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An L^2 approach to viscous flow in the half space with free elastic surface

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Abstract
We consider a linearized fluid-structure interaction problem, namely the flow of an incompressible viscous fluid in the half space $\mathbb{R}^n_+$ such that on the lower boundary a function h satisfying an undamped Kirchhoff-type plate equation is coupled to the flow field. Originally, h describes the height of an underlying nonlinear free surface problem. Since the plate equation contains no damping term, this article uses L^2-theory yielding the existence of strong solutions on finite time intervals in the framework of homogeneous Bessel potential spaces. The proof is based on L^2-Fourier analysis and also deals with inhomogeneous boundary data of the velocity field on the "free boundary" $x_n = 0$.

Keywords Undamped Kirchhoff-type plate equation · free surface flow · linearized model

Mathematics Subject Classification 35Q30 · 35L30 · 76D05

1 Fluid-structure interaction

1.1 Introduction

We consider the following linear, coupled system

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$$
\begin{aligned}
\partial_t u - \mu\Delta u + \nabla p &= f && \text{in } \mathbb{R}^n_+ \times (0,T) \\
\mathrm{div} u &= 0 && \text{in } \mathbb{R}^n_+ \times (0,T) \\
u|_{x_n=0} &= (0,...,0,\partial_t h + g_1) && \text{on } \mathbb{R}^{n-1} \times (0,T) \\
\partial_t^2 h + \Delta^2 h &= -p + g_2 && \text{on } \mathbb{R}^{n-1} \times (0,T) \\
u|_{t=0} &= u_0 && \text{in } \mathbb{R}^n_+ \\
(h,\partial_t h)|_{t=0} &= (h_0,(u_0)_n) && \text{in } \mathbb{R}^{n-1},
\end{aligned}
\tag{1.1}
$$

where u denotes the velocity of an incompressible, viscous fluid in $\mathbb{R}^n_+$ and p its pressure coupled with a boundary function h satisfying an undamped Kirchhoff-type plate equation. This problem is a linearised version of the following more abstract system of equations

$$
\begin{aligned}
\partial_t u - \mathrm{div}(2\mu D(u) - pI) &= f - (u\cdot\nabla)u && \text{in } \Omega_t \times (0,T) \\
\mathrm{div} u &= 0 && \text{in } \Omega_t \times (0,T) \\
u &= (0,...,0,\partial_t h + g_1) && \text{on } \Gamma_t \times (0,T) \\
\partial_t^2 h + \Delta^2 h &= F + g_2 && \text{on } \Gamma_t \times (0,T) \\
u &= u_0 && \text{in } \Omega_0 \\
(h,\partial_t h)|_{t=0} &= (h_0,(u_0)_n) && \text{in } \Gamma_0
\end{aligned}
$$

in a time-depending domain $\Omega_t = \{x = (x_1,...,x_n) \in \mathbb{R}^n : x_n > h(x',t)\}$, $t \in (0,T)$, with lower free boundary, $\Gamma_t = \{x = (x',x_n) : x_n = h(x',t)$. Here, $D(u) = \frac{1}{2}(\nabla u + (\nabla u)^\top)$ is the deformation tensor, and

$$
F := \langle (2\mu D(u) - pI)n_t, -e_n\rangle, \quad n_t = \frac{(\nabla' h, -1)}{\sqrt{1+|\nabla' h|^2}},
$$

denotes the force exerted by the incompressible fluid in Ω_t onto the free boundary Γ_t with exterior normal n_t pointing downward. Moreover, at $t = 0$,

$$
\Omega_0 := \{x \in \mathbb{R}^n : x_n > h_0(x')\}, \quad \Gamma_0 := \{x \in \mathbb{R}^n : x_n = h_0(x')\}.
$$

Additionally, a body force g_2 exerts a force from the outside onto the boundary. Hence, the boundary is moving and its motion is tracked by the scalar function h describing the height of the boundary. On the boundary the fluid velocity, u, coincides with the velocity of the boundary, *i.e.,* we assume the no-slip condition for its tangential part whereas its normal component coincides with $\partial_t h$ (kinematic boundary condition); furthermore, for mathematical generality, we include an additional force g_1 satisfying the compatibility condition $g_1|_{t=0} = 0$, which also models the phenomenon of leaking. In the domain Ω_t, the motion of the fluid is described by the Navier-Stokes equations with constant viscosity $\mu > 0$.

