Traveling wave solutions for complex nonlinear space–time fractional order $(2 + 1)$ -dimensional Maccari dynamical system and Schrödinger equation with dual power law nonlinearity

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Abstract

The traveling wave solutions for complex $(2 + 1)$ -dimensional Maccari dynamical system and Schrödinger equation in both space and time fractional terms are studied in this paper. Using extended simple equation method, exact, periodic and soliton solutions in fractional form are extracted for both models under study confirming the effectiveness of the method. The obtained fractional results have a variety of applications in fibre optics, quantum mechanics, oceanic and optical sciences.

Keywords Fractional order Schrödinger equation · Extended simple equation method · $(2 + 1)$ -Dimensional Maccari dynamical · Traveling wave solutions

1 Introduction

Currently, there is much research interest in fractional calculus and its applications. Differential equations with fractional order have been used to model a variety of systems that are fractal in nature. In nonlinear sciences, the numerical and traveling wave solutions of nonlinear fractional order partial differential equations (FPDEs) have wide range of applications in electromagnetic waves, electrode electrolyte polarization, viscoelasticity, fiber optics, control theory, signal processing, solid physics, plasma physics, circuits and biogenetics [1–5]. FPDEs are a generalization of classical integer order time-space partial differential equations which gives an exact interpretation of nonlinear phenomena. A variety of powerful and efficient methods have been proposed in the literature to study solitonic solutions of FPDEs. Homotopy perturbation method has been used for a system of fractional differential equations [6]. Fractional autonomous differential equations have been studied with variation iteration method

[7, 8]. Method of separation of variables has been used to study the solutions of Caputo-type partial differential equations [9]. Fractional heat equations have been studied with Homotopy analysis method [10]. Variation iteration method has also been used to solve different FPDEs [11]. Fractional order Navier–Stoke equation has been solved using Laplace decompositions method [12]. Fractional wave-like heat equations have been solved using the decomposition method [13] and the optimal homotopy asymptotic method [14].

The recently proposed extended simple equation method (ESEM) [15] is rapidly expanding in use. This method is computationally direct and efficient in extracting new solutions. Although the ESEM has been applied to integer order PDEs, to the best of our knowledge, very few articles in the literature have applied this novel method in finding solutions to FPDEs. In this paper, we use the novel ESEM to find new solitary wave solution to complex nonlinear space–time fractional $(2 + 1)$ -dimensional Maccari dynamical system [16, 17] and Schrödinger equation with dual power law nonlinearity

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[18–20]. Maccari derived the $(2 + 1)$ -dimensional Maccari system from Kadomtsev–Petviashvili equation through the use of asymptotically exact reduction method which is a Fourier expansion and spatiotemporal re-scaling [21]. The Maccari system describes rogue waves in water wave dynamics and has extensive application in oceanic and optical sciences. Several solutions have been extracted for integer order Maccari equation using various methods [22–24]. However, solutions to space–time fractional order Maccari system are rarely studied. Schrödinger equation is applied in describing Bose–Einstein condensate in quantum field theory [25]. It also has a wide range of application in traveling waves theory [26] and fibre optics [27]. Many scholars have tried to find analytical or numerical solutions to different types of integer order Schrödinger equation using various methods [26–33]. However, we discuss a space–time fractional order Schrödinger equation with non-integer balance number in this paper which is rare in the literature.

This paper is organized as follows: Description of ESEM is presented in Sect. 2. We give the application of the ESEM method to construct new traveling wave solutions to space–time fractional order Maccari and Schrödinger wave equations in Sects. 3 and 4 respectively. The results and discussion are presented in Sect. 5. Finally, the conclusion is given in Sect. 6.

2 Description of the extended simple equation method (ESEM)

FPDE of the form

$$Q(q, D_t^\alpha q, D_\rho^\beta q, D_t^\eta q, \dots, D_t^\alpha D_t^\alpha q, D_t^\alpha D_\rho^\beta q, D_\rho^\beta D_\rho^\beta q, D_\rho^\beta D_t^\eta q, \dots) = 0, \quad \text{for } \alpha, \beta, \eta \in (0, 1], \quad (1)$$

where q is an unknown function, Q is a polynomial in q and its partial fractional derivatives which involve the highest order derivatives and nonlinear terms. Currently, there are several definitions for fractional derivative which include the Riemann–Liouville and Caputo definitions. These definitions are as follows:

– Riemann–Liouville definition:

$$D_x^\alpha f(x) = \frac{1}{\Gamma(n - \alpha)} \left(\frac{d}{dx} \right)^n \int_0^x (x - t)^{n - \alpha - 1} f(t) dt, \quad n - 1 < \alpha \leq n \quad (2)$$

– Caputo definition:

$$D_x^\alpha = \frac{1}{\Gamma(n - \alpha)} \int_0^x (x - t)^{n - \alpha - 1} f(t) dt, \quad n - 1 < \alpha \leq n \quad (3)$$

The properties of Riemann–Liouville and Caputo fractional definitions are given in [34, 35]. The problem with using the Riemann–Liouville and Caputo fractional definitions is that they do not satisfy the known formula of the derivative of the product of two functions, the quotient of two functions and the chain rule [36].

A new definition of fractional derivative known as the conformable fractional derivative has been proposed by Khalil et al. [36]. This definition that is compatible with the classical Riemann–Liouville and Caputo fractional definitions. In addition, it satisfied the known formula of the derivative of the product of two functions, the quotient of two functions and the chain rule. The mean value theorem, fractional power expansion and fractional Laplace transformation definitions have been given for the conformable fractional derivative [36, 37]. We use the conformable fractional derivative of order α defined as:

Definition 1 [36] Given a function $f : [0, \infty) \rightarrow R$. Then the conformable fractional derivative of f order α is defined by

$$(D_\alpha f)(t) = \lim_{\epsilon \rightarrow 0} \frac{f(t + \epsilon t^{1-\alpha}) - f(t)}{\epsilon} \quad (4)$$

for all $t > 0, \alpha \in (0, 1]$.

