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The Tension Between Allowing Student Struggle and Providing Support When Teaching Problem-Solving in Primary School Mathematics

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Abstract This article reports two primary school teachers' perceptions of the effectiveness of lessons based on a problem-solving intervention. The intervention included enabling and extending prompts, independent student struggle time initially and time to share problem-solving strategies at the end. The intervention had two versions: one included whole class prompts and teachers anticipated students' responses before teaching; the other without these features. Each teacher implemented two lessons in year 1/2 composite classes, with one lesson common. Teachers identified positive impacts of the intervention including providing support for students, extending students' thinking and providing positive challenge during problem-solving. Struggle time was believed to negatively impact some students' resilience and confidence; both teachers deviated from the intervention to reduce struggle time. Students used more problem-solving strategies when struggle time was included compared to when the teacher modelled an approach for solving. There was a tension for teachers between providing time for students to struggle and preserving some students' confidence. One teacher facilitated student share time in the middle of one lesson, allowing students to experience both struggle and success; this compromise could address the tension. Overall, the intervention was perceived to positively impact teaching practice.

Résumé Cet article s'intéresse à la perception qu'ont deux enseignants du primaire sur l'efficacité des leçons fondées sur une intervention ciblée sur la résolution de problèmes. L'intervention comprenait des questions incitatives visant l'activation et le prolongement, une période allouée initialement aux élèves pour composer de façon indépendante avec les difficultés ainsi qu'un temps à la fin pour faciliter le partage de stratégies en résolution de problèmes. L'intervention comportait deux versions: l'une comprenait des questions incitatives destinées à toute la classe alors que les enseignants anticipaient les réponses des élèves avant l'instruction; l'autre ne comportait aucun de ces aspects. Chaque enseignant a mis en œuvre deux leçons dans des classes composites formées d'élèves de la première et de la deuxième an-

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née, avec une leçon commune. Les enseignants ont identifié les effets positifs de l'intervention, notamment en ce qui concerne le soutien apporté aux élèves, l'approfondissement de la réflexion faite par les élèves et l'apport favorable d'un défi dans la résolution de problèmes. On a perçu le temps utilisé pour surmonter les difficultés comme ayant entraîné des conséquences négatives sur la résilience et la confiance de certains élèves; les deux enseignants ont délaissé l'intervention pour réduire le temps durant lequel les élèves se démenaient avec les difficultés. Les élèves ont eu davantage recours à des stratégies de résolution de problèmes lorsque le temps consacré à composer avec les difficultés était inclus que lorsque l'enseignant modélisait une approche visant la résolution. Les enseignants ont été tiraillés entre le fait d'allouer du temps aux élèves pour gérer les difficultés et le désir de préserver la confiance de certains d'entre eux. Un enseignant a facilité une période de partage entre les élèves au milieu d'une leçon, ce qui a permis à ceux-ci de vivre à la fois une expérience de gestion des difficultés et de réussite. Ce compromis pourrait aider à traiter de la question du tiraillement. Dans l'ensemble, l'intervention a été perçue comme ayant un effet positif sur la pratique de l'enseignement.

Keywords Primary mathematics · Struggle time · Teaching · Problem-solving strategies · Problem-solving prompts

Introduction

Problem-solving, involving solution of unfamiliar (Kilpatrick et al., 2001) and cognitively demanding problems (McCormick, 2016), is important in mathematics education (e.g. Liljedahl et al., 2016) and a twenty-first century skill that teachers should help students develop (English & Sriraman, 2009). Hence, there is motivation for primary school teachers to consider teaching approaches to support problem-solving.

The novel nature and open-endedness of problem-solving, teacher difficulties in choosing appropriate problems, knowing appropriate times to intervene to support problem-solving and dealing with student struggle were issues reported by year 1 teachers in three Australian schools (Cheeseman, 2018). In a survey of 108 Australian upper primary teachers, McCormick (2016) found that teachers' focus on teaching the mathematics for problems removed the problem-solving nature of many tasks. During problem-solving lessons, the extent and nature of teacher assistance is an important consideration, as well as problem choice, as they impact the problem-solving nature of tasks and the potential for students to engage with problems. The study reported in this article investigated two Australian primary teachers' implementation of lessons based on a problem-solving intervention.

Problem-Solving in the Curriculum

Education is state-based in Australia, although based on the Australian curriculum (ACARA, n.d.). The Victorian mathematics curriculum had four proficiencies (*problem-solving*, *understanding*, *fluency* and *reasoning*), developed across three content strands (*number and algebra*; *measurement and geometry*; *statistics and probability*) at the time of this study (VCAA, n.d.). Problem-solving was identified as "the ability of students to make choices, interpret, formulate, model and investigate problem situations, select and use technological functions and communicate solutions effectively" (VCAA, n.d., para. 4), aligned with Schoenfeld (1992) who suggested students should use higher-order thinking skills and select solution methods when problem-solving. The Victorian curriculum provided an imperative for teachers to develop students' problem-solving strategies noting that "capable problem-solvers [i.e., students] solve unfamiliar and meaningful problems, create their own investigations, use prior knowledge and strategies to aide in their solving of problems and check the viability of their solutions" (VCAA, n.d., para 4). Importantly, students should choose and use strategies to solve unfamiliar problems. Although problem-solving was an expectation in the Australian curriculum, Stacey et al. (2015) reviewed mathematics pedagogies and resources in Australian school education and found problem-solving was often overlooked due to perceived lack of teaching resources and time. Cheeseman (2018) and McCormick (2016) also noted time constraints and accessibility to resources as barriers to problem-solving.

Problem-Solving Process and Structure of Lessons

Problem-solving interventions require two considerations: the structure and features to support development of problem-solving strategies. Mason et al. (1985) provided a three-phase problem-solving process (i.e. *entry*, *attack* and *review*) incorporating problem-solving strategies, aligned with Pólya (1945). The *entry* phase involves understanding the problem and deciding on appropriate strategies; this may include trialling cases (*specialise*) to decide on a strategy (Mason et al., 1985). In the *attack* phase, numbers may be varied systematically to search for a pattern and *generalising* is introduced. The *review* phase involves checking for both accuracy and transferability of strategies to other problems. Mason et al. noted ineffective problem-solving strategies can cause a student to be "stuck", resulting in *distilling* (i.e. articulating difficulty) or *mulling* (i.e. abandon problem momentarily to think and hopefully arrive at a successful strategy). Inherent in *distilling* and *mulling* is that finding a strategy is the remit of the student, rather than provided by the teacher. Given that students may gain insight into a problem while solving (Liljedahl, 2004), teachers should allow students time to engage with problems to develop strategies for solving.

Sullivan et al. (2016) reported a successful intervention with four phases ("posing the problem"; "differentiating the problem" (i.e. individual work before teacher provides a prompt to increase/decrease problem difficulty); "reviewing student activity on the problem", including sharing strategies; "consolidating the learning" (i.e. solving-related problems)). Teachers ($n = 34$) solved problems before classroom use to better anticipate students' responses. They were provided with enabling prompts (e.g. Could pictures help?) to support "differentiating the problem" as the ability to provide suitable prompts to individuals was identified as important. Provision of suitable prompts requires teachers to respond at a student's point-of-need, based on their thinking. NCTM (2014) also noted that teachers should "elicit and use evidence of student thinking" (p. 10). Twenty-six teachers in Sullivan et al. (2016) believed the intervention structure (outlined above) was successful, noting the importance of phase two (i.e. students attempt problem, prior to teacher use of prompts) and phase four (i.e. students explain thinking and identify efficient problem-solving strategies). All but five of the teachers would plan future lessons using the intervention, highlighting its efficacy.