The linearised version with fixed boundary is obtained after transforming the equations to a problem in $\mathbb{R}^n_+$ and ignoring all nonlinear terms appearing in this way as well as the convective term $(u\cdot\nabla)u$. The first part of the term F vanishes due to $\mathrm{div} u = 0$ and $u' = (u_1,\dots,u_{n-1}) = 0$ on $\partial\mathbb{R}^n_+$ so that also $\partial u_n/\partial x_n = 0$

on $\partial\mathbb{R}^n_+$, and only the pressure term remains, see $(1.1)_4$. For more details on this transformation see [8, Chapter 2], [6] and [11].

There is a lot of literature concerning fluids interacting with an elastic plate. Often an additional damping term like $-\partial_t\Delta h$ is introduced. This term allows the equation to have maximal L^p–regularity as considered by Denk and Saal, see [8], and [5, Chapter 4] in a space-time periodic setting. Weak solutions in L^2-spaces are constructed by Chambolle *et al.* [6] when adding a viscous damping term of either the form $\Delta^2\partial_t h$ or $-\Delta\partial_{tt}h$ in the plate equation. Similar results have been shown by Grandmont [11] in the case of a three-dimensional cavity with one part of the boundary being elastic and the other one being rigid; weak solutions are shown to exist until a possible time where intersections of the two boundary parts occur. However, the focus in [11] is on the behavior of solutions in the limit of a vanishing damping term and on a uniform positive lower bound of the time of existence.

Fluid structure interaction problems with a classical nonlinear von Kármán shallow shell allowing for both transversal and lateral displacements are considered by Chueshov and Ryzhkova [7]. Lengeler and Růžička [13] discussed the case of a linearly elastic Koiter shell instead of a flat plate and hence replaced the operator Δ^2 by an operator better suited for non-flat boundaries.

In the case of a bounded domain and no damping we still have the existence of a contraction semigroup, see Badra and Takahashi [1, Proposition 3.4]. In [4] Casanova, Grandmont and Hillairet construct weak solutions in a 2D periodic layer-type domain. For local-in-time strong solutions in this latter setting we refer to Beiraõ da Veiga [3]. Moreover, strong solutions are found by Badra and Takahashi [2] using a non-analytic semigroup of Gevrey class. Finally, in [10] the present authors construct weak solutions to the fluid-structure interaction problem of a viscous fluid coupled with a damped elastic plate under the nonlinear Coulomb boundary friction condition.

However, in our case of an unbounded domain even the existence of an L^2 -semigroup is doubtful, which is why we will make more basic considerations to solve the equation in the case of non-vanishing initial data. The methods used here admit a solution in the case $p = 2$ only since the corresponding multipliers are bounded but they are not Fourier multipliers for $p \neq 2$. Also note that although we show the existence of solutions on any finite time interval, we must exclude the case $T = \infty$ since the Fourier transform of a solution of the undamped plate equation, $\partial_t^2 h + \Delta^2 h = 0$, is given by terms involving cosine and sine functions which are not L^2–integrable on $(0, \infty)$. Adding a damping term as in [8], this issue is solved and allows the use of solution spaces with exponential weights. Additionally, in [8] the damping term also guarantees that the solution belongs to an inhomogeneous Sobolev space, which is not possible in the present undamped case.

This work is structured as follows: First we introduce the relevant solution spaces. Secondly, we show existence of solutions in the case of vanishing initial data using partial Fourier transforms. Finally, we reconstruct the initial data. Here we do not use abstract semigroup theory, as usually done, but the specific form of the undamped plate equation.

1.2 Solution spaces

One drawback of working with an undamped plate equation is that - unlike in [8] - we no longer obtain solutions $u \in L^2(0,T;H^2(\mathbb{R}^n_+))$ where $H^2(\mathbb{R}^n_+))$ denotes the usual inhomogeneous Sobolev space of order 2 over L^2; rather, we have to work in homogeneous spaces, *i.e.*, $u \in L^2(0,T;\dot{H}^2(\mathbb{R}^n_+))$. Since there are non-equivalent definitions of homogeneous Sobolev spaces and Bessel potential spaces, we choose a notion that fits well with our method of choice to obtain solutions, namely via the partial Fourier transform.

