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Blended Learning and Student-centered Active Learning Environment: a Case Study with STEM Undergraduate Students

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Abstract This paper describes an embedded case study of "blended" teaching integrated with traditional lessons in a Student-Centered Active Learning Environment and social activities on the platform. The didactic phenomena were designed by creating learning environments, artifacts, and teaching/learning sequences in authentic educational contexts. We aim at improving the task design of a mathematics lesson with an impact on students' performance in mathematics. Quantitative results show considerable benefits in the evolution of the use and coordination of several systems of semiotic representation. As a result, a better predisposition to the study of the subject seems to appear; moreover, the satisfaction test shows the achievement of alternative teaching methodologies for most of the students.

Résumé Cet article décrit une étude de cas intégrée d'enseignement «en mode hybride» combiné à des cours traditionnels dans un environnement d'apprentissage pratique centré sur l'étudiant ainsi que des activités sociales sur la plate-forme. Les phénomènes didactiques ont été conçus en créant des milieux d'apprentissage, des artefacts, et des séquences d'enseignement et d'apprentissage dans des contextes pédagogiques authentiques. Ainsi, il semble se développer une meilleure prédisposition à l'étude de la matière; de plus, le questionnaire mesurant le taux de satisfaction indique que les méthodes d'enseignement alternatives fonctionnent pour la plupart des étudiants. Nous visons à améliorer l'élaboration de la tâche associée à un cours de mathématiques de façon à avoir une incidence sur la performance des étudiants en mathématiques. Les résultats quantitatifs démontrent des avantages importants en ce qui concerne les progrès dans l'utilisation et la coordination de différents systèmes de représentation sémiotique.

Keywords Blended learning · Scaffolding · Peer-led team learning · Just-in-time teaching · Vygotskian perspective

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Introduction

Why do so many students have difficulties in mathematics if it is supported by the earliest form of intelligence we have? One of the difficulties in learning mathematics is the impossibility of conceptualization based on meanings referring to a concrete reality. On the one hand, every mathematical concept uses representations because there are no "objects" to exhibit; conceptualization needs to go through representative registers. According to Duval (1993), comprehension in mathematics presupposes the coordination of at least two registers of semiotic representation. Such coordination is not natural in students. These difficulties are found not only in students beginning in mathematics, as some research in mathematics education shows (D'Amore, 2000; Duval, 1993; Sbaragli & Santi, 2011). Some difficulties continue to be encountered in secondary school students and are also found in students attending the first year of STEM courses. Although they decide to attend a scientific faculty, some students do not have strong mathematical skills. Others do not have a "good relationship" with mathematics.

These studies start from some considerations on the obstacles students encountered during the first year of university, mainly in so-called service courses. In these courses, passing the mathematics exam is a sin to expiate rather than a resource for further studies. Some students have difficulties related to problems in mathematics (Zan, 2007) and epistemological obstacles (Brousseau, 1976), misconceptions established by inadequate teaching practices in the primary school (Sbaragli & Santi, 2011), and, in the upper secondary school, Functions, (Tall & Vinner, 1981), Infinity (Arrigo & D'Amore, 1999), Limits (Bagni, 1999), and Inequations (Bazzini & Tsamir, 2001). Also, many pieces of research have shown difficulties related to different uses of the language, such as the relationship between verbal and formal language (Ferrari, 2003). Other challenges are due to the linguistic specificity of the mathematical text (Branchetti & Viale, 2015; D'Amore, 2000), the use of different systems of semiotic representation (Duval, 1993) and gestures (Arzarello, 2006), and the formulation of the texts of the problems (Zan, 2012).

The research related to university teaching has been spreading over the last decade in Europe (a network of INDRUM researchers has been set up to deal only with this type of research); however, it is not very articulated in Italy. Only a small number of studies have been conducted in this direction. On the other hand, these studies are more widespread internationally, for example, in Australia (Palmer et al., 2015; Marginson et al., 2013) and the USA (Langen & Dekkers, 2005).

Nevertheless, the transition to university is still a hot topic in the mathematics education research community because, despite it being researched extensively and despite many attempts to address and alleviate these difficulties (bridging courses, support centers, etc.), problems persist (Kouvela et al., 2018). The highlighted difficulties acquire particular importance in heterogeneous classes, considered in educational planning. Moreover, the dominant form of thought of the students is the algorithmic one (Fandiño Pinilla, 2008), and the approach to problem-solving seems to be more based on the analogy with similar problems, where the students try to remember if they have already seen that type of problem. Therefore, the study of mathematics is often associated with memorizing techniques. So, by delegating to the algorithm form, the student renounces the "responsibility" of setting up a resolving process based on a conceptual approach to the discipline and often associates formalized procedures to the very nature of mathematics.

These phenomena are well known and widely studied by researchers in mathematics education, especially on the part of the French School that has been more concerned with studying the effects of the didactical contract and are called "formal delegation and the need for formal justification" (Brousseau, 1998). These considerations come from looking for new ways to reach a more effective teaching–learning process. In the USA, many teaching methodologies have been tested, trying to improve the mathematics teaching–learning process at the university level. In particular, just-in-time teaching (JiTT) and peer-led team learning (PLTL) have been implemented with positive effects on student performance (Novak, 2011; Preszler, 2009).

JiTT is a pedagogical approach that consists of taking advantage in the classroom of feedback from the activities that students carry out at home, intending to improve teaching effectiveness, optimize classroom time, and improve student motivation. PLTL is a model of teaching undergraduate science, math, and engineering courses that introduce peer-led workshops as an integral part of a course. Students who have done well in a class (for instance, General Chemistry) are recruited to become peer leaders.

Although the two methodologies have been applied effectively, methodological uniqueness does not always ensure learning success because not all students learn in the same way (D'Amore & Sbaragli, 2011; Weber et al., 2020).

Starting from these considerations, in this article, we describe a case study to see if JiTT and PLTL methodologies can be used together. We aim at improving the task design of a mathematics lesson with an impact on students' performance in mathematics, trying to foster a greater motivational and affective disposition towards the subject and an adequate instructional scaffolding to make the university student work in their proximal development zone. The experimentation is carried out with mechanical and management engineering students who attend Calculus 2 classes. This course was always conducted according to a traditional didactic approach: frontal lessons were alternated with written and oral tests at the end of the first semester, mainly assessing students' knowledge. We proposed experimentation with blended learning. JiTT and PLTL methodologies are integrated with the support of a social platform into a student-centered learning environment, i.e., a Student-Centered Active Learning Environment with Upside-down Pedagogies (SCALE-UP). We ask ourselves the research questions: integrating these teaching methods, is there an improvement in students' mathematical skills? Does it improve students' approach to studying the discipline, enhancing good motivational and affective dispositions?

The experimentation results seem to show an improvement in the performance of the experimental class and a considerable increase in students' interest and motivation. Also, many students, who initially had deficiencies on some topics, could achieve a positive result on the exam by working in the proximal development zone (Vygotsky, 1978) and with adequate scaffolding (Bruner, 1975).

This article is a prequel to other articles by the author about teaching mathematics in STEM courses and interdisciplinarity in mathematics education (Branchetti et al., 2018; Capone, 2022). Thanks to this preliminary, we have been able to compare how mathematics teaching in STEM courses changed before, during, and after the COVID pandemic (Branchetti et al. 2021; Capone & Lepore, 2020; Capone & Lepore, 2022). The paper is structured as follows. The "Teaching Methodologies" section will illustrate the teaching methodologies, methods applied, the social platform used, and the SCALE-UP learning environment, also regarded as the existing literature (related work). In the "Conceptual Framework" section, we will illustrate the theoretical background. In the "The Case Study" section, we will describe the case study. The "Data Analysis and Results" section will show the quantitative and qualitative analyses of the results obtained. Finally, the "Discussion and Conclusions" section will conclude and offer some future work ideas.

