Predicting the elastic properties of closed-cell aluminum foams: a mesoscopic geometric modeling approach

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Abstract

The biggest challenge in predicting the elastic properties of closed-cell aluminum foams is capturing the actual geometrical and topological characteristics precisely and efficiently such that the model can be used to predict the mechanical properties of the real foam accurately. This paper presents a mesoscopic modeling approach for constructing a geometric model that captures these characteristics and can be used to predict its elastic properties. The modeling approach introduces a new method for finding the number of seeds based on the cell diameter distribution and an algorithm for computing and assigning the irregular cell wall thickness based on reverse bubble growth. Results from the foam models developed in this work were found to have better accuracy in capturing the geometrical and topological characteristics of the real foam. All foam models generated by the proposed modeling approach have the same cell size distribution and irregular cell wall distribution as the real foam. The models have cells with around 14 faces with 5 edges on each face which is similar to real metal foams and naturally occurring foams. Numerical results from the foam models showed a better accuracy in predicting the relative Young's modulus of the real foam than the generic Laguerre–Voronoi, Kelvin, and Grenstedt models.

Keywords Aluminum foams · Elastic properties · Cell wall thickness · Seed number · Geometric modeling · Laguerre–Voronoi tessellations

1 Introduction

The exigence in materials with high stiffness and yet a low weight makes metallic foams the most used materials in the design of lightweight structures and energy absorbers [1]. Metallic foams come in different shapes and sizes and can be open-cell or closed-cell, regular or stochastic with a wide range of base materials such as aluminum, titanium, and copper, which can be produced by different manufacturing techniques [2]. Foams mechanical properties are heavily dependent on the mechanical properties of the base material and the geometry of the microstructure. Experimental study on the property-microstructure relationship of foams is limited due to the difficulty in the manufacture of foams with the prescribed microstructure

[3]. Mesoscale geometric modeling is suited for investigating these relationships.

Over the years, geometric modeling of foams has been studied by researchers to replicate the foam geometry and understand their mechanical behavior, and the methods used can be categorized into three: single cell repetition [2, 4, 5], tessellations [6–14], and image reconstruction [15–18]. The main challenge in the modeling of closed-cell foams is the ability to capture the actual geometrical and topological characteristic precisely and efficiently such that the model can be used to predict the mechanical properties of the real foam accurately. Investigations into the microstructure of real foams revealed that the cells were irregular polyhedrons with approximately 14 faces and each face having approximately 5 edges [6, 19–21]. Many researchers opted for Voronoi models due to their

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favorable features of randomness and to some extent simulate the process of foaming and the foam microstructure [22]. The yield criteria of closed-cell aluminum foams were investigated by Zhang et al. [23] using Poisson–Voronoi model. The effect of cell shape and size irregularity on the mechanical response of foams has been investigated by Tang et al. [11] using a Hardcore–Voronoi model. Laguerre–Voronoi tessellation was used by Wejrzanowski et al. [10] and Nie et al. [14] to investigate the influence of pore size variations on the specific surface area of open cell foams.

The elastic properties of real foams have been proven to be dependent on their geometrical and topological characteristics [1, 2, 24, 25]. The effects of cell wall thickness variation, cell size, and cell shape distributions on the elastic properties have been demonstrated by researchers [9, 11, 26]. However, apart from models generated by image reconstruction techniques, a model that captures the actual microstructural diversity of closed-cell foams and can be used to accurately predict its elastic properties has not yet been developed. Therefore, the objective of this work is to develop a foam model that captures the actual geometrical and topological characteristics of the real foam such as cell size distribution, irregular cell wall thickness, number of faces per cell, and number of edges per face. The foam model should be able to support the prediction of the elastic properties of the real foam. The key idea is to generate a model with consideration of microstructural diversity using Laguerre–Voronoi tessellation with the integration of realistic cell size and irregular cell wall thickness distributions acquired from real foam.

This paper is structured as follows. The foam material and the construction of the generic 3D Laguerre–Voronoi model is described. Secondly, the overview of the proposed modeling approach is presented. Thirdly, an approach for determining the number of seeds is presented. Fourthly, an algorithm for computation and assignment of realistic irregular cell wall thickness based on reverse bubble growth is presented. Lastly, the accuracy of the proposed modeling approach is demonstrated.