Theorem 1 [36] Let $\alpha \in (0, 1]$ and f, g be α -differentiable at a point $t > 0$. The

1. $D_\alpha(af + bf) = a(D_\alpha f) + b(D_\alpha g)$, for all $a, b \in R$
2. $D_\alpha(t^p) = pt^{p-\alpha}$, for all $p \in R$
3. $D_\alpha(\lambda) = 0$, for all constant functions $f(t) = \lambda$
4. $D_\alpha(fg) = f(D_\alpha g) + g(D_\alpha f)$
5. $D_\alpha(f/g) = \frac{g(D_\alpha f) - f(D_\alpha g)}{g^2}$
6. If, in addition, f is differentiable, then $(D_\alpha f)(t) = t^{1-\alpha} \frac{df}{dt}(t)$

These properties will be used in the next section.

Equation (1) can be evaluated by means of ESEM method, using the traveling wave:

$$q(\tau, \rho, t) = Q(\xi), \quad \xi = \frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \rho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha}, \quad (5)$$

where σ_1, σ_2 and σ_3 are nonzero constants.

Using the properties of the conformable fractional derivative in Theorem 1, the traveling wave Eq. (5) is used to reduce Eq. (1) to ordinary differential equation (ODE):

$$Q(q, q', q'', q''', \dots) = 0, \quad (6)$$

where Q is a polynomial in $q(\xi)$ and its derivatives with respect to ξ .

The solution of Eq. (6) is of the form,

$$q(\xi) = \sum_{k=-M}^M b_j Y^j(\xi), \quad (7)$$

where M_j , ($j = -M, -M + 1, \dots, -1, 0, 1, \dots, M$) are arbitrary constants to be determined, such that $b_M \neq 0$ and $Y(\xi)$ is an unknown function to be determined later, such that Y satisfies the ansatz equation

$$Y' = h_0 + h_1 Y + h_2 Y^2 + h_3 Y^3, \quad (8)$$

where h_0, h_1, h_2, h_3 are arbitrary constants.

When $h_0 = h_3 = 0$ the ansatz Eq. (8) has the following solution:

$$Y(\xi) = \frac{h_1 \exp(h_1(\xi + \xi_0))}{1 - h_2 \exp(h_1(\xi + \xi_0))}, \quad h_1 > 0, \quad (9)$$

$$Y(\xi) = \frac{-h_1 \exp(h_1(\xi + \xi_0))}{1 + h_2 \exp(h_1(\xi + \xi_0))}, \quad h_1 > 0. \quad (10)$$

When $h_1 = h_3 = 0$, the ansatz Eq. (8) has the following solutions:

$$Y(\xi) = \frac{\sqrt{h_0 h_2}}{h_2} \tan(\sqrt{h_0 h_2}(\xi + \xi_0)), \quad h_0 h_2 > 0, \quad (11)$$

$$Y(\xi) = \frac{\sqrt{-h_0 h_2}}{h_2} \tanh(\sqrt{-h_0 h_2}(\xi + \xi_0)), \quad h_0 h_2 < 0. \quad (12)$$

When $h_0 = h_2 = 0$, the ansatz Eq. (8) has the solutions:

$$Y(\xi) = \pm \frac{\sqrt{-h_1}}{\sqrt{\exp(-2(\xi + \xi_0)h_1) + h_3}}, \quad h_1 < 0, \quad (13)$$

$$Y(\xi) = \pm \frac{\exp((\xi + \xi_0)h_1)\sqrt{h_1}}{\sqrt{1 - \exp(-2(\xi + \xi_0)h_1 h_3)}}, \quad h_1 > 0. \quad (14)$$

The M in Eq. (7) is a positive integer determined by balancing the highest order derivatives and the nonlinear terms in Eq. (6). Substituting Eq. (7) along with Eq. (8) into Eq. (6) and setting coefficients of $Y^j(\xi)$ to zero, we obtained a system of algebraic equations and calculate parameters $b_{-1}, b_0, b_1, b_2, h_0, h_1, h_2, h_3, \sigma_1, \sigma_2, \sigma_3$. Substituting all the parameter values into Eq. (7) along with $Y(\xi)$ which is a general solution of Eq. (8), we obtained the required the solutions of Eq. (1).

3 The space–time fractional (2 + 1)-dimensional Maccari equation

We apply ESEM to find new exact solutions to the space–time fractional Maccari equations [16, 17]:

$$\begin{aligned} iD_t^\alpha q + D_\tau^{2\beta} q + qr &= 0, \\ D_t^\alpha r + D_\theta^\eta q + D_\tau^\beta(|q|^2) &= 0. \end{aligned} \quad (15)$$

Equation (15) is complex, therefore, it admits the traveling wave transformations Eq. (5) and the following:

$$\begin{aligned} q(\tau, \theta, t) &= Q(\xi) \exp(i\theta), \quad i = \sqrt{-1}, \quad r(\tau, \theta, t) = R(\xi), \\ \theta &= \frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \theta^\eta}{\eta} + \frac{\theta_3 t^\alpha}{\alpha}. \end{aligned} \quad (16)$$

