Fault tolerant consensus control of multi-agent systems with switching topology

Maryam Fattahi¹ · A. Afshar¹

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Abstract

This work studies fault tolerant consensus control of multi-agent systems with switching topology. It is assumed that some agents in some intervals don't receive information from their neighbors, but the overall communication topology remains jointly connected. Initially, additive input fault with one-side Lipschitz constraint is considered. Next intermittent failures in the transmission of data between agents and controller is discussed in control architectures of central and distributed. To achieve consensus in such conditions, Lyapunov functional method is used. Finally, to verify the developed approach, a numerical example will be given.

Keywords Fault tolerant consensus · Multi-agent systems · Switching topology · Additive fault · Intermittent failure · Lyapunov functional

1 Introduction

Recently field of distributed coordination of MAS has been an attractive topic for many mathematicians and engineers. This is due to successful applications of MAS in many areas such as rendezvous or formation of multi-vehicles, swarming, flocking ... [1–5]. These applications can be considered as a special case of consensus problem.

Here communication network is introduced to enable data exchange among agents. Communication topology may become switching due to many reasons such as movement of agents or communication link failures. This subject has been studied by many authors, for example [6] proposed a decentralized adaptive consensus algorithm in networks with jointly connected topologies. In [7] the problem of consensus of MAS consisting of a set of identical MIMO LTI systems under a time-varying network has been studied. In [8] the coordinated output regulation problem for multiple heterogeneous linear systems under switching topologies has been discussed. In [9] the adaptive consensus problem of networked mechanical systems

with time varying delay and jointly-connected topologies was studied.

On the other hand, fault in MAS can happen due to many reasons such as unpredictable disturbance in system, actuator failure, communication link failure and intentional injection of false signal to agents. Such problems will result in poor performance of system. Among published papers in this area the following articles are more relevant: in [10] distributed robust consensus of MAS with nonlinear dynamics and input fault has been studied. Shen et al. [11] studied the consensus tracking problem for a class of MAS with time-varying actuator fault. Kazerooni and Khorasani [12] analyzed the stability and consensus problem of a team of double-integrator unmanned vehicles subject to actuator faults. In [13] distributed occasional fault tolerant consensus of MAS with time-varying delay has been discussed. In [14] consensus tracking problem in MAS with partial loss of effectiveness of actuator was investigated. Zuo et al. [15] discussed adaptive fault-tolerant tracking control of MAS with actuator fault. In [16] distributed

✉ Maryam Fattahi, m.fattahi@aut.ac.ir; A. Afshar, aafshar@aut.ac.ir | ¹Department of Electrical Engineering, Amirkabir University, Hafez Street, Tehran, Iran.

tracking problem of linear higher-order MAS with missing control input in some intervals has been studied.

In all of this works communication topology is fix. However in practical applications of MAS, input fault may occur while the topology of system is switching. In this category there are few related works, for instance [17] studied robust consensus tracking control problem for linear leader–follower systems with switching directed networks. In this work the interaction network of all subsystems contains directed spanning trees. In [18] the distributed fault-tolerant control problem for a class of high-order MAS with switching undirected topologies has been studied. In [19] a cooperative actuator fault accommodation strategy for a team of linear time-invariant MAS with switching topology and directed communication network has been studied.

To the best of author's knowledge, until now, there is no paper consisting fault and switching topology with above stated problems in the dynamic of MAS. This is hold for both central and distributed architectures of controller. One vital question that arises is whether it is possible to propose theorems that guarantee the consensus of system under such conditions. This problem has been the main motivation to develop this work. To summarize, the main innovations of this paper are as follows:

1. Assumption of additive fault with nominal input that satisfies one-side Lipchitz constraint while topology of system is jointly connected.
2. In the category of occasional failure in the data exchange between controller and agents, the formulation of problem depends on the controller architecture. This problem hasn't been considered in any previous works.
3. As special case of II, the communication network failure may lead to some agents to be isolated in some intervals. This problem has also been considered in this work.

Motivated by these points, in this paper we propose fault tolerant consensus criteria for MAS with switching topology. In Sect. 2, the required definition and lemma is introduced. In Sect. 3, the model of system with additive fault is developed. Next, the problem of fault in the data exchange between controller and agents is formulated in both control architectures of central and distributed. Finally, in Sect. 4, a simulation example is given to show the advantages of the proposed approach.

2 Preliminaries

In this work we use some mathematical symbols with the following descriptions:

I_N :	identity matrix of dimension N ,
1 :	A column vector with matrix elements equal to one,
$A \otimes B$:	Kronecker product of matrices A and B
$\ x\ $:	2-norm of a matrix,
$\lambda_{\min}(A)$:	minimum eigenvalues of A ,
$\lambda_{\max}(A)$:	maximum eigenvalues of A ,

And for a directed graph G :

$V = \{v_1, v_2, \dots, v_n\}$	The set of nodes,
$\varepsilon = v \times v$	The set of directed edges,
$A_{adj} = [a_{ij}]_{N \times N}$	Adjacency matrix with $a_{ij} > 0$ if and only if $a_{ij} \in \varepsilon$,
$L = [l_{ij}]_{N \times N}$, $l_{ij} = \begin{cases} -a_{ij} & i \neq j \\ \sum_{i=1}^N a_{ik} & i = j \end{cases}$	Laplacian matrix

Also, a union graph of a collection of simple graphs $G^\sigma, \sigma = 1, \dots, N$ is jointly connected if and only if overall graph $G = \bigcup_{\sigma=1}^N G^\sigma$ that is the set of all possible directed topologies, is connected.