2 Materials

2.1 Foam material

The foam material used in this work was ALPORAS (Shinko Wire, Amagasaki, Japan) made from pure Aluminum. The cells are polyhedrons, and the cell diameter follows a log-normal distribution. The average number of faces per cell and the number of edges per face were 13.68 and 4.99 respectively [19]. The cell wall thickness is normally distributed with a mean of 0.1825 mm and a standard

deviation of 0.0919 mm. The relative density range was 10.60–12.38% [19]. The average values of the geometric parameters are listed in Table 1. The microstructure and cell shape were shown in Fig. 1.

3 Generic Laguerre–Voronoi model

This section describes the construction of generic Laguerre–Voronoi (LV) models as used by researchers and analyzes the geometrical and topological properties of the resulting foam model.

3.1 Construction of the generic Laguerre–Voronoi model

Researchers have predominantly used Voronoi tessellations in various fields in the geometric design ranging from the division of spaces [27, 28] to cellular and polycrystalline structures [29, 30]. Laguerre–Voronoi (LV) tessellation is a kind of Voronoi tessellation with weights attached to each seed that determine the size and shape of the cell it produces. It is evident that Laguerre–Voronoi tessellation can produce foam models with specific geometrical properties that are closer to the real foam due to its ability to provide control over the size and the shape of the cells [7, 10]. LV tessellation can be described as follows; for any point p_i in a set $P = \{p_1, p_2, p_3, \dots, p_n\}$, a weight r_i which is the sphere radius is provided to get a weight set $r = \{r_1, r_2, \dots, r_n\}$, and the distance between p_i and any point q is given by

$$d_L(p_i, q) = \left\{ \|p_i - q\|^2 - r_i^2 \right\}^{1/2} \quad (1)$$

where $d_L(p_i, q)$ is the Laguerre distance (or power distance).

A cell corresponding to point p_i is defined as

$$v_L(p_i) = \{p | p \in \mathbb{R}^3, d_L(p, p_i) < d_L(p, p_j), i \neq j\} \quad (2)$$

A Laguerre–Voronoi diagram (LVD) consists of a set of cells which are convex polyhedrons without overlaps interconnected in a topological manner [14]. In the

Table 1 Geometric parameters of the specimen [19]

Parameters	Average value
h_1 (mm)	5.12
h_2 (mm)	3.88
l (mm)	1.63
Number of faces per cell	13.68
Number of edges per face	4.99

Fig. 1 The cellular microstructure of the specimen (the red arrows indicate the missing or broken cell walls) [19]

Fig. 2 Model with 655 cells and the mean cell diameter of 5.0 mm

geometric modeling of aluminum foams using generic LVD the number of seeds is usually randomly selected [8, 10, 11, 14] or computed by dividing the volume of the domain V with the average radius ($\bar{r}$) of the cells [23, 31] this can be written as:

$$N_0 = \frac{V}{\left(\frac{4\pi}{3}\right)\bar{r}^3} \quad (3)$$

Figure 2 shows a cubic model with a side length of 35 mm (L35) generated using 655 seeds. The model has a mean cell diameter of 5.00 mm with a standard deviation of 0.99 mm. The vertices, edges, and faces are retained as the cell boundaries. This method is the general procedure used in generating Laguerre–Voronoi models

[6, 14]. The seed weights are usually generated from packing circles (2D) or hard spheres in (3D), this further imposes a constraint on the LVD such that each cell covers its circle or sphere.

