Analysis, dynamics and adaptive control synchronization of a novel chaotic 3-D system

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Abstract

In this paper, a new 3D chaotic dissipative system is introduced. Basics dynamical characteristics and properties are studied such as equilibrium points, Lyapunov exponent spectrum, Kaplan–Yorke dimension. Dynamics of the new 3D chaotic system is investigated also numerically using largest Lyapunov exponents spectrum and bifurcation diagrams. We derive new control results via adaptive control method based on Lyapunov stability theory and adaptive control theory for globally and exponentially synchronizing the identical 3-D novel chaotic systems with unknown system parameters. The results are validated by numerical simulation using Matlab.

Keywords Chaotic system · Lyapunov exponent · Bifurcation diagram · Adaptive control

1 Introduction

A large part of nonlinear dynamical systems are chaotic systems which are sensitive to initial conditions. This sensitivity is among the basic ideas of chaos and the most visible signature of its behavior. This phenomenon is ancient but was highlighted by Lorenz using the butterfly effect [1]. For a dynamical system, sensitivity to initial conditions is quantified by the Lyapunov exponents which measure the exponential divergence of nearby trajectories. In general, when a Lyapunov exponent is positive and the sum of the exponents of Lyapunov is negative, we will say that the system is chaotic and has a strange attractor. Lorenz discovered the first 3D chaotic system [1] in 1963 while he was modelling weather patterns with a 3-D model. The three-dimensional chaotic system has been a focal point of study for many researchers due to its applications. A great interest in the literature in the last few decades in the modelling of new chaotic systems. Many 3-D chaotic systems have been discovered such as Rössler system [2], Chen system [3], Lü system [4], Liu-Chen system [5], Zhu system [6], Sprott system [7], Vaidyanathan system [8], Elhadj [9]...etc. In recent years, many

methods of synchronization for chaotic systems have been introduced and applied such as; OYG method [10], back-stepping design method [11], sliding mode control [12], passive control [13], fuzzy control [14], nonlinear active control [15, 16], projective synchronization [17], function projective synchronization [18], global synchronization [19, 20]...etc. Adaptive control method is used when parameters are unknown or initially uncertain. In an adaptive method, control law and parameter update law are designed in such a way that the chaotic response system to behave like chaotic drive systems. As a result, adaptive scheme maintains consistent performance of a system in the presence of uncertainty as well as variations in plant parameters. This control method differs from other control methods because it does not require advance information about the limits on these uncertain or time-varying parameters because it relates to a control law that changes itself. Recently, many papers are available on synchronization of chaotic systems using this method of control [20–31]. This paper is organized as follows: The second section deals with the description and some properties of the novel chaotic system. After these; Next two sections deal with dynamics of the novel chaotic system and the synchronization

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problem for globally and exponentially synchronizing the identical 3-D novel chaotic systems using adaptive control law respectively, also, numerical simulations using MATLAB have been shown to illustrate our results for the new chaotic system. A conclusion is given in the last section.

2 Description of the novel chaotic system

In this research work, we propose a new 3D chaotic system obtained by simple modification of the system introduced in [32], which is given in system form as:

$$\begin{cases} \frac{dx}{dt} = a(y - x) + byz, \\ \frac{dy}{dt} = (c - a)x + cy - xz, \\ \frac{dz}{dt} = xy - z. \end{cases} \quad (1)$$

where a, b, c are positive real parameters with $b \neq 1$. In this paper, we shall show that the system (1) is chaotic when the parameters system a, b and c take the values:

$$a = 15, b = 8/3, c = 10. \quad (2)$$

2.1 Basic properties

In this section, some basic properties of system (1) are given. We start by equilibrium points of the system and check their stability at initial values of parameters a, b, c .

