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CORRECTION

Correction to: Majorization Results for Subclasses of Starlike Functions Based on the Sine and Cosine Functions

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Very recently, Tang et al. (original article) have studied two majorization results for the subclasses S_s^* and S_c^* of starlike functions, which are, respectively, connected with the sine and cosine functions. Now, we correct the definition of the class S_c^* and the corresponding majorization result (Theorem 2.2 in the original article).

Definition and main result

In the original version of this article, the authors introduced the following class S_c^*:

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$$S_c^* := \left\{ f : f \in \mathcal{A} \quad \text{and} \quad \frac{zf'(z)}{f(z)} \prec 1 + \cos z \right\}. \tag{1.5}$$

and obtained the corresponding majorization result (Theorem 2.2 in the original article).

Theorem 2.2 *Let the function $f \in \mathcal{A}$ and suppose that $g \in S_c^*$. If $f(z) \ll g(z)$, then, for $|z| \leqq r_2$,*

$$|f'(z)| \leqq |g'(z)|,$$

where r_2 is the smallest positive root of the following equation:

$$(1 - r^2)(1 - \cosh r) - 2r = 0. \tag{2.8}$$

Unfortunately, the authors found some minor mistakes (the definition of the class S_c^* and Theorem 2 in the original article) and correct them as follows.

1. The definition of the class S_c^* is corrected as

$$S_c^* := \left\{ f : f \in \mathcal{A} \quad \text{and} \quad \frac{zf'(z)}{f(z)} \prec \cos z \right\}. \tag{1.5$'$}$$

2. Base on the above definition (1.5′), we have rewritten (Theorem 2.2 in the original article) as Theorem 2.2′ and give the proof.

Theorem 2.2′ *Let the function $f \in \mathcal{A}$ and suppose that $g \in S_c^*$. If $f(z) \ll g(z)$, then, for $|z| \leqq r_2'$,*

$$|f'(z)| \leqq |g'(z)|,$$

where r_2' is the smallest positive root of the following equation:

$$(1 - r^2)\cos r - 2r = 0. \tag{2.8$'$}$$

Proof Since $g \in S_c^*$, from (1.5′) and the subordination relationship, we have

$$\frac{zg'(z)}{g(z)} = \cos\left(\omega(z)\right), \tag{2.9$'$}$$

where $\omega(z)$ is defined as (2.2) of Theorem 1 in the original article.

Similar to (2.3) (see Theorem 1 in the original article), we easily verify that

$$\cos r \leqq \cos R \leqq |\cos(\omega(z))| = |\cos(\mathrm{Re}^{it})| \leqq \cosh R \leqq \cosh r, \tag{2.10$'$}$$

where $\omega(z) = \mathrm{Re}^{it}$ with $R \leqq |z| = r < 1$ and $-\pi \leqq t \leqq \pi$.

Combining (2.9′) and (2.10′), it is easy to get that

$$\left|\frac{g(z)}{g'(z)}\right| \leqq \frac{r}{\cos r}. \tag{2.11′}$$

Applying (2.6) (see Theorem 1 in the original article) as well as (2.11′) in (2.5), (see Theorem 1 in the original article), we immediately show that

$$|f'(z)| \leqq \left[|\varphi(z)| + \frac{1-|\varphi(z)|^2}{1-r^2} \cdot \frac{r}{\cos r}\right]|g'(z)|. \tag{2.12′}$$

Next, according to (2.7) (see Theorem 1 in the original article) and just as the proof of Theorem 1 in the orignal article, we can deduce the required result (2.8′). Hence we have completed the proof of Theorem 2.2′. □

Remark In the original version of this paper, replacing $\varphi(z)=1+\cos z$ by $\varphi(z)=\cos z$ in the introduction (Page 4 in the original article), and the concluding remarks (Page 7 in the original article).

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