3.2 Geometric parameters of the generic Laguerre–Voronoi model

The geometric parameters of the model were extracted and analyzed, and some discrepancies were observed. The results indicated that the generic LV model has a mean cell diameter of 4.81 mm with a standard deviation of 0.96 mm. Another set of models were generated with a mean sphere diameter of 3, 4, 5, 6, 7, 8, 9 and 10 mm, with a standard deviation of 1 mm. Diameter distribution of less than 3 mm was not used as it is considered to be a micro-porosity [19]. The mean diameter of the spheres and the mean diameters of the cells were compared and shown in Fig. 3. It was observed that with a constant standard deviation, the convergence of the diameter increases as the domain size increases. When the cubic domain size was 25 mm, the mean diameter of the spheres and the cells was 3.00 mm and 2.97 mm respectively. While with a domain size of 65 mm the mean diameter of the spheres and the cells was 7.00 mm and 7.01 mm respectively. This discrepancy between the mean diameters of the randomly close-packed spheres used in Laguerre Tessellations and the resulting cells has been reported by a number of researchers [10, 14], and they proposed equations to quantify the error. The modeling approach presented in this paper guaranteed that both the spheres and the cells have the same mean diameter distribution (see Fig. 10).

A set of 10 generic LV models were generated using a dense packing of spheres with a constant mean diameter of 4.25 mm, while the relative standard deviation

Fig. 3 Mean diameter of spheres and mean diameter of foam cells versus the model size

Fig. 4 Mean cell diameter versus relative standard deviation (RSD) at a constant sphere diameter of 4.25 mm

(RSD) was varied from 0.15 to 0.33 at intervals of 0.2. The model dimensions were $30 \times 30 \times 30$ mm, and the number of seeds was 672 based on Eq. (3). The mean diameter and the standard deviation of the cells are shown in Fig. 4. The results indicated that the deviation between the sphere diameters and the cell diameters increases with the increase in the relative standard deviation (RSD), the RSD controls the dispersion of the spheres and subsequently the foam cells. The deviation at RSD of 0.15 and 0.33 was 2.25% and 7.48% respectively. Similarly, the proposed modeling approach in this study guaranteed the standard deviation of the spheres and cells to be the same (see Fig. 10).

4 An overview of the proposed modeling approach

Considering the discrepancies in the geometrical parameters of the “generic LV model” described in Sect. 3. In this paper, we proposed a new modeling approach based on a closed-cell aluminum foam with logarithmic normal cell diameter distribution. The approach is aimed at developing a foam model that captures the actual geometrical and topological characteristics of the real foam such as cell size distribution, irregular cell wall thickness, number of faces per cell, number of edges per face. To satisfy the cell size distribution criteria, dense packing of spheres with the same diameter distribution as the real foam is generated using a mean cell diameter and a standard deviation. With the aim of matching the mean cell diameter of the spheres and the foam cells, an algorithm for finding the number of seeds (spheres) is proposed. A Laguerre–Voronoi tessellation is performed using the spheres center and radii as point set and weight set respectively. An algorithm for computing irregular cell wall thickness based on reverse bubble growth is proposed. The algorithm computes the irregular cell wall thickness distribution, and for each cell, a new cell is generated by shrinking the original cells to accommodate the cell wall thickness. The shrinking is done in a way that the volume of the cell satisfies the target relative density. The cells were further subtracted from a solid cube of the same size as the foam model. The procedure can be summarized in the following steps:

1. Select the required average cell diameter distribution of the foam model $d(\mu, \sigma)$, where μ and σ are the mean and standard deviation of the distribution.
2. Define a domain in $\mathbb{R}^3$ with length, width and height ($l \times w \times h$) such that $l, w, h \geq 7\mu$.
3. Compute the number of seeds N and generate a dense packing of spheres with the pre-determined diameter distribution $d(\mu, \sigma)$.
4. Perform a Laguerre–Voronoi tessellation using the spheres centers and radii as point set and weight set respectively.
5. Extract cell faces and vertices of the Laguerre–Voronoi diagram (LVD) and assign the irregular cell wall thickness distribution using the proposed algorithm.
6. Perform a Boolean subtraction of the cells from a bulk of the same dimension.

The condition in step 2 is designed to avoid size effects as proposed in [2] and demonstrated experimentally by [19]. Figure 5 shows the schematic diagram of the modeling approach developed in this work. The process for determining the number of seeds is discussed

Fig. 5 Schematic diagram of the modeling procedure: **a** foam cell diameter data, **b** determination of number of seeds and packing of the spheres with pre-determine diameter distribution, **c** LV model,

d cell modifications and irregular cell wall thickness calculation, **e** boolean difference from solid of the same dimensions, **f** final foam model

Fig. 6 **a** Cell size distribution of models with different number of seeds. **b** The red circle enlarged

in Sect. 5, the Laguerre–Voronoi tessellation using the spheres in Sect. 6 and the algorithm for calculating the irregular cell wall thickness is discussed in Sect. 7.