Substituting (16) in (15) give:

$$i(\sigma_3 + 2\sigma_1 \theta_1)Q' - (\sigma_3 + \sigma_1^2)Q + \sigma_1^2 Q'' + QR = 0, \quad (17)$$

$$(\sigma_2 + \sigma_3)R' + \sigma_1(Q^2)' = 0. \quad (18)$$

Integrating Eq. (18) and setting the constant of integration to zero give:

$$R = \left(\frac{-\sigma_1}{\sigma_2 + \sigma_3} \right) Q^2. \quad (19)$$

The real and imaginary part of Eq. (17) can be separated as follows:

$$\sigma_1^2 Q'' - (\sigma_1^2 + \sigma_3)Q + QR = 0, \quad (20)$$

$$(\sigma_3 + 2\sigma_1 \theta_1)Q' = 0. \quad (21)$$

Substituting Eq. (19) in Eq. (20) gives:

$$\sigma_1^2 Q'' - (\sigma_1^2 + \sigma_3)Q - \left(\frac{\sigma}{\sigma_2 + \sigma_3} \right) Q^3 = 0. \quad (22)$$

Applying the homogeneous balance principle on Eq. (22) we get $M = 1$. From Eq. (7), the solution can be written as:

$$Q = b_{-1}Y + b_0 + b_1Y. \quad (23)$$

Where Y satisfies the well-known Riccati equation

$$Y' = h_0 + h_1 Y + h_2 Y^2 + h_3 Y^3. \quad (24)$$

Substituting Eqs. (23) and (24) in Eq. (22) we obtained a system of algebraic equations in parameters $b_{-1}, b_0, b_1, b_2, h_0, h_1, h_2, h_3, \sigma_1, \sigma_2, \sigma_3, \theta_1, \theta_2$ and θ_3 . We solve the system of algebraic equations for these parameters which gives the following solution cases:

For Case I: $h_3 = 0$

$$\begin{aligned} \text{Set 1: } b_1 &= 0, \quad \theta_3 = \frac{1}{2}(4h_0 h_2 \sigma_1^2 - h_1^2 \sigma_1^2 - 2\theta_1^2), \\ b_0 &= \pm \frac{h_1 \sqrt{\sigma_1(\sigma_2 + \sigma_3)}}{\sqrt{2}}, \quad b_{-1} = \pm h_0 \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}. \end{aligned}$$

Putting solution Set 1 in Eq. (23) gives the following solutions:

$$\begin{aligned}
q_1 = & \pm \frac{h_1 \sqrt{\sigma_1(\sigma_2 + \sigma_3)}}{\sqrt{2}} + \\
& + \frac{2h_2 h_0 \sqrt{\sigma_1(\sigma_2 + \sigma_3)}}{h_1 - \sqrt{4h_0 h_2 - h_1^2} \tan \left(\frac{\sqrt{4h_0 h_2 - h_1^2}}{2} \left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0 \right) \right)} \\
& \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{(4h_0 h_2 \sigma_1^2 - h_1^2 \sigma_1^2 - 2\theta_1^2) t^\alpha}{2\alpha} \right) \right), \quad \text{where } 4h_0 h_2 > h_1^2,
\end{aligned} \tag{25}$$

$$\begin{aligned}
r_1 = & \left(\frac{\sigma_1}{\sigma_2 + \sigma_3} \right) \\
& \times \left[\pm \frac{h_1 \sqrt{\sigma_1(\sigma_2 + \sigma_3)}}{\sqrt{2}} + \frac{2h_2 h_0 \sqrt{\sigma_1(\sigma_2 + \sigma_3)}}{h_1 - \sqrt{4h_0 h_2 - h_1^2} \tan \left(\frac{\sqrt{4h_0 h_2 - h_1^2}}{2} \left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0 \right) \right)} \right]^2.
\end{aligned} \tag{26}$$

Set2: $b_1 = \pm \frac{\sqrt{2\sigma_1(\sigma_2 + \sigma_3)}}{\sqrt{2}}$, $\theta_3 = \frac{1}{2}(4h_0 h_2 \sigma_1^2 - h_1^2 \sigma_1^2 - 2\theta_1^2)$,
 $b_0 = \pm \frac{h_1 \sqrt{\sigma_1(\sigma_2 + \sigma_3)}}{\sqrt{2}}$, $b_{-1} = 0$.

Putting solution Set 2 in Eq. (23) gives the following solutions:

$$\begin{aligned}
q_2 = & \pm \frac{h_1 \sqrt{\sigma_1(\sigma_2 + \sigma_3)}}{\sqrt{2}} \\
& + \frac{\sqrt{2\sigma_1(\sigma_2 + \sigma_3)} \left(h_1 - \sqrt{4h_0 h_2 - h_1^2} \tan \left(\frac{\sqrt{4h_0 h_2 - h_1^2}}{2} \left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0 \right) \right) \right)}{2} \\
& \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{(4h_0 h_2 \sigma_1^2 - h_1^2 \sigma_1^2 - 2\theta_1^2) t^\alpha}{2\alpha} \right) \right), \quad \text{where } 4h_0 h_2 > h_1^2,
\end{aligned} \tag{27}$$

$$\begin{aligned}
r_2 = & \left(\frac{\sigma_1}{\sigma_2 + \sigma_3} \right) \\
& \times \left(\pm \frac{h_1 \sqrt{\sigma_1(\sigma_2 + \sigma_3)}}{\sqrt{2}} + \frac{\sqrt{2\sigma_1(\sigma_2 + \sigma_3)} \left(h_1 - \sqrt{4h_0 h_2 - h_1^2} \tan \left(\frac{\sqrt{4h_0 h_2 - h_1^2}}{2} \left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0 \right) \right) \right)}{2} \right)^2.
\end{aligned} \tag{28}$$

For Case II: $h_0 = h_3 = 0$

Set 1: $\sigma_3 = \frac{2b_0^2 - h_1^2 \sigma_1 \sigma_2}{h_1^2 \sigma_1}$, $b_1 = \frac{2b_0 h_2}{h_1}$, $\theta_3 = \frac{1}{2}(-h_1^2 \sigma_1^2 - 2\theta_1^2)$, $b_{-1} = 0$.

