RESEARCH ARTICLE

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Power-law distribution in an urban traffic flow simulation

Daigo Umemoto[1] · Nobuyasu Ito[1,2]

Abstract

We found power-law behavior in the distribution of traffic on road segments in urban traffic simulations using digitized map of Kobe city in Japan as an example of an actual road network. As a comparison, we performed simulations using an artificial random road network and Manhattan-type road network. Similar power-law behavior was confirmed in the former, but not the latter. The behavior appeared robustly with or without traffic congestion, which suggests that its origin is not the interaction between vehicles. The power-law exponent was fitted using least squares method and obtained as -1.1 for Kobe city and the random road network, with optimization to avoid traffic congestion. The result did not change with the use of a different origin and destination distribution. From these results, one of the reasons that caused the power-law behavior was considered to be the randomness of the road network connection and edge lengths, whose fluctuations are obvious both in Kobe city and the random road network, unlike the grid network.

Keywords Traffic flow · Zipf's law

Introduction

Traffic models for a straight single lane have been established, whereas models on city-scale networks of connected roads have not reached the same level. Various urban traffic simulations have already been conducted that attempt to improve the flow of urban traffic, and it is simultaneously necessary to comprehend and control the behavior of simulation models because they are comprised of a very large number of parameters and conditions, as is the case for most social simulation models. Simulations that attempt many cases using high-performance computers shall

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provide novel measures to establish comprehensive models of social phenomena so that they can eventually be designed and controlled.

Power-law behavior, so-called Zipf's law, is observed in various phenomena both in society and nature, for example, language [1], income [2], population [3], hits on webpages [4] and earthquakes [5]. Such behavior is not expected from naive statistical law of large numbers and analytical exponential function; therefore, the Zipf-like power-law behavior is regarded as a significant and useful characterization of each phenomenon. Empirically, some roads have fixed small or large traffic volume, which resembles income or population disparity. This suggests that the population distribution of roads versus traffic volume (hereafter called traffic distribution) obeys a power-law in the same manner as income or population distribution.

In this paper, we report that a power-law actually appears in the traffic distribution of an urban traffic simulation performed using a digitized map of Kobe city in Japan as an example of an actual road network. By performing other simulations using artificially generated road networks, we conclude that the road network structure is one of the causes of the power-law appearance.

Kobe city traffic simulations

To perform urban traffic simulations, we used the traffic simulator called SUMO (Simulation of Urban MObility [6]) with a digitized road network of actual Kobe city which works in SUMO. The road network is shown in Fig. 1, and was obtained and edited in [7] from a digital road map provided by Zenrin Co., Ltd. The network includes highways, national roads, city roads with different speed limits, 100, 60 and 30 km/h, respectively. The SUMO default traffic signal pattern was used.

We assumed 17,500 vehicles during six hours of simulation time, which is equivalent to assuming that 70,000 vehicles appeared in 24 h. The road map was divided into six areas to generate non-uniform OD distribution, as shown in Fig. 1. To imitate the background traffic flow that travels across Kobe city from outside the city, the origin and destination (OD) distribution was set to be higher between area 1 (red) and area 4 (cyan) than it was between any other pair of areas; 8750 vehicles between area 1 and area 4, and 250 vehicles between other areas. The OD was randomized and uniformly distributed within each area. Simulations using these conditions were verified in [8] to have similar behavior to the empirical features of actual traffic.

Using an OD set, a script called "duarouter.py" included in the SUMO package, generated the shortest route with respect to time from its origin to destination, without considering traffic congestion. Shown as "none" in Fig. 2, the number of vehicles simulated in this initial routing continuously increased until the simulation ended as a result of severe congestion, which corresponds to a non-equilibrium state in which a constant fraction of entered vehicles never arrive at the destination until the simulation ends.