And finally the following Lemma and Definition is used in this work:

Lemma 1 [1] Zero is an eigenvalue of L and 1 is its relevant right eigenvector. Other eigenvalues of L have positive real parts. Moreover, G has a directed spanning tree if and only if zero is a simple eigenvalue of L .

Definition 1 [20] A positive constant τ is called the average dwell time for a switching signal $\sigma(t)$ if:

$$N^\sigma(t_2, t_1) \leq N_0 + \frac{t_2 - t_1}{\tau}$$

holds for all (t_2, t_1) and some scalar N_0 , where $N^\sigma(t_2, t_1)$ denotes the number of mode switches of a given switching signal $\sigma(t)$ over the interval (t_2, t_1) .

3 Main results

In this section our aim is to analysis a faulty MAS with switching topology. At first, it is assumed that there is additive nonlinear fault with nominal input. Next, this problem is studied when there is intermittent failures in the communication links between controller and agents.

3.1 Additive nonlinear fault

In this section a network of N -agents with the following dynamic is studied:

$$\dot{x}_i(t) = A_1 x_i(t) + B_1(u_i(t) + f_i(x_i, t)), \quad i = 1, \dots, N \quad (1)$$

where $x_i \in R^n$ is the state, $u_i \in R^p$ is the control input, $f_i(x_i(t), t)$ is the additive nonlinear fault, A, B are constant known matrices. It is assumed that nonlinear fault in controller satisfies one-side Lipschitz condition as:

$$(x_i(t) - x_j(t))^T (f_i(x_i, t) - f_i(x_j, t)) \leq \aleph_i \|x_i(t) - x_j(t)\|^2 \quad (2)$$

where $\aleph_i$ is an unknown scalar.

Remark 1 In most existing results in the area of fault tolerant control of MAS, constraints of Lipschitz condition, sector nonlinearity ... [21–23] have been considered. In such constraints, nonlinear term is bounded by a linear function that leads to conservatism in the representation of different nonlinear functions. Because most of the nonlinear functions are not bounded by linear functions, in order to represent them in more general form, the assumption of one-sided Lipschitz constraint has been considered in this work.

With $x = [x_1^T, x_2^T, \dots, x_N^T]^T$, $u = [u_1^T, u_2^T, \dots, u_N^T]^T$, $f = [f_1^T, f_2^T, \dots, f_N^T]^T$ system (1) is represented as:

$$\dot{x}(t) = (I_N \otimes A)x(t) + (I_N \otimes B)(u(t) + f(x(t), t)). \quad (3)$$

In order to achieve consensus, we should have:

$$\lim_{t \rightarrow \infty} (x_i(t) - x_j(t)) = 0, \quad \forall \quad i, j = 1, 2, \dots, N, \quad (4)$$

hence, consensus error between agents is defined as:

$$\zeta_i(t) = x_i(t) - x_j(t). \quad (5)$$

In order to achieve consensus in MAS (1) the following controller is proposed:

where G_c, G_d denote the graph of agents when graph is connected and disconnected, respectively. $K_1, K_2 \in R^{p \times n}$, η_i, γ_i are positive scalars, $\Lambda_1 = -P_1 B_1 K_1$, and P_1 is defined in (7).

Remark 2 In order to design distributed controller for MAS (1) in such conditions, the proposed controller consists of 2 terms. First term by itself has two parts: in first part adaptive mechanism has been proposed to relax the necessity of dependency of the controller gain on eigenvalues of Laplacian matrix of system. Hence, this adaptive control method results in distributed consensus for MAS. Second part is proposed to manage nonlinear fault of system. Here adaptive mechanism is again applied, because $\aleph_i$ is unknown and consequently it is estimated by using adaptive method. Second term is for isolated agents. It is similar to first term, but with this difference that isolated agents only use their own information.

For stability analysis of system (1) with controller (6), the following Lyapunov functional is selected:

$$V_1^\sigma = 1/2 \zeta(t)^T (L^\sigma \otimes P_1) \zeta(t) + \sum_{i=1}^N \frac{(\aleph_i - \hat{\aleph}_i)^2}{2} + \sum_{i=1}^N \frac{\bar{c}_i^{\sigma 2}}{2\eta_i}, \quad (7)$$

where $\bar{c}_i^\sigma = c_i - \hat{c}_i^\sigma$, $\hat{c}_i^\sigma = \frac{1}{\lambda_i^\sigma}$ and λ_i^σ denotes the non-zero eigenvalues of L^σ .