Teaching Methodologies

Teachers themselves are often looking for a methodology that can solve the difficulties of teaching–learning a subject. Many research pieces in pedagogy (Gallagher, 1994; Grossman et al., 2009, for example) have studied the teaching–learning processes and analyzed which strategic teaching actions to implement according to the concrete training situations the particular characteristics of the students. Also, in mathematics education, there are many types of research (Clements & Sarama, 2004; Davis, 2013; Steffe & Thompson, 2000), which underline the importance of the methodological choice to convey mathematical contents or to encourage good motivational and affective dispositions to the study of mathematics. In general, good teaching should promote the development of different and more autonomous learning processes, not only by transmission or reception but also by discovery, action, and problems.

On the other hand, the teacher should ensure a customizable educational program because not all students learn in the same way. Furthermore, s/he should promote and consolidate students' interest and motivation by creating a stimulating and innovative learning environment. D'Amore pointed out that sensational mistakes can be made by making the teaching choice fall on a single methodology. He classified these errors into pedagogical, epistemological, didactic, and semiotic errors (D'Amore, 2016). Using a unique method, one risks falling into what Raymond Duval (1993) calls the "cognitive paradox of learning": the student will identify the mathematical object with its representation (in the best case, if the methodological tool was successful). The abstract mathematical object will be unreachable for that student (Duval, 1993). Therefore, mathematical learning will not have taken place at all.

Thus, among the many active methodologies widespread in mathematics education, the choice fell on blended learning, including, next to the traditional lesson, the integration of JiTT and PLTL methodologies, taking advantage of technological innovations, and an adapted learning environment. The e-learning approach allowed students to relate continuously with the teacher and tutors and activate peer-education dynamics, thanks to a constant peer-to-peer debate between all the class group members (Capone et al. 2017). This research hypothesizes that a change in university mathematics teaching methodology can decisively influence learning, especially in degree courses. The discipline has a "service" role compared to other study fields. It allows us to act on the motivation and partially stem the difficulties due to a teaching contract that sees the student imitative and oriented to reproduction procedures.

In the authors' opinion, through collaboration with peers and students' direct involvement, also mediated by technology, it is possible to reactivate more significant and lasting learning processes. So, this study experiments with a hybrid teaching method that blended different teaching methods (Fig. 1), focusing on students and their epistemological needs.

In the following, we will describe, in detail, the JiTT, PLTL, Edmodo social platform, and SCALE-UP learning environment.

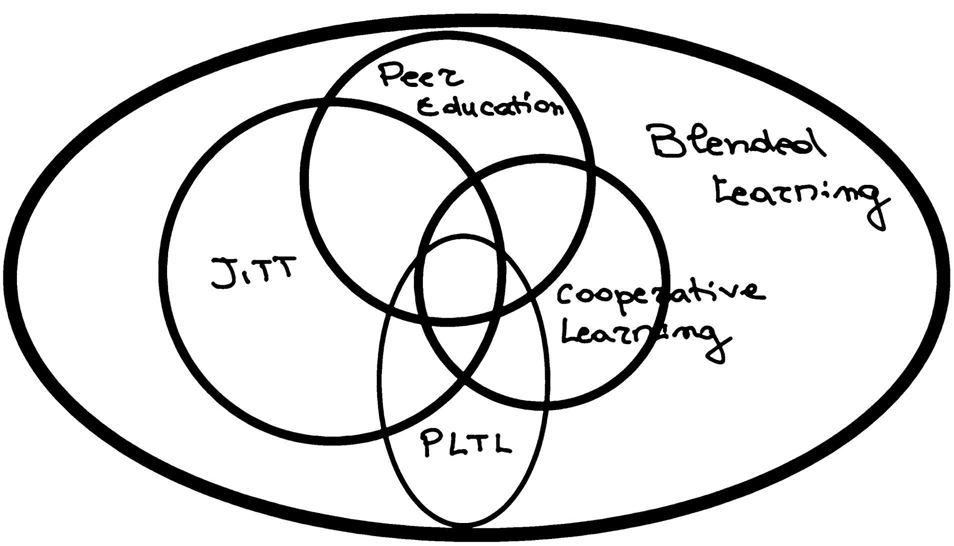

Fig. 1 Blended methodologies

Just-in-time Teaching

Just-in-time teaching is a pedagogical approach that consists of using in the classroom the feedback from the activities that students carry out at home to improve teaching effectiveness, optimize classroom time, and enhance student motivation. JiTT was first used in the late 1990s in Physics university courses in the USA, but its use has spread to many other academic disciplines. Following the methodological indications of JiTT, students are assigned some exercises, known as “Warmup exercises”, “Preflight checks”, or “Checkpoints” (Doyle, 1988), regarding the activities carried out in class. They are preparatory to the next lesson through an e-learning platform. The students work on the task at home and share it with each other by posting the exercises on the class’s page. The teacher can read students’ answers before the lesson to plan the classroom activities. S/he can start from the doubts, misconceptions, and difficulties that emerged from the discussion. This is one way to focus the university lesson on the student, which is rarely the case in daily teaching practice and promotes interactive learning. The following Table 1 clarifies this.

A JiTT teaching cycle can be outlined as follows:

JiTT’s activities also consider the motivational factors that influence students’ behavior. Theorists of motivational belief undertake the constructivist position that “the process of conceptual change is influenced by personal, motivational, social and historical processes, thus supporting a warm model of individual conceptual change” (Pintrich et al., 1993, p. 197). The research has shown that university students who report that their teaching material is more interesting, relevant, and valuable are more likely to use more in-depth processing strategies such as metacognitive processing and control strategies. “At the level of class and task, there are several features that could increase students’ situational interest—such as challenge, choice, novelty, fantasy, and surprise”. (Malone et al. 1987).

Teachers and students become a teaching–learning team, ready to begin the lesson with an awareness of the class’s mental status, making the learning experience as relevant as possible to a particular class at a specific time. What happens in the classroom consists of a mix of preplanned activities and creative improvisation suggested by student responses and guided by in-class reactions to thoughts and opinions in those responses (Novak, 2011).

All JiTT instructions are mediated by the teacher’s presence in synchronous or asynchronous mode. Web materials serve as a communication and organizational tool as a pedagogical resource. Since a large part of the students’ learning occurs outside the classroom, JiTT professionals see their pedagogical strategy as a feedback loop between teaching and learning and school and outside school experiences. Essential factors in students’ educational success are student–student interaction, i.e., peer-learning, student–teacher interaction, and active time: these three factors are immensely enhanced by technology. Web-based interaction prepares students and teachers for the subsequent classroom interaction, gives students some control over their learning, and enriches classroom social contacts.

Peer-led Team Education

Peer-led team learning (PLTL) is a model of teaching undergraduate science, math, and engineering courses that introduces peer-led workshops as an important part of a class. Students who have done well in a class (for instance, General Chemistry) are recruited to become peer leaders. Peer leaders meet with small groups of six to ten students each week for one to two hours to discuss, debate, and engage in problem-solving related to the course material.