In a study with year 1 and 2 students ($n = 75$), Russo and Hopkins (2017) found lesson structure did not impact students' problem-solving outcomes. Some students preferred *problem-first* (i.e. challenging problem, class discussion of strategies, worksheets and teacher-led summary), while others preferred *teach-first* (i.e. discussion of mathematical content, worksheets, challenging problem and teacher-led summary). Given positive responses to consistent features reported for both teachers and students, there is potential in having consistent features in problem-solving lessons, supporting development of a problem-solving intervention to be implemented across a range of lessons.

Effective Problem-Solving Interventions

Features that contribute to successful problem-solving include time for teachers to help students understand language in a problem and teacher anticipation of students' responses (Sullivan et al., 2016); time for student *struggle* before teachers provide enabling/extending prompts (Cheeseman et al., 2017; Sullivan et al., 2016); and time for students to collaborate and share knowledge (Ingram et al., 2016).

NCTM (2014) noted teachers should include tasks “that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies” (p. 10) and provide “students, individually and collectively, with opportunities and supports to engage with productive struggle” (p. 10). This call for student struggle supported inclusion of time where students are not provided with strategies by either the teacher or peers; hence, there should be time for individual struggle. Ingram et al. (2020) noted the importance of students exploring problems without assistance initially, even though some students may struggle. Problem-solving interventions that included time to *struggle* with problems improved students’ problem-solving skills (Roche et al., 2013). Sullivan et al. (2014) found that primary students ($n = 758$) preferred to *struggle* before being given solutions, with 88% preferring problems of equal or greater difficulty to the problems in their study, highlighting students’ desire for challenging problems. Teachers in Russo et al. (2022) generally noted the importance of providing prompts after students had time to “grapple” with problems; struggle time was valued to enable students to understand problems and attempt solutions prior to teacher support through prompts.

Ingram et al. (2016) reported strategies that contributed to student persistence in problem-solving, following a survey of ten teachers who trialled strategies that encouraged students (aged 9–11) to persist with challenging problems. Strategies included encouraging collaboration; providing access to materials; encouraging students to reflect on their ability to persist; and posing probing questions to direct student thinking; as well as implementing extending prompts (i.e. encourage finding more responses; looking for patterns in results) and enabling prompts (i.e. support students to understand and attempt problems). Russo et al. (2022) also identified that teacher provision of enabling prompts (i.e. related to problem, but requiring fewer steps or of lower complexity) and extending prompts could cater for a range of students. Cheeseman et al. (2017) reported year 3–6 teachers ($n = 37$) who found extending and enabling prompts supported students to understand and solve problems and extend thinking. Clear expectations of students (e.g. recording thinking, independent or group work) are beneficial, also time for teachers to pay attention to students’ explanations and teacher questions to understand students’ thinking (Roche et al., 2013). The discussion above highlights a range of considerations in developing a problem-solving intervention for mathematics. Following is a problem-solving intervention and a report on its implementation in two classes.

The research question is: How did teachers implement the problem-solving intervention and what were the implications for teaching and learning of problem-solving?

Method

The literature review identified ten features to include in a problem-solving intervention (first column, Table 1). The first-named researcher developed the intervention, with two versions, based on these ten features and planned four lessons using the intervention (Table 2).

Teachers in one coeducational primary school in Victoria, Australia, were invited to participate by the first-named researcher, who taught at the school; hence, convenience sampling was used. Prior to this, ethics was approved by the University of Melbourne and permission approved from the Department of Education and Training Victoria and school principal. Two teachers, Beth and Rachel (pseudonyms), volunteered and provided information about the study to their classes (composite classes, year 1/2 students, aged 6–8 years). Seven students from Beth’s class and eight students from Rachel’s class volunteered to participate, with permission from parents/guardians. Each teacher implemented two problem-solving lessons with student data collected from participants.

Table 1 Problem-solving intervention features and rationale (adapted from Stewart (2020))

Intervention feature	Implementation considerations	Rationale		Intervention*	
		Students	Teachers	WCP	IP
1. Challenging problem	Lesson focus is to promote problem-solving	Choose and use problem-solving strategies to solve a challenging problem	Teacher expectation that lesson focus is students' choice and use of problem-solving strategies	Yes	Yes
2. Introduce challenging problem	Class discussion about problem. Teacher advice regarding not providing strategies for solving	Understand the problem and devise an individual strategy for solving	Teacher expectation about how to introduce the problem	Yes	Yes
3. Struggle time	Time allocated for students to *struggle* with problems	Learn to persist	Observe students and decide on relevant prompts	Yes	Yes
4. Students use problem-solving strategy and record their work	Time allocated for students to use and record problem-solving strategies	Express and organise thinking	Understand students' choices and use of problem-solving strategies	Yes	Yes
5. Problem-solving strategy sharing	Time allocated for students to share problem-solving strategies	Verbalise thinking to peers and determine efficient problem-solving strategies	Understand students' thinking about, and use of, problem-solving strategies	Yes	Yes
6. Teacher anticipates students' responses	Teachers predict students' responses prior to the lesson	Targeted prompts to better support students to engage with the problem	Recognise mathematical features and assumptions that are specific to the problem and hence be prepared to prompt students appropriately	Yes	No
7. Enabling and extending prompts	Prompts increase or decrease the difficulty of the problem	Receive support to engage with the problem or to extend their thinking	Support students to engage with the problem and extend their thinking	Yes	Yes
8. General prompts	Prompts to support and extend the problem	Receive support to engage with the problem or to extend their thinking	Support students to engage with the problem and extend students' thinking. Can also be used in other problem-solving lessons	Yes	Yes
9. Problem-specific prompts	Prompts to support and extend the problem	Receive support to engage with the problem or to extend their thinking	Support students to engage with the problem and extend students' thinking. Prompts target specific misconceptions or provide specific guidance	Yes	Yes
10. Whole class prompts	Two allocated times where teachers prompt their students as a class	Receive support to engage with the problem or to extend their thinking	Support and/or extend the entire class at the same time	Yes	No

*WCP whole class prompts included, and teachers anticipate students' responses, IP whole class prompts excluded, and teachers not expected to anticipate students' responses

Table 2 Overview of lessons: intervention-IP (Coin lesson) and intervention-WCP (Counter lesson, Cookie lesson and Over-and-Over lesson) (adapted from Stewart (2020))

Challenging problem	Problem assumption	Whole class prompt	Purpose	Intervention version
Coin problem: How many ways can you make 50c using coins?	1. Australian coins as currently used 2. The coins (50, 20, 10 and 5 cent coins) are known 3. The same value coin can be used once or several times	1. John said there were three solutions to the problem, is he correct? Why? Why not? (with visual including only options with like coins) 2. Kath said there were 13 ways you could make 50 cents. Is she correct? Why? Why not?	1. Visual is displayed to the entire class. Students understand that coin combinations can be used in solutions 2. Prompt students to search for more coin combinations	WCP
Cookie problem: Everyone got the same number of cookies. How many cookies may there have been?	1. Students comprehend the term, “same amount” 2. Each person gets a whole number of cookies 3. Students choose the number of people and cookies each person received	No	N/A	IP
Over-and-Over problem: I added the same number over and over until I reached 16. Which number may I have started with?	1. Students comprehend the terms, “same” and “over” 2. Students know their answers will be single numbers, not a number sentence	No	N/A	IP

Problem-Solving Intervention Development

Two versions of the intervention were created: intervention-WCP (whole class prompts included, teachers anticipate students' responses) and intervention-IP (whole class prompts excluded, teachers not expected to anticipate students' responses); this enabled the impact of whole class prompts and teacher anticipation of students' responses to be investigated. Table 1 summarises intervention features.

Problem-Solving Lesson Plans Based on Intervention

Four lessons were planned with four problems. Each problem:

- Supported use of a range of problem-solving strategies.
- Catered for a range of abilities as students can look for patterns or rules.
- Required finding a number of cases for the answer.