2.2 Equilibrium points

Putting equations of system (1) equal to zero i.e.,

$$\begin{cases} a(y - x) + byz = 0, \\ (c - a)x + cy - xz = 0, \\ xy - z = 0. \end{cases} \quad (3)$$

gives the five equilibrium points:

$$\begin{cases} p_0 = (0, 0, 0), \\ p_{1,3} = \left(\pm \frac{\sqrt{2}}{4a}(s - 2a) \sqrt{\frac{a}{b^2c}(s - 2ab + 4bc)}, \mp \frac{\sqrt{2}}{2} \sqrt{\frac{a}{b^2c}(s - 2ab + 4bc)}, -\frac{s}{2b} \right), \\ p_{2,4} = \left(\pm \frac{\sqrt{2}}{4a}(k - 2a) \sqrt{\frac{a}{b^2c}(k - 2ab + 4bc)}, \mp \frac{\sqrt{2}}{2} \sqrt{\frac{a}{b^2c}(k - 2ab + 4bc)}, -\frac{k}{2b} \right), \end{cases} \quad (4)$$

where

$$\begin{aligned} s &= a + \sqrt{a^2b^2 - 2a^2b + a^2 - 2ab^2c + 6abc + b^2c^2} \\ &\quad + ab - bc, \\ k &= a - \sqrt{a^2b^2 - 2a^2b + a^2 - 2ab^2c + 6abc + b^2c^2} \\ &\quad + ab - bc. \end{aligned} \quad (5)$$

2.3 Stability

In order to check the stability of the equilibrium points we derive the jacobian matrix at a point $p(x, y, z)$ of the system (1).

$$J(p) = \begin{pmatrix} -a & a + bz & by \\ c - a - z & c & -x \\ y & x & -1 \end{pmatrix} \quad (6)$$

For p_0 , we obtain $J(p) = \begin{pmatrix} -a & a & 0 \\ c - a & c & 0 \\ 0 & 0 & -1 \end{pmatrix}$ with characteristic polynomial equation $(\lambda + 1)(\lambda^2 + (a - c)\lambda - 2ac + a^2) = 0$, which has three eigenvalues:

$$\lambda_1 = -1, \lambda_2 = 11.419, \lambda_3 = -16.419. \quad (7)$$

Since all the eigenvalues are real Hartma-Grobman theorem implies that p_0 is a saddle point and unstable according to Lyapunov theorem of stability.

Using the same methods, the eigenvalues of the Jacobien at p_1 and p_3 are:

$$\lambda_1 = 6.6861, \lambda_2 = 4.9619, \lambda_3 = -17.648. \quad (8)$$

then p_1 and p_3 are two unstable saddle points.

The eigenvalues of the Jacobien at p_2 and p_4 are:

$$\begin{aligned} \lambda_1 &= 1.8884 + 3.3001i, \lambda_2 = 1.8884 - 3.3001i, \\ \lambda_3 &= -9.7768 \end{aligned} \quad (9)$$

then p_2 and p_4 are two unstable saddle-focus because none of the eigenvalues have real part zero and λ_1, λ_2 are complex.

2.4 Dissipativity

A dissipative dynamical system satisfy the condition:

$$\dot{V} = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} < 0 \quad (10)$$

In the case of the system (1), we have:

$$\dot{V} = c - (a + 1). \quad (11)$$

For $a = 15, b = 8/3, c = 10$ we obtain $\dot{V} = -6 < 0$ and therefore dissipativity condition holds on this system.

Also,

$$\frac{dV}{dt} = e^{c-(a+1)} \simeq 0.0024788. \quad (12)$$

then the volume of the attractor decreases by a factor of 0.0024788.

2.5 Lyapunov exponents and Kaplan–Yorke dimension

Lyapunov exponents are used to measure the exponential rates of divergence and convergence of nearby trajectories, which is an important characteristic to judge the system whether is chaotic or not. the existence of at least one positive Lyapunov exponent implies that the system is chaotic.

For the chosen parameters values (2), the Lyapunov exponents of the novel chaotic system (1) are obtained using the Wolf algorithm [33] in Matlab as:

$$L_1 = 0.65581, L_2 = -0.00085397, L_3 = -6.655. \quad (13)$$

The Lyapunov exponents spectrum is shown in Fig. 1.

Since the spectrum of Lyapunov exponents (13) has a positive term L_1 , it follows that the 3-D novel chaotic system (1) is chaotic. The Kaplan–Yorke dimension of system (1) is calculated as:

$$D_{KL} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0984. \quad (14)$$

3 Dynamics of the system

In this section, the dynamical behaviors of the system (1) are investigated numerically using the largest lyapunov exponents spectrum and bifurcation diagrams .