To make vehicles avoid congested roads, a script called "duaiterate.py" included in the SUMO package generated a better route based on the travel time calculated using the previous simulation result. By applying these optimized
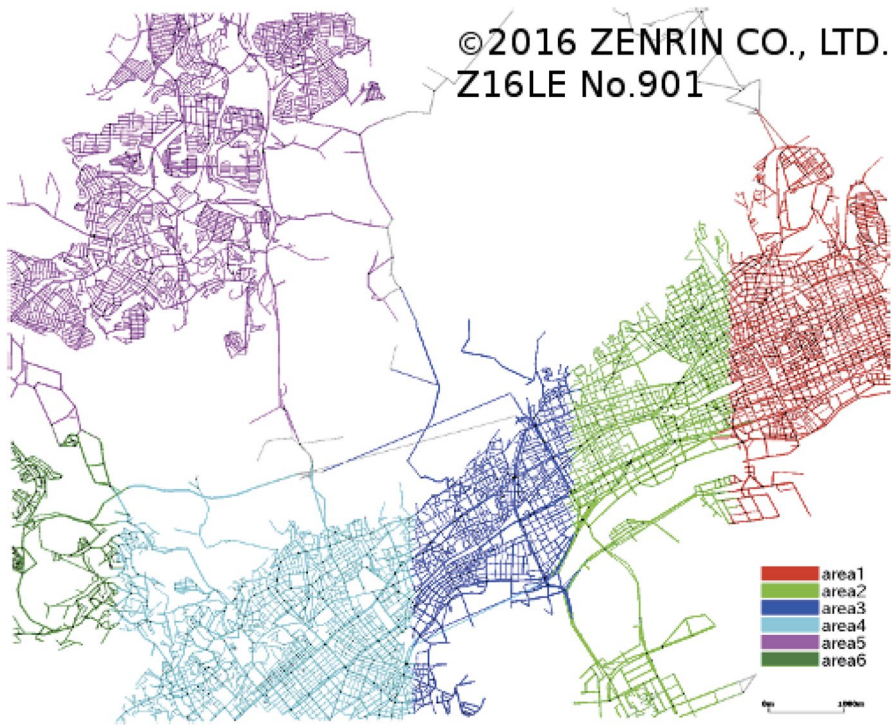
Fig. 1 Road network of Kobe city, sourced from [8]. Six separated areas are shown in different colors
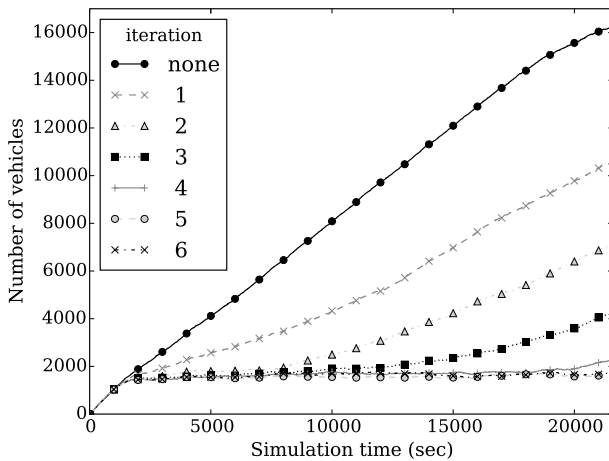

Fig. 2 Number of running vehicles vs. simulation time. The initial route (solid curve with black circles) shows a continuous increase of running vehicles, which represents an unrealistic scenario in which most vehicles do not reach the destination within the simulation time. By iterating six times, the time evolution converged into a flat shape, except for the beginning of the simulation, which we assess to be an equilibrium state and a realistic scenario

routes, shown as "1" in Fig. 2, a greater fraction of vehicles than that in the previous simulation reached their destination, which is a more realistic scenario. By repeating these iterations, the time evolution converged into a flat shape with constant simulated vehicles shown as "6" in Fig. 2, which represents an equilibrium state in which the same number of vehicles that entered the simulation reached the destination and disappeared. We assess that this equilibrium state is sufficiently realistic because all vehicles are considered to reach the destination within a finite time. The appropriate number of iterations was obtained as six because the time evolution after five and six iterations obeyed an almost identical flat shape.

For each simulation, the number of vehicles, N_V, that passed each road was recorded. The number of roads, N_R, were accumulated using logarithmic binning and divided by bin width ΔN_V to obtain $N_R/\Delta N_V$ as a normalization. Figure 3(left) shows the result of plotting them with respect to N_V, which we call traffic distribution. Clearly, a power-law distribution appeared across the entire region. The traffic distribution after six iterations is also shown in Fig. 3(right). The power-law behavior remained, with a smooth cutoff appearing at the right end of the distribution.