The suggested problems (Table 2) were anticipated to be challenging for the students as they required interpretation of a worded problem, determination of a strategy and provision of an answer that fully addressed all components of the problem. Fülöp (2021) defines a problem as "a challenge for which the participant has a goal, but does not have direct access to a method or an algorithm that gives the solution" (p. 1312). For the *Coin problem*, the students had not yet studied money, so they utilised understanding from outside the classroom; the *Cookie problem* was open-ended, so the students needed to produce an answer to address the requirements of the problem. Students had to select and use strategies to solve these non-routine problems; the intention was that the teacher would provide prompts only, rather than suggest or model approaches for solving. Teachers were provided with one lesson based on intervention-WCP (i.e. *Coin*) and three on intervention-IP (i.e. *Counter*, *Cookie* and *Over-and-Over*). Both teachers taught the Coin problem and one lesson based on intervention-IP (Rachel *Cookie*; Beth *Over-and-Over*). Table 2 shows the lessons taught and some assumptions behind problems.

Figure 1 shows pages 1 and 2 of intervention-WCP populated with "How many ways can you make 50c using coins"? Fig. 2 shows pages 1 and 2 of intervention-IP populated with "Everyone got the same number of cookies. How many cookies may there have been"? Problem-specific additions from researcher are in different fonts (e.g. title of task in Fig. 1). Boxes indicate features in Table 1.

For each lesson, the following were added to the intervention:

- Challenging problem
- Individual prompts
- General enabling prompts (e.g. Could you use a table?)
- General extending prompts (e.g. Are there more answers? How do you know?)
- Problem-specific enabling prompts (e.g. Could drawing an array help?)
- Problem-specific extending prompts (e.g. How does changing the number of people or the amount of cookies they receive change your answer?).
- Whole class prompts specific to the challenging problem (only for intervention-WCP)

Draft lessons were discussed, and teachers invited to make changes; neither teacher suggested changes to the intervention structure.

Task: How many ways can you make 50 cents using coins?			
Time	Activity	What the students are doing	What the teacher is doing
8 minutes	Introduction of task		Introducing the task. Ensuring students understand task. Not showing students how to solve the problem. Explaining to students that they need to individually record their problem-solving strategies.
15 minutes	Solving the task.	Discussing, solving the task and individually recording problem-solving strategies.	Roaming, observing and thinking about which groups/students need to be given prompts without intervening.
5 minutes	Introducing prompt 1 to class.		Discussing and displaying John's solution to the problem: John said there were 3 ways you could make 50 cents. Is he correct? Why? Why not?
10 minutes	Solving the task.	Discussing, solving the task and individually recording problem-solving strategies.	Giving students enabling prompts where necessary. Giving students extending prompts where necessary.
2 minutes	Introducing prompt 2 to class.		Discussing Kath's solution: "Kath said there were 13 ways to make 50 cents. Is this correct? Why? Why not?
10 minutes	Solving the task.	Discussing, solving the task and individually recording problem-solving strategies.	Giving students enabling and extending prompts where necessary. Selecting students who have used efficient problem-solving strategies and notifying them that they will share their problem-solving strategies with the class.
10 minutes	Share time.	Sharers share problem solving strategies. Class engage in discussions.	Ensuring students are stay on track and speaking about the problem-solving strategies they've used.

Challenging problem

Introduce challenging problem

Students use problem-solving strategy and record their work

Struggle time

Whole class prompts

Enabling and extending prompts

Problem-solving strategy sharing

Teachers anticipates student responses

	General problem-solving prompts	Problem-specific prompts
Enabling Prompts	Could you use a table? Can you think of an easier question like this? Can you think of a problem like this? Could you have a guess? Could you make a list? Could you act it out? Could you start with the answer and work backwards? Could you draw a picture? Could materials help?	**Prompts that help organise ideas** Could "cut and paste" pictures of coins help? Could fake coins help? **Prompts that complete a step towards solving the problem** Make 50 cents using only 10 cent coins. **Prompts that reduce the size of numbers** Could you use a strategy to work out how many ways you can make 10c or 20c?
Extending Prompts	Are there more answers? How do you know? How could someone find all of the answers? Is there another way that you could solve this problem? How many different ways could you solve this problem? What pattern can you see? How would you teach someone how to solve this problem?	**Prompts that make the problem harder** What strategies could you use to make $1, $2, $10, 70 cents or $150 in as many ways as possible? What if you could no longer include 20 cent coins in your ways to make 50 cents? **Prompts that encourage more ways to solve the problem** **Prompts that encourage more possible answers** Can you use more than one coin in your combination to make 50 cents? **Prompts to find a pattern or rule** How can you explain to John that he is wrong? If you were finding out how many ways you can make $1 would you just double your answer? Why? Why not?

General prompts

Problem-specific prompts

Fig. 1 Problem-solving intervention-WCP (whole class prompts), page 1 and page 2 populated with *Coin problem* (Stewart, 2020)

Task: Everyone got the same number of cookies. How many cookies may there have been? **Challenging problem**			
Time	Activity	What the students are doing	What the teacher is doing
10 minutes	Introduction of task		Introducing the task. Ensuring students understand task. Not showing students how to solve the problem. Explaining to students that they need to individually record their problem-solving strategies. **Introduce challenging problem**
15 minutes	Solving of task.	Discussing, solving the task and individually recording problem-solving strategies.	Roaming, observing and thinking about which groups/students need to be given prompts without intervening. **Students use problem-solving strategy and record their work** **Struggle time**
25 minutes			Giving students enabling prompts where necessary. Giving students extending prompts where necessary. **Enabling and extending prompts** Selecting students who have used efficient problem-solving strategies and notifying them that they will share their problem-solving strategies with the class.
10 minutes	Share time.	Sharers share problem solving strategies. Class engage in discussions. **Problem-solving strategy sharing**	Ensuring students are stay on track and speaking about the problem-solving strategies they've used.

	General problem-solving prompts	Problem-specific prompts
Enabling Prompts	**General prompts** Could you use a table? Could pictures help? Can you think of an easier question like this? Can you think of a problem like this? Could you have a guess? Could you make a list? Could you act it out? Could you start with the answer and work backwards? Could you draw a picture? Could materials help?	**Prompts that help organise ideas** Could counters help? Could drawing an array help? **Problem-specific prompts**
		Prompts that complete a step towards solving the problem What if there were 3 people? What if each person got 2 cookies? Could 12 be an answer? - Does checking this number help you find others?
		Prompts that reduce the size of numbers
Extending Prompts	Are there more answers? How do you know? How could someone find all of the answers? Is there another way that you could solve this problem? How many different ways could you solve this problem? What pattern can you see? How would you teach someone how to solve this problem?	**Prompts that make the problem harder** What is one person didn't get the same number of cookies as everyone else?
		Prompts that encourage more ways to solve the problem
		Prompts that encourage more possible answers Could you change the number of people? Could you change the number of cookies that each person gets?
		Prompts to find a pattern or rule How does changing the amount of people or the amount of cookies they receive change your answer?

Fig. 2 Problem-solving intervention-IP (individual prompts), page 1 and page 2 populated with *Cookie problem* (Stewart, 2020)

Teacher Choices—Lessons Based on Intervention

Teachers were invited to add prompts to lessons; none was added to the *Coin problem*. Each teacher added one prompt to their second lesson (Rachel, "Could you use pictures of cookies?" (RL2); Beth, "Could you skip count?" (BL2)). Rachel altered the *Cookie problem* wording to "Everyone (in our class) got the same number of cookies. How many cookies may there have been?", potentially changing the problem by indicating the number of students is the number in the class. Despite this, Rachel implemented the problem as intended by directing students to choose the number of students for the problem.

For the *Coin problem*, teachers selected from two student recording sheets, one with visuals (i.e. showing coins) and one without. Both teachers chose visuals, suggesting this scaffolding was perceived to be helpful. Enabling/extending prompts could be provided to students verbally or through individual prompts printed on paper strips; Rachel chose verbal prompts only, while Beth used both types.

