Compensation of asynchronous time of flight measurements in a long baseline navigation

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Abstract

In a long baseline (LBL) acoustic positioning system, a navigation subject estimates its own position at a time based on the ranges between itself and the LBL transponders. To obtain the ranges, each transponder sends acoustic wave to the subjects receiver. Here, a range equals to time required by the wave to travel between these two points multiplied by the wave propagation speed. This method is known as the time of flight (ToF) measurement. One of the ideals in carrying out a ToF is that the navigation subject would remain still throughout the measurement. Corollary, position estimation based on the ToF holds on the same ideal. However, an acoustic wave propagates in a very low speed. Due to this characteristic, the displacement of a moving navigation subject like an autonomous underwater vehicle (AUV) may not be negligible and becomes a source of bias for the range estimations. The AUV motion would also lead to asynchronous ToFs, i.e. acoustic waves sent by transponders arrive at the AUV in different time epochs. In this paper, we present LBL navigation for an AUV that deals with uncertainty due to motion of the AUV. Here, state estimator is modeled while considering the bias for each ToF. Once we obtain the state space representation, we apply Kalman filter to estimate the AUV position and speed. By simulation, we demonstrate that the estimator follows the actual states with good accordance.

Keywords Autonomous underwater vehicle · Inference diagram · Long baseline · Sonar · Underwater acoustics

1 Introduction

Initial development of concepts and applications of underwater acoustic positioning systems can be traced back to 1960s [1, 2]. For underwater applications, these systems are preferable since position references from the more convenient global positioning systems (GPS) are not available in the water due to the attenuation of electromagnetic waves [3, 4]. Among these underwater systems, a long baseline (LBL) [5] system is worth considering since it shares similarity with a GPS in terms of configuration and working principles.

In a LBL system, a navigation subject estimates its own position using the ranges between itself and each of LBL transponders. To obtain a range, the subject receives an

acoustic wave sent by a transponder. This exchange can be arranged in a two-way travel time (OWTT) or one-way travel-time (OWTT) configuration [6]. Here, a range equals to time required by the wave to travel between these two points multiplied by the wave propagation speed. This method is known as the time of flight (ToF) measurement [7]. Using several ToFs as references, the subject's position could be then determined by applying certain technique such as time-of-arrival (ToA) [8] or time-difference-of-arrival (TDoA) [9].

An unbiased ToF measurement holds to the following ideals. First, the wave propagates in a line of sight (LoS), i.e. in a straight path. Second, the navigation subject remains still throughout the measurement. Corollary, applying either TDoA or ToA hold the same ideal. Otherwise, a ToF

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will result in a *pseudorange*, i.e. a range plus its bias(es). Regarding to the first one, biased ToF measurements due to non-LoS scenario may occur both in underwater acoustic [10] and electromagnetic [11] based systems.

On the other hand, a ToF differing from the second ideal would virtually affect only the acoustic based systems. This is because the propagation speed of an acoustic wave could be very low (around 1500 m/s) compared to the speed of an electromagnetic wave (3×10^8 m/s). Due to this characteristic, a moving navigation subject like an autonomous underwater vehicle (AUV) may have displaced while carrying out a ToF. Furthermore, ToFs may be completed in different time epochs considering the range differences among the transponders with respect to the AUV. Hence, applying either ToA or TDoA to determine position should also deal with this uncertainty.

The asynchronous ToFs in LBL navigation have been addressed in the literature. In [12–14], the AUV processes the ToFs are in a sequence with known queuing times. On the other hand, in [15] this uncertainty is a consequence of the slow acoustic wave propagation speed that occurs in a scenario where a LBL with 10^3 m² order of area is being used by an AUV with a considerable fast speed.

In this paper, we present LBL navigation for an AUV that deals with uncertainty due to motion of the AUV. Here, we model the state estimator while considering range bias that occurs in each ToF. Similar to [15], we also consider the clock offset as the source of range uncertainty while set a departure by addressing it now as a time-varying case. Once we obtain the systems state space representation, we apply the Kalman filter to estimate the AUV position and speed.

1.1 Notation

In this paper, **I** and **0** denote identity and zero matrices, respectively. Depending on the context of usages, their dimensions would adjust accordingly.