PLTL can be understood in the context of cognitive science. It is consistent with social constructivism and Vygotsky’s ideas (1932) because students are asked to construct their understanding under a more capable peer’s supervision. They can be said to be learning within the zone of proximal development.

Table 1 JiTT teaching cycle

1	Teacher	Uploads the course material from the lesson and adds exercises to the e-learning platform as a stimulus for the next lesson
2	Student	Carries out the homework, compares himself with his peers by posting a possible solution for the exercises on the platform. Other students participate in the online discussion and express how the activities are carried out
3	Teacher	Examines the exercises carried out, paying particular attention to the errors that have emerged and to the misconceptions
4	Teacher	During the lesson, mentions items that emerged from the student's homework as brainstorming to start
5	Student	Through the exercises, self-assessed their training

PLTL initially involved chemistry classes and was tested in many US countries. The methodology has been proven successful in various STEM courses (Báez-Galib et al., 2005; Lewis & Lewis, 2005, for example). There are many examples of the application of the methodology in mathematics classes. For example, Liou-Mark et al. (2010) applied PLTL to PreCalculus courses, and they found that significant improvements in the performance of students attending the classes were achieved.

The methodology aims to facilitate students' learning of the discipline by fostering interaction between the course students while helping each other in education (Gosser, 2011). Overall, Gosser (2011) found that, in a large number of studies, the average percentage of students receiving a positive assessment was 15% higher for students participating in PLTL groups than for students not participating in such groups. Remedial or enhancement courses are activated in many Italian universities, mainly linked to introductory classes. However, in the Italian context, there is no specific reference to the methodology of PLTL in the literature, although classes are, in some cases, organized with the same aims as PLTL. Our experimentation is inspired by research results in mathematics education in the USA, adapting the PLTL to the Italian university context. The PLTL methodology can be understood in cognitive sciences and follows social constructivism, Vygotsky's ideas (1932), and Bruner's instructional scaffolding (1975).

Edmodo

Edmodo (https://edmodo.com) is a social platform designed by Nick Borg and Jeff in 2008 for teaching, including students, teachers, and parents. Edmodo's home page is very similar to Facebook's home page. The working environment allows greater privacy because teachers can create a virtual classroom and allow only registered provided automatically generated code and periodically change. Students can create a profile in Edmodo and interact with the class and all the group members. In Edmodo, there are many tools to share files and ideas with other students and teachers. Edmodo allows users to upload documents, links, videos, and images stored in the library section. The site also offers the possibility to share materials between teachers who want to form a group and stay in touch. Participants can share media content such as homework, get their teachers' suggestions, quizzes, notices, notes, and vote on polls (Jarc, 2010).

Researchers have investigated that social networks have a profound impact on the way students learn and collaborate (Capone et al., 2018); research results also attest to the satisfaction of many teachers who work in a collaborative environment increasing their educational professionalism. Some teachers noted that Edmodo has strengthened the relationships among pupils and led to a more solid class (Mills & Chandra, 2011). As a result, Edmodo can also be a tool that allows teachers to conduct a course more easily (Witherspoon, 2011).

Learning Environment

SCALE-UP is a learning environment specifically created to facilitate active and collaborative learning among students (Gaffney et al., 2008). The name SCALE-UP originally stood for "Student-Centered Activities for Large Enrolment Undergraduate Physics", but given its characteristics, many institutions have used it for courses in different disciplines. The acronym has been changed to "Student-Centered Active Learning Environment with Upside-down Pedagogies". The basic idea is that students are given a stimulus to reflect on (an exercise, a multiple-choice test, a task to look for mistakes made by other students). Teachers act as a coach and scaffolder, intervening appropriately in the groups without being intrusive, answering students' questions that may arise while carrying out a task and make a comparison between the various groups at the end of each activity.

The spaces are carefully designed to facilitate interactions between students working on generally short tasks. In these spaces, students are engaged in hands-on learning activities or simulations. They sit around a table in order to interact face-to-face and work in small groups. A decade of research indicates significant learning improvements (Dori & Belcher, 2004). It was decided to adopt this model, taking up the experiments made at the Massachusetts Institute of Technology in Boston and adapting it to the course's specific needs.

Conceptual Framework

The teaching methodologies that have been used for this experimentation have in common the Vygotskian perspective of teaching (Vygotsky, 1978) and Bruner's idea of didactic scaffolding (Bruner, 1984). Scaffolding is the help given to a student during the whole learning process, and it is specific for each student; this didactic approach allows to experiment with student-centered learning. Vygotsky stresses that the community plays a fundamental role in creating meaning and developing cognition because cognitive skills and thought patterns are established in the social and cultural contexts of the learning environment (Vygotsky, 1978). Two significant ideas in Vygotsky's work are the roots of PLTL. First, interaction with a more informed and qualified peer or teacher effectively develops skills and strategies. In other words, knowledge is scaffolding with the support of a more experienced and successful student. Secondly, tasks and activities should be designed to improve their performance levels. Vygotsky proposed the concept of the proximal development zone (ZPD), which is the difference between the independent problem-solving skills of students (lower end of the ZPD) and their development potential (upper end of the ZPD) with the guidance of more experienced peers. The scaffolding allows the student to work in their proximal development zone, gradually subtracting as the students becomes more and more autonomous.

Three essential features of scaffolding can facilitate learning (Wood & Wood, 1996). The first feature is the interaction between the student and the expert. This interaction should be collaborative for it to be effective.

The second is that learning should occur in the student's proximal development zone. To do this, the teacher must continuously check the student's level of knowledge. The third feature of scaffolding is that the expert's support and guidance are gradually removed as the student becomes more competent until the student becomes independent. In agreement with the fact that "every function of cultural development appears first on the social and then on the psychological level, firstly among people as an interpsychological category, then within the student as intrapsychological category" (Vygotskij, 1987, p. 11), we tried to enhance peer comparison.

The social platform supported collaborative learning among students and favored constant interaction with the teacher, who could take a supportive role. Moreover, the idea of working just in time requires the student's active involvement in the lesson, who, from being a passive receiver of information, can become an active builder of their knowledge, creating a sort of community of practice (Wenger, 1996). This method, which has its roots in constructivism (Hurd, 2009), aims to build shared collective knowledge, a way of living, working, and studying, based mainly on sharing and overcoming individualism and competition. Ability becomes means to create collectively, following the method of social constructivism. Learning, coming from this perspective, is understood as:

1. Creation of meaning: from a lifelong learning perspective, our experience is significant. The experience becomes substantial when reflecting on it.
2. Identity development: learning is a process that allows us to interact, participate, and define our space/role in a community.

3. Belonging to a community: to change, to recognize oneself, or to leave, the individual must know his community, identifying with it or not, making their contribution.
4. Result of practice in a community: the union between know-how and competence.

According to Argyris and Schön (1978), who worked on individual and collective learning, a teaching action aims to train students to problematize the act, reflect, analyze, give meaning to their daily practice, and develop a very important competence: learning to learn. A community of practices is established and consolidated only through confrontation de visu. The online approach can favor and intensify establishing a community of practice. It is consolidated only through the direct interaction of the community members, triggering a continuous negotiation of practices and meanings and making the training/learning path a place of exchange. Reifications or realization of tasks is carried out collaboratively; active and collaborative participation is equal and personalized; negotiation of meanings or continuous reflection of the group on what has been done (Wenger, 1996).