Substituting solution Set 1 in Case II into Eq. (23) gives the following solutions:

$$q_3 = b_0 + \frac{2b_0 h_2 \exp\left(h_1\left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0\right)\right)}{1 - c_2 \exp\left(h_1\left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0\right)\right)} \times \exp\left(i\left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} - \frac{(h_1^2 \sigma_1^2 + 2\theta_1^2)t^\alpha}{2\alpha}\right)\right), \quad h_1 > 0, \quad (29)$$

$$r_3 = \left(\frac{\sigma_1}{\sigma_2 + \sigma_3}\right) \left(b_0 + \frac{2b_0 h_2 \exp\left(h_1\left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0\right)\right)}{(1 - c_2 \exp\left(h_1\left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0\right)\right))}\right)^2. \quad (30)$$

$$q_4 = b_0 - \frac{2b_0 h_2 \exp\left(h_1\left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0\right)\right)}{1 - c_2 \exp\left(h_1\left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0\right)\right)} \times \exp\left(i\left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} - \frac{(h_1^2 \sigma_1^2 + 2\theta_1^2)t^\alpha}{2\alpha}\right)\right), \quad h_1 < 0, \quad (31)$$

$$r_4 = \left(\frac{\sigma_1}{\sigma_2 + \sigma_3}\right) \left(b_0 - \frac{2b_0 h_2 \exp\left(h_1\left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0\right)\right)}{(1 - c_2 \exp\left(h_1\left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0\right)\right))}\right)^2. \quad (32)$$

For Case III: $h_1 = h_3 = 0$

Set 1: $b_1 = 0$, $\theta_3 = 2h_0 h_2 \sigma_1^2 - \theta_1^3$, $b_0 = 0$, $b_{-1} = \pm \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}$.

Substituting solution Set 1 in Case III into Eq. (23) gives the following solutions:

$$q_5 = \left(\frac{\pm \sqrt{h_0 h_2} \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}}{\tan(\sqrt{h_0 h_2}(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))}\right) \times \exp\left(i\left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{(\theta_3)t^\alpha}{\alpha}\right)\right), \quad h_0 h_2 > 0, \quad (33)$$

$$r_5 = \left(\frac{\sigma_1}{\sigma_2 + \sigma_3}\right) \left(\frac{\pm \sqrt{h_0 h_2} \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}}{\tan(\sqrt{h_0 h_2}(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))}\right)^2. \quad (34)$$

$$q_6 = \left(\frac{\pm \sqrt{-h_0 h_2} \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}}{\tanh(\sqrt{-h_0 h_2}(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))}\right) \times \exp\left(i\left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{(\theta_3)t^\alpha}{\alpha}\right)\right), \quad h_0 h_2 < 0, \quad (35)$$

$$r_6 = \left(\frac{\sigma_1}{\sigma_2 + \sigma_3}\right) \left(\frac{\pm \sqrt{-h_0 h_2} \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}}{\tanh(\sqrt{-h_0 h_2}(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))}\right)^2. \quad (36)$$

Set 2: $b_1 = \pm h_2 \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}$, $\theta_3 = 2h_0 h_2 \sigma_1(\sigma_2 + \sigma_3)$, $b_0 = 0$, $b_{-1} = 0$.

Substituting solution Set 2 in Case III into Eq. (23) gives the following solutions:

$$q_7 = \left(\frac{\pm h_2 \sqrt{2\sigma_1(\sigma_2 + \sigma_3)} \sqrt{h_0 h_2} \tan(\sqrt{h_0 h_2}(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))}{h_2}\right) \times \exp\left(i\left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{(2h_0 h_2 \sigma_1(\sigma_2 + \sigma_3))t^\alpha}{\alpha}\right)\right), \quad h_0 h_2 > 0, \quad (37)$$

$$r_7 = \left(\frac{\sigma_1}{\sigma_2 + \sigma_3}\right) \times \left(\frac{\pm h_2 \sqrt{2\sigma_1(\sigma_2 + \sigma_3)} \sqrt{h_0 h_2} \tan(\sqrt{h_0 h_2}(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))}{h_2}\right)^2. \quad (38)$$

$$q_8 = \left(\frac{\pm h_2 \sqrt{2\sigma_1(\sigma_2 + \sigma_3)} \sqrt{-h_0 h_2} \tanh(\sqrt{-h_0 h_2} (\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))}{h_2} \right) \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{(2h_0 h_2 \sigma_1 (\sigma_2 + \sigma_3)) t^\alpha}{\alpha} \right) \right), \quad h_0 h_2 > 0, \quad (39)$$

$$r_8 = \left(\frac{\sigma_1}{\sigma_2 + \sigma_3} \right) \times \left(\frac{\pm h_2 \sqrt{2\sigma_1(\sigma_2 + \sigma_3)} \sqrt{-h_0 h_2} \tanh(\sqrt{-h_0 h_2} (\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))}{h_2} \right)^2. \quad (40)$$

Set 3: $b_2 = \pm h_2 \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}$, $\theta_3 = 8h_0 h_3 \sigma_1^2 - \theta_1^3$, $b_0 = 0$, $b_{-1} = \pm h_0 \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}$.