2 Problem statement

We consider ToF measurements involving a LBL formed by five transponders with known and fix position, $\mathbf{r}_1, \dots, \mathbf{r}_5 \in \mathbb{R}^3$ and an AUV, as depicted in Fig. 1. The motion of the AUV along trajectory AB is represented by yellow-dotted green circles. To estimate its position at time t , namely $\mathbf{r}(t)$, the AUV is supposed to receive acoustic waves from all transponders at the same time, represented by the red dashed arrows. However, the acoustic waves arrive in different time epochs as results of the AUV motion, as represented by the solid black arrows.

Fig. 1 ToF measurements involving a LBL with five transponders and an AUV

The above scenario would be addressed with the following assumptions. First, the AUV movement is a linear motion. Second, information about senders identity and send time is encoded into each transmitted wave [16]. Third, the dynamics and uncertainties of the transponders are all identical. Fourth, the AUV position is represented by its receivers coordinate frame.

In extension to our previous work [17], the following model derivation would consider the kinematics of the AUV in three dimensions. To mitigate the vertical dilution of precision (VDOP) [18]—a type of uncertainty that arises in height/depth estimation using the GPS/LBL, one of the transponders is to be mounted to the bottom of a vessel, i.e. placed above the AUV.

2.1 Vehicle kinematics

The dynamics of position and speed of the AUV is given by

$$\dot{\mathbf{r}}(t) = \mathbf{v}(t), \quad (1)$$

where $\mathbf{v}(t) \in \mathbb{R}^3$ denotes the AUV velocity, and

$$\dot{\mathbf{v}}(t) = \mathbf{0}, \quad (2)$$

where $\mathbf{0} \in \mathbb{R}^3$ i.e. the AUV is assumed moving in a linear motion. It is of interest to express the system in a discrete form, thus we write (1) as

$$\mathbf{p}(k+1) = \mathbf{p}(k) + \tau \mathbf{v}(k), \quad (3)$$

where $k \in \mathbb{N}$ and $\tau > 0$ denote the sampling time and period, respectively, and $t := k\tau$. Accordingly, we may now write (2) as

$$\mathbf{v}(k+1) = \mathbf{v}(k). \quad (4)$$

2.2 Pseudorange

The pseudorange resulted by a biased ToF range measurement between the AUV with transponder j ($j = 1, \dots, 5$) at time sampling k can be expressed as:

$$\|\mathbf{r}(k) - \mathbf{r}_j\| = c \cdot (t_j(k) - (t_0(k)) + c\theta + \delta_j(k), \quad (5)$$

where c denotes the acoustic wave speed, $t_0(k)$ and $t_j(k)$ denote the acoustic waves send and receive times k , respectively; $\theta(k)$ denotes transponder's clock offset; and $\delta_j(k)$ is a Gaussian additive noise representing pseudorange j uncertainty due to the AUV movement. One may rewrite (5) as

$$d_j(k) = \|\mathbf{r}(k) - \mathbf{r}_j\| + c\theta + \delta_j(k), \quad (6)$$

where $d_j(k) = c \cdot (t_j(k) - (t_0(k)))$. Furthermore, a time-varying clock-offset can be modeled as:

$$\theta(k+1) = \theta(k) + \tau_c \alpha(k) + w(k), \quad (7)$$

where $\alpha(k)$ and τ_c denotes the clock skew—the instantaneous clock drift rate [19, p. 7], and the clock-offset sampling

respectively, and (7)–(8) as

$$x_3(k+1) = x_3(k) + \tau_c x_4(k) + w(k) \quad (11)$$

$$x_4(k+1) = x_4(k) + \eta(k), \quad (12)$$

respectively.

2.4 Pseudorange differences

Since the AUV states are to be estimated based on ToFs, it is then of interest to obtain explicit relations between $d_j(k)$ and $\mathbf{x}_i(k)$ in (6). In [20], this is obtained by computing pseudorange difference, i.e.:

$$d_i(k) - d_j(k),$$

where i also denotes the index of the pseudorange ($i = 1, \dots, 5$) but $i \neq j$. This technique is introduced in [8] as an approach to the ToA location. For L pseudoranges, this computation will result in $L(L-1)/2$ possible combinations.