5 Determination of the number of seeds

The number of seeds used in generating foam models has been of interest to researchers due to its influence on the geometrical properties of the foam and hence the mechanical properties. A high number of cells results in a higher number of faces in the domain. The deformation

mechanism of closed-cell foam is linked to the bending and stretching of the cell wall (faces) [2]. Increasing the number of seeds will increase the number of faces which will make the foam stiffer. Many researchers chose a number randomly [8, 10, 11, 14] or derived from filling the given volume with cells of a given radius [23, 31] as shown in Eq. (3). The L35 model shown in Fig. 2 was generated with a number of seeds computed from Eq. (3). The geometrical properties of the model deviate from the real foam. The mean cell diameter of the model was 4.81 mm with a standard deviation of 0.96 mm, while the real foam has a mean cell diameter was 5.00 mm with a standard deviation of 0.99 mm. To understand the relationship between the number of seeds and the mean cell diameter, two more models were generated with the same diameter distribution, but with 605 and 555 seeds, the mean cell diameter of the models was found to be 4.93 mm and 5.08 mm respectively. This indicated that the best value of the number of seeds for this model is somewhere between 605 and 555. So with a guess of 580 seeds, the model gives a result of 5.00 mm. The fit for the cell diameter distribution of all the four models is shown in Fig. 6.

It can be seen that the diameter distribution of the model generated with 580 seeds converges with that of the real foam. Similar trends were also observed on models generated with cubic domain sizes of 50 mm (L50) and 65 mm (L65). The L50 Models had 1910, 1810, 1710 and 1610 seeds, while the L65 models had 4195, 3800, 3750 and 3600 seeds. The number of seeds that resulted in cell diameter distribution closest to the experimental data and the spheres used in generating the LV diagram was 1710 and 3750 for the L50 and L65 respectively. This indicated that the number of seeds based on the cell diameter distribution is always less than the one calculated by Eq. (3). The result is shown in Table 2.

In this work, an algorithm of finding the number of seeds based on the cell diameter distribution is proposed, such that the mean diameter of the spheres and the cells will be equal and at the same time guarantee the number of faces and edges to be within the range of a real foam. First, the initial number of seed N_0 is calculated based on Eq. (3), then an exhaustive search is imposed in the descending order, for each loop the

LVD is constructed, and the cell diameter distribution is measured. A model is accepted if its diameter distribution is within 10% of the target, else the number of seeds is reduced by unity. This seems to be a greedy approach, but it guarantees the value of the number of seeds. It became evident that the number of seeds used in the generic Voronoi tessellations is an overestimation because it assumes the sphere (and the cells) to be monodispersed that is they are of the same size, and the standard deviation is zero. The result from the iterative algorithm was analyzed, and multivariable non-linear regression was used to find the relationship between the parameters (number of seeds N , volume V , mean diameter μ , and mean diameter standard deviation σ) using IBM SPSS statistic package. Equation (4) gives the relationship between the parameters

$$N = \frac{V}{\pi/6(\mu^3 + \sigma^3 + c_1\mu\sigma^2 + c_2\mu^2\sigma)} \quad (4)$$

where c_1 and c_2 are constants with the values of 3.076 and -0.030 respectively.

Figure 7 shows the number of seeds based on the Eqs. (3) and (4) and the algorithm. Equation (4) provides

Fig. 7 Determination of the number of seeds

Table 2 Geometric parameters comparison

Parameters	Number of seeds Eq. (3)	Number of seeds (observed)	Mean cell diameter (mm)	Number of faces per cell	Number of edges per face
Real foam	–	–	5.00	14.00	5.00
Model L35	655	580	5.00	13.46	5.13
Model L50	1910	1710	4.99	14.10	5.17
Model L65	4195	3750	4.99	14.42	5.18

the best approximation to the results of the exhaustive search algorithm, and therefore Eq. (4) will be used in determining the number of seeds through the rest of this work.