Substituting solution Set 3 in Case III into Eq. (23) gives the following solutions:

$$r_9 = \left(\frac{\sigma_1}{\sigma_2 + \sigma_3} \right) \left(\frac{\sqrt{h_0 h_2} \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}}{\tan(\sqrt{h_0 h_2} (\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))} \right) \quad (42)$$

$$q_9 = \frac{\sqrt{h_0 h_2} \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}}{\tan(\sqrt{h_0 h_2} (\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))} + \frac{h_2 \sqrt{2\sigma_1(\sigma_2 + \sigma_3)} \sqrt{h_0 h_2} \tan(\sqrt{h_0 h_2} (\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))}{h_2} \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{(8h_0 h_3 \sigma_1^2 - \theta_1^3) t^\alpha}{\alpha} \right) \right), \quad h_0 h_2 > 0, \quad (41)$$

$$q_{10} = \frac{\sqrt{-h_0 h_2} \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}}{\tanh(\sqrt{h_0 h_2} (\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))} + \frac{h_2 \sqrt{2\sigma_1(\sigma_2 + \sigma_3)} \sqrt{-h_0 h_2} \tanh(\sqrt{-h_0 h_2} (\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))}{h_2} \quad (44)$$

$$\times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{(8h_0 h_3 \sigma_1^2 - \theta_1^3) t^\alpha}{\alpha} \right) \right), \quad h_0 h_2 < 0, \quad (45)$$

$$r_{10} = \left(\frac{\sigma_1}{\sigma_2 + \sigma_3} \right) \left(\frac{\sqrt{-h_0 h_2} \sqrt{2\sigma_1(\sigma_2 + \sigma_3)}}{\tanh(\sqrt{-h_0 h_2} (\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))} + \frac{h_2 \sqrt{2\sigma_1(\sigma_2 + \sigma_3)} \sqrt{-h_0 h_2} \tanh(\sqrt{-h_0 h_2} (\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} + \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0))}{h_2} \right)^2. \quad (46)$$

4 Complex Schrödinger equation with dual power law nonlinearity

The complex Schrödinger equation with dual power law nonlinearity is given by

$$iD_t^\alpha \phi + \frac{1}{2}(D_\tau^{2\beta} \phi + D_\rho^{2\eta} \phi) + (|\phi|^{2a} + \kappa|\phi|^{4a})\phi = 0, \quad (47)$$

where $\phi(\tau, \rho, t)$ is the complex amplitude of the wave form and a is constant. This equation has applications in nonlinear optics, quantum mechanics, plasma physics and super conductivity. Various authors have discussed the integer order of this equation using various methods [18–20]

Eq. (47) admits the following traveling wave transformation

$$(\sigma_1^2 + \sigma_2^2)\phi'' - (\theta_1^2 + \theta_2^2 + 2\theta_3)\phi + 2\phi^{2a+1} + 2\kappa\phi^{4a+1} = 0, \quad (49)$$

if we set $\sigma_3 = \theta_1\sigma_1 + \theta_2\sigma_2$. The homogeneous balancing of Eq. (49) gives $M = \frac{1}{2a}$.

Let the solution of Eq. (49) be

$$\Phi(\xi) = c(Y(\xi))^{\frac{1}{2a}}. \quad (50)$$

When Eq. (50) and (8) are substituted into Eq. (49) we obtain a system of equation in $h_0, h_1, h_2, \sigma_1, \sigma_2, \theta_1, \theta_2, \theta_3$. The solution to the system leads to the following solution cases:

For Case I: $h_0 = h_3 = 0$

$$\text{Set 1: } \theta_2 = \sqrt{\frac{-2\kappa(a^2+2a+1)(\sigma_1^2+2\sigma_3)-2a-1}{2\kappa(a^2+2a+1)}},$$

$$\sigma_2 = \pm \frac{-h_1^2\sigma_1^2\kappa(a+1)^2-2a^2(2a+1)}{\sqrt{h_1^2\kappa(a+1)^2}}, h_2 = \frac{2h_1\kappa(a+1)(c^{2a})}{2a+1}.$$

Putting solution Set 2 and the solution of Eq. (24) in Eq. (50) we get the following solutions:

$$\phi_1 = c \left(\frac{h_1 \exp(h_1(\frac{\sigma_1\tau^\beta}{\beta} + \frac{\sigma_2\rho^\eta}{\eta} - \frac{(\theta_1\sigma_1+\theta_2\sigma_2)t^\alpha}{\alpha} + \xi_0))}{1 - \left(\frac{2h_1\kappa(a+1)c^{2a} \exp(h_1(\xi+\xi_0))}{2a+1}\right) \exp(h_1(\frac{\sigma_1\tau^\beta}{\beta} + \frac{\sigma_2\rho^\eta}{\eta} - \frac{(\theta_1\sigma_1+\theta_2\sigma_2)t^\alpha}{\alpha} + \xi_0))} \right)^{\frac{1}{2a}}$$

$$\times \exp\left(i\left(\frac{\theta_1\tau^\beta}{\beta} + \frac{\theta_2\rho^\eta}{\eta} + \frac{\theta_3t^\alpha}{\alpha}\right)\right), \quad h_1 > 0, \quad (51)$$

$$\phi_2 = c \left(\frac{-h_1 \exp(h_1(\frac{\sigma_1\tau^\beta}{\beta} + \frac{\sigma_2\rho^\eta}{\eta} - \frac{(\theta_1\sigma_1+\theta_2\sigma_2)t^\alpha}{\alpha} + \xi_0))}{1 + \left(\frac{2h_1\kappa(a+1)c^{2a} \exp(h_1(\xi+\xi_0))}{2a+1}\right) \exp(h_1(\frac{\sigma_1\tau^\beta}{\beta} + \frac{\sigma_2\rho^\eta}{\eta} - \frac{(\theta_1\sigma_1+\theta_2\sigma_2)t^\alpha}{\alpha} + \xi_0))} \right)^{\frac{1}{2a}}$$

$$\times \exp\left(i\left(\frac{\theta_1\tau^\beta}{\beta} + \frac{\theta_2\rho^\eta}{\eta} + \frac{\theta_3t^\alpha}{\alpha}\right)\right), \quad h_1 < 0. \quad (52)$$

$$\phi(\tau, \rho, t) = \Phi(\xi) \exp(i\theta),$$

$$\xi = \frac{\sigma_1\tau^\beta}{\beta} + \frac{\sigma_2\rho^\eta}{\eta} - \frac{\sigma_3t^\alpha}{\alpha},$$

$$\theta = \frac{\theta_1\tau^\beta}{\beta} + \frac{\theta_2\rho^\eta}{\eta} + \frac{\theta_3t^\alpha}{\alpha}, \quad (48)$$

where $\sigma_1, \sigma_2, \sigma_3, \theta_1, \theta_2, \theta_3$ are constants and $\alpha(0 < \alpha \leq 1)$, $\beta(0 < \beta \leq 1)$, $\eta(0 < \eta \leq 1)$, are the fractional orders.