Methodology

The study was a qualitative case study of two teachers' implementation of a problem-solving intervention; Yin (2015) noted case study enabled in-depth understanding of the complexity of a phenomenon in its real-life context. Corbett-Whittier and Hamilton (2013) support use of case studies in educational settings to allow for collection of context-specific and meaningful data, suggesting educational contexts are too complex for widespread generalisations due to diversity of settings, individuals and pedagogies.

This study used a case study approach to gain rich data about the two teachers' implementation of the problem-solving intervention. Teachers' implementation of the problem-solving intervention was the phenomenon of interest with a focus on their perspectives and reflections. A range of research instruments were used to gain multiple perspectives on the intervention implementation.

Lesson observation notes about teachers and students' responses to each lesson were recorded and students' work samples were collected and photographed by the first-named researcher at the end of each lesson. Two semi-structured interviews were conducted with each teacher, before and after the lessons. Interview 1 focussed on teachers' views on (and previous experience with) problem-solving and introduced the *Coin problem.* Possible student solutions were discussed to assist teachers in anticipating students' responses (i.e. feature of intervention-WCP). Interview 2 focussed on each teacher's reflections on their two lessons. Photos of students' work samples were available for teachers to refer to when discussing students' responses to the lessons.

Data sets (i.e. interview transcripts, student work samples, lesson observation notes) were examined using content analysis. Weber (1990) noted content analysis as systematically and objectively making valid inferences by examining texts that represent social communication. To implement content analysis, coding was used. Data sets were read three times each to familiarise the researcher with their content, followed by a combination of deductive and inductive coding. Features of the intervention identified in the problem-solving literature (Table 1) guided the deductive analysis to ensure key features of the intervention were the focus of the analysis. Inductive coding was used to reduce bias and ensure that new insights that arose from the data were identified. Data sets were revisited multiple times to check that findings correctly reflected the data.

Deductive Coding

Data sets were coded based on words, phrases and portions of the data related to one or more of the problem-solving intervention features in Table 3. Codes were combined and findings were drawn from these combined codes. Literature was revisited multiple times throughout this process.

Table 3 Features of problem-solving interventions identified from literature

Problem-solving intervention features
1. Provide a challenging problem (Sullivan et al., 2016)
2. Introduce challenging problem (Sullivan et al., 2016)
3. Struggle time (Ingram et al., 2016)
4. Problem-solving strategy recording (Sullivan et al., 2016)
5. Problem-solving strategy sharing (Sullivan et al., 2016)
6. Teachers anticipate student responses (Sullivan et al., 2016) (only in lesson structure-WCP)
7. Extending and enabling problem-solving prompts (Cheeseman et al., 2017)
8. General problem-solving prompts
9. Problem-specific prompts
10. Whole class prompts (only in lesson structure-WCP)

Table 4 shows comments from Rachel relating to *struggle time*. Several codes were combined related to "Teaching strategies Rachel uses to reduce the amount of struggle for students". For example, Rachel noted she would "model an example" (R2040) if she was to teach the lesson again, which inherently suggested a reduction in *struggle time* (Feature 3) as students would be provided with a successful problem-solving strategy by the teacher. This comment provides an example where the teacher did not use the term *struggle time*, but this could be inferred. Additional codes include those where Rachel reported prompting students (e.g. by identifying links with prior knowledge) where the intervention suggested students work without teacher support.

Inductive Coding

Inductive coding was based on Krippendoff's (2004) six components of content analysis where meaning is developed from data through the process of considering each component and revisiting components as the researcher reviews and rereads the text; there is an expectation that components be reviewed considering new learning from data. In Table 5, each separate component is described, including the role of the researcher. Following the deductive and inductive coding, five themes were identified.

Table 4 Sample comments relating to struggle: Rachel (interviews)

Teaching strategies Rachel used to reduce the amount of struggle for students:
• "Even just discuss with the kids, you know, how many people they could have…" (R2040)
• "Um, so tryna [trying to] create those links between what we already know so they're not quite stumped and stuck on what they think they don't understand and then practice." (R1116)
• "Model an example" (R2040)
• "So, I was meant to roam without intervening in the first fifteen minutes for their exploration but there were a few kids who I could see were just weren't gonna [going to] have a go so I jumped in before I should have." (R2022)
• "This question matches up quite well with, ah the fact that we're doing multiplication… they're using strategies that we're using in class at the moment and it really suits the question" (R2010)

Table 5 Krippendorff's components of content analysis including researcher's procedure

Krippendorff's content analysis component	Description of Krippendorff's content analysis component	What the researcher did
Unitising	Determining sections of the data sets that are of interest to the analysis	Familiarised self with the content of each of the data sets and documented interesting points (e.g. It appears that both teachers used a range of general and problem-specific prompts in their respective lessons) which related to the research questions
Sampling	Focussing on examples of sections of the data that are of interest and represent a range of other sections	Continued to read sections of the data sets to familiarise self with content and documented interesting points which related to the research questions
Recording coding	Coding relevant words and phrases from the data sets	Found words, phrases and portions of the results that linked to the interesting points that related to the research questions
Reducing data to manageable representations	Summarising data	Linked codes with similar meaning or that related to the same concept
Abductively inferring contextual phenomena	Analysing	Drew inferences to create findings
Narrating the answer to the research question	Recording findings	Recorded the findings in the results

Results and Discussion: Problem-Solving

Teacher deviations from the intervention mainly centred on students' abilities to understand the requirements of a problem, develop problem-solving strategies and develop appreciation of efficiency of approaches through class discussion of strategies. Five themes were identified related to problem-solving: *struggle*, *teachers anticipate student responses*, *prompts*, *student share time* and *problem-solving practice: teachers.*

Theme One: Struggle

Time for students to *struggle* with a problem was an intervention feature included in each lesson plan, both during the problem introduction, where teachers were asked not to model approaches for solving, and for the first 15 min of independent student work. In interviews, both teachers identified inclusion of *struggle* time as problematic for some students. The following sections explain teachers' views on *struggle* and provide insight into students' responses to *struggle*.

Prior to teaching, neither teacher identified that *struggle* would support students' problem-solving. Despite having discussed the intervention with the researcher, thus being aware that *struggle* was an intervention feature, Rachel did not mention *struggle* in interview one. Beth discussed student *struggle* as a negative consequence of lessons that were too fast paced and focussed on students with good understanding of mathematics, noting "… I think that's the problem in maths, that a lot of teachers just move onto the next [i.e., mathematical topic, problem, or lesson] and the next. And this kid, you can see… Losing their confidence. Losing their enjoyment…" (B1072–B1074). Student *struggle* was believed to contribute to loss of student confidence and enjoyment in mathematics, so something to be minimised. Beth identified two teaching strategies to reduce her students' *struggle*, both of which scaffolded students:

- Providing students with a random problem (e.g. $2+2$) that they can confidently answer when they have difficulty with a problem
- Modelling an example to provide a strategy for solving the problem.

Both can be good teaching strategies; however, the point at which they are implemented will impact the extent to which students have a chance to *struggle* and the problem-solving nature of a problem.

Although the intervention purposefully included *struggle* in the initial stages of problem-solving, both teachers deviated to reduce the *struggle* experienced by students. Reducing the amount of struggle in lessons was consistent with interview responses, where the benefits of *struggle* for supporting students' problem-solving were not identified. Although Rachel reduced the suggested *struggle* time, she did include some struggle time and the observed benefits of this in students' work are discussed in the next section. Table 6 summarises foci of the intervention, teacher deviations and the impact of deviations.