Based on the proposed Lyapunov functional in (7) one can obtain that:

$$\begin{aligned} (\phi_1 = 1/2 \min_{\sigma} (\lambda_{\min}(L^\sigma)) \lambda_{\min}(P_1)) \|\zeta\|^2 &\leq V_1^\sigma \leq \phi_2 \\ &= 1/2 \max_{\sigma} (\lambda_{\max}(L^\sigma)) \lambda_{\max}(P_1) \|\zeta\|^2. \end{aligned} \quad (8)$$

Now, the time-derivative of V_1^σ is computed when $i \in G_c$:

$$\begin{aligned} \dot{V}_1^\sigma &= 1/2 \zeta^T(t) (L^\sigma \otimes P_1 A) \zeta(t) + 1/2 \zeta^T(t) (L^\sigma \otimes A^T P_1) \zeta(t) \\ &+ \zeta^T(t) (CL^{\sigma 2} \otimes P_1 B K) \zeta(t) - \sum_{i=1}^N \hat{\aleph}_i \zeta_i^T(t) (L^\sigma \otimes P_1 B) \zeta_i(t) + \zeta^T(t) (L^\sigma \otimes P_1 B) f \\ &- \sum_{i=1}^N (\aleph_i - \hat{\aleph}_i) \|\zeta_i(t)^T P_1 B \zeta_i(t)\| + \sum_{i=1}^N \bar{c}_i^\sigma \left[-\gamma_i c_i + \sum_{j=1}^N a_{ij}^\sigma \zeta_i(t)^T \Lambda \sum_{j=1}^N a_{ij}^\sigma \zeta_j(t) \right], \end{aligned} \quad (9)$$

$$u_i(t) = \begin{cases} c_i K_1 \sum_{j=1}^N a_{ij} (x_j(t) - x_i(t)) + H_1(x_i(t)), \\ \dot{c}_i(t) = \eta_i \left[-\gamma_i c_i(t) + (\sum_{j=1}^N a_{ij}^\sigma (x_i(t) - x_j(t))^T \Lambda_1 \sum_{j=1}^N a_{ij}^\sigma (x_i(t) - x_j(t)) \right] \\ H_1(x_i(t)) = -\hat{\aleph}_i (x_i(t) - x_j(t)), \\ \dot{\hat{\aleph}}_i(t) = \|(x_i(t) - x_j(t))^T (P_1 B(x_i(t) - x_j(t)))\|, i \in G_c(t), \\ K_2 x_i(t) + H_2(x_i(t)) \\ H_2(x_i(t)) = -\hat{\aleph}_i x_i(t) \\ \dot{\hat{\aleph}}_i(t) = \|x_i(t)^T P_2 B x_i(t)\|, i \in G_d(t) \end{cases} \quad (6)$$

where $C = [c_1, c_2, \dots, c_N]^T$.
Since:

Now, we define $U = [1/\sqrt{N} Y_1]$, $U^T = \begin{bmatrix} 1/\sqrt{N} \\ Y_2 \end{bmatrix}$ with $U^T L U = \text{diag}\{0, \lambda_2, \dots, \lambda_N\}$ and $\bar{\zeta} = (U^T \otimes I)\zeta$. By these new variables, equation in (13) is equal to:

$$\begin{aligned} \dot{V}_1^\sigma + \alpha_1 V_1^\sigma &\leq +1/2\zeta^T(t)(L^\sigma \otimes P_1 A)\zeta(t) + 1/2\zeta^T(t)(L^\sigma \otimes A^T P_1)\zeta(t) \\ &\quad - \sum_{i=1}^N \gamma_i c_i \bar{c}_i^\sigma - \zeta^T(t)(\hat{c}_i^\sigma L^{\sigma 2} \otimes P_1 B K_1)\zeta(t-d(t)) + \alpha_1 \left(\zeta^T(t)(L^\sigma \otimes P_1)\zeta(t) + \sum_{i=1}^N \frac{(\mathfrak{N}_i - \hat{\mathfrak{N}}_i)^2}{2} + \sum_{i=1}^N \frac{\bar{c}_i^{\sigma 2}}{2\eta_i} \right) \\ &= - \sum_{i=1}^N \gamma_i c_i \bar{c}_i^\sigma + \alpha_1 \left(\sum_{i=1}^N \frac{(\mathfrak{N}_i - \hat{\mathfrak{N}}_i)^2}{2} + \sum_{i=1}^N \frac{\bar{c}_i^{\sigma 2}}{2\eta_i} \right) + \sum_{i=1}^N \lambda_i (\bar{\zeta}_i^T(t)(1/2A^T P_1 + 1/2P_1 A + \alpha_1 P_1 - \hat{c}_i^\sigma \lambda_i^\sigma P_1 B K_1)\bar{\zeta}_i(t) \\ &= - \sum_{i=1}^N \gamma_i c_i \bar{c}_i^\sigma + \alpha_1 \left(\sum_{i=1}^N \frac{(\mathfrak{N}_i - \hat{\mathfrak{N}}_i)^2}{2} + \sum_{i=1}^N \frac{\bar{c}_i^{\sigma 2}}{2\eta_i} \right) \\ &\quad + \sum_{i=1}^N \lambda_i^\sigma (\bar{\zeta}_i^T(t)(1/2A^T P_1 + 1/2P_1 A + \alpha_1 P_1 - P_1 B K_1)\bar{\zeta}_i(t), \end{aligned} \quad (14)$$