Putting Eq. (48) in (47) give the following ODE and separating it into real and imaginary parts gives:

$$\text{Set 2: } \theta_2 = -\frac{\sqrt{-2\theta_1^2\kappa-4\theta_3\kappa-2\theta_1^2\kappa a^2-4\theta_3\kappa a^2-8\theta_3\kappa a-2a-1}}{\sqrt{2\sqrt{\kappa a^2+2\kappa a+\kappa}}},$$

$$h_0 = \frac{2h_1\kappa(a+1)c^{2a}}{2a+1},$$

$$\sigma_2 = \pm \frac{\sqrt{-h_1^2\sigma_1^2\kappa a^2-2h_1^2\sigma_1^2\kappa a-h_1^2\sigma_1^2\kappa-4a^3-2a^2}}{\sqrt{h_1^2\kappa a^2+2h_1^2\kappa a+h_1^2\kappa}}.$$

Putting solution Set 2 and the solution of Eq. (24) in Eq. (50) we get the following solutions:

$$\phi_3 = c \left(\frac{h_1 \exp(h_1(\frac{\sigma_1\tau^\beta}{\beta} + \frac{\sigma_2\rho^\eta}{\eta} - \frac{(\theta_1\sigma_1+\theta_2\sigma_2)t^\alpha}{\alpha} + \xi_0))}{1 - \left(\frac{2h_1\kappa(a+1)c^{2a}}{2a+1}\right) \exp(h_1(\frac{\sigma_1\tau^\beta}{\beta} + \frac{\sigma_2\rho^\eta}{\eta} - \frac{(\theta_1\sigma_1+\theta_2\sigma_2)t^\alpha}{\alpha} + \xi_0))} \right)^{\frac{1}{2a}}$$

$$\times \exp\left(i\left(\frac{\theta_1\tau^\beta}{\beta} + \frac{\theta_2\rho^\eta}{\eta} + \frac{\theta_3t^\alpha}{\alpha}\right)\right), \quad h_1 > 0, \quad (53)$$

$$\phi_4 = c \left(\frac{-h_1 \exp(h_1(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} - \frac{(\theta_1 \sigma_1 + \theta_2 \sigma_2) t^\alpha}{\alpha} + \xi_0))}{1 + \left(\frac{2h_1 \kappa(a+1)c^{2a}}{2a+1} \right) \exp(h_1(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} - \frac{(\theta_1 \sigma_1 + \theta_2 \sigma_2) t^\alpha}{\alpha} + \xi_0))} \right)^{\frac{1}{2a}} \\ \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{\theta_3 t^\alpha}{\alpha} \right) \right), \quad h_1 < 0. \quad (54)$$

For Case II: $h_1 = h_3 = 0$

Set 1: $\kappa = 0, \theta_2 = \pm \sqrt{-\theta_1^2 - 2\theta_3}, \sigma_2 = i\sigma_1$

Putting solution Set 1 in Case II and the solution of Eq. (24) in Eq. (50) we get the following solutions:

$$\phi_5 = c \left(\frac{\sqrt{h_0 h_2}}{h_2} \tan \left(\sqrt{h_0 h_2} \left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} - \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0 \right) \right) \right)^{\frac{1}{2a}} \\ \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{\theta_3 t^\alpha}{\alpha} \right) \right) \quad h_0 h_2 > 0, \quad (55)$$

$$\phi_6 = c \left(\frac{-\sqrt{-h_0 h_2}}{h_2} \tanh \left(\sqrt{-h_0 h_2} \left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} - \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0 \right) \right) \right)^{\frac{1}{2a}} \\ \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{\theta_3 t^\alpha}{\alpha} \right) \right) \quad h_0 h_2 < 0. \quad (56)$$

Set 2: $\kappa = 0, \theta_2 = \pm \sqrt{-\theta_1^2 - 2\theta_3}, \sigma_2 = -i\sigma_1$

Putting solution Set 2 in Case II and the solution of Eq. (24) in Eq. (50) we get the following solutions:

$$\phi_7 = c \left(\frac{\sqrt{h_0 h_2}}{h_2} \tan \left(\sqrt{h_0 h_2} \left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} - \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0 \right) \right) \right)^{\frac{1}{2a}} \\ \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{\theta_3 t^\alpha}{\alpha} \right) \right) \quad h_0 h_2 > 0, \quad (57)$$

$$\phi_8 = c \left(\frac{-\sqrt{-h_0 h_2}}{h_2} \tanh \left(\sqrt{-h_0 h_2} \left(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} - \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0 \right) \right) \right)^{\frac{1}{2a}} \\ \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{\theta_3 t^\alpha}{\alpha} \right) \right) \quad h_0 h_2 < 0. \quad (58)$$

For Case III: $h_0 = h_2 = 0$

Set 1: $\kappa = 0, \theta_2 = -\sqrt{-\theta_1^2 - 2\theta_3}, \sigma_2 = \pm i\sigma_1$

