Mathematical modeling and performance evaluation of A-pan crystallization system in a sugar industry

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Abstract

In this paper, an effort has been made to formulate a mathematical model of A-pan crystallization system of a sugar plant using fuzzy reliability approach. The sugar plant comprises eight subsystems. A-pan crystallization system is one of the most important representative among sugar plant systems. The A-pan crystallization system has four subsystems arranged in a series. The configuration of first and second subsystems is 2-out-of-2: G with two cold standby while third and fourth subsystems are in single-unit configuration. A mathematical model has been proposed by considering exponential distribution for failure and repair rates. By considering fuzzy reliability approach and Markov birth–death model differential equations have been derived. These equations are then solved by Runge–Kutta method of fourth order using MATLAB (Ode 45 function) to obtain the fuzzy availability. The results of the proposed model are beneficial for system designers.

Keywords A-pan crystallization system · Markov process · Fuzzy availability · Runge–Kutta method

1 Introduction

Due to the population explosion in last century sugar consumption is rapidly increasing throughout the world. According to Meriot [1] sugar production by centrifugal/structured sugar plants in India is more than 84% of the total production of sugar. One important ration of any sugar plant is the assurance of high availability for maximum production. To achieve higher availability of a system it is mandatory that all of its subsystem attain higher reliability. In this study, an effort has been made to analyze the availability of A-pan crystallization system of a sugar plant. The existing literature like Adamyan and David [2], Gupta et al. [3], Mehmood and Lu [4], Garg and Sharma [5], Kumar and Mudgil [6], Loganathan et al. [7], and Saini and Kumar [8] shows that a lot of techniques have been used to analyze the performance of the industrial systems in terms of reliability and availability such as Reliability block diagram, semi-Markov process, Markovian approach and

fault tree analysis. However, in above techniques all operating states have been considered that system either work in full capacity or completely fail but in many industrial systems this condition does not seems realistic. Therefore, here an effort has been made to analyze the system in all reduced states between failed and operative states, i.e., in fuzzy states. In previous studies including Nailwal and Singh [9] and Neeraj and Barak [10], it is also observed lot of computational work has been carried out to obtain the availability. Here, Runge–Kutta method of fourth order has been opted to obtain the numerical solution of differential difference equations. A lot of successful applications of fuzzy reliability approach and Runge–Kutta method has been reported in literature. The concept of fuzzy set theory has been coined by Zadeh [11]. He described the importance of fuzzy sets in the development of scientific and industrial systems. The concept of component failure possibility rather than failure probability was introduced by Kaufmann [12]. He presented a lot of applications of

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fuzzy sets in various fields like hardware/software reliability, risk analysis, etc. Srinath [13] used Markovian approach for availability analysis considering constant failure and repair rates. Arora and Kumar [14] performed availability analysis of power generation system. The recurrence relations for availability and MTBF have been derived by considering constant failure and repair rates. Barabady and Kumar [15] carried out performance evaluation of a repairable system in terms of system reliability and availability. Kiureghian and Ditlevson [16] examined the availability, reliability and downtime of system with repairable constituents. Garg et al. [17] proposed a mathematical model of a repairable block board manufacturing system using a birth–death Markov Process. The differential equations have been solved for the steady-state performance evaluation. Rahman et al. [18] studied root causes for failure of a division wall super heater tube of a coal-fired power station. Kumar et al. [19] developed a simulation model for performance evaluation of urea decomposition unit in fertilizer plant. Kumar and Tewari [20] suggested a mathematical model for performance evaluation of CO₂ cooling system. Adhikary et al. [21] accomplished RAM investigation of coal-fired thermal power plants. Goyal and Grover [22] comprises a comprehensive bibliography on effectiveness measurement of manufacturing systems. Kumar [23] used Markov approach to analyze an availability simulation model for power generation system. Khanduja et al. [24] designated a performance improvement model of crystallization unit of a sugar plant using MA and GA. Hojjati-Emami et al. [25] performed reliability prediction for the vehicles equipped with advanced driver assistance systems and passive safety systems. Aggarwal et al. [26] discussed the performance analysis and optimization of a butter oil production system using MA and Runge–Kutta method to calculate the mean time between failure (MTBF). Kumar et al. [27] developed a stochastic model for casting process and performed sensitivity analysis for various reliability measures. Kadyan and Kumar [28] analyzed the availability and profit of feeding system in sugar manufacturing plant. Aggarwal et al. [29] formulated a mathematical model and obtained the results for reliability of the serial processes in feeding system. Kumar and Tewari [30] used PSO technique for performance analysis and optimization of CSDGB filling system of a beverage plant. Kadyan and Kumar [31] analyzed the availability based operational behavior of B-Pan crystallization system in the sugar industry. Kumar and Saini [32] developed a mathematical model of sugar plant as a whole system. Recently, Dahiya et al. [33] analyzed a feeding system of sugar plant subject to coverage factor. Dahiya et al. [33, 34] evaluated the fuzzy availability of a harvesting system using fuzzy reliability approach.