Struggle Promoted a Range of Problem-Solving Strategies

Beth modelled successful problem-solving strategies (number sentences; look for a pattern) in both lessons, contrasting with Rachel's approach which incorporated some *struggle* time and students had to find and use their own strategies. Teacher modelling of strategies and excluding specific *struggle* time resulted in fewer problem-solving strategies used by Beth's students; most used the strategy demonstrated. Table 7 shows that for the *Coin problem* (RL1/BL1) Beth's students used the two strategies demonstrated (i.e. 2 and 3), while Rachel's students used one more strategy (i.e. 1, 2 and 3). The largest number of strategies (6) was used by Rachel's students for the *Cookie problem* (RL2). Rachel prompted

Table 6 Summary of the deviations made by the two teachers in all lessons based on the intervention (adapted from Stewart (2020))

Intervention focus	Rachel – Lesson 1 (*Coin* problem-WCP)		Rachel – Lesson 2 (*Cookie* problem-IP)		Beth – Lesson 1 (*Coin* problem-WCP)		Beth – Lesson 2 (*Over-and-Over* problem-IP)	
	Deviation	Impact	Deviation	Impact	Deviation	Impact	Deviation	Impact
Ensure students understand the problem, without providing approaches for solving	Assumptions behind problem not explicitly stated	Students may be unclear about the goal of the problem	Adjusted problem initially which limited potential for problem-solving strategies. In response to students' uncertainty, restated original problem	Students may be unclear about the goal of the problem	Assumptions behind problem not explicitly stated Modelled problem approach	Students may be unclear about the goal of the problem Students may copy teacher's approach	Assumptions behind problem not explicitly stated Modelled problem approach	Students may be unclear about the goal of the problem Students may copy teacher's approach
Monitor students as they attempt the problem to determine who needs enabling and extending prompts	Prompted students too early, rather than providing time for *struggle*	Limited ability to capitalise on *struggle* time to develop problem-solving skills	Prompted students too early, rather than provide time for *struggle*	Limited ability to capitalise on *struggle* time to develop problem-solving skills	Prompted students too early, rather than providing time for *struggle*	Limited ability to capitalise on *struggle* time to develop problem-solving skills	Prompted students too early, rather than provide time for *struggle*	Limited ability to capitalise on *struggle* time to develop problem-solving skills
Introduce whole class prompts 1 and 2	Whole class prompts not used	Limited opportunities for students to understand the scope of the problem	NA	NA	Whole class prompts not used	Limited opportunities for students to understand the scope of the problem	NA	NA

Table 6 (continued)

Intervention focus	Rachel – Lesson 1 (*Coin* problem-WCP)		Rachel – Lesson 2 (*Cookie* problem-IP)		Beth – Lesson 1 (*Coin* problem-WCP)		Beth – Lesson 2 (*Over-and-Over* problem-IP)	
	Deviation	Impact	Deviation	Impact	Deviation	Impact	Deviation	Impact
Provide enabling and extending prompts where necessary	No deviation	NA	Focused on share time in the middle of the lesson, with limited prompting used	Students learn to apply strategies from their peers, rather than respond to prompts and develop their own strategies	No deviation	N/A	No deviation	N/A
Identify students with efficient problem-solving strategies to share their approaches with the class	Teacher invited all students to share strategies, not just those with efficient strategies	Class discussion did not focus on efficient strategies	Teacher invited all students to share strategies, not just those with efficient strategies	Class discussion did not focus on efficient strategies	Teacher invited all students to share strategies, not just those with efficient strategies	Class discussion did not focus on efficient strategies	Teacher invited all students to share strategies, not just those with efficient strategies	Class discussion did not focus on efficient strategies

Table 7 Frequency of problem-solving strategies in lessons (adapted from Stewart (2020))

Problem-solving strategies	Frequency of problem-solving strategies			
	RL1 ($n=7$)	RL2 ($n=8$)	BL1 ($n=7$)	BL2 ($n=7$)
Draw a picture	4 (#1, #2, #4, #5)	6 (#2, #3, #4, #6, #7, #8)	0	4 (#1, #2, #6, #7)
Number sentences	4 (#3, #5, #6, #7)	1 (#5)	7 (All)	7 (All)
Look for a pattern	6 (#1, #2, #3, #5, #6, #7)	2 (#1, #3)	7 (All)	0
Write a story	0	5 (#1, #2, #3, #5, #6)	0	0
Highlighting key word	0	1 (#5)	0	0
Working out an answer in more than one way	0	2 (#2, #5)	0	1 (#6)
Total number of strategies used in the lesson	3	6	2	3

#Student

Fig. 3 Beth's number sentence visual for introduction to *Coin problem* (BL1) (Stewart, 2020)

1. ________ + ________ = 50c
2. ________ + ________ + ________ = 50c

some students early in this lesson, noting that she "definitely deviated [i.e., from lesson] and sort of pushed, nudged them [i.e., the students] in the right direction" as she observed students were not attempting the problem. Although struggle time was reduced through providing a prompt, Rachel did not model a strategy and included some dedicated *struggle* time. A greater range of strategies were demonstrated in lessons without teacher modelling and where class *struggle time* was included, so these intervention features supported students in choosing and using strategies.

Impact of Reducing Struggle

Neither Beth nor Rachel included the recommended *struggle* time (Tables 4 and 6), prompting students earlier than suggested. When introducing the *Coin problem* (BL1), Beth modelled the strategy of using a number sentence (Fig. 3). However, the examples may have detracted from consideration of the real-life context constrained by Australian currency (5c, 10c, 20c, 50c); the first number sentence is impossible. In this case, teacher modelling, intending to reduce *struggle*, may have hindered students' problem-solving as it focussed on a numerical problem of adding two numbers to give 50, rather than consider the real-life constraints of adding coins to give 50c.

Beth identified the impact of some of her choices to reduce struggle. In introducing *Over-and-Over* (BL2), Beth focussed on the words "same" and "over" to reach "16"; some students focussed on the number 16 and added a range of numbers, rather than the same number, to equal 16 (e.g. student 2 in Fig. 4).

"We talked about the key words being the "number", "sixteen". We talked about repeated… the "same number" … really reinforced at the start of the lesson and some students … They've been focussed on one element of that… like the "sixteen" being the important thing but the "repeated" being not so important" (B2099).

Not providing time for students to struggle with the problem prior to class discussion resulted in some students not understanding the problem. This might be counterintuitive for teachers who

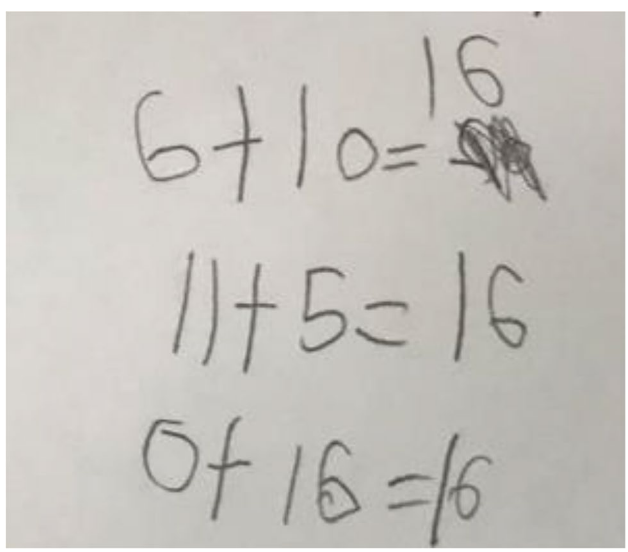

Fig. 4 Incorrect solutions as numbers not repeated: student 2 (BL2)

view struggle as negative, rather than supporting students to develop understanding of a problem, to support problem-solving.