Fig. 1 Lyapunov exponents spectrum of system (1) for $a = 15$; $b = 8/3$; $c = 10$

3.1 Fix $a = 8/3$; $c = 10$; and vary a

Figure 2 shows the spectrum of largest Lyapunov exponents of system (1) with respect to parameter a : Obviously, when $a \in [12, 22]$; the behaviour of system (1) are either chaotic, periodic or converge to a one of the equilibria. When $a \in [12.5, 16]$; the maximum Lyapunov exponent are positive, implying that the new system (1) is chaotic in this range of parameters . Figure 3 shows the strange attractor when $a = 15$. For $a \in [16.4, 16.8]$; the maximum Lyapunov exponent almost always equals zero, implying that the new system (1) has a periodic orbit. Figure 4 shows the periodic orbit when $a = 16.5$. The maximum Lyapunov exponent is negative for $a \in [18.5, 22]$ which means that the trajectories of the new system (1) is fall to converge to

Fig. 2 Variation of the largest Lyapunov exponent of system (1) versus the parameter $a \in [12, 22]$; and $b = 8/3$; $c = 10$

Fig. 3 The strange attractor of the new system (1) when $a = 15$; $b = 8/3$; $c = 10$

Fig. 4 The periodic orbit of the new system (1) when $a = 16.5$; $b = 8/3$; $c = 10$

Fig. 5 Bifurcation diagram of system (1) versus the parameter $a \in [12, 22]$; and $b = 8/3$; $c = 10$

equilibria. This dynamics is confirmed by the bifurcation diagram in Fig. 5.

3.2 Fix $a = 15$; $c = 10$; and vary b

Figure 6 shows the largest Lyapunov exponents spectrum of the system (1) with respect to parameter b : Obviously, when $b \in [0, 20]$; the behavior of the system (1) is in chaotic state for all the values of this range except some values where the system become in periodic state. This dynamics is confirmed by the bifurcation diagram in Fig. 7.

Fig. 6 Variation of the largest Lyapunov exponent of system (1) versus the parameter $b \in [0, 20]$; and $a = 15$; $c = 10$

Fig. 7 Bifurcation diagram of system (1) versus the parameter $b \in [0, 20]$; and $a = 15$; $c = 10$

3.3 Fix $a = 15$; $b = 8/3$; and vary c

Figures 8 and 9 shows the spectrum of largest Lyapunov exponents and the bifurcation diagram of the system (1) versus the parameter $c \in [5, 13]$; and $a = 15$; $b = 8/3$. In $c \in [5, 13]$; the behavior of the system (1) is in either converge to a one of equilibria, periodic or chaotic. Figures 10 and 11 depicts the chaotic attractor of the novel system (1) in 3-D view, while in Figs. 12, 13, 14, 15, 16, 17, 18 and 19, the 2-D projections of the strange chaotic attractor of the novel chaotic system (1) on (x, y) , (y, x) , (x, z) , (z, x) , (y, z) , (z, y) planes, are shown, respectively.

Fig. 8 Variation of the largest Lyapunov exponent of system (1) versus the parameter $c \in [5, 13]$; and $a = 15$; $b = 8/3$

Fig. 9 Bifurcation diagram of system (1) versus the parameter $c \in [5, 13]$; and $a = 15$; $b = 8/3$

Fig. 10 3D view on the x - z - y space of the chaotic attractor of the novel system (1)

Fig. 11 3D view on the y - z - x space of the chaotic attractor of the novel system (1)

Fig. 12 Projection on x - y plane of the chaotic attractor of system (1) for: $a = 15$; $b = 8/3$; $c = 10$

4 Adaptive synchronization of the identical 3-D novel chaotic systems

In this section, we derive an adaptive control law for globally and exponentially synchronizing the identical 3-D novel chaotic systems with unknown system parameters.