Keeping in view the above facts and figures in mind, in this paper, an effort has been made to formulate a mathematical model of A-pan crystallization system of a sugar plant using fuzzy reliability approach. The objective of this study is to help the sugar industry management persons to evaluate the performance of the sugar plant by developing reliability model for A-pan crystallization system. A-pan crystallization system is one of the most important representative among sugar plant systems. The A-pan crystallization system has four subsystems arranged in a series. The configuration of first and second subsystems is 2-out-of-2: G with two cold standby while third and fourth subsystems are in single-unit configuration. A mathematical model has been proposed by considering exponential distribution for failure and repair rates. By considering fuzzy reliability approach and Markov birth–death model differential difference equations have been derived. These equations are then solved by Runge–Kutta method of fourth order using MATLAB (Ode 45 function) to obtain the fuzzy availability. Numerical and graphical results are also found to elucidate the effect of subsystems.

The whole manuscript has been organized in six sections. The first section is introductory in nature. A detailed literature review and gap of research is discussed here. In Sect. 2, system description, assumptions, notations, possible states and state transition diagram are appended. In Sect. 3, mathematical model has been developed for A-pan crystallization system and differential difference equation are derived. Expressions for fuzzy availability, fuzzy busy period and fuzzy profit analysis are also carried out in this section. In Sect. 4, the effect of various failure rates and repair rates on availability and profit is deliberated through tables and graphs. Conclusion drawn from analysis is discussed in Sect. 5.

2 System description

The A-pan crystallization system has four subsystems arranged in a series. In first subsystem crystallizer (subsystem A) semi-solid form of the liquid is frenzied to evaporate remaining water. The frenzied process is performed in a long duration of time on slow heating. After conversion of semi-solid form of the liquid into magma it is sent to centrifugal machine (subsystem B) to separate out sugar crystals. The subsystem C cooled and graded the sugar crystals. Finally, sugar is weighted and bagging in subsystem D. If there is some amount of sugar in power form, than it is again processed through crystallization. The detailed description of subsystems are as follows (for detailed description see Kumar [35]).

2.1 Crystallizer (subsystem A)

Sahu [36] described crystallization as a process in which masecutes are slowly stirred while they cool from pan dropping temperature to surrounding temperature. Progressive cooling reduces the solubility of sugar and forced to crystallization. This whole procedure is carried out in special equipment called crystallizer. Crystallizers are U shaped or circular in diameter, open and horizontal containers. The massecuite is cooled by air and cooling action is very slow. In present study, subsystem-A consist of four identical crystallizer unit working in the 2-out-of-2: G with two cold standby configuration. In this subsystem, semi-solid form of the liquid is frenzied to evaporate remaining water. The frenzied process is performed in a long duration of time on slow heating. In the unavailability of two operative units the subsystem fails and resulted as the complete failure of the A-pan crystallization system. It is connected in series configuration with other subsystems B, C, and D.

2.2 Centrifugal machine (subsystem B)

Sahu [36] explained that masticates produced from crystallizer send to rotating type equipment know to be a centrifuge. In present study, present system also consists of four identical units working in the 2-out-of-2: G with two cold standby configuration. This subsystem, after conversion of semi-solid form of the liquid into magma, separates out sugar crystals. In the unavailability of two operative units the subsystem fails and resulted as the complete failure of the A-pan crystallization system. It is connected in series configuration with other subsystems A, C, and D.