Moreover, a social platform seems to contribute to the negotiation of meanings. However, it does not entirely replace the physical interaction between the community members: it would seem like the online environment works better if it is in complementarity with something solidly constituted offline, i.e., in presence (Cohen & Prusak, 2001).

In short, in the social dimension of learning, communities of learners can be considered, in a Vygotskian perspective, as multiple "zones of proximal development". Here, mutual peer tutoring, fueled by cognitive scaffolding, creates "team choreography" (Le Boterf, 1999), which guides, without directing, the learner's naive theories. The learner is taught to revisit his knowledge and reflect on his experiences. He is facilitated in solving problems in a situation of impasse. He is supported in knowledge construction, development of skills, and competencies to achieve training objectives centered on his needs. Interactive processes among community agents become an engine that promotes communication and sharing of knowledge, skills, expertise, and openness to multiple perspectives. The diversity of knowledge, experience, and skills within a working collective represents a potential for broader and richer action through the valorization of all types of intelligence and personal talents; at the same time, this diversity legitimizes diversity and understanding of differences. Scaffolding is therefore not only cognitive but also affective-motivational and relational-social. Affective scaffolding encourages, promotes, and approves the student to approach expert practice. It stimulates active participation, interest, and creativity, acting positively on the sense of confidence, on feelings of self-esteem and self-efficacy, and on empowerment aimed at commitment and responsibility, therefore on the motivation to learn.

The Case Study

We designed Calculus II's course as a case study for the proposed methodology. The case study has ideographic and nomothetic intentions (Yin, 1981, 2013). This section describes the course's design, showing its structure and how the methods were mapped to it.

Context

The experimental context is the mathematical class attended by the students of the first year of Mechanical Engineering and Management Engineering at the University of Salerno. The course was carried out during the second semester of the first year after students had attended and/or taken a Calculus 1 exam. The course has been restructured following the constructive alignment suggested by Biggs and Tang (2010).

The course included 90 h of lessons, divided into 54 h of theory lessons and 36 h of training; also 24 h of exercises with the tutors, dividing the students into two sub-groups, 12 additional hours of activities for students who have reported a short evaluation at the first test. The course has been designed considering the Teaching Council's indications and the Lisbon descriptors.

The educational goals are subdivided into Knowledge and Understanding, Applying Knowledge and Understanding—Engineering Analysis, Applying knowledge and understanding—engineering design; Making judgments—engineering practice; and Communication skills and transversal skills.

- About Knowledge and Understanding, the aims of understanding the terminology used in mathematical analysis; knowledge of demonstration methods; knowledge of the fundamental concepts of mathematical analysis. Knowledge related to integral functions of a variable, numerical series, sequences and series of functions, functions of several variables, differential equations, multiple integrations, curves, curves integrals, surfaces and surface integrals, and vector fields.
- About Applying Knowledge and Understanding—Engineering Analysis, the aims are applying the theorems and the rules studied to solve problems; building methods and troubleshooting procedures; know how to process and communicate information using a formal linguistic log; applying knowledge of the concepts and methods of calculus and mathematical tools to solve differential equations, integral curves, and integral and surface integrals; perform series and integral calculations; and calculate maximum and minimum functions of two variables, applying knowledge to develop demonstrations of certain theorems consistently.
- About Applying knowledge and understanding—engineering design, the aims are applying knowledge to find the most appropriate methods to solve a math problem and be able to find optimizations in solving a math problem.
- About Making judgments—engineering practice, the aim is applying the acquired knowledge to contexts different from those presented during the course.
- About Communication skills—transversal skills, the aim is to learn more about the topics covered by teaching materials other than those proposed during the course.
- About Learning skills—transversal skills, the aims are learn how to decipher the topics discussed using teaching materials other than those proposed during the course and develop a positive attitude towards math based on respect for truth and availability to seek motivation and to clarify its validity.

In the following, the contents of the course have been scheduled (Table 2).

Table 2 Contents of the course schedule

Content	Time (in hours)	
	Theory	Exercise
Linear algebra	8	6
Differential equations	6	4
Functions of several variables	6	4
3D geometry, curves, and integral curves	6	4
Double integrals	6	4
Triple integrals	4	2
Differential forms	6	4
Surface and surface integrals	6	4
Function series	6	4

Table 3 A learning unit schedule

Learning unit	Competencies		
	Learning outcomes	**Educational goals**	**Indicators**
Surfaces and surfaces integral	Identify regular surfaces	Understand the definition of a parametric surface and sketch the surface	Find a tangent plane to a parametric surface
	Identify parametric equations for surfaces	Find a set of parametric equations to represent a surface	Evaluate surface integrals
	Identify strategies to find a tangent plane to a parametric surface	Find a normal vector and a tangent plane to a parametric surface	Evaluate a flux integral
	Identify strategies to solve problems	Find the area of a parametric surface	
	Use differential calculus tools in the description and modeling of various phenomena	Evaluate a surface integral as a double integral	
	Identify strategies to find the surface area	Evaluate a surface integral for a parametric surface	
	Identify strategies to evaluate a surface integral	Determine the orientation of a surface	
	Identify strategies to evaluate a flux integral	Determine the orientation of a surface	

The teaching path has been divided into learning units, and the competence goals and training targets have been highlighted through appropriate competence indicators for each unit. In the following, Table 3 shows an example of the learning unit concerning surfaces and integral surfaces.

Participants

The participants in the educational experiment are 112 students attending the first year of Mechanical Engineering and Management Engineering at the University of Salerno. Also involved in the experiment were the course teacher, two experts of the discipline for the exercises, and three students attending the master's degree in engineering as tutors. The background of the students is heterogeneous. As shown in Fig. 2, most of them attended scientific high school (about 80%). In Italy, the scientific high school provides a more in-depth study of mathematics than other schools (132 h of mathematics in the last year), while in the classical high school (grammar), only 66 h in the last year. In the technical institutes, 132 h are foreseen. In professional institutes, mathematics subjects are dealt with more lightly because students generally decide to work after the last year of high school and not enroll at university. Although students already attended a Calculus class in the first semester, their background also affects the following examinations.

Besides, about 24% of the course students had not yet taken the Calculus 1 exam.

The research was conducted in compliance with the code of ethics. All experimental data, both the students' verification tests and the answers to the questionnaire, were treated anonymously.

Methods

The methodology is based on a single case study to identify intervention strategies in the specific didactic situation. Direct observation, understood as student–teacher interaction through information technologies, collected empirical data on teaching effectiveness. The approach was ideographic based on qualitative methods through a questionnaire to students and from students' interactions with the e-learning platform, and nomothetic based on quantitative data emerging from the results of the tests.

This is a non-comparative observational case study where an attempt is made to respond to a problematic situation. We aim to verify whether the comparative use of JiTT (just-in-time teaching) and

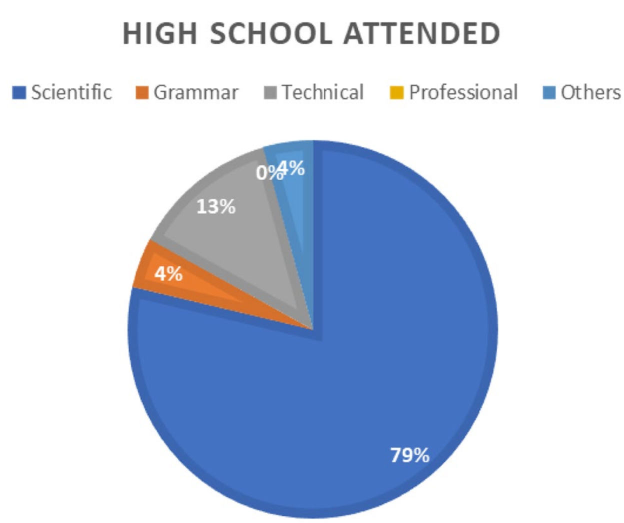

Fig. 2 High school attended by the students

PLTL (peer-led team learning) in mathematics education can lead students to acquire mathematical competence and satisfy the teaching action.