Although Rachel included *struggle* time at the start of lessons, she provided enabling prompts earlier than suggested in both lessons (Tables 4 and 6). Some students' negative response to *struggle* at the start of lessons impacted her decision to reduce *struggle* ("I could see frustration from a lot of kids especially in the first ten minutes", R2042). Individual *struggle* was also reduced through students observing successful strategies used by peers; Rachel noted "when they saw another kid doing a strategy… they had that light bulb [moment] where they went "Oh, they're doing 'groups of' so I might draw groups or I'll do an array…."" (R2010). Rachel credited this for some students' success in problem-solving. It is not only the teacher who determines *struggle* time, particularly when students work in groups. An intervention feature was that students have individual *struggle* time, prior to discussion of strategies with their teacher or peers. If the intention of problem-solving is purely finding a correct answer, then observing and applying a successful strategy would result in success. However, given the imperative in the curriculum for developing "capable problem-solvers" and the importance of the problem-solving process (e.g. Pólya, 1945), students must be able to choose and use strategies, as well as find a correct answer.

Understanding the goal of struggle time did not always result in inclusion of *struggle* time in teaching. Rachel acknowledged students were "meant to be in a state of frustration and confusion" (R2042) during *struggle* time, however suggested that reduced *struggle* time could foster students' perseverance with problems and maintain engagement, thus avoiding students' "I don't understand it so I'm not gonna do it attitude" (R2042). This suggested that unless students could solve problems relatively quickly, they would disengage with mathematics. There was inherent *struggle* in the *Coin problem* due to the need for students to consider calculations with money, a topic yet to be taught. Rachel noted student 3 crossed out her working and did not persevere "…she completely gave up and refused almost to participate because she didn't understand…" (R2064–R2066). Although Rachel attributed student 3's inability to solve the problem to lack of understanding of money, the student recorded three correct responses, namely, 5 10's is 50, 20c + 20c + 10c is 50c and 20c + 10c + 10c + 10 = 50c (Fig. 5), providing evidence of use of a successful strategy to partially solve the problem.

Rachel suggested her extension students, with good mathematical understanding, did not respond positively to *struggle* as "they couldn't figure out how to approach this [i.e., the problem]" (R2004).

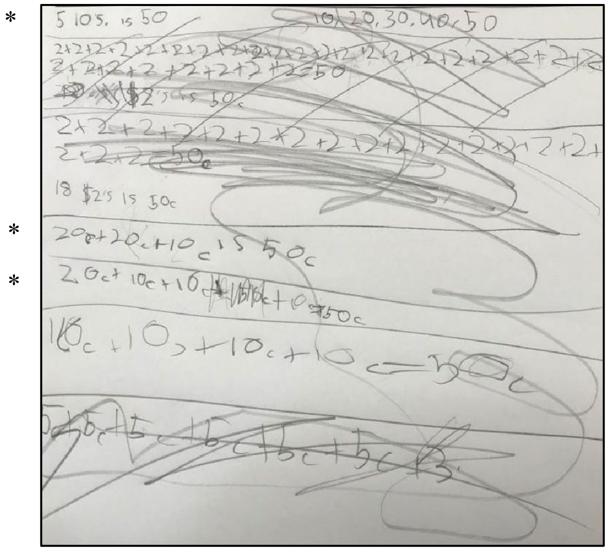

Fig. 5 Three correct solutions for *Coin problem* (*)—(RL1) (adapted from Stewart (2020))

These students, who may be used to solving problems quickly, may be perturbed by having to *struggle* and not finding a correct answer immediately. In contrast, students who find mathematics difficult were expected to be inclined to persist with problems as they were "…used to maybe failing or not doing as well" (R2002), so anticipating *struggle* in mathematics. Due to perceived student resilience issues during the *Coin problem*, Rachel indicated that if she taught the lesson again she would support students when introducing the problem by "model(ling) an example" (R2040) or having a "discuss(ion) with the kids" (R2040). In this case, reducing struggle is aligned with maintaining students' perseverance with problems. Although both teachers wanted to reduce struggle for students with good understanding and those who found mathematics difficult, a potential benefit of providing the opportunity to *struggle* might be that students are more willing to *struggle* as it becomes a normal part of problem-solving.

Overall, *struggle* time was not recognised as contributing to students' problem-solving skills by Beth or Rachel, but instead negatively impacting students' resilience and confidence in mathematics; hence, both teachers deviated to reduce the amount of *struggle* their students experienced. For teachers used to scaffolding students' learning, the notion of *struggle* may be counterintuitive, as it is related to inability to solve a problem rather than thinking time to engage with a problem and consider strategies. Cheeseman (2018) also found many year 1 teachers did not see the benefit of students *struggling* with problems. One conclusion could be that teachers do not want students in the early years of primary school to *struggle* in mathematics, to remain positive about learning mathematics. Although our study was limited to two teachers and their classes, the increased range of problem-solving strategies used by the class where struggle was included suggests that struggle time could be an important consideration for teachers to support students' problem-solving. If problem-solving includes having opportunities to select solution methods and use higher-order thinking (Schoenfeld, 1992), particularly in the entry phase where problems are introduced and students decide on strategies for solving (Mason, et al., 1985), then struggle time is important. Our findings suggest that some teachers may find dedicated *struggle* time difficult to implement as they perceive it to be negative and to result in less student confidence and resilience in mathematics.

Theme 2: Teachers Anticipate Student Responses

Two different implementations of the intervention (i.e. teachers either did or did not anticipate students' responses prior to teaching) provided insight into whether a teacher's anticipation of students' responses supported teachers in promoting effective problem-solving by students.

In the first lesson (RL1 and BL1, intervention-WCP), each teacher anticipated students' responses to the *Coin problem* prior to teaching and there were few misunderstandings identified in students' work (Table 8), highlighting the efficacy of this approach. This builds on Sullivan et al. (2016) who found that year 3 and 4 teachers were better prepared to support students if they anticipated students' possible responses before a problem was implemented. The current study, with year 1 and 2 students, suggests this approach might also be beneficial for supporting problem-solving of younger children.

In the second lessons (*Over-and-Over*, BL2; *Cookie*, RL2), teachers did not anticipate students' responses prior to teaching (intervention-IP). Many of Beth's students produced some incorrect responses (Table 8), suggesting there were misunderstandings about the goal of the problem. The teacher introduction, where Beth modelled a strategy (Table 5 and 6), may have contributed, as students largely replicated her approach rather than decide on a strategy to solve. Beth did not discuss the assumption behind the problem (i.e. same number needed to be added to make 16); the need for this discussion may have been apparent if students' responses had been anticipated. Rachel's students did not demonstrate any misunderstanding of the problem in RL2 which may be due to the nature of *Cookie* (i.e. open-ended and simple wording). It seemed that for straightforward problems the students could understand the problem and work towards a solution; however, where the problem was more complex (e.g. BL2), then possibly

Table 8 Problem misunderstandings identified in student work samples for all lessons based on the intervention (adapted from Stewart (2020))

Teacher and lesson number	Code	Problem	Students' misunderstanding	Number of students' work samples with each misunderstanding	Number of student work samples that included one or more of the misunderstandings
Rachel, lesson 1	RL1	*Coin problem*: How many ways can you make 50 cents using coins?	Included coin worth zero cents Included coins that do not exist Included two cent coins Drew coins for no reason	[2 (out of 7) (student #4, #6)] [2 (out of 7) (#3, #7)] [1 (out of 7) (#7)] [1 (out of 7) (#4)]	4 (out of 7) (student #3, #4, #6 and #7)
Rachel, lesson 2	RL2	Everyone got the same number of cookies. How many cookies may there have been?	None	None	None
Beth, lesson 1	BL1	*Coin problem*: How many ways can you make 50 cents using coins?	Included coin worth zero cents	5 (out of 7) (#1, #3, #4, #5, #6)	5 (out of 7) (#1, #3, #4, #5, #6)
			Included coins that do not exist	1 (out of 7) (#6)	
Beth, lesson 2	BL2	*Over-and-Over problem*: I added the same number over and over until I reached 16. What number may I have started with (BL2)?	Added a range of numbers to equal sixteen	6 (out of 7) (#1, #2, #4, #5, #6, #7)	6 (out of 7) (#1, #2, #4, #5, #6, #7)
In all four lessons		N/A	N/A	18	15 (out of 21)

having the teacher anticipate students' answers might have highlighted potential student difficulties and helped the teacher to orchestrate the lesson to support students' problem-solving.