Let $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier transform and its inverse, respectively. Elements of $\mathbb{R}^n$ are often written in the form $x = (x', x_n)$ with $x' \in \mathbb{R}^{n-1}$, $x_n \in \mathbb{R}$, and the phase variable ξ as (ξ', ξ_n). Similarly, vector fields $u = (u_1, \dots, u_n)$ are splitted into $u' = (u_1, \dots, u_{n-1})$ and u_n; in particular, we write $0' = (0, \dots, 0) \in \mathbb{R}^{n-1}$.

Definition 1.1 Given $s \in \mathbb{R}$ we define the homogeneous Sobolev space $\dot{H}^s(\mathbb{R}^n)$ as

$$\dot{H}^s(\mathbb{R}^n) := \{u \in Z'(\mathbb{R}^n) : |\xi|^s \hat{u} \in L^2(\mathbb{R}^n)\},$$

equipped with the norm

$$\|u\|_{\dot{H}^s(\mathbb{R}^n)} = \| \mathcal{F}^{-1}(|\xi|^s \hat{u})\|_{L^2(\mathbb{R}^n)}.$$

Here $Z'(\mathbb{R}^n) := \mathcal{S}'(\mathbb{R}^n)/P(\mathbb{R}^n)$ where $P(\mathbb{R}^n)$ denotes the space of all polynomials on $\mathbb{R}^n$ and $\mathcal{S}'(\mathbb{R}^n)$ denotes the set of Schwartz distributions. For the domain $\mathbb{R}^n_+$ we define the homogeneous Sobolev space $\dot{H}^s(\mathbb{R}^n_+)$ via restriction:

$$\dot{H}^s(\mathbb{R}^n_+) := \left\{u|_{\mathbb{R}^n_+} : u \in \dot{H}^s(\mathbb{R}^n)\right\},$$
$$\|u\|_{\dot{H}^s(\mathbb{R}^n_+)} := \inf\left\{\|v\|_{\dot{H}^s(\mathbb{R}^n)} : v|_{\mathbb{R}^n_+} = u\right\}.$$

We have the following useful properties that we will frequently use.

Proposition 1.2 *Let $s \in \mathbb{R}$ and let $\Lambda = (-\Delta)^{1/2}$ denote the operator defined by $\Lambda u = \mathcal{F}^{-1}(|\xi|\mathcal{F}u)$.*

i) $\dot{H}^0(\mathbb{R}^n) = L^2(\mathbb{R}^n)$.
ii) The mapping $\Lambda^\omega \mathcal{F} : \dot{H}^s(\mathbb{R}^n) \to \dot{H}^{s-\omega}(\mathbb{R}^n)$ is an isomorphism and maps $Z'(\mathbb{R}^n)$ onto itself for all $\omega \in \mathbb{R}$.
iii) If $s > \frac{1}{2}$ then there is a bounded linear mapping $\mathrm{tr} : \dot{H}^s(\mathbb{R}^n_+) \to \dot{H}^{s-\frac{1}{2}}(\partial\mathbb{R}^n_+) \cong \dot{H}^{s-\frac{1}{2}}(\mathbb{R}^{n-1})$, the trace operator, such that $\mathrm{tr}(\phi)(x') = \phi(x', 0)$ for all $\phi \in \mathcal{S}(\mathbb{R}^n)$ with $0 \notin \mathrm{supp}\mathcal{F}(\phi)$.

For a proof of iii) see Theorem 2.1 in [12]. For i) and ii) as well as for more information about homogeneous spaces see [14, Chapter 5].

Concerning Bochner spaces with respect to time over a Banach space X we write $b \in \dot{H}^k(0,T;X)$ meaning that $\partial_t^k b \in L^2(0,T;X)$ for $k \in \mathbb{N}$, $T \in (0,\infty]$. In most cases, X will be a homogeneous Bessel potential space $\dot{H}^s(\Omega)$ where $\Omega = \mathbb{R}^n, \mathbb{R}^n_+$ or $\Gamma := \mathbb{R}^{n-1} = \partial\mathbb{R}^n_+$. Since the Fourier transform is used only on $\mathbb{R}^{n-1}$, it will, for the sake of simplicity, be denoted by $\mathcal{F}$ rather than $\mathcal{F}'$, its inverse by $\mathcal{F}^{-1}$ rather than $(\mathcal{F}')^{-1}$. However, the phase variable still is ξ'. For the one-dimensional Fourier transform with respect to time the phase variable will be called τ.