$$\zeta^T(t)(CL^{\sigma 2} \otimes P_1 B K)\zeta(t) = \sum_{i=1}^N (\bar{c}_i^\sigma + \hat{c}_i^\sigma) \sum_{j=1}^N a_{ij}^\sigma \zeta_j(t)^T \Lambda \sum_{j=1}^N a_{ij}^\sigma \zeta_j(t), \quad (10)$$

and based on (2) one can obtain that:

$$\zeta_i^T(t)(L^\sigma \otimes P_1 B)f_i(x_i(t), t) \leq \mathfrak{N}_i \left\| \zeta_i^T(t)(L^\sigma \otimes P_1 B)\zeta_i(t) \right\|, \quad (11)$$

substituting (10–11) into (9) yields:

$$\begin{aligned} \dot{V}_1^\sigma &\leq 1/2\zeta^T(t)(L^\sigma \otimes P_1 A)\zeta(t) + 1/2\zeta^T(t)(L^\sigma \otimes A^T P_1)\zeta(t) \\ &\quad - \sum_{i=1}^N \mathfrak{N}_i \zeta_i^T(t)(L^\sigma \otimes P_1 B)\zeta_i(t) - \sum_{i=1}^N \mathfrak{N}_i \left\| \zeta_i^T(t)(L^\sigma \otimes P_1 B)\zeta_i(t) \right\| \\ &\quad - \sum_{i=1}^N (\mathfrak{N}_i - \hat{\mathfrak{N}}_i) \left\| \zeta_i(t)^T P_1 B \zeta_i(t) \right\| \\ &\quad + \sum_{i=1}^N -\gamma_i c_i \bar{c}_i^\sigma - \zeta^T(t)(\hat{c}_i^\sigma L^{\sigma 2} \otimes P_1 B K_1)\zeta(t), \end{aligned} \quad (12)$$

that results:

$$\begin{aligned} \dot{V}_1^\sigma &\leq 1/2\zeta^T(t)(L^\sigma \otimes P_1 A)\zeta(t) + 1/2\zeta^T(t)(L^\sigma \otimes A^T P_1)\zeta(t) \\ &\quad + \sum_{i=1}^N -\gamma_i c_i \bar{c}_i^\sigma - \zeta^T(t)(\hat{c}_i^\sigma L^{\sigma 2} \otimes P_1 B K)\zeta(t-d(t)). \end{aligned} \quad (13)$$

by removing similar terms in (14), one can obtain that:

$$\begin{aligned} \dot{V}_1^\sigma + \alpha_1 V_1^\sigma &\leq - \sum_{i=1}^N \gamma_i c_i \bar{c}_i^\sigma + \alpha_1 \left(\sum_{i=1}^N \frac{(\mathfrak{N}_i - \hat{\mathfrak{N}}_i)^2}{2} + \sum_{i=1}^N \frac{\bar{c}_i^{\sigma 2}}{2\eta_i} \right) \\ &\quad + \zeta(t)^T (1/2A^T P_1 + 1/2P_1 A + \alpha_1 P_1 - P_1 B K_1)\zeta(t). \end{aligned} \quad (15)$$

it is obvious that if the following LMI is feasible:

$$1/2A^T P_1 + 1/2P_1 A + \alpha_1 P_1 - P_1 B K_1 < 0, \quad (16)$$

one can get:

$$\dot{V}_1^\sigma + \alpha_1 V_1^\sigma < - \sum_{i=1}^N \gamma_i / 2\hat{c}_i^{\sigma 2} + \alpha_1 \left(\sum_{i=1}^N \frac{(\mathfrak{N}_i - \hat{\mathfrak{N}}_i)^2}{2} + \sum_{i=1}^N \frac{\bar{c}_i^{\sigma 2}}{2\eta_i} \right), \quad (17)$$

from (17), it can be deduced that system (1) with controller (6) reach to consensus with exponential rate $\mu_1 < \alpha_1$, thus (17) can be rewritten as:

$$\begin{aligned} \dot{V}_1^\sigma &< -\mu_1 V_1^\sigma - (\alpha_1 - \mu_1) V_1^\sigma \\ &\quad - \sum_{i=1}^N \gamma_i / 2\hat{c}_i^{\sigma 2} + \alpha_1 \left(\sum_{i=1}^N \frac{(\mathfrak{N}_i - \hat{\mathfrak{N}}_i)^2}{2} + \sum_{i=1}^N \frac{\bar{c}_i^{\sigma 2}}{2\eta_i} \right), \end{aligned} \quad (18)$$

also, based on Lyapunov functional in (7) one has:

$$1/2\lambda_2 \lambda_{\min}(P_1) \|\zeta\|^2 \leq V_1^\sigma, \quad (19)$$

substituting (19) into (18) yields:

$$\dot{V}_1^\sigma < -\mu_1 V_1^\sigma - \frac{(\alpha_1 - \mu_1)}{2} \lambda_2 \lambda_{\min}(P_1) \|\zeta\|^2 - \sum_{i=1}^N \gamma_i / 2\hat{c}_i^{\sigma 2} + \alpha_1 \left(\sum_{i=1}^N \frac{(\mathfrak{N}_i - \hat{\mathfrak{N}}_i)^2}{2} + \sum_{i=1}^N \frac{\bar{c}_i^{\sigma 2}}{2\eta_i} \right), \quad (20)$$