6 Construction of LVD over a packing of polydisperse spheres

Haphazardness of the foam manufacturing process results in products with dispersion in the cell diameters. The dispersion can be described by the relative standard deviation (RSD). ALPORAS foam cell diameter distribution has been recorded to have an RSD of 0.15–0.22 [19]. RSD is the ratio of the standard deviation to the mean of a distribution. The cell volume distribution in real foams has been suggested to be log-normal [32, 33]. Therefore, the generation of the cells with a pre-defined log-normal distribution of volume in the computations can be performed using the mean diameter distribution of a real foam obtained using image-processing techniques [23].

In this work, the random close-packing of spheres is generated using the mean diameter distribution and the derived number of seeds. A modified collective rearrangement algorithm called ‘force-biased algorithm’ [34] is used to generate the sphere packing. The algorithm produces isotropic sphere packing with a very high density. The Laguerre–Voronoi Tessellation is carried out using NEPER [35], a multiscale polycrystal generation software package. NEPER can simulate grain growth with options for considering sphere or circle packing as a template.

7 Calculating the irregular cell wall thickness

The cell wall thickness is one of the most prominent properties of the foam. The irregular cell wall thickness is responsible for the foam mechanical properties, and it is directly linked with the porosity of the foam. Researchers have investigated its effect on the mechanical response of foams [3, 6, 26, 31]. Many researchers opted for modeling the cell walls in the finite element modeling stage with shell elements and assigning uniform thickness as:

$$t = \frac{V}{\sum S} \times \frac{\rho^*}{\rho_s}$$

where t is the uniform thickness of the shell elements, V is the volume of the bulk, S is the surface area of each

element, ρ^* is the density of the foam, and ρ_s is the density of the bulk. This simplification provides an easy control over the relative density of the model and lowers the analysis computation time, but it overestimates the mechanical properties of the real foam [6].

The use of variable thickness in foam models is limited due to its complexity and often high computation time during analysis. Li et al. [26] studied the cell wall thickness variation on the elastic properties of two-dimensional foams, and the result indicated that increasing the cell wall randomness decreases the elastic modulus. The effect of cell wall thickness variation on the stiffness of closed-cell foams was investigated by Chen et al. [6], the thickness variation was introduced by grouping shell elements on the same face and assigning different thickness to different groups, so that shell elements from the same cell face have the same thickness. The relative density of the model was then calculated, and the thickness was scaled by a factor so that the relative density of the models matched a prescribed value. The results indicated that both Young's modulus and shear modulus decreases with increasing cell wall thickness variations.

In this study, we proposed a new method for generating the irregular cell walls along with their thickness distribution while considering the overall relative density of the model all in the geometric modeling stage. The process is based on reverse bubble growth, which mimics the manufacturing process of the foam. Foaming agents generates bubbles in molten aluminum and expands to create pores which keep growing until it stabilizes, as the bubble grows the relative density of the foam decreases. The LVD model described above shows the foam model after complete expansion. The proposed method involves reversing the bubble growth process, i.e. shrinking cells to accommodate the cell wall thickness based on the distribution acquired from the real foam.

The process is implemented in an algorithm whose objective is to generate irregular cell wall thickness distribution at a specific relative density. The algorithm produces irregular cell wall thickness distribution similar to the real aluminum foam structure. In this work, the foam cells are considered as independent polyhedrons, therefore, a divide-and-conquer method that deals with each cell independently is proposed. If all the cells are shrunk, a gap between adjacent cells is created, the volume fraction of this gap is the relative density of the foam. In the analysis of a single cell, after obtaining the centroid, a cell can be represented as a group of pyramids. The cell walls are the base of the pyramids, and the apex of all the pyramids meets at the centroid. Applying the divide-and-conquer approach to each pyramid, if the ratio of the wall thickness of each pyramid to the volume of the pyramid is equal to the target relative density, then the

Fig. 8 **a** A cell divided into pyramids. **b** A single pyramid showing the shrinking distance (l)

relative density of the total wall thickness of all the pyramids that makes a single cell is equal to the target relative density of that cell. It should be noted that when the two pyramids are identical, their wall thickness will be the same. Fortunately, in Laguerre–Voronoi tessellation such a situation is infrequent.