1.3 Main results

Now we can formulate the main result on existence of solutions. Recall that $\Gamma = \mathbb{R}^{n-1} = \partial\mathbb{R}^n_+$.

Theorem 1.3 *Let* $T \in (0, \infty)$ *and*

$$\begin{aligned} &u_0 \in \dot{H}^2\big(\mathbb{R}^n_+\big),\ \operatorname{div} u_0 = 0,\ u_0'|_{\partial\mathbb{R}^n_+} = 0, \quad f \in L^2((0,T)\times\mathbb{R}^n_+),\\ &g_1 \in L^2(0,T;\dot{H}^{3/2}(\Gamma)) \cap \dot{H}^1(0,T;\dot{H}^{-1/2}(\Gamma)),\ g_1(0) = 0,\\ &g_2 \in L^2(0,T;\dot{H}^{1/2}(\Gamma)), \quad h_0 \in \dot{H}^{7/2}(\Gamma) \end{aligned}$$

be given. Then the system

$$\begin{aligned} \partial_t u - \mu\Delta u + \nabla p &= f && \text{in } \mathbb{R}^n_+ \times (0,T)\\ \operatorname{div} u &= 0 && \text{in } \mathbb{R}^n_+ \times (0,T)\\ (u', u_n)|_{x_n=0} &= (0', \partial_t h + g_1) && \text{on } \Gamma \times (0,T)\\ \partial_t^2 h + \Delta^2 h &= -p + g_2 && \text{on } \Gamma \times (0,T)\\ u(0) &= u_0 && \text{in } \mathbb{R}^n_+\\ (h(0), \partial_t h(0)) &= (h_0, (u_0)_n) && \text{in } \Gamma \end{aligned} \tag{1.2}$$

admits a solution

$$\begin{aligned} &u \in L^2(0,T;\dot{H}^2(\mathbb{R}^n_+)) \cap \dot{H}^1(0,T;L^2(\mathbb{R}^n_+)),\\ &p \in L^2(0,T;\dot{H}^1(\mathbb{R}^n_+)),\\ &h \in \dot{H}^1(0,T;\dot{H}^{3/2}(\Gamma)) \cap \dot{H}^2(0,T;\dot{H}^{-1/2}(\Gamma)). \end{aligned}$$

Furthermore, we have the estimate

$$\begin{aligned} &\|u\|_{L^2(0,T;\dot{H}^2(\mathbb{R}^n_+))\cap\dot{H}^1(0,T;L^2(\mathbb{R}^n_+))} + \|p\|_{L^2(0,T;\dot{H}^1(\mathbb{R}^n_+))}\\ &\quad + \|h\|_{\dot{H}^1(0,T;\dot{H}^{3/2}(\Gamma))\cap\dot{H}^2(0,T;\dot{H}^{-1/2}(\Gamma))}\\ &\le C\big(\|f\|_{L^2((0,T)\times\mathbb{R}^n_+)} + \|g_1\|_{L^2(0,T;\dot{H}^{3/2}(\Gamma))\cap\dot{H}^1(0,T;\dot{H}^{-1/2}(\Gamma))}\\ &\quad + \|g_2\|_{L^2(0,T;\dot{H}^{1/2}(\Gamma))}\big) + CT^{1/2}\big(\|u_0\|_{\dot{H}^2(\mathbb{R}^n_+)} + \|h_0\|_{\dot{H}^{7/2}(\Gamma)}\big). \end{aligned} \tag{1.3}$$

Note that $T \in (0, \infty)$ is arbitrary, but that C in (1.3) is independent of T.

2 Proofs

2.1 The case of vanishing initial data

First we assume $f=0$ in $(1.1)_1$ as well as homogeneous initial data in $(1.1)_5$, *i.e.*, $u(0)=u_0=0$. Since $g_1(0)=0$ we extend the functions $g_1 \in L^2(0,\infty;\dot{H}^{3/2}(\Gamma)) \cap \dot{H}^1(0,\infty;\dot{H}^{-1/2}(\Gamma))$ and $g_2 \in L^2(0,\infty;\dot{H}^{1/2}(\Gamma))$ in time by zero to corresponding functions defined on $\mathbb{R}$. Then we apply the Fourier transform with respect to time and $x' \in \mathbb{R}^{n-1}$ to get for $v=\hat{u}$ and $\hat{h}$ the system