therefore, if the following inequality holds:

$$\|\zeta\|^2 < \frac{1}{\lambda_2 \lambda_{\min}(P_1)(\alpha_1 - \mu_1)} \times \left(-\sum_{i=1}^N \gamma_i / 2\hat{c}_i^{\sigma^2} + \alpha_1 \left(\sum_{i=1}^N \frac{(\hat{x}_i - \hat{x}_i)^2}{2} + \sum_{i=1}^N \frac{\bar{c}_i^{\sigma^2}}{2\eta_i} \right) \right), \quad (21)$$

one can obtain that:

$$\dot{V}_1^\sigma < -\mu_1 V_1^\sigma. \quad (22)$$

□

Until now we have proved exponential consensus of MAS (1) in intervals that graph is connected. For stability analysis of $\zeta(t)$ when $i \in G_d$, we choose similar Lyapunov functional with (7) as:

$$V_2^\sigma = V_1^\sigma(P_2), \quad (23)$$

similar to above procedure for derivative of V_1^σ in $i \in G_c$, the stable exponential behavior of $\zeta(t)$ is obtained if the following LMI is satisfied:

$$1/2A^T P_2 + 1/2P_2 A + \alpha_2 P_2 - P_2 B K_2 < 0. \quad (24)$$

Until now, the exponential consensus of MAS (1) have been proved for each switching interval, separately. Now consensus of agents in all intervals is studied. It is assumed that switching intervals of system are $t_1, t_2, \dots, t_{N\sigma}$. On the other hand based on (8) and (22) one can write:

$$V_1(t_i) \leq e^{-\mu_1(t-t_{i-1})}(\phi_2/\phi_1)V_1(t_{i-1}), \quad (25)$$

based on N^σ in Definition 1, one has:

$$V_1(t) \leq e^{-\mu_1(t-t_0)}(\phi_2/\phi_1)^{N^\sigma} V_1(t_0), \quad (26)$$

that can be rewritten as:

$$V_1(t) \leq e^{-(\mu_1 - \ln(\phi_2/\phi_1)/\tau)(t-t_0)}(\phi_2/\phi_1)^{N_0} V_1(t_0). \quad (27)$$

□

It should be noticed that (25–27) can be satisfied by $V_2(t)$.

The following Theorem shows that consensus for MAS (1) can be achieved if some conditions are satisfied.

Theorem 1 Consensus in MAS (1) with control input (6) is achieved, if:

1. There exist positive definite matrices P_1, P_2 such that LMIs in (16) and (24) to be feasible.
2. Average dwell time of system satisfies the following inequalities:

$$\tau > \ln(\phi_2(V_1^\sigma)/\phi_1(V_1^\sigma))/\mu_1,$$

$$\tau > \ln(\phi_2(V_2^\sigma)/\phi_1(V_2^\sigma))/\mu_2.$$

where ϕ_1, ϕ_2 are defined in (8).

3.2 Intermittent failure in the communication link between controller and agents

Consider the following MAS:

$$\dot{x}_i(t) = A_1 x_i(t) + B_1 u_i(t), \quad i = 1, \dots, N \quad (28)$$

As previously mentioned, fault considered in this section is intermittent failure in the communication link between controller and agents. In the architecture of central control for MAS, all the required controller calculations are done in the central control unit (CCU). When bilateral communication links between agents and CCU is failed, CCU can't receive any information from agents and also agents can't receive control signal. However, in the architecture of distributed control, the required controller calculations are done in control unit of each agent by using information from neighbor agents. When communication links between agent and its neighbors is failed, controller can't receive any information from other agents and consequently the behavior of agent is similar to a single system.

Based on above discussion, for a centralized control strategy this problem is formulated as:

$$u_i(t) = \begin{cases} c_i K_3 \sum_{j=1}^N a_{ij}(x_j(t) - x_i(t)) \\ \dot{c}_i(t) = \eta_i \left[-\gamma_i c_i(t) + \left(\sum_{j=1}^N a_{ij}^\sigma(x_j(t) - x_i(t))^\top A_3 \sum_{j=1}^N a_{ij}^\sigma(x_i(t) - x_j(t)) \right) \right], & t_k \leq t < t_{k+h} \\ 0 & t_{k+h} \leq t < t_{k+h+l}, \quad k, h, l \in N, \end{cases} \quad (29)$$

and for a distributed one, it can be formulated as:

$$u_i(t) = \begin{cases} c_i K_5 \sum_{j=1}^N a_{ij}(x_j(t) - x_i(t)) \\ \dot{c}_i(t) = \eta_i \left[-\gamma_i c_i(t) + \left(\sum_{j=1}^N a_{ij}^\sigma(x_i(t) - x_j(t))^\top \Lambda_5 \sum_{j=1}^N a_{ij}^\sigma(x_i(t) - x_j(t)) \right) \right], & t_k \leq t < t_{k+h} \text{ and } i \in G_c. \\ K_6 x_i(t), & t_{k+h} \leq t < t_{k+h+l} \text{ or } i \in G_d. \end{cases} \quad (30)$$

With $x = [x_1^T, x_2^T, \dots, x_N^T]^T$, $u = [u_1^T, u_2^T, \dots, u_N^T]^T$, system (28) with failure (29) is equal to:

$$\dot{x}(t) = \begin{cases} (I_N \otimes A + L \otimes CBK_3)x(t), & t_k \leq t < t_{k+h} \\ 0 & t_{k+h} \leq t < t_{k+h+l}, \end{cases} \quad (31)$$

and system (28) with failure (30) is equal to:

$$\dot{x}(t) = \begin{cases} (I_N \otimes A + L \otimes CBK_3)x(t), & t_k \leq t < t_{k+h} \\ (A + BK)x(t) & t_{k+h} \leq t < t_{k+h+l}, \text{ or } i \in G_d. \end{cases} \quad (32)$$

In the next subsections system (31) and (32) will be studied.

