1.072668	5.126896
	0.1										− 1.297024	0.176623	1.077742	
	0.15										− 1.435636	0.192434	1.083042	
		0.05									− 1.092510	0.147080	1.068807	6.975803
		0.15									− 1.003900	0.149289	1.065257	
		0.3									− 0.868999	0.153045	1.058479	
			0.1								− 1.087927	0.143764	1.069423	8.050529
			0.3								− 1.092510	0.147080	1.068807	
			0.5								− 1.094794	0.148538	1.068534	
				0.1							− 1.097378	0.077744	1.068490	6.578669
				0.3							− 1.088090	0.209307	1.069094	
				0.5							− 1.080369	0.316414	1.069593	
					− 0.1						− 0.975248	0.151864	0.859721	13.778157
					0						− 1.031886	0.151805	0.961256	
					0.1						− 1.092510	0.147080	1.068807	
						− 0.1					− 0.933702	0.194337	1.086643	5.512076
						0					− 0.962657	0.184468	1.083427	
						0.1					− 0.990127	0.175650	1.080360	
							− 0.5				− 1.084062	0.146221	0.887828	6.587887
							0				− 1.089174	0.146745	0.995922	
							0.5				− 1.092510	0.147080	1.068807	
								− 0.1			− 1.134575	0.070776	1.066143	8.490989
								0			− 1.129362	0.091889	1.066400	
								0.1			− 1.122686	0.108377	1.066791	
									− 0.5		− 1.088743	0.142518	1.069113	8.959809
									0		− 1.091502	0.145855	1.068889	
									0.5		− 1.093932	0.148815	1.068692	
										− 1	− 1.236197	0.151849	1.058132	5.996309
										0	− 1.141312	0.148763	1.065218	
										1	− 1.042657	0.145293	1.072436	

Table 4 The values of $C_f(Re_x)^{0.5}$, $Nu_x(Re_x)^{-0.5}$ and $Sh_x(Re_x)^{-0.5}$ for various values of $\beta_v, Q_H, \Lambda_2, M, \phi, \phi_d, \Lambda, Bi, S, \beta_t, \beta_c$, with SWCNT + water 50 °C mixture

M	ϕ	ϕ_d	Λ	Bi	S	β_t	β_c	β_v	Q_H	Λ_2	$C_f(Re_x)^{0.5}$	$Nu_x(Re_x)^{-0.5}$	$Sh_x(Re_x)^{-0.5}$	Time execution
0.5											− 0.910148	0.113508	0.727146	3.300013
1.5											− 1.034844	0.129243	0.721428	
2.5											− 1.149741	0.139920	0.716362	
	0.05										− 0.973880	0.122185	0.724195	3.184508
	0.1										− 1.079336	0.138831	0.725802	
	0.15										− 1.203118	0.157699	0.727455	
		0.05									− 0.900306	0.110221	0.722942	3.342873
		0.15									− 0.829076	0.113673	0.721535	
		0.3									− 0.721449	0.119716	0.718785	
			0.1								− 0.900900	0.111258	0.722924	3.340596
			0.3								− 0.900306	0.110221	0.722942	
			0.5								− 0.899915	0.109390	0.722946	
				0.1							− 0.908319	0.061951	0.722587	3.367223
				0.3							− 0.893748	0.148866	0.723230	
				0.5							− 0.883669	0.206835	0.723673	
					− 0.1						− 0.840745	0.137427	0.648636	3.223023
					0						− 0.870110	0.128063	0.685169	
					0.1						− 0.900306	0.110221	0.722942	
						− 0.1					− 0.798575	0.175899	0.727605	3.051422
						0					− 0.816178	0.163268	0.726809	
						0.1					− 0.833265	0.151515	0.726032	
							− 0.5				− 0.896510	0.109521	0.639954	3.604010
							0				− 0.898776	0.109940	0.689205	
							0.5				− 0.900306	0.110221	0.722942	
								− 0.1			− 0.865352	− 0.063702	0.726410	3.688504
								0			− 0.891361	− 0.001130	0.724755	
								0.1			− 0.901430	0.037546	0.723917	
									− 0.5		− 0.881065	0.085413	0.723908	3.360447
									0		− 0.892153	0.099552	0.723351	
									0.5		− 0.900306	0.110221	0.722942	
										− 1	− 1.045359	0.121954	0.716955	3.221397
										0	− 0.951303	0.114681	0.720839	
										1	− 0.845632	0.104988	0.725193	

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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