2.3 Grader (subsystem C)

Kumar [35] describes this subsystem as a series system which consists of hopper cooler and sugar grader. In

this subsystem, the crystals are cooled and cut in small/medium pieces. It is connected in series configuration with other subsystems. It is a single unit system. Its failure causes the complete system failure.

2.4 Weighment and bagging (subsystem D)

Kumar [35] defined it as a system where sugar weighted and packed in bags. It is also connected in series configuration with other subsystems. It is also single unit system. Its failure causes the complete system failure.

Assumptions

1. All failure time and repair time random variables are exponentially distributed.
2. The failed unit after repair becomes as good as new.
3. The system may be any of the states between fully operative to complete failed. The value of coverage factor lies between 0 and 1.
4. Sufficient repair facility always remains available to perform various repair activities.
5. No simultaneous failures occurs.

2.5 Fuzzy availability

Kumar and Saini [37] stated a fuzzy stochastic semi-Markov model $\{(S_n, T_n), n \in N\}$ having 'n' states with transition time. Let $U = \{S_1, S_2, \dots, S_n\}$ represent the population of discourse. On this population, we define a fuzzy success state $S, S = \{(S_i, \mu_S(S_i)); i = 1, 2, \dots, n\}$ and a fuzzy failure state $F, F = \{(S_i, \mu_F(S_i)); i = 1, 2, \dots, n\}$, where $\mu_S(S_i)$ and $\mu_F(S_i)$ are trapezoidal fuzzy numbers, respectively. The fuzzy availability of the system is defined as: $A(t) = \sum_{i=1}^k \mu_S(S_i)P_i(t)$, where k denotes the operative states (Fig. 1).

System states

The system may be any one of the following states:

Fig. 1 Reliability block diagram

$S_1(A_{2,2,0}, B_{2,2,0}, C, D); S_2(A_{2,1,1}, B_{2,2,0}, C, D); S_3(A_{2,2,0}, B_{2,1,1}, C, D); S_4(A_{2,2,0}, B_{2,0,2}, C, D); S_5(A_{2,0,2}, B_{2,2,0}, C, D); S_6(A_{2,0,2}, B_{2,1,1}, C, D); S_7(A_{2,1,1}, B_{2,1,1}, C, D); S_8(A_{2,1,1}, B_{2,0,2}, C, D); S_9(A_{2,0,2}, B_{2,0,2}, C, D); S_{10}(A_{2,2,0}, B_{2,2,0}, C, D); S_{11}(A_{2,2,0}, B_{2,2,0}, C, d); S_{12}(A_{2,1,1}, B_{2,2,0}, C, d); S_{13}(A_{2,1,1}, B_{2,2,0}, c, D); S_{14}(A_{2,0,2}, B_{2,2,0}, c, D); S_{15}(a, B_{2,2,0}, C, D); S_{16}(A_{2,0,2}, B_{2,2,0}, C, d); S_{17}(A_{2,0,2}, B_{2,1,1}, c, D); S_{18}(a, B_{2,1,1}, C, D); S_{19}(A_{2,0,2}, B_{2,1,1}, C, d); S_{20}(A_{2,1,1}, B_{2,1,1}, C, d); S_{21}(A_{2,1,1}, B_{2,1,1}, c, D); S_{22}(A_{2,1,1}, b, C, D); S_{23}(A_{2,1,1}, B_{2,0,2}, c, D); S_{24}(A_{2,1,1}, B_{2,2,0}, C, d); S_{25}(A_{2,1,1}, b, C, D); S_{26}(A_{2,2,0}, B_{2,0,2}, C, d); S_{27}(A_{2,2,0}, B_{2,0,2}, c, D); S_{28}(A_{2,2,0}, B_{2,1,1}, C, d); S_{29}(A_{2,2,0}, B_{2,1,1}, c, D); S_{30}(a, B_{2,0,2}, C, D); S_{31}(A_{2,0,2}, B_{2,0,2}, C, d); S_{32}(A_{2,0,2}, B_{2,0,2}, c, D); S_{33}(A_{2,0,2}, b, C, D).$

Notations

$X_{(OU, SU, FUR)}$: States of the subsystem A and B.