Theme 3: Prompts

The range of prompts enabled teacher choice, with teachers including some provided in the lessons and additional prompts added in-the-moment while teaching (Table 9). Rachel used more prompts than Beth across both lessons (14, 22; 10, 9). Teacher use of a range of prompts suggested they were useful in the problem-solving lessons. Neither teacher used all provided prompts, indicating that teachers made decisions about appropriateness. Table 10 shows the *Coin* prompts, including those provided in the lesson but not used by either teacher. Neither teacher used general enabling prompts to foster an approach for solving (e.g. "Could you make a list?"). However, they utilised general enabling prompts that encouraged students to read and understand the problem (e.g. Can you think of an easier question like this?).

Both teachers provided a rationale for use of a range of prompts, with decisions focussed on scaffolding students. Rachel identified that specific and general prompts enabled her to scaffold students' solution of problems, rather than model one given approach for solving or direct students to a particular strategy; choice of problem-solving strategy remained with the students. Beth noted an enabling prompt ("What if the number you reached was 6?") to scaffold a student having difficulty with *Over-and-Over*, reducing the size of the number (i.e. 6 cf 16) to create a simpler, but related problem. This supported the student "to realise that she could do more for the sixteen" (B2083) and that additional solutions existed for the original problem. Prompts provided suggested phrases to support problem-solving with Rachel noting they "were fantastic because there were some kids that I didn't want to lead directly but I didn't know exactly how to tell them where to go without giving them an answer" (R2012). In this case, prompts provided Rachel with pedagogical support by providing phrases that scaffolded students' problem-solving. Cheeseman et al. (2017) noted the importance of extending and enabling prompts for supporting students' problem-solving and our study highlights that prompts can also support teaching of problem-solving, by providing teachers with phrases to prompt students in the problem-solving process. Teachers' repertoires of enabling and extending prompts for problem-solving were enhanced through the intervention.

Rachel kept prompts "general" to minimise the scaffolding provided to students, so they had to determine solution strategies, contrasting with the approach of modelling strategies. Rachel's use of "general" was different to the categorisation of general prompts in the intervention; for Rachel, "general" prompts referred to questions that promoted student reflection and she noted "I tried to keep it really general. I tried to keep a lot of my conversation in questions, so it was pushing them to do the thinking" (R2050). Rachel identified affordances of prompts, particularly the benefit for students in determining strategies, for example, "Could you change the number of cookies each person gets?" (R2044) was identified as effective for encouraging consideration of the full extent of the *Cookie problem*, rather than assume one cookie per person (i.e. approach used by most of her students initially).

Table 9 Summary table of general and problem-specific prompts identified in all lesson observation data

	Number of available prompts	Number of prompts used by teachers	Additional prompts added by teacher	Number of prompts not used
BL1	29	4	6	25
RL1	29	6	8	23
BL2	25	6	3	19
RL2	31	10	12	21

Table 10 Prompts in *Coin problem* lessons (adapted from Stewart (2020))

Type	Rachel	*n*	Beth	*n*	Prompts that were not used by either teacher
General enabling	*Reminding students of the question	1	*Teacher underlined key words in question	1	Could you use a table?
	*Reminded about reading the key words	2	* "Mmm think a bit more."	1	Could pictures help?
			* "Did you finish that one?"	1	Could you think of an easier question like this?
					Can you think of a problem like this?
					Could you have a guess?
					Could you make a list?
					Could you act it out?
					Could you start with the answer and work backwards?
					Could you draw a picture?
					Could materials help?
General extending	"Could you try it another way?"	10	"Are there any more ways?"	6	Are there more answers?
					How do you know?
					How could someone find all of the answers?
					What patterns can you see?
					How would you teach someone how to solve this problem?
Problem-specific enabling	* "Can you skip count?"	1	* "Don't get gold coins."	1	Could cut and paste pictures of gold coins help?
	* "Could you use a 100 s chart?"	1	Teacher handed out coins	1	Could you use a strategy to work out how many ways you can make 10c or 20c?
	* "Could you count by another number?"	1	* "Do you need 6 10 s?"	1	
	* "Can you add those for me?"	1	*Asked a student who was making 20c to make 50c	1	
	* "Do we have 90 cent coins?"	1	"What could you count by?"	1	
	"Use the same strategy with a different coin."	1	"Do you know how to skip count?"	2	
	"That's $50. Try and use your strategy to make 50 cents."	1			
	* "Let's count on from 10."	1			
	"Could you use 10 s?"	1			
	"Use the coins."	1			

Recognising affordances of prompts will assist teachers in choosing appropriate prompts to scaffold or extend students at different stages of the problem-solving process. A challenge for teachers is identifying when specific prompts are most effective, particularly when presented with several enabling and extending prompts and selecting prompts in-the-moment in response to students' discussion, work or questions in class. Effectiveness of prompts could relate to helping students understand a problem, choose a strategy and reflect on a solution or the capacity to extend students through an additional challenge. Both teachers identified the purposes of prompts and reflected on their use of prompts, which suggested they were also assessing the effectiveness of prompts to achieve goals, such as scaffolding students.

Timing of prompts can impact the extent of scaffolding students receive at each stage of a problem-solving lesson, or the extent to which students are extended beyond the scope of the original problem (Sullivan et al., 2016). Timing is inextricably linked to selection of prompts, as to have a well-timed prompt a teacher needs to select an appropriate prompt that helps students move forward with their problem-solving; it can also relate to a decision not to use a prompt at a particular point in a lesson. Both teachers provided some prompts earlier than suggested, deviating from the intervention to scaffold students rather than allow students the *struggle* time suggested. Two possible reasons for this could be that the planned *struggle* time was too long, and teachers noted it was not productive, or else teachers wanted students to solve problems relatively quickly and did not see benefit in students spending time struggling.

Enabling prompts support students who either have misunderstood a problem or are not making progress. Strategic use of enabling prompts was demonstrated when Rachel was observed prompting students who were unclear about the *Cookie* problem to reread the question. There were also instances where selection of prompts did not support students who misunderstood a problem. Beth prompted students to draw pictures for *Over-and-Over*, which can be a useful strategy for many problems, but not helpful for supporting understanding of this problem; the given prompts in the intervention targeted the goal of the problem (e.g. Have you added the same number over and over to reach 16?). Timing of extending prompts was also important, as providing such a prompt too early can result in students focussing on the extension problem, without answering the original problem. Figure 6 shows an example where Beth gave an extending prompt (i.e. to increase the total) to student 6, encouraging consideration of an extension problem before completing the original problem. Prior to this, the student had found six (out of the 13) correct responses (i.e. numbers 1–6). Once the new total (i.e. $1) was provided, the student did not complete the original problem. Some students prompted in ways that did not appear to target their misconceptions or guide them to fully answer problems; in the case of student 6, the prompt did not address the incorrect thinking behind responses 7 and 8. Selection and timing of prompts here impacted the extent to which the student engaged successfully in the problem-solving lesson.

Theme 4: Student Share Time

The intervention included 10 min share time at the end of each lesson to foster both sharing of problem-solving strategies and discussion of the effectiveness and efficacy of these strategies. Neither teacher included this full 10 min of share time; however, Rachel facilitated an additional share time in the middle of the second lesson. Neither teacher asked students who used effective strategies to share their problem-solving strategies in the share time sections of their lessons; thus, the opportunity for students to learn about effective strategies from peers was missed.