$$\begin{aligned}
(i\tau + \mu|\xi'|^2)v' - \mu\partial_n^2 v' + i\xi'\hat{p} &= 0 && \text{in } \mathbb{R}\times\mathbb{R}^n_+\\
(i\tau + \mu|\xi'|^2)v_n - \mu\partial_n^2 v_n + \partial_n\hat{p} &= 0 && \text{in } \mathbb{R}\times\mathbb{R}^n_+\\
i\xi'\cdot v' + \partial_n v_n &= 0 && \text{in } \mathbb{R}\times\mathbb{R}^n_+\\
(v', v_n)|_{x_n=0} &= (0', i\tau\hat{h} + \hat{g}_1) && \text{on } \mathbb{R}\times\Gamma\\
(-\tau^2 + |\xi'|^4)\hat{h} &= -\hat{p}|_{x_n=0} + \hat{g}_2 && \text{on } \mathbb{R}\times\Gamma.
\end{aligned} \tag{2.1}$$

From $(2.1)_{1,2,3}$ we deduce $\Delta p = 0$ or, in other words,

$$\partial_n^2\hat{p} - |\xi'|^2\hat{p} = 0,$$

which is solved in $\mathcal{S}'(\mathbb{R}^{n-1})$ by

$$\hat{p} = \gamma e^{-|\xi'|x_n} \qquad \text{for } a.a.\ x_n > 0 \tag{2.2}$$

with a function $\gamma = \gamma(\xi',\tau) \in \mathbb{C}$. Now we can solve $(2.1)_1$ for v'; the generic solution is given by

$$\begin{aligned}
v' &= \alpha' e^{-Bx_n} - \frac{\xi'\gamma}{\tau}e^{-Ax_n}, \quad \alpha' \in \mathbb{C}^{n-1},\\
A &= |\xi'|,\ \ B := \sqrt{i\tau\mu^{-1} + |\xi'|^2}\ \ (\mathrm{Re}B > 0).
\end{aligned} \tag{2.3}$$

Indeed,

$$\begin{aligned}
(i\tau + \mu|\xi'|^2)e^{-Bx_n} - \mu\partial_n^2 e^{-Bx_n} &= (i\tau + \mu|\xi'|^2)e^{-Bx_n} - \mu B^2 e^{-Bx_n} = 0,\\
((i\tau + \mu|\xi'|^2) - \mu|\xi'|^2)\tfrac{-\xi'\gamma}{\tau}e^{-Ax_n} &= -i\xi'\gamma e^{-Ax_n} = -i\xi'\hat{p}.
\end{aligned}$$

Concerning α' $(2.1)_4$ implies that

$$\alpha' = \frac{\xi'\gamma}{\tau}.$$

So for now we have

$$v' = \frac{\xi'\gamma}{\tau}(e^{-Bx_n} - e^{-Ax_n}). \tag{2.4}$$

Using this representation we consider $(2.1)_3$ to determine $\partial_n v_n$:

$$\partial_n v_n = -i\xi' \cdot v' = |\xi'|^2 \frac{\gamma}{i\tau}(e^{-Bx_n} - e^{-Ax_n}).$$

This implies

$$\partial_n^2 v_n = A^2 \frac{\gamma}{i\tau}(-Be^{-Bx_n} + Ae^{-Ax_n}).$$

We exploit this representation to solve $(2.1)_2$ and get that

$$v_n = \frac{1}{i\tau + \mu A^2}(-\partial_n \hat{p} + \mu \partial_n^2 v_n) = A\frac{\gamma}{i\tau}e^{-Ax_n} - \frac{1}{B}\frac{\gamma}{i\tau}A^2 e^{-Bx_n}. \tag{2.5}$$

In particular, $v_n|_{x_n=0} = \frac{\gamma A}{i\tau} - \frac{\gamma}{i\tau B}A^2$. Now we determine two identities for $\hat{h}$ by using $(2.1)_4$ and $(2.1)_5$:

$$\begin{aligned} \hat{h} &= \frac{1}{i\tau}(-\hat{g}_1 + v_n|_{x_n=0}) = \frac{-\hat{g}_1}{i\tau} - \frac{\gamma A}{\tau^2} + \frac{\gamma A^2}{\tau^2 B}, \\ \hat{h} &= \frac{-\hat{p}|_{x_n=0} + \hat{g}_2}{A^4 - \tau^2} = \frac{-\gamma + \hat{g}_2}{A^4 - \tau^2}. \end{aligned} \tag{2.6}$$