3.2.1 Failure in central control strategy

For the stability analysis of system (31) in $t_k \leq t < t_{k+h}$ (intervals with no communication link failure), we choose the following Lyapunov functional:

$$V_3 = 1/2 \zeta(t)^T (L \otimes P_3) \zeta(t) + \sum_{i=1}^n \frac{\bar{c}_i^2}{2\eta_i}, \quad (33)$$

similar to previous analysis in Sect. 3.1, if the following LMI is feasible:

$$1/2A^T P_3 + 1/2P_3 A + \alpha_3 P_3 - P_3 B K_3 < 0 \quad (34)$$

for:

$$\|\zeta\|^2 < \frac{1}{\lambda_2 \lambda_{\min}(P_3)(\alpha_3 - \mu_3)} \left(-\sum_{i=1}^N \gamma_i / 2\hat{c}_i^2 + \alpha_3 \left(\sum_{i=1}^N \frac{\bar{c}_i^2}{2\eta_i} \right) \right) \quad (35)$$

one can obtain that:

$$\dot{V}_3 < -\mu_3 V_3, \mu_3 < \alpha_3. \quad (36)$$

For stability analysis of system (31) in $t_{k+h} \leq t < t_{k+h+l}$ (failure intervals), we choose the following Lyapunov functional:

$$V_4 = 1/2 \zeta(t)^T (L \otimes P_4) \zeta(t), \quad (37)$$

similar to previous analysis, if the following LMI is feasible:

$$1/2A^T P_4 + 1/2P_4 A + \alpha_4 P_4 < 0 \quad (38)$$

one can obtain that:

$$\dot{V}_4 < \alpha_4 V_4. \quad (39)$$

□

Remark 3 It should be noted, MAS in failure intervals can be stable ($\alpha_4 < 0$) or unstable ($\alpha_4 > 0$). On the other hand, from (36) it can be concluded that the energy of consensus error in non-failure intervals is exponentially decreasing. Thus for exponential consensus of overall MAS (31), increased energy of consensus error in failure intervals (depend on inherent behavior of system) should be compensated in non-failure intervals. The mathematic of this problem has been shown in the sequel.

The following Theorem shows that consensus for MAS (31) can be achieved if some conditions are satisfied.

Theorem 2 Consensus in MAS (28) with failure (29) is achieved if:

1. There exist $P_3, P_4 > 0$ such that matrix inequalities (34) and (38) to be feasible,
2. There exists $\rho > 1$ such that:

$$P_3 > \rho P_4,$$

3. There exists $\delta > 0$ such that:

$$V_3 \leq (\rho + \delta) V_4,$$

4. If N_{fail} and T_{fail} are respectively the number and total time of communication link failure in $t \in [t_1, t_2]$, there exist $\alpha^* > \alpha > 0$ such that:

$$-\mu_3(t - T_{fail}) + \alpha_4(T_{fail}) \leq \alpha^* t, ((\rho + \delta)/\rho)^{N_{fail}(t)} \leq e^{\alpha t}.$$

We note that if above conditions are satisfied, system will reach to consensus with exponential rate $\alpha^* - \alpha$.

Proof Choose the following switching Lyapunov functional:

$$V(t) = \begin{cases} V_3(t) & \text{if } t_k \leq t < t_{k+h} \\ V_4(t) & \text{if } t_{k+h} \leq t < t_{k+h+l} \end{cases}, \quad (40)$$

from (36) and (39) one has:

$$V(t) \leq \begin{cases} e^{-\mu_3(t-t_k)} V_3(t_k) & \text{if } t_k \leq t < t_{k+h} \\ e^{\alpha_4(t-t_{k+h})} V_4(t_{k+h}) & \text{if } t_{k+h} \leq t < t_{k+h+l} \end{cases} \quad (41)$$

for $t_0 \in [t_k, t_{k+h}]$:

$$\begin{aligned} V(t) &\leq e^{-\mu_3(t-t_k)} V_3(t_k) \leq e^{-\mu_3(t-t_k)} (\mu + \delta) V_4(t_k^-) \leq e^{-\mu_3(t-t_k)} (\mu + \delta) e^{\alpha_4(t_k-t_{k-\tau})} V_4(t_{k-\tau}) \\ &\leq e^{-\mu_3(t-t_k)} (\mu + \delta) / \mu e^{\alpha_4(t_k-t_{k-\tau})} V_3(t_{k-\tau}) \leq \dots \leq ((\mu + \delta) / \mu)^{N_{fail}} e^{-\mu_3(t-T_{fail}(t))} e^{\alpha_4(T_{fail}(t))} V(0) \end{aligned} \quad (42)$$