Both teachers commented on the need for more time for discussion and reflection on problem-solving strategies at the end of their lessons. Beth noted the need for more share time ("I wish we'd done it a little bit more", B2054) and Rachel noted her "end of lesson sharing was a bit too short" (R2082) and that "next time I'd probably give myself more time" (R2082). This highlights a tension in teaching between allowing students time to explore strategies and allocating sufficient time for class discussion of the efficacy of different strategies. After Rachel's additional share time in the middle of RL2 (see Table 4 and 6), her

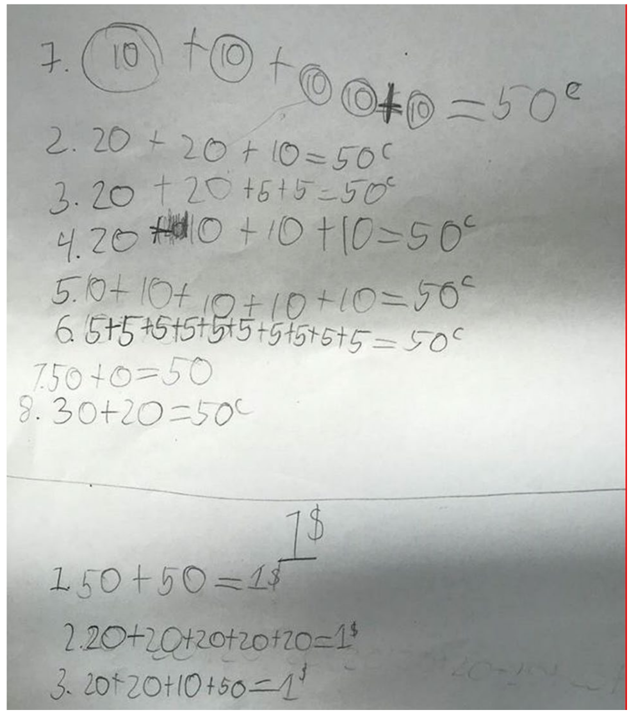

Fig. 6 Six correct (1–6) and two incorrect (8, 9) responses for original total and three correct responses for new total: student 6 (BL1)

students were observed being encouraged to apply strategies observed from peers ("If you're stuck please stay on the floor [i.e., for further group discussion]. If not go back to your table and find another way. Think about the strategies people shared and use one that might work for you."). Rachel reflected on the success of encouraging students to apply strategies observed during mid-lesson share time and suggested that "the kids that maybe didn't have as many strategies or many ways to approach it could sort of steal ideas from people who were sharing" (R2080), so the perceived benefit was that students could learn successful strategies from their peers. This deviation from the intervention appeared to have a positive impact on Rachel's students' problem-solving in the lesson.

Neither teacher reported on why they did not choose specific students to share their problem-solving strategies during share time. To ensure that share time encourages students to reflect upon their choice of problem-solving strategies, having a range of students share their approaches and discuss their effectiveness is imperative. It is unknown whether teachers did not see benefits in choosing specific students or whether this was due to perceived time constraints.

Although both teachers reported that share time at the end of their lessons was too short, their reflections highlighted that they would increase share time in future problem-solving lessons. Ingram et al. (2016) highlighted the role of student sharing and collaboration in successful problem-solving and both teachers recognised that they needed to allow more time for sharing. The intervention made share time explicit; this may bring the need for inclusion of share time to the attention of teachers.

Theme 5: Problem-Solving Practice: Teachers

Both Rachel and Beth indicated that involvement in the study had positively impacted their teaching of problem-solving. The specific impact Beth mentioned was that she would encourage students to use more manipulatives and pictures in her problem-solving lessons in the future to make mathematics more concrete for her students. “When you’ve got the manipulatives or when you’ve got the students drawing it, they can see it. It’s something concrete it’s not just… a blur of numbers” (B2341).

Rachel identified positive aspects of the intervention, including students being flexible in their approaches to solving, noting one student who “modelled her thinking in… three different ways and was able to articulate it … fantastic” (R2070). She also appreciated the extending and enabling prompts which helped her to guide her students’ thinking. Rachel noted that she would implement lessons based on the intervention in the future; one such lesson, focussed on multiplication, had already been planned. For this multiplication lesson, Rachel planned to reduce the amount of *struggle* suggested in the intervention by giving students a problem with different totals depending on whether students’ mathematical understanding was categorised as low (total of 10), middle (total of 30) or high (total of 50 or 100). Not expecting all students to grapple with the more difficult total (i.e. 50 or 100) may impact the potential for students to develop problem-solving skills.

Teachers’ comments suggested that the intervention had a positive impact on their teaching of problem-solving, in particular the availability of prompts. Including specific features in the problem-solving intervention and naming them could foster teacher recognition that these features are important considerations.

Conclusion

Both teachers were able to implement the intervention, including enabling and extending prompts in their lessons. The teachers gave few comments about whether the intervention improved their students’ problem-solving abilities, but they both noted challenges experienced by students, predominantly related to *struggle*. Despite this, the study showed that there is potential for students to demonstrate a wider range of problem-solving strategies when provided with *struggle* time, compared to the situation where they are shown an approach for solving by the teacher or another student. Although Sullivan et al. (2014) found that most students in their study preferred to struggle with problems and solve difficult problems, the teachers in our study wanted to reduce students’ struggle. Their rationale for reducing struggle was mainly focussed on maintaining students’ confidence and resilience in tackling challenging problems. This tension between students’ expectation for challenging problems (and implicit expectation of struggle) and teachers’ desire to reduce struggle may result in problems being set that are not challenging enough for all students. McCormick (2016) noted the need for cognitively demanding problems, but if teachers view students’ struggle as negative then approaches to reduce struggle, for example by modelling a strategy, may reduce the cognitive demand of problems.

S*truggle* time was an integral feature of the intervention and teacher deviation to reduce struggle can impact the efficacy of the intervention. *Struggle* was intended to be positive, to promote development of persistence with problem-solving and to support individuals to consider strategies that might be used to solve problems where the answer is not evident immediately. *Struggle* time was not anticipated to be reduced; hence, for future implementation of the intervention, it would be beneficial to include an explicit rationale for *struggle time* to encourage teachers not to reduce this.

The intervention included numerous enabling and extending prompts, but teachers made selections as to which ones they included. Having a range of prompts enabled teacher choice to target the scaffolding for individuals and to extend students. However, it may be beneficial to provide a limited number of general enabling or extending prompts so that students can develop familiarity with these prompts over time and teachers could encourage students to ask themselves these prompts, thus facilitating problem-solving.

The fact that the teachers did not always implement the intervention as intended in the lessons could be viewed in a positive light. It implies that the two teachers were using their professional judgement about effective problem-solving practices during the lessons. Roche et al. (2013) identified the importance of struggle time for improving students’ problem-solving. Sharing in the middle of problem-solving lessons may help resolve the tension between allowing student struggle and providing support when teaching problem-solving.

Both teachers reflected on their teaching of problem-solving lessons through the experience of trialling lessons based on the intervention and planned to implement aspects of the intervention in the future. The teachers perceived that the intervention had an impact on their teaching of problem-solving, with share time in lessons identified as an aspect to increase in future problem-solving lessons. Given that Stacey et al. (2015) identified a perceived lack of problem-solving resources to support Australian teachers with problem-solving, the teachers' perception of the benefits of the intervention on their problem-solving practices provides promise for use of the intervention to support the teaching of problem-solving. Explicit statements about features of the intervention which should be included and those where teacher choice is expected, for example in selection of appropriate prompts, could support teachers in the implementation of problem-solving lessons.

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Declarations

Ethics Approval The study reported was part of the Master of Education research of the first-named author. The research had University Ethics approval and approval from Department of Education and Training, the school, teachers and parents/guardians of participating students.

Conflict of Interest The authors declare no competing interests.

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