Putting solution Set 1 in Case III and the solution of Eq. (24) in Eq. (50) we get the following solutions:

$$\phi_9 = c \left(\pm \frac{\sqrt{-h_1}}{\sqrt{\exp(-2(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} - \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0)h_1 + h_3)}} \right)^{\frac{1}{2a}} \\ \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{\theta_3 t^\alpha}{\alpha} \right) \right) \quad h_1 < 0, \quad (59)$$

$$\phi_{10} = c \left(\pm \frac{\sqrt{h_1} \exp((\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} - \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0)h_1)}{\sqrt{1 - \exp((-2(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} - \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0)h_1 h_3))}} \right)^{\frac{1}{2a}} \\ \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{\theta_3 t^\alpha}{\alpha} \right) \right) \quad h_1 > 0. \quad (60)$$

Set 2: $\kappa = 0, \theta_2 = \sqrt{-\theta_1^2 - 2\theta_3}, \sigma_2 = \pm i\sigma_1$

Putting solution Set 2 in Case III and the solution of Eq. (24) in Eq. (50) we get the following solutions:

$$\phi_{11} = c \left(\pm \frac{\sqrt{-h_1}}{\sqrt{\exp(-2(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} - \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0)h_1 + h_3)}} \right)^{\frac{1}{2a}} \\ \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{\theta_3 t^\alpha}{\alpha} \right) \right) \quad h_1 < 0, \quad (61)$$

$$\phi_{12} = c \left(\pm \frac{\sqrt{h_1} \exp((\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} - \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0)h_1)}{\sqrt{1 - \exp((-2(\frac{\sigma_1 \tau^\beta}{\beta} + \frac{\sigma_2 \varrho^\eta}{\eta} - \frac{\sigma_3 t^\alpha}{\alpha} + \xi_0)h_1 h_3))}} \right)^{\frac{1}{2a}} \\ \times \exp \left(i \left(\frac{\theta_1 \tau^\beta}{\beta} + \frac{\theta_2 \varrho^\eta}{\eta} + \frac{\theta_3 t^\alpha}{\alpha} \right) \right) \quad h_1 > 0. \quad (62)$$

5 Discussion

ESEM has been successfully used to extract different solutions for the space–time fractional $(2 + 1)$ -dimensional Maccari dynamical system and Schrödinger equation. These solutions are expressed in terms of fractional exact, periodic, and soliton solutions. For the Maccari dynamical system Eqs. (29), (30), (31), (32) are exact solutions, Eqs. (25), (26), (27), (28), (33), (34), (37), (38), (41) and (42) are periodic solutions and Eqs. (35), (36), (39), (40), (44) and (46) are soliton solutions. The solutions we found for the Maccari system are novel compared to integer order solutions that have been extracted in the literature [38–41].

These solutions have a large application in applied sciences such as coastal, ocean, and maritime engineering. Similarly for the Schrödinger equation Eqs. (51), (52), (53), (54), (59), (60), (61) and (62) are exact solutions, Eqs. (55) and (57) are periodic solution and Eqs. (56) and (58) are soliton solutions. The solutions that were obtained for the Schrödinger equation are novel compared to solutions extracted for other Schrödinger equations in the literature [15, 18, 20, 26–33]. By choosing different values for parameter a ($a \neq 0$), one can obtain many special solutions for the Schrödinger equation in different forms. These solutions show that ESEM is a powerful method that is effective and convenient. Also, ESEM gives more exact solutions compared to other methods. The solutions we obtained through this method are structurally different from other methods and generate many special solutions by choosing different fractional order for α , β , and η . The change of space–time fractional order changes the nature of the solution and have huge influence on the nonlinear propagation of the solution.

6 Conclusion

In this paper, we have successfully extracted new solitary solutions for complex nonlinear space–time fractional $(2 + 1)$ -dimensional Maccari dynamical system and Schrödinger equation with dual power law nonlinearity. These complex space–time fractional models were converted to nonlinear ODEs using a traveling wave transformation and the resulting ODEs were solved successfully using ESEM. By applying ESEM, variety of solutions which include exact, periodic and soliton solutions in fractional form are extracted for both models under study confirming the effectiveness of the method. The obtained fractional results have a variety of applications in fibre optics, quantum mechanics, oceanic and optical sciences since fractional orders models are more suitable for describing complex systems with memory and delay [42].

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interests.