Thus, the master system is given by the novel chaotic system dynamics:

$$\begin{cases} \frac{dx_1}{dt} = a(x_2 - x_1) + bx_2x_3, \\ \frac{dx_2}{dt} = (c - a)x_1 + cx_2 - x_1x_3, \\ \frac{dx_3}{dt} = x_1x_2 - x_3. \end{cases} \quad (15)$$

Fig. 13 Projection on y-x plane of the chaotic attractor of system (1) for: $a = 15; b = 8/3; c = 10$

Fig. 15 Projection on z-x plane of the chaotic attractor of system (1) for: $a = 15; b = 8/3; c = 10$

Fig. 14 Projection on x-z plane of the chaotic attractor of system (1) for: $a = 15; b = 8/3; c = 10$

Fig. 16 Projection on y-z plane of the chaotic attractor of system (1) for: $a = 15; b = 8/3; c = 10$

Also, the slave system is given by the novel chaotic system dynamics:

$$\begin{cases} \frac{dy_1}{dt} = a(y_2 - y_1) + by_2y_3 + u_1, \\ \frac{dy_2}{dt} = (c - a)y_1 + cy_2 - y_1y_3 + u_2, \\ \frac{dy_3}{dt} = y_1y_2 - y_3 + u_3. \end{cases} \quad (16)$$

In (15) and (16), the system parameters a, b, c are unknown and the design goal is to find an adaptive feedback controls u_1, u_2, u_3 using the states x_1, x_2, x_3 and estimates;

$a_1(t), b_1(t), c_1(t)$ of the unknown parameters a, b, c , respectively.

The synchronization error between the novel chaotic systems (15) and (16) is defined as:

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \quad (17)$$

(17) implies;

Fig. 17 Projection on z-y plane of the chaotic attractor of system (1) for: $a = 15; b = 8/3; c = 10$

Fig. 19 Projection on z-x plane of the chaotic attractor of system (1) for: $a = 15; b = 45; c = 10$

Fig. 18 Projection on x-z plane of the chaotic attractor of system (1) for: $a = 15; b = 45; c = 10$

$$\begin{cases} \dot{e}_1 = \dot{y}_1 - \dot{x}_1 \\ \dot{e}_2 = \dot{y}_2 - \dot{x}_2 \\ \dot{e}_3 = \dot{y}_3 - \dot{x}_3 \end{cases} \quad (18)$$

Thus, the synchronization error dynamics is obtained as:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + b(y_2 y_3 - x_2 x_3) + u_1 \\ \dot{e}_2 = (c - a)e_1 + ce_2 - y_1 y_3 + x_1 x_3 + u_2 \\ \dot{e}_3 = -e_3 + y_1 y_2 - x_1 x_2 + u_3 \end{cases} \quad (19)$$

We take the adaptive control law defined by:

$$\begin{cases} u_1 = -a_1(e_2 - e_1) - b_1(y_2 y_3 - x_2 x_3) - k_1 e_1 \\ u_2 = -(c_1 - a_1)e_1 - c_1 e_2 + y_1 y_3 - x_1 x_3 - k_2 e_2 \\ u_3 = e_3 - y_1 y_2 + x_1 x_2 - k_3 e_3 \end{cases} \quad (20)$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (20) into (19), we obtain the closed-loop error dynamics as:

$$\begin{cases} \dot{e}_1 = (a - a_1)(e_2 - e_1) + (b - b_1)(y_2 y_3 - x_2 x_3) - k_1 e_1 \\ \dot{e}_2 = -(a - a_1)e_1 + (c - c_1)(e_1 + e_2) - k_2 e_2 \\ \dot{e}_3 = -k_3 e_3 \end{cases} \quad (21)$$

The parameter estimation errors are defined as:

$$\begin{cases} e_a(t) = a - a_1(t), \\ e_c(t) = c - c_1(t), \\ e_b(t) = b - b_1(t). \end{cases} \quad (22)$$

Differentiating (22) with respect to t , we obtain:

$$\begin{cases} \frac{de_a(t)}{dt} = -\frac{da_1(t)}{dt}, \\ \frac{de_c(t)}{dt} = -\frac{dc_1(t)}{dt}, \\ \frac{de_b(t)}{dt} = -\frac{db_1(t)}{dt}. \end{cases} \quad (23)$$

By using (22), we rewrite the closed-loop system (21) as:

$$\begin{cases} \dot{e}_1 = e_a(e_2 - e_1) + e_b(y_2y_3 - x_2x_3) - k_1e_1, \\ \dot{e}_2 = -e_ae_1 + e_c(e_1 + e_2) - k_2e_2, \\ \dot{e}_3 = -k_3e_3. \end{cases} \quad (24)$$

We consider the quadratic Lyapunov function given by:

$$V(e_1, e_2, e_3, e_a, e_b, e_c) = \frac{1}{2} \left(\sum_{i=1}^3 k_i e_i^2 + e_a^2 + e_b^2 + e_c^2 \right) \quad (25)$$

which is a positive definite function on R^6 .