In our case, following the pseudorange-difference derivation steps in [20] are based on (6) and (9)–(12). This yields:

$$\begin{aligned} d_i(k+1) - d_j(k+1) &= \frac{d_i(k) + d_j(k)}{d_i(k+1) + d_j(k+1)} [d_i(k) - d_j(k)] \\ &\quad - 2\tau \frac{(\mathbf{r}_i^T - \mathbf{r}_j^T)}{d_i(k+1) + d_j(k+1)} \mathbf{x}_2(k) \\ &\quad + 2c \frac{[d_i(k+1) - d_i(k)] - [d_j(k+1) - d_j(k)]}{d_i(k+1) + d_j(k+1)} x_3(k) \\ &\quad + 2c \frac{(\tau_c(d_i(k+1) - d_j(k+1)))}{d_i(k+1) + d_j(k+1)} x_4(k) + \gamma(k) \end{aligned} \quad (13)$$

period, respectively, while $w(k)$ is additional Gauss noise for the clock offset. The dynamics of clock skew is modeled as

$$\alpha(k+1) = \alpha(k) + \tau_c \alpha(k) + \eta(k), \quad (8)$$

while $\eta(k)$ is additional Gauss noise for the clock skew.

2.3 State definition

To formulate a state-space representation of the system, we define

$\mathbf{r}(k) := \mathbf{x}_1(k)$, $\mathbf{v}(k) := \mathbf{x}_2(k)$, $\theta(k) := x_3(k)$, and $\alpha(k) := x_4(k)$, from which we may now rewrite (3)–(4) as

$$\mathbf{x}_1(k+1) = \mathbf{x}_1(k) + \tau \mathbf{x}_2(k) \quad (9)$$

$$\mathbf{x}_2(k+1) = \mathbf{x}_2(k), \quad (10)$$

where $\gamma(k)$ represents all terms related to the process noises. For more details in derivation of (13), the reader may consult [17].

We may now define the pseudorange differences as additional states, i.e.

$$d_1(k) - d_2(k) := x_5(k), \dots, d_4(k) - d_5(k) := x_{14}(k),$$

noticing that 5 pseudoranges will result in 10 combinations of pseudorange differences.

2.5 State space representation

Since all states have been modeled, we may now define a state vector

$$\mathbf{x}(k) = [\mathbf{x}_1(k) \ \mathbf{x}_2(k) \ x_3(k) \ x_4(k) \ x_5(k) \ \dots \ x_{14}(k)]^T \in \mathbb{R}^{3+3+1+1+10},$$

to express the systems state space representation as

$$\mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{w}(k) \quad (14)$$

$$\mathbf{y}(k+1) = \mathbf{C}(k+1)\mathbf{x}(k+1), \quad (15)$$

where $\mathbf{A}(k)$, $\mathbf{w}(k)$, $\mathbf{y}(k)$, and $\mathbf{C}(k)$ denotes the state matrix, process noise vector, output vector, and observation matrix, respectively. With an exception to the size, the structure of $\mathbf{A}(k)$ in (14) is similar to the one in [17], i.e.:

$$\mathbf{A}(k) = \begin{bmatrix} \mathbf{I} & \tau\mathbf{I} & 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 & 0 \\ 0 & 0 & 1 & \tau_c & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & \mathbf{A}_{52}(k) & \mathbf{A}_{53}(k) & \mathbf{A}_{54}(k) & \mathbf{A}_{55}(k) \end{bmatrix} \in \mathbb{R}^{18 \times 18},$$

where:

$$\mathbf{A}_{52}(k) = -2\tau \left[\frac{\mathbf{r}_1^T - \mathbf{r}_2^T}{d_1(k+1) + d_2(k+1)} \quad \dots \quad \frac{\mathbf{r}_4^T - \mathbf{r}_5^T}{d_4(k+1) + d_5(k+1)} \right]^T \in \mathbb{R}^{10 \times 3},$$

$$\mathbf{A}_{53}(k) = 2c \left[\frac{[d_1(k+1) - d_2(k+1)] - [d_1(k) - d_2(k)]}{d_1(k+1) + d_2(k+1)} \quad \dots \quad \frac{[d_4(k+1) - d_5(k+1)] - [d_4(k) - d_5(k)]}{d_4(k+1) + d_5(k+1)} \right]^T \in \mathbb{R}^{10},$$

$$\mathbf{A}_{54}(k) = 2c\tau_c \left[\frac{d_1(k+1) - d_2(k+1)}{d_1(k+1) + d_2(k+1)} \quad \dots \quad \frac{d_4(k+1) - d_5(k+1)}{d_4(k+1) + d_5(k+1)} \right]^T \in \mathbb{R}^{10},$$

and

$$\mathbf{A}_{55}(k) = \text{diag} \left(\frac{d_1(k) + d_2(k)}{d_1(k+1) + d_2(k+1)}, \dots, \frac{d_4(k) + d_5(k)}{d_4(k+1) + d_5(k+1)} \right) \in \mathbb{R}^{10 \times 10}.$$