Algorithm steps:

1. Dividing cells into pyramids

Consider a single cell as shown in Fig. 8a, the centroid of the cell is O . For each cell walls, the vertices are projected (green lines) to the centroid to make a pyramid. The cell walls are the base of the pyramids, and the apex of all the pyramids in a particular cell meets at the centroid.

2. Shrinking of a single pyramid

Consider a single pyramid, as shown in Fig. 8b, the base area of the pyramid is s shown in light gray, the height is h , the volume of the pyramid V_1 is $\frac{1}{3}sh$, the shrink distance of the pyramid is l , and the new volume of pyramid V_2 after shrinking is $\frac{1}{3} \frac{s(h-l)^3}{h^2}$.

3. The shrinking distance

Let the relative density of the pyramid equal to the target relative density, $(V_1 - V_2)/V_1 = \bar{\rho}_t$ then the shrinking distance (thickness) becomes $l = h - h(1 - \bar{\rho}_t)^{1/3}$.

4. Set minimum thickness

From the derived thickness expression, it can be seen that the wall thickness is irregular due to the difference in h of each pyramid. If this formula is used strictly, the

final relative density will match the target relative density, but considering the subsequent volume shrinking and the actual aluminum foam structure, there should be a minimum thickness (*minThickness*) so that the wall thickness is not less than this value. The introduction of *minThickness* will increase the relative density of the generated model. Also, the cell walls on the boundaries of the model are excluded in the shrinking operation in consideration of the subsequent Boolean operation. This will reduce the relative density of the generated model.

5. Iterative refinement

Because of the introduction of *minThickness* and the exclusion of cell walls at the boundaries, the relative density generated will deviate from the target relative density. To rectify this problem, an iterative refinement method is proposed so that the relative density of the resulting model is equal to the target relative density. The idea is to calculate the relative density of the current model and check for convergence with the target relative density. The convergence condition is that the absolute value between current relative density and the target relative density should not exceed 0.01% ($fabs(\bar{\rho} - \bar{\rho}_t) < 0.01\%$), '*fabs()*' is a function that returns the absolute value of a number without truncating the decimals. A new control parameter d is set, and the wall thickness generation formula is modified to $l = h - h(1 - \bar{\rho}_t)^{1/3} \times d_{initial}$, the wall thickness is changed iteratively by changing the value of d , until the relative density converges with the target relative density.

Iteration steps:

(a) Set the range of d

Set the range $[d_{min}, d_{max}]$ which is determined by the specific model size. For example, in a model of size 35 mm × 35 mm × 35 mm, d_{min} can be set to 0.8 and d_{max} is 1.2. The only constraint is that the value of d_{max} cannot be so large that the relative density of the model is greater than 1. In this paper, the initial value of d is set to 1. By comparing the current relative density and the target relative density, the value is either increased or decreased accordingly. For example, when the current relative density of the generated model is larger than the target relative density, d is decreased to reduce the relative density of the generated model.

(b) Convergence criteria

Specifically, if $\bar{\rho} < \bar{\rho}_t$, it means that d is too small, update d_{min} and increase d , $d = d + fabs(d_{max} - d) * 0.1$; if $\bar{\rho} > \bar{\rho}_t$, it means that d is too large, update d_{max} and decrease d , $d = d - fabs(d - d_{min}) * 0.1$. Where $\bar{\rho}$ is the current relative density, $\bar{\rho}_t$ is the target relative density. By continuously updating d and the range $[d_{min}, d_{max}]$, the convergence condition ($fabs(\bar{\rho} - \bar{\rho}_t) < 0.01\%$) can be achieved. An example with target relative densities (12.35% and 11.25%) is shown in Tables 3 and 4, respectively.