Combining these identities we solve for the still unknown term γ and see that

$$\gamma\left(-\frac{A}{\tau^2} + \frac{A^2}{B\tau^2} + \frac{1}{A^4 - \tau^2}\right) = \frac{\hat{g}_2}{A^4 - \tau^2} + \frac{\hat{g}_1}{i\tau}.$$

We define

$$\mathcal{N}(A, \tau) := \left(A^4 - \tau^2 - \frac{\tau^2}{A} + \mu i\tau A + \mu i\tau B\right)^{-1} \tag{2.7}$$

and use the identity

$$\tfrac{-\tau^2 B}{(B-A)A} \cdot (A^4 - \tau^2)\left(\left\{-\tfrac{A}{\tau^2} + \tfrac{A^2}{B\tau^2}\right\} + \tfrac{1}{A^4-\tau^2}\right) = (A^4 - \tau^2) - \tfrac{\tau^2 B}{(B-A)A}$$

where $\frac{-\tau^2 B}{(B-A)A} = -\frac{\tau^2}{A} + \mu i\tau(A + B)$. Now we conclude that

$$\gamma = \mathcal{N}(A, \tau)\frac{-\tau^2 B}{(B - A)A}\left(\hat{g}_2 + \frac{A^4 - \tau^2}{i\tau}\hat{g}_1\right) \tag{2.8}$$

$$= \mathcal{N}(A, \tau)\left(\frac{-\tau^2}{A} + \mu i\tau A + \mu i\tau B\right)\left(\hat{g}_2 + \frac{A^4 - \tau^2}{i\tau}\hat{g}_1\right). \tag{2.9}$$

Summarizing (2.2), (2.4), (2.5), (2.6), (2.8) and (2.9) we obtain a solution $\hat{p}, v = \hat{u}, \hat{h}$ in Fourier space by

$$
\begin{aligned}
\hat{p} &= \gamma e^{-Ax_n} \overset{(2.9)}{=} \left(\frac{-\tau^2}{A} + \mu i\tau A + \mu i\tau B\right)\mathcal{N}(A,\tau)\left(\hat{g}_2 + \frac{A^4-\tau^2}{i\tau}\hat{g}_1\right)e^{-Ax_n},\\
v' &= \frac{\xi'\gamma}{\tau}(e^{-Bx_n} - e^{-Ax_n})\\
&= \frac{i\xi'}{A}\frac{i\tau B}{A}\mathcal{N}(A,\tau)\left(\hat{g}_2 + \frac{A^4-\tau^2}{i\tau}\hat{g}_1\right)A\frac{e^{-Bx_n} - e^{-Ax_n}}{B-A},\\
v_n &= -\frac{i\tau B}{A}\mathcal{N}(A,\tau)\left(\hat{g}_2 + \frac{A^4-\tau^2}{i\tau}\hat{g}_1\right)A\frac{e^{-Bx_n} - e^{-Ax_n}}{B-A}\\
&\quad + i\tau\mathcal{N}(A,\tau)\left(\hat{g}_2 + \frac{A^4-\tau^2}{i\tau}\hat{g}_1\right)e^{-Bx_n},\\
\hat{h} &= \frac{-\hat{g}_1}{i\tau} - \frac{\gamma A}{\tau^2} + \frac{\gamma A^2}{\tau^2 B} = \mathcal{N}(A,\tau)\hat{g}_2 + \big((A^4-\tau^2)\mathcal{N}(A,\tau) - 1\big)\frac{1}{i\tau}\hat{g}_1.
\end{aligned}
\tag{2.10}
$$

Lemma 2.1 *The functions*

$$
\tau A\mathcal{N}(A,\tau),\quad \frac{\tau^2}{A}\mathcal{N}(A,\tau),\quad \tau^{3/2}\mathcal{N}(A,\tau),\quad (A^4-\tau^2)\mathcal{N}(A,\tau),\quad \tau B\mathcal{N}(A,\tau)
$$

are uniformly bounded with respect to $(A,\tau) \in (0,\infty)\times\mathbb{R}$.