Differentiating V along the trajectories of the systems (23) and (24), we obtain the following:

$$\begin{aligned} \dot{V} = & - \sum_{i=1}^3 k_i e_i^2 + e_a \left(-e_1^2 - \frac{da_1(t)}{dt} \right) \\ & + e_b \left((y_2y_3 - x_2x_3)e_1 - \frac{db_1(t)}{dt} \right) \\ & + e_c \left(e_1e_2 + e_2^2 - \frac{dc_1(t)}{dt} \right) \end{aligned} \quad (26)$$

In view of (26), we take the parameter update law as follows:

$$\begin{cases} \frac{da_1(t)}{dt} = -e_1^2 \\ \frac{db_1(t)}{dt} = (y_2y_3 - x_2x_3)e_1 \\ \frac{dc_1(t)}{dt} = e_1e_2 + e_2^2 \end{cases} \quad (27)$$

Substituting (27) into (26), we get

$$\dot{V} = - \sum_{i=1}^3 k_i e_i^2 \quad (28)$$

which is a negative definite function on R^3 . Hence, by the Lyapunov stability theory [34], it follows that $e_i(t) \rightarrow 0$ as $t \rightarrow \infty$ for $i = 1, 2, 3$.

Hence, we have proved the following theorem.

Theorem 1 The 3-D novel chaotic systems (15) and (16) with unknown parameters are globally and exponentially synchronized for all initial conditions by the adaptive feedback control law (20) and the parameter update law (27), where k_1, k_2, k_3 are positive constants.

4.1 Numerical simulations

We used the classical fourth-order Runge-Kutta method with step size $h = 10^{-6}$ to solve the systems of differential equations (15), (16) and (27), when the adaptive control law (20) is applied.

Fig. 20 Synchronization of the states $x_1(t)$ and $y_1(t)$

Fig. 21 Synchronization of the states $x_2(t)$ and $y_2(t)$

The parameter values of the novel 3-D chaotic system (15) are chosen as in the chaotic case (2). The positive gain constants are taken as $k_i = 2$, for $i = 1, 2, 3$.

The initial conditions of the novel chaotic systems (15) and (16), are chosen as: $x_1(0) = 5, x_2(0) = 20, x_3(0) = 6$ and $y_1(0) = -3, y_2(0) = 10, y_3(0) = 20$.

Furthermore, as initial conditions of the parameter estimates of the unknown parameters, we have chosen $a_1(0) = 20, b_1(0) = 10, c_1(0) = 40$.

In Figs. 20, 21 and 22, the synchronization of the states of the master system (15) and slave system (16) is depicted, when the adaptive control law (20) and parameter update law (27) are implemented. In Fig. 23, the time-history of the parameter estimates $a_1(t), b_1(t), c_1(t)$ is depicted.

Fig. 22 Synchronization of the states $x_3(t)$ and $y_3(t)$

Fig. 23 Time-history of the parameter estimates $a_1(t)$, $b_1(t)$, $c_1(t)$

5 Conclusion

This paper presented a new 3D chaotic system. Some basic properties of the new chaotic system are given, such as equilibrium points, Lyapunov exponent spectrum, Kaplan–Yorke dimension. The dynamics of the new 3D chaotic system are investigated also numerically using largest Lyapunov exponents spectrum and bifurcation diagrams. Also, we derive new control results via adaptive control method based on Lyapunov stability theory and adaptive control theory for globally and exponentially synchronizing the identical 3-D novel chaotic systems with unknown system parameters. The results are validated by numerical simulation using Matlab.

Compliance with ethical standards

Conflict of interest The authors declare that they have no competing interests

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