Meanwhile, the structure of (15) will be addressed in the forthcoming section.

2.6 System observability

We are about to determine the entries of $\mathbf{C}(k)$ in (15) in the sense of minimum sensors, i.e. the least numbers and kinds of sensors required by the system to be observable. For this purpose, we apply the so-called graphic approach [21] by drawing an inference diagram as shown in Fig. 2. Here, every state is represented by a node. For simplicity, the index k is excluded from the node.

Fig. 2 Inference diagram of the system states

By inspecting (9)–(12) while noticing that (13) forms $x_5(k), \dots, x_{14}(k)$, one would find that $x_2(k)$, $x_3(k)$, and $x_4(k)$ appear on the right side of other states. Accordingly, on Fig. 2 we draw arrows toward nodes x_2 , x_3 , and x_4 from other nodes. For example, we draw arrows toward node x_2 from nodes x_1 and $x_5, \dots, x_{14}$, since appears on the right side of (9) and (13). This means that information about $x_2(k)$ can be infer once information about $x_1(k)$ and $x_5(k), \dots, x_{14}(k)$ are available, implying that sensor deployment to measure $x_2(k)$ is not necessary. In this sense, $x_2(k)$ is assigned as non-sensor node. Applying the same reasoning, one would also find that x_3 and x_4 fall into the non-sensor node category.

Conversely, x_1 and from $x_5, \dots, x_{14}$ do not appear on the right side of any other equation. Accordingly, on Fig. 2 there is no incoming arrow toward nodes x_1 and $x_5, \dots, x_{14}$. This means that information about $x_1(k)$ and $x_5(k), \dots, x_{14}(k)$ can only gathered through measurements. Hence, x_1 and from $x_5, \dots, x_{14}$ are assigned as sensor nodes.

Figure 2 seems implying that (14) requires 10+1 sensors to be observable. Nonetheless, one should recall that $x_5(k), \dots, x_{14}(k)$ are originated from (6) for five pseudoranges. It means deploying the LBL, i.e. five acoustic transponders, suffices as a minimum requirement for the system's states observation. Accordingly, the matrix output in (15) can be written as:

$$\mathbf{C}(k) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \mathbf{I} \end{bmatrix} \in \mathbb{R}^{10 \times 18}, \quad (16)$$

where $\mathbf{I}$ here corresponds to $x_5(k), \dots, x_{14}(k)$.

2.7 Proposed solution

The system defined by (14)–(15) is linear, despite that some of the entries of $\mathbf{A}(k)$ are time-varying. Furthermore, all systems uncertainties are modeled as Gaussian. Therefore, we chose a standard Kalman filter to estimate the states.

3 Simulation

3.1 Setup and numerical values

We consider that the above scenario occurs in the middle of an extensive shallow sea. Here, five transponders with low update rate, i.e. 0.1 kHz [22], are deployed at coordinates $\mathbf{r}_1 = [0 \ 0 \ 0]^T$ m, $\mathbf{r}_2 = [0 \ 0 \ 100]^T$ m, $\mathbf{r}_3 = [2000 \ 0 \ 100]^T$ m, $\mathbf{r}_4 = [0 \ 2000 \ 101]^T$ m, and

$\mathbf{r}_5 = [2000 \ 2000 \ 99]^T$ m, respectively. Accordingly, the LBL update rate is set as the value of τ , i.e. 10 s.