Integration of JiTT and PLTL, Using Edmodo in SCALE-UP environment

This subsection will describe how JiTT and PLTL methodologies were used in a Calculus II class with engineering students. To the classic lecture, we added e-learning activities using the Edmodo platform described in the previous section. After the lesson, a summary was loaded on the platform each time. Besides, we said exercises of increasing difficulty, and students were invited to carry them out by posting the tasks or results. Often the students confronted each other about carrying out the exercises and pointed out the difficulties they encountered or commented on the other students' performance. In a non-invasive way, the teacher and tutors had the opportunity to read the comments and analyze the progress of the exercises. The teacher collected the most common errors or misconceptions highlighted by the words, which were often generated by an incorrect interpretation of the explanation in class or by the teacher's wrong approach to the subject. The next lesson took its cue from brainstorming about the misconceptions that emerged from the exercises, which clarified any dark points. Students often send private messages to teachers or tutors, especially the most difficult ones. It made it possible to monitor step by step the skills acquired by the students, the problems encountered, and the points to dwell on with more considerable attention. In the following figure, we show the home page of the class Calculus II.

Visually it has a central space where messages appear, enclosed between two service boxes, on the right and left, as in Fig. 3.

Within the group, communication can be one-to-many (teacher to all, student to all) or discreet, between teacher and student, or student and student. In addition to communicating with the teacher, students can send attachment documents that the teacher marks online and send back to the sender (even in one-to-one mode). Other useful tools that the platform offers are a shared library where one can store documents and images, a calendar where it is possible to mark the deadlines of homework assignments and the dates of checks, create quizzes, and manage evaluations while protecting privacy. The e-learning approach allowed students to interact with teachers and tutors daily and ongoing basis. It enabled them to engage in and develop peer education through dialogue between all class members.

Interaction in a virtual community was also helpful because it allowed the transition from a colloquial language register to a formal one. As shown in Fig. 4, the dialogue between students, even when using informal language, allowed the teacher to identify some difficulties in approaching the exercises.

In Fig. 4a, a student asks colleagues to confirm correctly that the exercise is being carried out.

> S1: Good evening, has anyone managed to set up this exercise, taken from the professor's handouts?
> S2: I also divided the domain into two domains. I made the integral on the domain D1 and then on the domain D2, but I'm not sure.
> S1: Let us see if the professor reads us and can help us.
> T1: I confirm that the exercise is correct. Tomorrow, the lesson will begin with correcting this exercise because I observed that many people had difficulties with the group chat.

In Fig. 4b, a student shows the execution of the exercise related to the resolution of a double integral; he shows how he identified the extremes of integration; although the exercise's performance is correct, the exercise's result is not correct.

> S1: How do I find the domain and thus the extremes of integration? I have done it this way and cannot find it.
> S2: The exercise setup seems correct to me. You have made some calculation errors.
> S3: Here is my procedure. The result is correct. (Student S3 uploads the correct answer.)

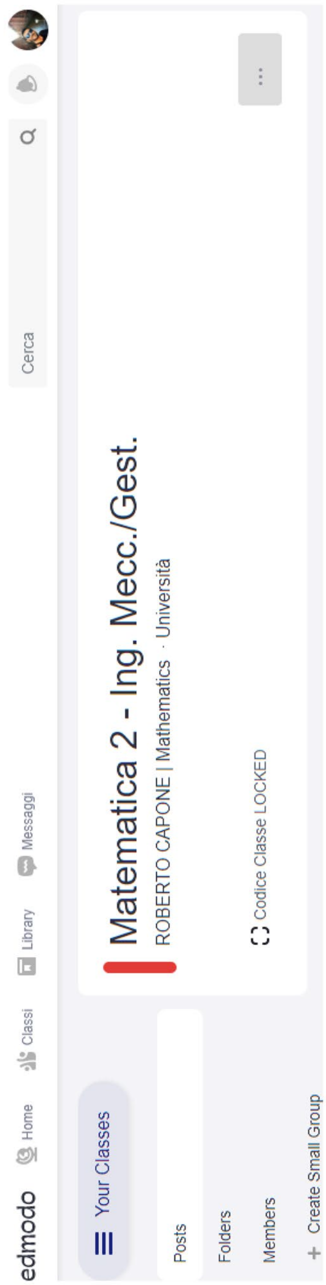

Fig. 3 Homepage—Calculus II (Matematica 2)

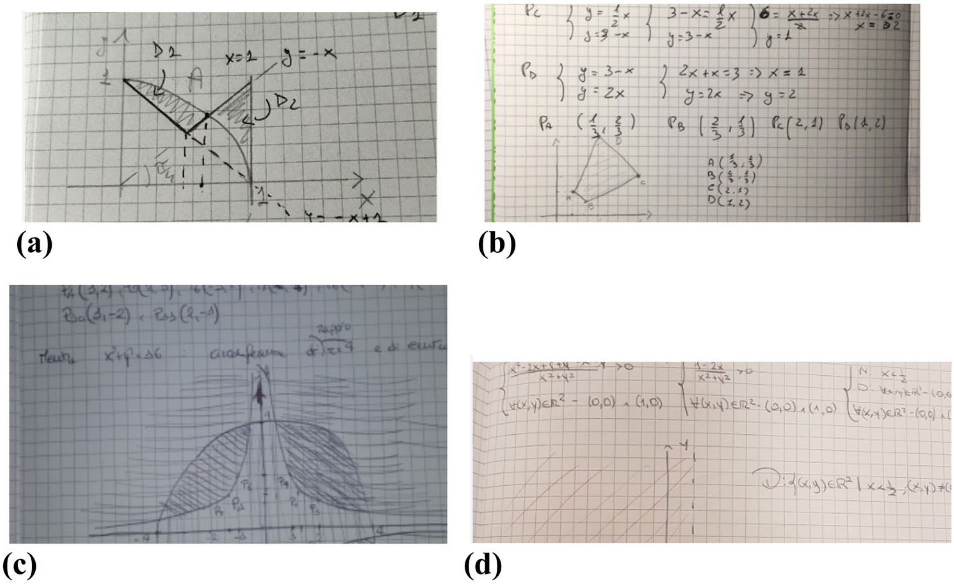

Fig. 4 a–d Some protocols from the Edmodo platform

In Fig. 4c, a student reports on the teacher's exercise and continues to carry it out in class.

S3: The professor asked me to publish the development of this domain done in the class because there were many students absent today. Good work! I hope it will be of help to you.

In Fig. 4d, a student reports on the execution of the exercise.

The students discussed the teachers' or tutors' resolution of exercises in e-learning mode. The students chose the activities to be carried out, like in Fig. 4a. The students ask for clarifications about activities they have encountered difficulties in, like in Fig. 4b. Students share not completed exercises on the platform as their teacher left them this task as a stimulus, like in Fig. 4c. Students share the platform's tutoring exercises to support absent colleagues, like in Fig. 4d. Thanks to all these interventions on the forum, the students' shared posts and comments identify "Just in time", the thematic cores deepened in the classroom, the misconceptions, and most common errors.