A, B, C, D: working states of all the four subsystems with full capacity

a, b, c, d: failed states of subsystems A, B, C, and D respectively

OU: No. of operative units

SU: no. of standby unit

FUR: no. of failed units under repair

λ_i ($1 \leq i \leq 4$): failure rates of subsystem A, B, C, and D respectively

β_i ($1 \leq i \leq 4$): repair rates of subsystem A, B, C, and D respectively

S_i : i th state of the system

α : coverage factor having values $0 \leq \alpha \leq 1$. A fully restored state achieve coverage factor value α .

$P_i(t)$: it denotes the probability that the system is in i th state at time t .

3 Mathematical modeling

The mathematical modeling of A-pan crystallization system is carried out by using Markov birth–death process. The differential equations based on the mathematical model are as follows:

$$\begin{aligned} P_1(t + \Delta t) &= [1 - 2\alpha\lambda_2 - 2\alpha\lambda_1 \\ &\quad - (1 - \alpha)\lambda_3 - (1 - \alpha)\lambda_4]P_1(t)\Delta t \\ &\quad + \beta_1P_2(t)\Delta t + \beta_2P_3(t)\Delta t + \beta_4P_{11}(t)\Delta t + \beta_3P_{10}(t)\Delta t \\ \frac{P_1(t + \Delta t) - P_1(t)}{\Delta t} &= [-2\alpha\lambda_1 - 2\alpha\lambda_2 \\ &\quad - (1 - \alpha)\lambda_3 - (1 - \alpha)\lambda_4]P_1(t) \\ &\quad + \beta_1P_2(t) + \beta_2P_3(t) + \beta_3P_{10}(t) + \beta_4P_{11}(t) \end{aligned}$$

Taking limit $\Delta t \rightarrow 0$, we get

$$\begin{aligned} \frac{dP_1(t)}{dt} + [2\alpha\lambda_1 + 2\alpha\lambda_2 + (1 - \alpha)\lambda_3 + (1 - \alpha)\lambda_4]P_1(t) \\ = \beta_1P_2(t) + \beta_2P_3(t) + \beta_3P_{10}(t) + \beta_4P_{11}(t) \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{dP_2(t)}{dt} + [\beta_1 + 2\alpha\lambda_1 + 2\alpha\lambda_2 + (1 - \alpha)\lambda_3 + (1 - \alpha)\lambda_4]P_2(t) \\ = \beta_1P_5(t) + \beta_2P_7(t) + \beta_3P_{13}(t) + \beta_4P_{12}(t) + 2\alpha\lambda_1P_1(t) \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{dP_3(t)}{dt} + [\beta_2 + 2\alpha\lambda_1 + 2\alpha\lambda_2 + (1 - \alpha)\lambda_3 + (1 - \alpha)\lambda_4]P_3(t) \\ = \beta_3P_{29}(t) + \beta_4P_{28}(t) + \beta_1P_7(t) + \beta_2P_4(t) + 2\alpha\lambda_2P_1(t) \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{dP_4(t)}{dt} + [\beta_2 + 2\alpha\lambda_1 + 2(1 - \alpha)\lambda_2 + (1 - \alpha)\lambda_3 + (1 - \alpha)\lambda_4]P_4(t) \\ = \beta_1P_8(t) + \beta_2P_{25}(t) + \beta_4P_{26}(t) + \beta_3P_{27}(t) + 2\alpha\lambda_2P_3(t) \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{dP_5(t)}{dt} + [\beta_1 + 2\alpha\lambda_2 + 2(1 - \alpha)\lambda_1 + (1 - \alpha)\lambda_4 + (1 - \alpha)\lambda_3]P_5(t) \\ = \beta_2P_6(t) + \beta_3P_{14}(t) + \beta_1P_{15}(t) + \beta_4P_{16}(t) + 2\alpha\lambda_1P_2(t) \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{dP_6(t)}{dt} + [\beta_2 + \beta_1 + 2\alpha\lambda_2 + (1 - \alpha)\lambda_3 + (1 - \alpha)\lambda_4 \\ + (1 - \alpha)\lambda_1]P_6(t) = \beta_3P_{17}(t) + \beta_1P_{18}(t) + \beta_2P_9(t) \\ + \beta_4P_{19}(t) + 2\alpha\lambda_2P_5(t) + 2\alpha\lambda_1P_7(t) \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{dP_7(t)}{dt} + [\beta_1 + \beta_2 + 2\alpha\lambda_2 + (1 - \alpha)\lambda_3 + (1 - \alpha)\lambda_4 \\ + 2\alpha\lambda_1]P_7(t) = \beta_1P_6(t) + \beta_3P_{21}(t) + \beta_2P_8(t) \\ + \beta_4P_{20}(t) + 2\alpha\lambda_2P_2(t) + 2\alpha\lambda_1P_3(t) \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{dP_8(t)}{dt} + [\beta_2 + \beta_1 + 2\alpha\lambda_1 + (1 - \alpha)\lambda_3 + (1 - \alpha)\lambda_4 \\ + 2(1 - \alpha)\lambda_2]P_8(t) = \beta_2P_{22}(t) + \beta_3P_{23}(t) \\ + \beta_4P_{24}(t) + \beta_1P_9(t) + 2\alpha\lambda_2P_7(t) + 2\alpha\lambda_1P_4(t) \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{dP_9(t)}{dt} + [\beta_2 + \beta_1 + 2(1 - \alpha)\lambda_2 + (1 - \alpha)\lambda_3 + (1 - \alpha)\lambda_4 \\ + 2(1 - \alpha)\lambda_1]P_9(t) = \beta_1P_{30}(t) + \beta_4P_{31}(t) + \beta_3P_{32}(t) \\ + \beta_2P_{33}(t) + 2\alpha\lambda_2P_6(t) + 2\alpha\lambda_1P_8(t) \end{aligned} \quad (9)$$