The new cells that were generated by shrinking the former cells are subtracted from a solid with the same dimension as the foam model. The Boolean subtraction

Table 4 Iteration parameters for target relative density $\bar{\rho}_t$ of 11.25% and minThickness of 0.05

Current relative density $\bar{\rho}$	Control parameter d	d_{min}	d_{max}
–	1.0000	0.8000	1.2000
0.10586	1.0000	1.0000	1.2000
0.10790	1.0200	1.0200	1.2000
0.10974	1.0380	1.0380	1.2000
0.11138	1.0542	1.0542	1.2000
0.11286	1.0687	1.0542	1.0687
0.11270	1.0672	1.0542	1.0672
0.11256	1.0658	1.0542	1.0658

automatically deletes the cell walls on the boundaries which were excluded in the shrinking process because they intersect with the solid boundaries. The result is a foam model with irregular cell wall distribution having a relative density within 0.01% of the target density.

8 Results and discussion

8.1 Representativeness of the model and the distributions of geometric parameters

Foam geometric models are used in finite element analyses to investigate the properties of the real foam. The representativeness of a model depends highly on geometric properties. For example, in a structural analysis the stiffness and strength are directly related to the cell wall (faces) resistance to deformation, selecting the appropriate number of seeds to create the cells will determine the number of faces in the model which affects the stiffness of

Table 3 Iteration parameters for target relative density $\bar{\rho}_t$ of 12.35% and minThickness of 0.05

Current relative density $\bar{\rho}$	Control parameter d	d_{min}	d_{max}
–	1.0000	0.8000	1.2000
0.11486	1.0000	1.0000	1.2000
0.11707	1.0200	1.0200	1.2000
0.11905	1.0380	1.0380	1.2000
0.12083	1.0542	1.0542	1.2000
0.12242	1.0687	1.0687	1.2000
0.12386	1.0818	1.0687	1.8180
0.12370	1.0804	1.0687	1.8040
0.12357	1.0781	1.0687	1.0781

Table 5 Geometric parameters of foam models

Model	Mean (mm)	RSD	L (mm)	N_0	N
N01	4.25	0.15	35.00	1067	997
N02	4.30	0.17	35.00	1030	946
N03	4.35	0.19	35.00	995	894
N04	4.40	0.21	35.00	961	845
N05	4.45	0.23	35.00	929	797
N06	4.50	0.25	35.00	899	748
N07	4.55	0.27	35.00	869	704
N08	4.60	0.29	35.00	841	661
N09	4.65	0.31	35.00	814	619
N10	4.70	0.33	35.00	789	579

Fig. 9 Mean cell diameter versus RSD for generic LV models and proposed models

the foam. Similarly, in a thermal analysis, the key parameter is the surface area of the faces, a high number of faces result in high surface area and vice versa. A model that captures the actual geometric parameters of the real foam thus becomes a representative of the real foam and can be used in any analysis.

The modeling approach proposed in this work was used in generating a set of models with specific geometric properties as shown in Table 5. The models have a mean cell diameter ranging from 4.25 to 4.70 mm with an interval of 0.05 mm, the relative standard deviation (RSD) varies from 0.15 to 0.33, and the corresponding standard deviation was calculated. The size of the domain (cube) was kept constant at 35 mm, which satisfied the condition given in step 2 of the modeling procedure.

Generic LV models were also constructed using the generic number of seed (N_0), and the proposed models were constructed using the number of seeds (N) presented in this work. The cell diameter distribution of the models was computed and shown in Figs. 9 and 10. The average number of faces per cell and edges per side is shown in Fig. 11. The mean cell diameter of the generic LV models varies linearly with the real foam mean cell diameter at RSD below 0.2, and becomes slightly irregular at RSD above 0.2, at 0.3 it becomes random. The number of faces decreases as the RSD increases, while the number of edges is approximately constant.

The proposed modeling approach thus produced precisely the cell size of the real foam; this cell size convergence is of prominent importance as it strongly affects the mechanical properties of the foam model [3, 7, 36, 37].

The modeling approach presented in this work, can produce models with same average cell diameter of the real foam with a number of faces per cell of approximately 14 and number of edges (sides) per face of 5 which agrees

Fig. 10 Cell diameter standard deviation versus RSD for generic LV models and proposed models

with real foams based on literature [6, 19–21] and naturally occurring foams like vegetable cells [20].