The aim has been to create a learning environment, a research community, and a living laboratory that stimulates ideas but, above all, motivates the students to ensure their educational success. Moreover, we tried to renounce the expressions "acquisition of knowledge" and "transfer of learning" and to think about learning in terms of legitimate peripheral participation (Lave & Wenger, 1991), to socially organized activities, understanding the educational process as social reconstruction starting from the biological forms of behavior (Vygotskij, 1987).

In addition to social interaction through the platform, students benefited from hours of practice with teaching assistants. Furthermore, small group activities were organized for students who did not achieve positive results in the first test. Experienced students supported these groups (generally 4 or 5 groups of 4 people), creating a social workshop to develop dynamics, experiment activities, design, share, and improve self-esteem and relational and communicative skills. In the following picture (Fig. 5), on the left, students work in groups in a circular arrangement during a PLTL laboratory at the Didactics of Mathematics Laboratory of the University of Salerno; on the right, a classic collection of students following a lecture.

Fig. 5 To the left, a laboratory of SCALE-UP at the University of Salerno; to the right, a traditional classroom for the students

Exercises and problems were submitted to students inspired by those carried out in the classroom: students did their homework in groups, often asking more experienced colleagues' intervention. The bond of similarity perceived among the students involved in these educational interventions was based on their effectiveness. Feeling some commonality with the other people involved, sharing similar problems or everyday experiences with them, "seeing each other again" in other people's actions/situations, etc. favored educational communication's credibility and effectiveness. On the other hand, it changed students' behavior and attitudes. Peer leaders have been seen as models to reread their experiences and acquire knowledge and skills of various kinds.

Data Analysis and Results

This section shows and discusses the case study results, analyzed from a quantitative and qualitative point of view.

Formative and Summative Assessment

The teaching activity was monitored continuously, not only through computer systems but also through two tests that have had a function of formative evaluation and summative evaluation. It made it possible to identify some students' difficulties early, especially in mathematical competence. Those students

2A - Find the domain of the following function and represent it in a Cartesian plan

$$f(x,y) = 2^{\sqrt{x^2+3y^2-1}}$$

2B - Identify and sketch the level curves (or contours) for the function in (1;0).

2C - Find the relative and absolute maximums and minimums of the function.

2D - Write the equation (both algebraic and parametric) of the line contained in the plane tangent to the function in (1;1) and which is incident to the line of parametric equations and through A (1;2;1):

$$\begin{cases} x = 3 + k \\ y = k \\ z = 3 + k \end{cases}$$

Fig. 6 A question in the first test

1A - Draw the part of plane A limited by $y = x^2, y = 2x^2$ and by $x = y^2, x = 3y^2$;
1B - Solve the following double integral

$$\iint_A \frac{1}{x^2 y^2}\, dxdy$$

where A is the domain previously drawn
1C - Calculate the same integral using Gauss-Green's formulas.

Fig. 7 A question of the second test

who showed widespread fundamental deficiencies after the first test suggested that the study was supplemented with two additional hours of lessons, managed using the cooperative learning methodology. These integrative hours took place in a SCALE-UP learning environment. The students were divided into small groups and worked with a tutor who supported the students throughout the recovery of basic skills. The learning space was designed to facilitate student interaction and group interaction. The students' results regarding essential mathematical competencies were collected following the first didactic interventions. It was obtained following the logic of mastery learning (Block & Burns, 1976), according to which all students can achieve basic goals in each discipline in enough time and with appropriate methodological changes. Two mid-term tests were given during the semester. The first test was on the topics of the first part of the course (functions of several variables, linear algebra, differential equations, curves, and integral curves). The second test was on the second part of the course (differential forms, multiple integrals, surface integrals, function series).

Below is an example of a question proposed to the students in the first test (Fig. 6).

The following is an example of a question proposed to the students in a second test (Fig. 7).

The following is an example of a question proposed to an examination test (Fig. 8).

As can be seen from these examples of questions given to students during the tests and the exam, we tried to verify the skills acquired by the student or "the proven ability to use knowledge, skills and personal and methodological abilities to solve unknown problem situations" (EQF, 2005). The student's skills were assessed considering the following dimensions: resources, i.e., the student's basic knowledge and skills; interpretation structures, i.e., how the student reads and interprets problematic situations; structures in action, i.e., how the student reacts to a problem; self-regulatory structures, i.e., how the student learns from experience and adapts his strategies to the stresses coming from the context (Trinchero, 2012). In the specific case of the last question, the dimensions of competence evaluated are as follows in Table 4.

The assessment of competencies was carried out based on the following assessment heading (Table 5 describes the four levels of competence).

Table 4 Evaluated competencies schedule

Resources	Know the notion of the domain of a function, intersection with the coordinated axes, relative maximum and minimum, concavity and convexity, flexion
Interpretation structures	Know how to understand that the solution of the problem is not in the application of an algorithm, but in a rethinking about the graph of the function
Action structures	Know how to build the graph of a function when the analytical form is not known already but for which is possible to derive its characteristics
Self-regulating structures	Know how to evaluate strategies compared with the owed objectives and data

Table 5 The four levels of competence

A+/A **Advanced**	B **High**	C **Medium**	D **Beginning**	NS **Low**
The student:	**The student:**	**The student:**	**The student:**	**The student:**
schedules the sequence of the calculation procedures themselves	Uses the analysis skills, representing them also in graphic form	uses the analysis tools, representing them also in graphic form with some uncertainties. S/He identifies the strategies to solve problems	uses the analysis methods, not always representing them in graphic form, although with some uncertainty;	does not have the necessary competencies to take the following exams. The student is asked to retake the test again
S/He accurately describes all the sequences examined and identifies their possible use	S/He identifies the most appropriate strategies to solve problems	S/He uses the tools of differential calculus in the description and modeling of various phenomena	identifies strategies to solve problems by making a few procedural errors;	
S/He masters the tools of infinitesimal calculus	Make proper use of differential calculus tools in describing and modeling various phenomena		uses the tools of differential calculation, not always managing to describe the model present in nature	
He makes appropriate use of the specific language of the discipline				

Table 6 First test results

Grade	Number of students	Percentage
NS	36	32.4%
D	24	21.6%
C	35	31.5%
B	12	10.8%
A	4	3.6%

Table 7 Second test results

Grade	Number of students	Percentage
NS	30	31.9%
D	11	11.7%
C	22	23.4%
B	17	18.1%
A	14	14.9%

The first test showed a rather high number of students who did not reach sufficient levels of competence in the items being studied, and only a few students acquired an advanced level, as shown in Table 6.

The second test showed (in Table 7) a marked increase in the number of students who successfully passed the test, highlighting advanced skills.

The results were compared to the results of 117 students in the previous course of the same cohort, using the same assessment test format. It was found that 81% of students passed the exam in the winter session compared to 60% in the previous cohort. In addition, 58% achieved good results compared to 32% in the last year. The reported data seem to confirm a ten-year research record showing significant improvements in learning following the implementation of student-centered learning pathways (Beichner et al., 2006; Dori & Belcher, 2004).

These results seem to be following other research: Gavrin (2010) reported that 80% of the students in his JiTT class responded "yes" to "Do the JiTT exercises help you to be well prepared for the lecture?" versus 21% affirmative to the same question in "other classes". He found a 58% vs. 18% split on "staying focused", a 59% vs. 18% split on "feeling like an active participant", and a 71% vs. 21% split on "finding classroom time useful" (Gavrin, 2010).