$$\begin{aligned} P_{10}(t + \Delta t) &= [1 - \beta_3]P_{10}(t)\Delta t + (1 - \alpha)\lambda_3P_1(t)\Delta t \\ \frac{P_{10}(t + \Delta t) - P_{10}(t)}{\Delta t} &= -\beta_3P_{10}(t) + (1 - \alpha)\lambda_3P_1(t) \end{aligned} \quad (10)$$

Taking limit $\Delta t \rightarrow 0$, we get

$$\begin{aligned} \frac{dP_{10}(t)}{dt} + \beta_3P_{10}(t) &= (1 - \alpha)\lambda_3P_1(t) \\ \frac{dP_{11}(t)}{dt} + \beta_4P_{11}(t) &= \lambda_4(1 - \alpha)P_1(t) \end{aligned} \quad (11)$$

$$\frac{dP_{12}(t)}{dt} + \beta_4P_{12}(t) = \lambda_4(1 - \alpha)P_2(t) \quad (12)$$

$$\frac{dP_{13}(t)}{dt} + \beta_3 P_{13}(t) = \lambda_3(1 - \alpha)P_2(t) \quad (13) \quad \frac{dP_{30}(t)}{dt} + \beta_1 P_{30}(t) = 2\lambda_1(1 - \alpha)P_9(t) \quad (30)$$

$$\frac{dP_{14}(t)}{dt} + \beta_3 P_{14}(t) = \lambda_3(1 - \alpha)P_5(t) \quad (14) \quad \frac{dP_{31}(t)}{dt} + \beta_4 P_{31}(t) = \lambda_4(1 - \alpha)P_9(t) \quad (31)$$

$$\frac{dP_{15}(t)}{dt} + \beta_1 P_{15}(t) = \lambda_1(1 - \alpha)P_5(t) \quad (15) \quad \frac{dP_{32}(t)}{dt} + \beta_3 P_{32}(t) = \lambda_3(1 - \alpha)P_9(t) \quad (32)$$

$$\frac{dP_{16}(t)}{dt} + \beta_4 P_{16}(t) = \lambda_4(1 - \alpha)P_5(t) \quad (16) \quad \frac{dP_{33}(t)}{dt} + \beta_2 P_{33}(t) = 2\lambda_2(1 - \alpha)P_9(t) \quad (33)$$