References

- Islam T, Akbar MA, Azad AK (2018) Traveling wave solutions to some nonlinear fractional partial differential equations through the rational (G'/G) -expansion method. *J Ocean Eng Sci* 3:76–81
- Hilfer R (2000) Applications of fractional calculus in physics. Word Scientific Company, Singapore
- Luchko AY, Gorenflo R (1998) The initial value problem for some fractional differential equations with the Caputo derivative. Pre print series A08-98, Fachbereich Mathematik und Informatik, Freie Universität, Berlin
- Arshad M, Lu D, Wang J (2017) Fractional sub-equation method for a generalized space-time fractional Fisher equation with variable coefficients. *Nonlinear Sci Lett A* 8:162–170
- Singh J, Rashidi MM, Kumar D, Swroop R (2016) A fractional model of a dynamical Brusselator reaction-diffusion system arising in triple collision and enzymatic reactions. *Nonlinear Eng* 5:277–285
- Abdulaziz O, Hashim I, Momani S (2008) Solving systems of fractional differential equations by homotopy-perturbation method. *Phys Lett A* 372:451–459
- Yulita Molliq R, Noorani MSM, Hashim I (2009) Variational iteration method for fractional heat- and wave-like equations. *Nonlinear Anal Real World Appl* 10:1854–1869
- He J-H (2000) Variational iteration method for autonomous ordinary differential systems. *Appl Math Comput* 114:115–123
- Gorguis A, Chan WKB (2008) Heat equation and its comparative solutions. *Comput Math Appl* 55:2973–2980
- Xu H (2008) Analysis of a time fractional wave-like equation with the homotopy analysis method. *J Phys Lett A* 372:1250–1255
- Ozis T, Agrseven D (2008) He's homotopy perturbation method for solving heat-like and wave-like equations with variable coefficients. *Phys Lett A* 372:5944–5950
- Kumar S, Kumar D, Abbasbandy S, Rashidi MM (2014) Analytical solution of fractional Navier–Stokes equation by using modified Laplace decomposition method. *Ain Shams Eng J* 5:569–574
- Momani S (2005) Analytical approximate solution for fractional heat-like and wave-like equations with variable coefficients using the decomposition method. *Appl Math Comput* 165:459–472
- Sarwar S, Iqbal S (2017) Exact solutions of the non-linear fractional Klein-Gordon equation using the optimal homotopy asymptotic method. *Nonlinear Sci Lett A* 8:365–373
- Ali A, Seadawy AR, Lu D (2017) Soliton solutions of the nonlinear Schrödinger equation with the dual power law nonlinearity and resonant nonlinear Schrödinger equation and their modulation instability analysis. *Optik* 145:79–88
- Asaraai A (2015) The application of modified F-expansion method solving the Maccari's system. *Br J Math Comput Sci* 11:1–13
- Nadia CC, Younis M (2016) New and more exact traveling wave solutions to integrable $(2 + 1)$ -dimensional Maccari system. *Nonlinear Dyn* 83:1395–1401
- Zhang LH, Si JG (2010) New soliton and periodic solutions of $(2 + 1)$ -dimensional nonlinear Schrödinger equation with dual-power law of nonlinearity. *Commun Nonlinear Sci Numer Simul* 15:2747–2754
- Seadawy AR, Arshad M, Lu D (2017) Stability analysis of new exact traveling wave solutions of new coupled KdV and new coupled Zakharov–Kuznetsov systems. *Eur Phys J Plus* 132:1–19
- Biswas A (2008) 1-Soliton solution of $(2 + 1)$ -dimensional nonlinear Schrödinger equation in dual-power law media. *Phys Lett A* 372:5941–5943
- Maccari A (1996) The Kadomtsev–Petviashvili equation as a source of integrable model equations. *J Math Phys* 37:6207–6212
- Wazwaz AM (2012) Solitary waves solutions for extended forms of quantum Zakharov–Kuznetsov equations. *Phys Scr* 85:65011–65019
- Kumar H, Chand F (2014) Exact traveling wave solutions of some nonlinear evolution Equations. *J Theor Appl Phys* 8:1–10
- Demiray ST, Pandir Y, Bulut H (2015) New solitary wave solutions of Maccari system. *Ocean Eng* 103:153–159

25. Kartashov YV, Malomed BA, Torner L (2011) Solitons in nonlinear lattices. *Rev Mod Phys* 83:247–305
26. Carles R, Dumas E, Sparber C (2010) Multiphase weakly nonlinear geometric optics for Schrödinger equations. *SIAM J Math Anal* 42:489–518
27. Thomas R, Kharif C, Manna M (2012) A nonlinear Schrödinger equation for water waves on finite depth with constant vorticity. *Phys Fluids* 24:395–416
28. Arshad M, Seadawy AR, Lu D (2017) Optical soliton solutions of the generalized higher-order nonlinear Schrödinger equations and their applications. *Opt Quant Electron* 2017:49–421
29. Arshad M, Seadawy AR, Lu D (2017) Exact bright-dark solitary wave solutions of the higher-order cubic-quintic nonlinear Schrödinger equation and its stability. *Optik* 138:40–49
30. Lu D, Seadawy AR, Arshad M (2017) Applications of extended simple equation method on unstable nonlinear Schrödinger equations. *Optik* 140:136–144
31. Nasreen N, Lu D, Arshad M (2018) Optical soliton solutions of nonlinear Schrödinger equation with second order spatiotemporal dispersion and its modulation instability. *Optik* 161:221–229
32. Arshad M, Seadawy AR, Lu D (2017) Bright-dark solitary wave solutions of generalized higher-order nonlinear Schrödinger equation and its applications in optics. *J Electromagn Waves Appl* 31:1–10
33. Seadawy AR, Arshad M, Lu D (2018) Elliptic function solutions and travelling wave solutions of nonlinear Dodd-Bullough–Mikhailov, two-dimensional Sine–Gordon and coupled Schrödinger–KdV dynamical models. *Results Phys* 10:995–1005
34. Li ZB, He J-H (2010) Fractional complex transformation for fractional differential equations. *Math Comput Appl* 15:970–973
35. He J-H, Li ZB (2012) Converting fractional differential equations into partial differential equations. *Therm Sci* 16:331–334
36. Khalil R, Al Horani M, Yousef A, Sababheh M (2014) A new definition of fractional derivative. *J Comput Appl Math* 264:65–70
37. Abdeljawad T (2015) On conformable fractional calculus. *J Comput Appl Math* 279:57–66
38. Rostamy D, Zabihi F (2012) Exact solutions for different coupled nonlinear Maccari’s systems. *Nonlinear Stud* 19:229–239
39. Bulut H, Yel G, Haci M (2017) Baskonus Novel structure to the coupled nonlinear maccari’s system by using modified trial equation method. *Adv Math Mod Appl* 2:14–19
40. Cenesiz Y, Tasbozan O, Kurt A (2017) Functional Variable Method for conformable fractional modified KdV-ZK equation and Maccari system. *Tbilisi Math J* 10:117–125
41. Yuan F, Rao J, Porsezian K, Mihalache D, He J (2016) Various exact rational solutions of the two-dimensional Maccari’s system. *Rom J Phys* 61:378–399
42. Ku A (2013) Mahata on fractional Schrödinger equation and its application. *Int J Sci Eng Res* 4:1–5

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