3 - The following vector field is given.

$$F(x,y) = \sqrt{x^2 - y^2}\,\hat{\imath} - ylog\left(x + \sqrt{x^2 - y^2}\right)\hat{\jmath}$$

3A - Draw the domain and study if it is conservative;

3B - find the potential function for the vector field;

3C - finds the work across the curve of parametric equations (2t; t-1) with $t \in [1; 2]$

Fig. 8 A question of examination test

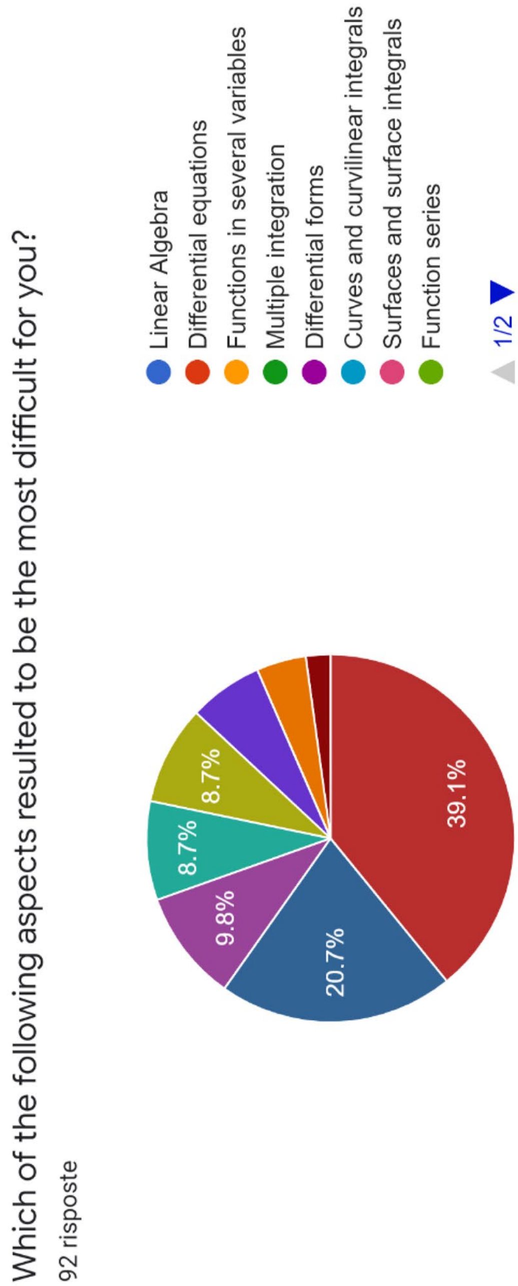

Fig. 9 The difficulties of the subjects divided into topics

Satisfaction Questionnaire

At the end of the course, students were given a questionnaire to understand the difficulties encountered. When asked, “Have you experienced problems studying the course topics”, 63.8% state that they have located difficulties. The encountered difficulties concerned all the subjects with a higher incidence (39.1%) to study surfaces and surface integrals (Fig. 9). From the given test, the student’s satisfaction with the teaching strategies adopted and their cheerful disposition to the discipline study emerge, even though most claim to have encountered difficulties studying the discipline.

The following question was asked: in your opinion, why did you encounter difficulties during the course?

The following graph (Fig. 10) shows the motivations most frequently cited by students due to the difficulties they encountered (90 students answered the question).

The answers given by the students were as follows: The topics are difficult (34 students); we had little time to study the subjects (43 students); past deficiencies (28 students); the teacher was not always clear (10 students).

Although there were difficulties, many students highlighted that it was handy to give some space to the exercises and clarify the doubts that emerged on the platform in class.

In the following, there are some of the students’ answers to the satisfaction test:

> S1: The lessons certainly have their difficulties, and that’s obvious. However, I appreciated how the teacher presented the topics and the time spent on the exercises in the classroom so that any doubts could be clarified in class. In most cases, the lessons went smoothly.
> S2: I felt at ease, despite the difficulty of the topics dealt with; it was a pity not to be able to deepen them, but anyway, I am satisfied because the study of these topics gave me a wider view of the faculty I am attending.
> S3: The course was not very simple due to the complexity of some topics but made easy by the teaching materials provided by the teacher and tutors.

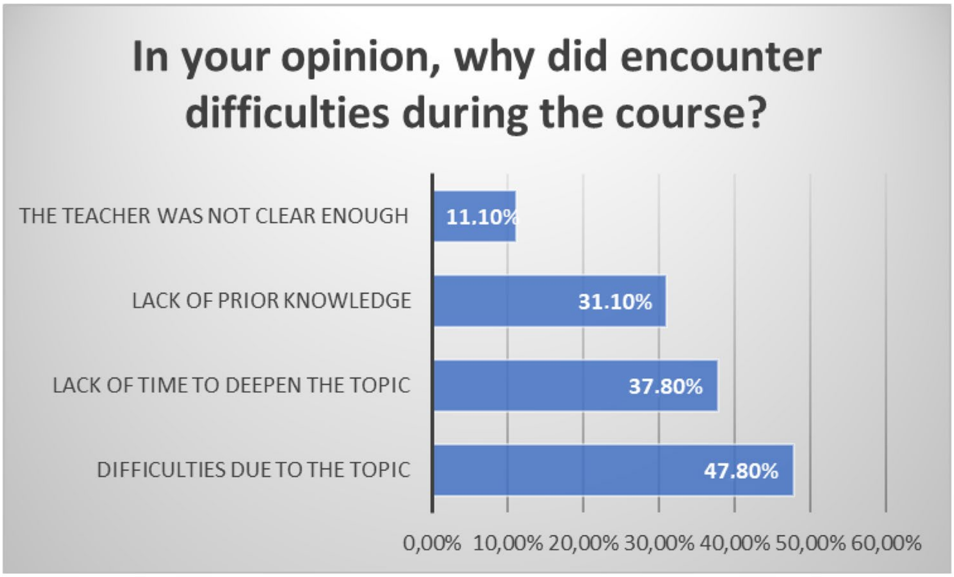

Fig. 10 The motivations most frequently cited by students respond to the difficulties they encounter

At the end of the course, therefore, a better attitude to the study of the subject seems to emerge; from the satisfaction test, the appreciation of the use of alternative teaching methods appears in most students (79% appreciate the didactic innovations of the course very much, 10% enjoy much, 7% did not influence the educational success decisively, 4% prefer more traditional teaching).

Another important element of university teaching in STEM courses is the early drop-out of students due to low levels of knowledge in mathematics. An element that could be related to the effectiveness of the methodologies adopted is the reduction of dropouts compared to the previous year.

The dropout (4%) has been lower than the percentage of the previous year (10%).

Discussion and Conclusions

In this paper, the attempt to integrate the educational advantages of using the just-in-time teaching (JiTT) methodology with a blended online and in-class format teaching approach (Novak et al. 1999) in a Student-Centered Active Learning Environment with Upside-down Pedagogies (SCALE-UP) with the peer-led team learning methodology was examined. The students with whom the experiment was conducted attended the first year of engineering. Many of them considered mathematics an obstacle to further studies because of the subject's difficulties and were not adequately motivated. We have focused on this research on these two aspects, studying and experimenting with alternative methodologies to frontal lessons to overcome obstacles of the discipline and encourage students to study mathematics. We have taken our starting point from methodologies already successfully tested in the USA, believing that one methodology does not always manage to be inclusive of all students to help them overcome the different and multiple difficulties of approaching mathematics.