$$\frac{dP_{17}(t)}{dt} + \beta_3 P_{17}(t) = \lambda_3(1 - \alpha)P_6(t) \quad (17) \quad \text{with initial condition:} \quad P_j(0) = \begin{cases} 1, & \text{if } j = 1 \\ 0, & \text{if } j \neq 1 \end{cases} \quad (34)$$

$$\frac{dP_{18}(t)}{dt} + \beta_1 P_{18}(t) = 2\lambda_1(1 - \alpha)P_6(t) \quad (18)$$

$$\frac{dP_{19}(t)}{dt} + \beta_4 P_{19}(t) = \lambda_4(1 - \alpha)P_6(t) \quad (19)$$

$$\frac{dP_{20}(t)}{dt} + \beta_4 P_{20}(t) = \lambda_4(1 - \alpha)P_7(t) \quad (20)$$

$$\frac{dP_{21}(t)}{dt} + \beta_3 P_{21}(t) = \lambda_3(1 - \alpha)P_7(t) \quad (21)$$

$$\frac{dP_{22}(t)}{dt} + \beta_2 P_{22}(t) = 2\lambda_2(1 - \alpha)P_8(t) \quad (22) \quad A_F = \sum_{i=1}^n P_i(t) \quad (35)$$

$$\frac{dP_{23}(t)}{dt} + \beta_3 P_{23}(t) = \lambda_3(1 - \alpha)P_8(t) \quad (23) \quad B_F = \sum_{i=10}^{11} \frac{1}{10} P_i(t) + \sum_{k=12,13,28,29} \frac{2}{10} P_k(t) + \sum_{m=14-16,21}^{25-27} \frac{3}{10} P_m(t)$$

$$\frac{dP_{24}(t)}{dt} + \beta_4 P_{24}(t) = \lambda_4(1 - \alpha)P_8(t) \quad (24) \quad + \sum_{n=17-20}^{22-24} \frac{4}{10} P_n(t) + \sum_{l=30}^{33} \frac{5}{10} P_l(t) \quad (36)$$

$$\frac{dP_{25}(t)}{dt} + \beta_2 P_{25}(t) = 2\lambda_2(1 - \alpha)P_4(t) \quad (25)$$

$$\frac{dP_{26}(t)}{dt} + \beta_4 P_{26}(t) = \lambda_4(1 - \alpha)P_4(t) \quad (26) \quad \text{The profit incurred to the system model in steady state can be obtained as}$$

$$\frac{dP_{27}(t)}{dt} + \beta_3 P_{27}(t) = 2\lambda_3(1 - \alpha)P_4(t) \quad (27) \quad P_F = K_0 * A_F - K_1 * B_F \quad (37)$$

$$\frac{dP_{28}(t)}{dt} + \beta_4 P_{28}(t) = \lambda_4(1 - \alpha)P_3(t) \quad (28) \quad \text{where } K_0, \text{ revenue per unit up-time of the system; } K_1, \text{ cost per unit time for which server is busy in repair activities.}$$

$$\frac{dP_{29}(t)}{dt} + \beta_3 P_{29}(t) = \lambda_3(1 - \alpha)P_3(t) \quad (29)$$

4 Numerical and graphical results

In this section, for a fixed set of values of failure and repair rates given as follows:

Set-1: $\lambda_1 = 0.02$, $\lambda_2 = 0.05$, $\lambda_3 = 0.03$, $\lambda_4 = 0.023$, $\beta_1 = 0.61$, $\beta_2 = 0.8$, $\beta_3 = 0.72$, $\beta_4 = 0.45$ the numerical values of fuzzy availability and profit function have been obtained by considering different values of coverage

Fig. 2 State transition diagram

Table 1 Effect of failure rate of various subsystems on the profit function of plant with respect to time at $\alpha = 0.9$