and for $t_0 \in [t_{k+h}, t_{k+h+l}]$ one has:

$$\begin{aligned} V(t) &\leq e^{\alpha_4(t-t_{k+h})} V_4(t_{k+h}) \leq e^{\alpha_4(t-t_{k+h})} (1/\mu) V_3(t_{k+h}^-) \leq e^{\alpha_4(t-t_{k+h})} (1/\mu) ((e^{-\mu_3(t_{k+h}-t_{k+h-\tau})} V_3(t_{k+h-\tau})) \\ &\leq e^{\alpha_4(t-t_{k+h})} ((\mu + \delta) / \mu) ((e^{-\mu_3(t_{k+h}-t_{k+h-\tau})} V_4(t_{k+h-\tau})) \leq \dots \leq ((\mu + \delta) / \mu)^{N_{fail}+1} e^{-\alpha_1(t-T_{fail}(t))} e^{\alpha_4(T_{fail}(t))} V(0) \end{aligned} \quad (43)$$

finally, from (42) and (43) one can obtain that:

$$V(t) \leq e^{-(\alpha^* - \alpha)t} V(0). \quad (44)$$

□

3.2.2 Failure in distributed control strategy

For stability analysis of system (32) in $t_k \leq t < t_{k+h}$, we choose the following Lyapunov functional:

$$V_5^\sigma = V_3^\sigma(P_5), \quad (45)$$

similar to previous analysis if the following LMI is feasible:

$$1/2A^T P_5 + 1/2P_5 A + \alpha_5 P_5 - P_5 B K_5 < 0, \quad (46)$$

for:

$$\|\xi\|^2 < \frac{1}{\lambda_2 \lambda_{\min}(P_5)(\alpha_5 - \mu_5)} \left(-\sum_{i=1}^N \gamma_i / 2\hat{c}_i^{\sigma^2} + \alpha_5 \left(\sum_{i=1}^N \frac{\bar{c}_i^{\sigma^2}}{2\eta_i} \right) \right), \quad (47)$$

one can get:

$$\dot{V}_5^\sigma < -\mu_5 V_5^\sigma, \mu_5 < \alpha_5. \quad (48)$$

For stability analysis of system (32) in $t_{k+h} \leq t < t_{k+h+l}$ or $i \in G_d$, we choose the following Lyapunov functional:

$$V_6^\sigma = V_4^\sigma(P_6), \quad (49)$$

that results if the following LMI is feasible:

$$1/2A^T P_6 + 1/2P_6 A + \alpha_6 P_6 - P_6 B K_6 < 0 \quad (50)$$

one can obtain that:

$$\dot{V}_6^\sigma < -\mu_6 V_6^\sigma, \mu_6 < \alpha_6. \quad (51)$$

□

The following Theorem shows that consensus for MAS (32) can be achieved if some conditions are satisfied.

Theorem 3 Consensus in MAS (28) with failure (31) is achieved, if:

1. There exist positive definite matrices P_5, P_6 such that matrix inequalities (46) and (50) to be feasible,

2. There exists $\rho_1 > 1$ such that:

$$P_5 > \rho_1 P_6,$$

3. There exists $\delta_1 > 0$ such that:

$$V_5 \leq (\rho_1 + \delta_1) V_6,$$

4. If N_{fail} and T_{fail} are respectively the number and total time of failures of communication link in $t \in [t_1, t_2]$ there exist $\alpha_1^* > \alpha_2^* > 0$ such that:

$$-\mu_5(t - T_{fail}) + \mu_6(T_{fail}) \leq \alpha_1^* t, ((\rho_1 + \delta_1) / \rho_1)^{N_{fail}(t)} \leq e^{\alpha_2^* t}.$$

5. Average dwell time of system satisfies the following inequalities:

$$\tau > \ln(\phi_2(V_5^\sigma) / \phi_1(V_5^\sigma)) / \mu_5,$$

$$\tau > \ln(\phi_2(V_6^\sigma) / \phi_1(V_6^\sigma)) / \mu_6.$$

We note that if above conditions are satisfied, finally system with exponential rate $\alpha_1^* - \alpha_2^*$ reach to consensus. Also, $\phi_2(V_5^\sigma)$, $\phi_1(V_5^\sigma)$, $\phi_2(V_6^\sigma)$, $\phi_1(V_6^\sigma)$ are similar to (8).

Proof See proof in Theorems 1 and 2. □

Remark 4 It should be noticed, condition 1 of the proposed Theorems is LMI condition that can be easily solved by MATLAB LMI toolbox. Conditions 2–4 of Theorems 2, 3 are constraints that satisfy stability of system in the conditions of intermittent failure of communication link between agents and controller. Although these conditions are conservative, however they are required in stability analysis of systems with the mentioned failure (for instance, see [24]). The physical concept of these three conditions is that the energy reduction of system in non-failure intervals can overcome increasing energy of system in failure intervals and consequently the whole system will be stable. Also

Fig. 1 communication graph of system (52) **Fig. 2** Evolution of consensus of states $x_{1i}, i = 1, \dots, 6$ in the controlled system (52)

condition 5 of Theorem 3 is constraint on the inherent switching topology of system.