Therefore, we have integrated JiTT and PLTL. In themselves, these two methodologies also included using an e-learning teaching that we thought to implement through the social platform Edmodo, both for its graphical interface and ease of use through the application available on mobile phones. These methodologies have been integrated harmoniously because their common cultural matrix can be found in Vygotsky's and Bruner's theories. The experiments were carried out during the whole semester, collecting quantitative and qualitative data.

The qualitative analysis based on the students' answers shows considerable enthusiasm for using technological tools for continuous comparison with teachers and colleagues and supporting study.

From qualitative and quantitative data analyses alike, it seems that behavioral engagement is also positively associated with the realistic, practical, and guided discovery aspects of the task design, the activity structure, and the use of mobile technology. Additionally, working in a team also appears to have had a positive effect. Mathematics confidence is positively associated with real, guided, and practical tasks, with the use of technology, also appearing influential. Not surprisingly, both transformative and computational technology use is most significantly related to confidence using technology, with the variety of technologies noted as adding to flexibility and adaptability. In conjunction with the task design, the transformative and computational use of technology appears to have the most influence on students' attitudes to using technology for learning mathematics.

In particular, the integration of JiTT and PLTL methodologies seems to provide learning outcomes coherent with the theses supported by the exponents of social constructivism and Vygotsky's ideas, as students show to improve their skills under the guidance of a more capable peer acting in the proximal development zone. From the findings in the table related to the second test, it should be noted that there is an improvement in students' performance in problem-solving; an increase in conceptual learning can also be seen (Fandiño Pinilla, 2008).

Although the small sample does not allow us to draw statistically significant conclusions, a definite improvement in the performance of the experimental class and a considerable increase in students' interest and motivation were observed. As also confirmed by recent studies (Novak, 2011), JiTT seems to have activated learning that has respected the needs of the students, respected the time, and increased motivation to study. Thus, this first experimentation has successfully stimulated curiosity for new and more in-depth investigations.

JiTT seems to be encouraging all the class students to:

- participate more actively and reflectively in the learning/teaching process;
- share their experiences and points of view with colleagues on the course and with more experienced people; and
- follow the lessons more actively and consistently in the classroom.

JiTT seems to have encouraged the experimenter teacher to:

- show interest in the helpful mistake's students make and offer corrective support;
- take advantage of students' mistakes to plan a more effective educational intervention in the classroom;
- interact more directly with students;
- amplify the time available for their lessons through synchronous and asynchronous interventions; and
- encourage the student community to help each other.

Other qualifying aspects of the experience, to be further verified, have simplified the recovery by those absent from the lessons thanks to the e-learning method and availability of contents.

The SCALE-UP learning environment has made it possible for 45% of the students who had failed at the first test to make up for the fundamental shortcomings, increasing improvement.

The PLTL has contributed a lot to the recovery of basic skills because students were constantly confronted with their peers in line with Vygotsky's research and pedagogical research on educational scaffolding methods.

One of the fundamental aspects highlighted by the students in the interviews is that the methodologies adopted to put the student at the center, his needs, and the educational activities of the teacher have been adapted to theirs. This seems to have spurred them to participate in both presence and online courses. Strategic management of these core components consists of the analysis, decisions, and course of action that an organization or individual that operates programs or classes needs to create and sustain to ensure the quality of learning. Active participation in the lessons has improved student performance, as evidenced in other studies, as evidenced by the results of the tests. The teacher's presence in discussions has facilitated learning and clarifying any learners' questions (instructor participation in class discussion). Therefore, attention to the discussion, especially conceptualizing, has been very important in learning processes. For example, in mathematics, it is justified every time the constitution of a mathematician is involved, from the outside (real-world mathematization) or the inside of mathematics (theory of functions).

The approach to the discussion was understood as an attempt to provide a set of tools for analysis and planning by the experienced teacher without reducing the responsibility of the pupils. This Vygotskyan process referred to interactions between subjects (teachers and pupils) who played different roles that must be preserved and valued in the teaching–learning activity. An important aspect was encouraging peer learning within a teaching community (also virtual) by creating working groups or encouraging the accessible aggregation of students in working groups through team-based learning methods. Like Vygotsky suggested, peer communication also allowed internalizing cognitive processes implicit in interactions and provided new patterns that influenced individual thinking, emphasizing the proximal development zone. The quantitative data concerned improving the skills found by comparing the results of the tests carried out during the semester and a satisfaction test. From the beginning, the experimentation shows greater participation of the students in the educational dialogue and the class because the lesson was inspired by their training needs, the difficulties encountered in carrying out the homework, and the need to respond to their request to overcome specific cognitive barriers.

The data show an improvement of the student's performance compared to students of the previous year's cohort. The satisfaction test shows a better affective disposition of the students towards the study of mathematics. After these tests, the experimentation also continues with students, integrating further stimuli to the study, such as augmented reality for studying functions in two variables. This first experiment successfully stimulated curiosity for new and more in-depth investigations. Although the smallness of the sample does not allow us to draw statistically significant conclusions, a clear improvement in the performance of the experimental class and a considerable increase in student interest and motivation have been observed. The success of the didactic intervention is also highlighted by the small number of dropouts (4%), lower than the percentage of the previous year (10%). Other qualifying aspects of the experience, not negligible, to be subjected to further testing, were facilitation of reintegration by the absents from the lessons thanks to the e-learning method of delivery of some contents and the attention paid to students with special educational needs who were able to fill their gaps and felt involved in the educational activity.

The educational findings can be summarized as follows:

- students' ability to solve problems is improved.
- their conceptual understanding is increased
- their attitudes are better
- failure rates (especially for women and minorities) are drastically reduced

Thanks to SCALE-UP, scaffolding is intellectual, technical, organizational support, emotional, cognitive, and metacognitive. Emotional because it aims to stimulate the learner to learn, encourage them, and encourage them to overcome any barriers of a motivational nature. Meta-cognitive is intended to support the learner in acquiring specific knowledge or competence and developing metacognitive skills that will enable them to learn. In this way, a Calculus course has been of service to the student and colleagues in subsequent classes.

The research findings showed that blended learning by integrating several teaching methodologies can provide students with the tools for a better predisposition to learning. In addition, the integration of JiTT and PLTL in a SCALE-UP learning environment seems to confirm the idea that social interactions are at the origin of the construction of individual skills and that possessing individual skills of a certain complexity allows the individual to subsequently participate in more complex social interactions, which in turn will enable the construction of skills of higher complexity.

This article is a prequel to other articles by the author on teaching mathematics in STEM courses. This study's qualitative and quantitative data show that student participation, student-to-student interaction, and teacher-to-student interaction have increased thanks to teaching methodologies such as JiTT and PLTL in a SCALE-UP learning environment. The social platform seems to have influenced this process. It was thus decided in the following years to use an adaptive e-learning platform to support students in finding the material and increase the levels of participation, engagement, and motivation.

This preliminary feasibility study provided the basis for more detailed studies based on the analysis of factors such as participation, engagement, and motivation and the impact on improving students' competencies. This preliminary study aimed to compare how mathematics teaching in STEM courses changed before, during, and after the COVID pandemic (Branchetti et al. 2021; Capone & Lepore, 2020, 2021, 2022).

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