8.2 Elastic properties

A model generated with dimensions $35 \times 35 \times 35$ mm was generated with a mean cell diameter distribution of 5 mm and RSD of 0.2. This model will be used for verification against the experimental data [19, 38]. After irregular cell wall thickness assignment and the Boolean subtraction from a bulk cube of the same dimensions using the algorithm presented in this work, the remaining vertices and faces (cell wall) of the model were read and meshed using a four-node tetrahedral element in Hypermesh. During the meshing process, any edge less than 0.01 mm is

Fig. 11 Geometric parameters modeled in the current work

Fig. 12 3D finite element loading condition with model between rigid bodies: fixed (green) and movable (red)

either grown, collapsed or deleted. The number of element ranges from 8×10^6 to 10×10^6 .

A quasi-static compression analysis was carried out in the finite element package ABAQUS [39]. The analysis was carried out on a Desktop computer with quadcore Intel i5 2.70 GHz CPU and a 16 GB memory. The mass scaling and parallelization features of ABAQUS were exploited and the computation time ranged from 12 to 72 h depending on the model's relative density. The cell wall of the model was taken to be aluminum with a Young modulus of 70 GPa, density 2700 kg/m^3 , Poisson ratio 0.3, Yield stress 50 MPa. The loading condition is shown in Fig. 12. Table 6 shows the parameters of the generic LV model [38] and the current work.

It was observed that the result from one generation of the microstructure to another with the same parameters differ. Table 7 shows the parameters of 3 models generated with the same parameters.

Relative Young's moduli result from experiments, Kelvin model, Grenestedt model, and the generic LV model are shown in Fig. 13 alongside results from the current work. These models have overestimated the relative moduli of the real foam due to the inefficient modeling process that fails to capture the stochastic nature and geometrical properties of the real foam. The foam models presented in the current work predicts the relative moduli of the real foam due to the combined effect of weighted Voronoi tessellation, the number of seeds and irregular thickness distribution and assignment developed and presented in this work.

Table 6 Parameters of the generic LV model [38] and the LV model in the current work

Parameters	Ref. [38]	Current work
Number of seeds	Equation (3)	Equation (4)
Cell diameter distribution	Gamma distribution	Lognormal distribution
Relative standard deviation	0.87	0.15–0.35
Model size	$35 \times 35 \times 35 \text{ mm}$	$35 \times 35 \times 35 \text{ mm}$
Finite element	Shell	Solid
Cell wall thickness	Uniform	Irregular (distributed)

Table 7 Properties of 3 models generated with the same parameters

Parameters	Model A	Model B	Model C
Number of cells	997	997	997
Average cell diameter (mm)	4.25	4.25	4.25
Relative standard deviation (mm)	0.64	0.64	0.64
Model size (mm)	$35 \times 35 \times 35$	$35 \times 35 \times 35$	$35 \times 35 \times 35$
Average number of faces	13.86	13.92	13.89
Average number of edges	5.15	5.16	5.15
Relative Young's modulus (%)	1.12	1.15	1.14

Fig. 13 Relative moduli versus relative density

9 Conclusion

In this study, a new mesoscopic geometric modeling approach was presented for closed-cell aluminum foams based on Laguerre–Voronoi tessellation. The approach introduced a process for finding the number of seeds along with a robust algorithm for realistic irregular cell wall thickness computation based on reverse bubble growth. The approach considers the microstructural diversity of the foam and creates a model that captures the actual geometrical and topological characteristic of

metal foams by integrating realistic cell size and irregular cell wall thickness distributions in the modeling procedure such that the foam model has the same distributions as the real foam. The modeling approach was verified by comparing the cell size distribution, irregular cell wall thickness distribution, and the relative Young's modulus of the models with the real foam. Models generated by the proposed modeling approach have the same cell size and irregular cell wall thickness distribution as the real foam and shows a better accuracy in predicting the relative Young's modulus of the real foam than the generic Laguerre–Voronoi, Kelvin, and Grenestedt models.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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