The AUV was set to start initially at $\mathbf{r}(1) = [340 \ 300 \ 5]^T$ m and expected to finish at $\mathbf{r}(n) = [340 \ 300 \ 60]^T$ m with initial speed $\mathbf{v}(0) = [4 \ 0 \ 0]^T$ m/s, while following a trajectory shown in Fig. 3. Accordingly, initial values for position and velocity estimations are set to be $\hat{\mathbf{r}}(0) = [300 \ 300 \ 5]^T$ m and $\hat{\mathbf{v}}(0) = [4 \ 0 \ 0]^T$ m/s, respectively. Meanwhile, the ToF uncertainties due to the AUV movement are depicted in Fig. 4.

Regarding to the clock uncertainties, the values of τ_c , $\theta(0)$, and $\alpha(0)$ are set to be 900 s, 0.03 s, and 3×10^{-5} s/s, respectively. As one may notice, the clock offset initial value is equivalent to 45 m deviation of a ToF. Meanwhile, $w(t)$ —is set to be exhibited during $t_0(k)$, and $\eta(k)$ are depicted in Fig. 5.

For the Kalman Filter, the *a priori* error covariance matrix is set initially to be $\text{diag}(5 \cdot \mathbf{I}, 10^{-1} \cdot \mathbf{I}, 10^{-2}, 10^{-10}, 5 \cdot \mathbf{I}) \in \mathbb{R}^{18 \times 18}$, where the entries denote the error

Fig. 3 The trajectory AUV should follow

Fig. 4 The range uncertainties due to the AUV movement, modeled as white noises

Fig. 5 The additional noises for clock offset and clock skew

Fig. 6 Errors of the AUV position estimation in three dimensions

covariance of the position, speed, clock offset, clock skew, and range differences, respectively. Subsequently, the noise covariance matrix is set to be $\text{diag}(10^{-4} \cdot \mathbf{I}, 10^{-6}, 10^{-4} \cdot \mathbf{I}, 10^{-5}, 10^{-12}, 10^{-13}, 10^{-6} \cdot \mathbf{I}) \in \mathbb{R}^{18 \times 18}$. As one may notice, smaller values are chosen for error noise covariance of the vertical position and speed entries. Finally, the measurement noise covariance matrix is set to be $\text{diag}(10^{-8} \cdot \mathbf{I}) \in \mathbb{R}^{10 \times 10}$.

3.2 Results and discussion

The simulation is run for $n = 336$, i.e. 3360 s. The errors of the AUV position estimation in three dimensions are shown in Fig. 6, while the errors of the AUV velocity estimation are shown in Fig. 7, also in three dimensions. Despite exhibiting large errors at some points, in general, the estimation can follow the actual position trajectory in considerable good accuracy. Similar results are also obtained in

velocity estimations. On the other hand, noticeable white noise in z axis is due to the fact that AUV velocity in this axis are zero in most occasions and very small in other ones.

Furthermore, compensation of the clock offset and clock skew are shown in Fig. 8. As one may notice, the peak of the clock offset contribute to more than 600 m of pseudorange. Nonetheless, the clock offset and clock skew decay over the time and eventually converge to small number.

Overall, the state estimator can follow the actual position and speed of the AUV while dealing with asynchronous ToFs and clock uncertainties. As indicated in Fig. 6, for position estimation, estimator reached its convergence after 2500 s for x axis. Better results are obtained for y and z axes, where the convergence are reached after 1000 s. However, a more proper tuning on the Kalman filter seem necessary such the estimation converge faster.

Fig. 7 Errors of the AUV velocity estimation in three dimensions **Fig. 8** Compensations of the LBL transponders' clock offset and clock skew

4 Conclusion

In this paper, we have presented compensation of asynchronous ToFs in LBL navigation for an AUV. We derived model that considering the movement during the ToF as a source of uncertainties. By using graphic approach, we demonstrated the observability of the systems and minimum sensors required for the system realization. By simulation, we show that the derived model and the estimator can follow the actual position and speed of the AUV in a good accordance.

Since the update rate of the LBL is considerable very low, inclusion of a much faster sensor like inertial measurement unit (IMU) to the system becomes a sensible future work. In such configuration, the AUV would use an IMU as the main navigation aid. Noticing that an IMU tends to accumulate errors [23], the LBL would be

assigned as its periodic corrector. On the other hand, how the estimator would deal with possible occasions when a ToF measurement fails would be also worth to investigate in the future.

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Compliance with ethical standards

Conflict of interest On behalf of all authors, the corresponding author solemnly declares that there are no conflicts of interest

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