The appearance of the power-law without any iterations implies that this behavior was independent of traffic congestions; the shortest path from the origin to destination simply caused the power-law distribution. Additionally, the power-law exponent was fitted using the least squares method for the fitting region 10^0–10^2 and was obtained as -1.4 without iterations and -1.1 with iterations. It increased after the iterations, which implies that the traffic distribution became closer to a uniform distribution as a result of the iterations.
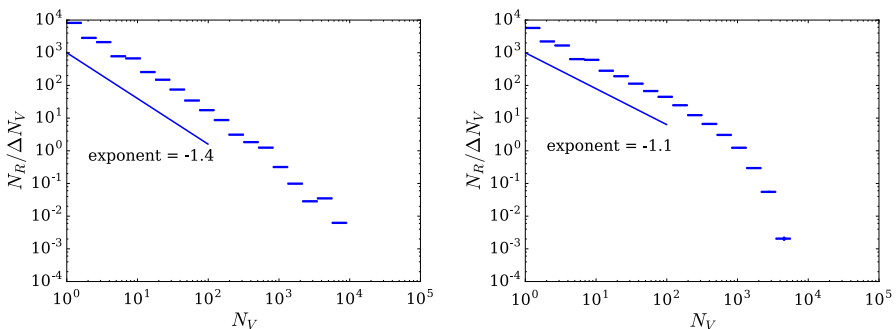

Fig. 3 Left: traffic distribution in Kobe city derived from routes with the shortest travel time, without considering traffic congestion. The x-axis is the number of vehicles that went through each road segment, whereas the y-axis is the number of roads per unit vehicle. Right: same as left, but after six iterations. The distribution was re-binned so that any apparent bin widths were the same when plotted on a logarithmic scale. To normalize the bin contents, N_R were divided by their widths ΔN_V. Error bars are given but are almost invisible, except for the right end of the distribution in the right panel. The errors were obtained as the standard errors of five simulations with different random ODs. In each panel, the power-law line is drawn with the same exponent obtained from the best least square fit to enable visual comparison
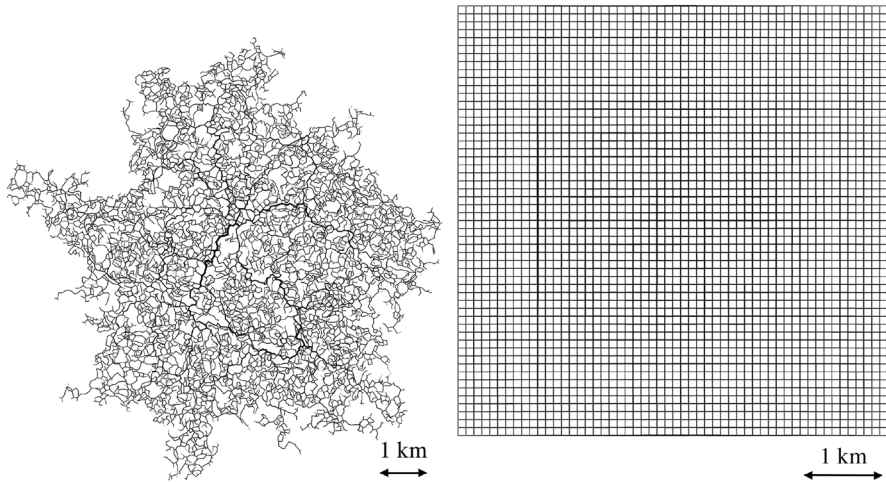
Fig. 4 Left: random road network. Right: grid road network

Table 1 Random parameters	Parameter	Value
	rand.iterations	15,000
	rand.bidi-probability	1.0
	rand.max-distance	1000.0
	rand.min-distance	50.0
	rand.min-angle	1.5
	rand.num-tries	3
	rand.connectivity	0.95
	rand.neightbor-dist1,2,3,4,5	Default

Results of the artificial condition and networks

To determine the condition that caused the power-law distribution, simulations were performed using two additional road networks and compared with the Kobe city simulation. The first road network, shown in Fig. 4(left), was a randomly generated by executing the "netgenerate" command included in the SUMO package. The parameters of the command were set as shown in Table 1. The second road network, shown in Fig. 4(right), was a grid network, which was also generated using the aforementioned command. A grid of 50×50 square blocks was arranged, with sides of 100 m. These two road networks are hereafter called the random network and grid network, respectively. No traffic signals were set in either map.