Time in days	Set-1: initial observations	$\lambda_1 = 0.02$ to $\lambda_1 = 0.07$	$\lambda_2 = 0.05$ to $\lambda_2 = 0.2$	$\lambda_3 = 0.03$ to $\lambda_3 = 0.10$	$\lambda_4 = 0.023$ to $\lambda_4 = 0.063$
20	9905.92	9898.251	9846.437	9810.858	9818.942
40	9905.914	9898.237	9846.427	9810.852	9818.926
60	9905.914	9898.237	9846.427	9810.852	9818.926
80	9905.914	9898.237	9846.427	9810.852	9818.926
100	9905.914	9898.237	9846.427	9810.852	9818.926
120	9905.914	9898.237	9846.427	9810.852	9818.926
140	9905.914	9898.237	9846.427	9810.852	9818.926
160	9905.914	9898.237	9846.427	9810.852	9818.926

factor in the range $0 \leq \alpha \leq 1$. From Tables 1, 2, 3, 4 and Figs. 3, 4, 5 and 6, it is observed that fuzzy availability and fuzzy profit sharply declined as the value of coverage

factor decreases and time of operation increases. The system is highly available for use and profitable against the high value of coverage factor. From Tables 1, 2, 3 and

Table 2 Effect of failure rate of various subsystems on the profit function of plant with respect to time at $\alpha = 0.7$

Time in days	Set-1: initial observations	$\lambda_1 = 0.02$ to $\lambda_1 = 0.07$	$\lambda_2 = 0.05$ to $\lambda_2 = 0.2$	$\lambda_3 = 0.03$ to $\lambda_3 = 0.10$	$\lambda_4 = 0.023$ to $\lambda_4 = 0.063$
20	9724.772	9710.617	9608.394	9454.929	9477.488
40	9724.756	9710.579	9608.35	9454.915	9477.455
60	9724.756	9710.579	9608.35	9454.915	9477.455
80	9724.756	9710.579	9608.35	9454.915	9477.455
100	9724.756	9710.579	9608.35	9454.915	9477.455
120	9724.756	9710.579	9608.35	9454.915	9477.455
140	9724.756	9710.579	9608.35	9454.915	9477.455
160	9724.756	9710.579	9608.35	9454.915	9477.455

Table 3 Effect of failure rate of various subsystems on the profit function of plant with respect to time at $\alpha = 0.5$

Time in days	Set-1: initial observations	$\lambda_1 = 0.02$ to $\lambda_1 = 0.07$	$\lambda_2 = 0.05$ to $\lambda_2 = 0.2$	$\lambda_3 = 0.03$ to $\lambda_3 = 0.10$	$\lambda_4 = 0.023$ to $\lambda_4 = 0.063$
20	9551.82	9160.514	9444.344	9125.438	9160.514
40	9551.797	9160.473	9444.258	9125.419	9160.473
60	9551.797	9160.473	9444.258	9125.419	9160.473
80	9551.797	9160.473	9444.258	9125.419	9160.473
100	9551.797	9160.473	9444.258	9125.419	9160.473
120	9551.797	9160.473	9444.258	9125.419	9160.473
140	9551.797	9160.473	9444.258	9125.419	9160.473
160	9551.797	9160.473	9444.258	9125.419	9160.473

Table 4 Effect of failure rate of various subsystems on the profit function of plant with respect to time at $\alpha = 0.3$

Time in days	Set-1: initial observations	$\lambda_1 = 0.02$ to $\lambda_1 = 0.07$	$\lambda_2 = 0.05$ to $\lambda_2 = 0.2$	$\lambda_3 = 0.03$ to $\lambda_3 = 0.10$	$\lambda_4 = 0.023$ to $\lambda_4 = 0.063$
20	9385.743	8864.783	9326.7	8818.855	8864.783
40	9385.715	8864.74	9326.611	8818.834	8864.74
60	9385.715	8864.74	9326.611	8818.834	8864.74
80	9385.715	8864.74	9326.611	8818.834	8864.74
100	9385.715	8864.74	9326.611	8818.834	8864.74
120	9385.715	8864.74	9326.611	8818.834	8864.74
140	9385.715	8864.74	9326.611	8818.834	8864.74
160	9385.715	8864.74	9326.611	8818.834	8864.74

Fig. 3 Effect of variation in failure rate of crystallizer on fuzzy availability with respect to time (days) **Fig. 4** Effect of variation in failure rate of A-type centrifugal on fuzzy availability with respect to time (days)