Remark 5 From above Theorems, it can be concluded that the assumption of equal failure time for agents is not required and each agent can have its own different failure time.

Remark 6 It should be noticed, because the dimension of LMI conditions depend only on the system order (n), not the number of agents (N), it is easy to extend results for large scale MAS.

Remark 7 In this work, we have considered same P in all switching intervals. To reach more feasible region in LMI conditions, different P_i can be defined in each switching interval.

Remark 8 Compare to results in [25] that studied consensus tracking of MAS with one-sided Lipschitz constraint, for solving LMIs in [25] one should know the bound of nonlinear term of system. The prediction of constant of nonlinear term ($\mathfrak{N}_i$) is very difficult in general practices. However, in this work, applying adaptive control law, the achieved results don't depend on the constant of nonlinear term of system.

Fig. 3 Evolution of consensus of states $x_{2i}, i = 1, \dots, 6$ in the controlled system (52) **Fig. 4** Evolution of consensus of states $x_{3i}, i = 1, \dots, 6$ in the controlled system (52)

Remark 9 Compare to results in [16, 26, 27] that studied controller failure in MAS, the presented work has some novelties such as:

1. Results in [16, 26, 27] are dependent on the minimum eigenvalues of Laplacian matrix of system which lead to a central analysis method for MAS. However, in this paper, applying adaptive mechanism in controller

Fig. 5 Evolution of consensus of states $x_{1i}, i = 1, \dots, 6$ in the controlled system (52)

Fig. 8 Evolution of consensus of states $x_{1i}, i = 1, \dots, 6$ in the controlled system (52)

Fig. 6 Evolution of consensus of states $x_{2i}, i = 1, \dots, 6$ in the controlled system (52)

Fig. 9 Evolution of consensus of states $x_{2i}, i = 1, \dots, 6$ in the controlled system (52)

Fig. 7 Evolution of consensus of states $x_{3i}, i = 1, \dots, 6$ in the controlled system (52)

Fig. 10 Evolution of consensus of states $x_{3i}, i = 1, \dots, 6$ in the controlled system (52)

design, results don't depend on the minimum eigenvalues of Laplacian matrix of system

2. In [16, 26, 27] authors only considered central control method in conditions of failure of communication

links. However this work has studied intermittent failure in both architectures of central and distributed control.

3. In [16, 26, 27], it is assumed that topology of system in all intervals is connected. However, communication links between agents may be disconnected such that in some intervals there exists an agent that is isolated from other agents. This issue has been considered in this work.

4 Numerical simulation

Example Consider the following MAS with communication graph as Fig. 1:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, \dots, N \quad (52)$$

with:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -5 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

and additive nonlinear fault:

$$f(x(t), t) = [0 \ 1/x_2 \ 0.5/x_3]^T,$$

it is obvious that $f(x(t), t)$ satisfies one-side Lipchitz constraint. To achieve consensus for system (52) with above fault, Theorem 1 is used. It is easy to show that with gains calculated for controller (6) as given below, LMIs in (16) and (24) are feasible:

$$K_1 = K_2 = [-2 \ 0 \ 0],$$

also, it can be seen that for $\tau > 3.1$ conditions of Theorem 1 are feasible. With the obtained gains, we have simulated system (52). The results in Figs. 2, 3 and 4 show that consensus has been achieved.

Now intermittent failure in communication links between agents and controller is considered. Assume that $N_{fail} = 4$, $T_{fail} = 25\%$, at first the strategy of central controller is considered. To reach consensus in MAS (52) with these assumptions, we can use our result in Theorem 2. Assuming $\rho, \delta, \mu_3, \alpha_4$ respectively as 1.4, 0.1, 0.38, 0.26, and the gain of controller in (29) as:

$$K_3 = [-3.5 \ 1 \ 1],$$

conditions of Theorem 2 are feasible. With the obtained gain, we have simulated system (52). The results of the simulation of MAS (52) in Figs. 5, 6 and 7 show that we could reach to consensus. Now assume that control strategy is distributed. To reach consensus in MAS (52) in such

conditions, we use the obtained result in Theorem 3. Assuming $\rho, \delta, \mu_5, \mu_6, \tau$ respectively as 1.7, 0.2, 0.33, 0.17, 4.1, and the gain of controller in (30) as:

$$K_5 = K_6 = [-5 \ 1.5 \ 1],$$

conditions of Theorem 3 are feasible. The results of the simulation of MAS (52) in Figs. 8, 9 and 10 show that consensus has been achieved.

It should be noticed, for obtaining unknown variables in condition 4 of Theorems 2 and 3, one should use trial and error method.

5 Conclusion

This work studied fault tolerant consensus of Multi-Agent Systems with switching topology. It was assumed that some agents in some intervals were isolated and don't receive information from their neighbors. At first, additive faults with nominal input were assumed. The nonlinear fault of agents satisfied one-side Lipchitz constraint. Then, occasional failure in the transmission of data between controller and agents in both architectures of central and distributed control was studied, separately. Applying Lyapunov functional method, consensus criteria were established to show the exponential behavior of system. Finally, for evaluating results, a numerical example has been solved.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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