To compare simulations among the three maps using a common condition, a uniform OD distribution was used across the entire map. Uniformity was with respect to edges. To provide conditions similar to those of the Kobe city simulation, the

simulation time was fixed to six hours, and one vehicle was inserted every one second of the simulation time for simplicity; therefore, 21,600 vehicles were simulated using each map, which is a similar value to the total vehicle number 17,500 used in the previous section. The OD distribution was generated by a script called "randomtrips.py", included in the SUMO package.

The simulation results for the three maps with five iterations on each are shown in Fig. 5. Clearly, a power-law distribution appeared only in Kobe city and the random road network. Additionally, in the Kobe city simulation, the modification from non-uniform to uniform OD did not affect the results, because we confirmed that a smooth cutoff appeared when the number of iterations increased in the Kobe city map, as seen in the last section. The exponent was fitted to be −1.4 without iterations and −1.1 with iterations, which is the same value with non-uniform OD.

Summary and discussion

In this paper, we performed an urban traffic simulation using a digitized map of the actual Kobe city road network. Power-law behavior appeared in the traffic distribution when using a relatively realistic OD set with imitated background traffic. When applying route iterations, a smooth cutoff appeared at the right end of the traffic distribution, while the left half of the distribution kept obeying a power-law with a larger exponent than without iterations.

To determine what type of factor caused the power-law, we also performed simulations with uniform OD distribution over Kobe city, random and grid networks. The traffic distribution continued to obey a power-law for Kobe city and the random road network, whereas it did not for the grid network. For Kobe city, the power-law exponent was fitted to be the same value with different ODs and with or without iterations.

The results that showed that the power-law did not appear in the grid map suggest that one of the causes of the power-law behavior is the road network structure. Additionally, the power-law distribution appeared clearly without any iterations, as shown in Fig. 3, which suggests that choosing the shortest routes in the network caused the power-law distribution, while the interactions among vehicles
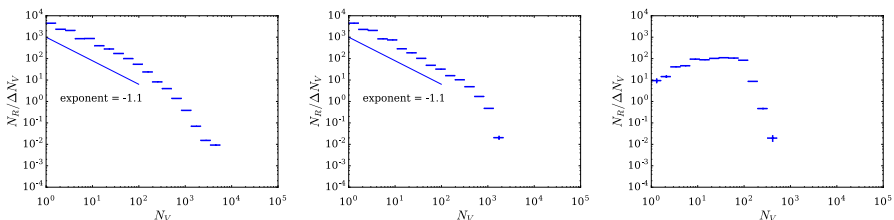

Fig. 5 Same as Fig. 3, but for Kobe city (left), random map (middle) and grid map (right). OD is uniformly distributed across each entire map, and the results after five iterations are shown. For Kobe city and the random map, a power-law distribution appeared, but this was not the case for the grid map. The exponents were fitted to be −1.1 both in Kobe city and the random map, after five iterations

are not likely to be a reason. Furthermore, the result that the power-law exponent did not change with different ODs implies that the exponent is characterized by the statistics of the road network structure. The power-law behavior that we observed is expected to be confirmed in actual traffic distributions extracted from big data.

In realistic road networks, because of asymmetric length or connection in the network, there is always a shortest route from the origin to a destination, unlike a grid network that has numerous alternative routes that have exactly the same travel distance. It is also natural to consider that there are common road segments that are frequently included in such shortest routes, such as highways and bypasses. With the exception of the grid network, these bypass segments are limited in number, and gather traffic volumes, which make them appear at the right end of the traffic distribution. Conversely, alternative segments that are larger in number share the traffic volume and appear at the left end of the traffic distribution. This inverse proportion shall appear as a power-law in the traffic distribution, with an exponent similar to -1.0. To verify this speculation, network analysis is required to determine the shortest paths exhaustively over actual cities and artificially generated road networks, which we will perform in future research.

The same power-law exponent with different ODs also implies that the exponent is independent of the simulation scale. The difference between a uniform and non-uniform OD distribution in Kobe city was the existence of imitated background traffic from outside the city. Even for the uniform OD, area 3 in Kobe city had background traffic from area 2 and area 4 in Fig. 1, for example, which is a similar scenario to an entire map with background traffic. Therefore, considering a larger amount of background traffic corresponds to considering a smaller simulation scale. This resembles the scale-free feature of fractal geometry, which also implies that the fractalness of actual road networks caused the power-law. Therefore, the comparison of simulations using a planned or self-organized road network of actual cities shall be another step in future research to determine the origin of the power-law behavior.

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