Fig. 5 Effect of variation in failure rate of grader on fuzzy availability with respect to time (days)

Fig. 6 Effect of variation in failure rate of weighing and bagging on fuzzy availability with respect to time (days)

4, a variation in the profit incurred by system model is observed between 7 and 13% w.r.t. variations in coverage factor. The effect of variation in various failure rates by changing $\lambda_1 = 0.02$ to $\lambda_1 = 0.07$, $\lambda_2 = 0.05$ to $\lambda_2 = 0.2$, $\lambda_3 = 0.03$ to $\lambda_3 = 0.10$ and $\lambda_4 = 0.023$ to $\lambda_4 = 0.063$ on fuzzy availability and profit are shown numerically and graphically. The variation in fuzzy availability and profit w.r.to failure rate (λ_1) shown in Fig. 3 and Tables 1, 2, 3, 4 and it is revealed that there is approximately 7% difference in availability and profit w.r.to (α) and 0.15% variation $\alpha = 0.9$ by changing $\lambda_1 = 0.02$ to $\lambda_1 = 0.07$. The variation in fuzzy availability and profit w.r.to failure rate (λ_2) shown in Fig. 4 and Tables 1, 2, 3, 4 and it is revealed that there is approximately 7% difference in availability and profit w.r.to (α) and 0.15% variation $\alpha = 0.9$ by changing

$\lambda_2 = 0.05$ to $\lambda_2 = 0.2$. The variation in fuzzy availability and profit w.r.to failure rate (λ_3) shown in Fig. 5 and Tables 1, 2, 3, 4 and it is revealed that there is approximately 13% difference in availability and profit w.r.to (α) and 1.1% variation $\alpha = 0.9$ by changing $\lambda_3 = 0.03$ to $\lambda_3 = 0.1$. The variation in fuzzy availability and profit w.r.to failure rate (λ_4) shown in Fig. 6 and Tables 1, 2, 3, 4 and it is revealed that there is approximately 13% difference in availability and profit w.r.to (α) and 7% variation $\alpha = 0.1$ by changing $\lambda_4 = 0.023$ to $\lambda_4 = 0.063$.

5 Conclusion

In this section, effect of various failure rates of subsystems on fuzzy availability and profit function has been observed for arbitrary values of parameters and costs in Tables 1, 2, 3, 4 and Figs. 3, 4, 5 and 6 respectively. It is revealed from the study that availability and profit of A-pan crystallization system go on decreasing with the increase of coverage factor (α) and failure rates of the subsystems A, B, C and D shown in Tables 1, 2, 3, 4 and Figs. 3, 4, 5 and 6. However, the effect of failure rates of subsystems C and D is much more because these are single unit systems. The use of redundant system is proved very helpful in improving the availability and profit of subsystem A and B. So, it is recommended that by controlling the failure rates of subsystem C and D or by providing the redundant unit for operation in standby to subsystem C and D the fuzzy availability and profit can be improved. As a concluding remark, it is suggested to system designers that A-pan crystallization system of a sugar industry can play dynamic role in improving fuzzy availability and profit of whole sugar industry by providing proper attention to subsystem C and subsystem D (Table 5).

Table 5 Effect of failure rate of various subsystems on the profit function of plant with respect to time at $\alpha = 0.1$

Time in days	Set-1: initial observations	$\lambda_1 = 0.02$ to $\lambda_1 = 0.07$	$\lambda_2 = 0.05$ to $\lambda_2 = 0.2$	$\lambda_3 = 0.03$ to $\lambda_3 = 0.10$	$\lambda_4 = 0.023$ to $\lambda_4 = 0.063$
20	9225.281	8587.487	9216.05	8532.143	8587.487
40	9225.25	8587.446	9216.007	8532.12	8587.446
60	9225.25	8587.446	9216.007	8532.12	8587.446
80	9225.25	8587.446	9216.007	8532.12	8587.446
100	9225.25	8587.446	9216.007	8532.12	8587.446
120	9225.25	8587.446	9216.007	8532.12	8587.446
140	9225.25	8587.446	9216.007	8532.12	8587.446
160	9225.25	8587.446	9216.007	8532.12	8587.446

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Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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