Proof We note that $B = \mathrm{Re}B + \frac{i\tau}{2\mu\mathrm{Re}B}$ with $\mathrm{Re}B > 0$ and decompose $\mathcal{N}(A,\tau)^{-1}$ into its real and imaginary part:

$$
\begin{aligned}
\mathcal{N}(A,\tau)^{-1} &= A^4 - \tau^2 - \frac{\tau^2}{A} + \mu i\tau B + \mu i\tau A\\
&= A^4 - \tau^2\left(1 + \frac{1}{A} + \frac{1}{2\mathrm{Re}B}\right) + \mu i\tau(\mathrm{Re}B + A).
\end{aligned}
$$

First we prove that $\tau A\mathcal{N}(A,\tau)$ is bounded:

$$
|\tau A\mathcal{N}(A,\tau)| \le \frac{|\tau|A}{|\mathrm{Im}(\mathcal{N}(A,\tau)^{-1})|} \le \frac{|\tau|A}{|\tau|\mu(\mathrm{Re}B + A)} \le \frac{A}{\mu A} = \frac{1}{\mu}.
$$

Next we consider $\frac{\tau^2}{A}\mathcal{N}(A,\tau)$. If $|\tau| \le A^2$, we use the imaginary part of $\mathcal{N}(A,\tau)^{-1}$ to get as above the estimate

$$
\frac{\tau^2}{A}|\mathcal{N}(A,\tau)| \le \frac{|\tau|A}{|\mathrm{Im}(\mathcal{N}(A,\tau)^{-1})|} \le \frac{1}{\mu}.
$$

However, if $|\tau| \ge A^2$, we use the real part of $\mathcal{N}(A,\tau)^{-1}$ and are led to the estimate

$$\frac{\tau^2}{A}|\mathcal{N}(A,\tau)| \leq \frac{\tau^2}{A}\frac{1}{|\mathrm{Re}(\mathcal{N}(A,\tau)^{-1})|}$$
$$= \frac{\tau^2}{A}\frac{1}{\tau^2\left(1+\frac{1}{A}+\frac{1}{2\mathrm{Re}B}\right)-A^4} \leq \frac{\tau^2}{A}\frac{1}{\frac{\tau^2}{A}} = 1.$$

The function $\tau^{3/2}\mathcal{N}(A,\tau)$ is also bounded due to the previous considerations and Young's inequality $|\tau^{3/2}| = \left|\frac{\tau}{\sqrt{A}}\sqrt{\tau A}\right| \leq \frac{\tau^2}{2A} + \frac{|\tau|A}{2}$.

Due to $|B| \leq \mu^{-1/2}\sqrt{|\tau|} + A$ we deduce the boundedness of $\tau B\mathcal{N}(A,\tau)$ from the previous cases. Finally, the boundedness of $(A^4-\tau^2)\mathcal{N}(A,\tau)$ follows from the identity

$$(A^4-\tau^2)\mathcal{N}(A,\tau) = 1 - \left(\frac{-\tau^2}{A} + \mu i\tau A + \mu i\tau B\right)\mathcal{N}(A,\tau),$$

see (2.7), and the previous cases. □

To simplify the analysis of the multiplier functions, we reduce the question of their boundedness to a problem in $\mathbb{R}^{n-1}$ rather than in $\mathbb{R}^n_+$ by the following lemma.

Lemma 2.2

i) *Let* $f:\mathbb{R}\times\mathbb{R}^{n-1}\to\mathbb{C}$ *be measurable,* $c>0$ *and* $\chi\in\{cA, cB\}$, *see* (2.3). *Then we have the equivalence*

$$f(\tau,\xi')e^{-\chi x_n} \in L^2(\mathbb{R}\times\mathbb{R}^n_+) \iff \frac{1}{\sqrt{\mathrm{Re}\chi}}f(\tau,\xi') \in L^2(\mathbb{R}\times\mathbb{R}^{n-1})$$

with equivalent norms.

ii) *There exists a constant* $C>0$ *such that*

$$A\left|\frac{e^{-Bx_n}-e^{-Ax_n}}{B-A}\right| \leq Ce^{-\frac{1}{2}Ax_n}$$

for all $x_n, A>0$ *and* $\tau\in\mathbb{R}\backslash\{0\}$.

Proof i) The claim follows from the calculation

$$\int_{\mathbb{R}}\int_{\mathbb{R}^n_+}|f(\tau,\xi')e^{-\chi x_n}|^2\,\mathrm{d}\xi'\mathrm{d}x_n\mathrm{d}\tau$$
$$= \int_{\mathbb{R}}\int_{\mathbb{R}^{n-1}}\left(\int_0^\infty |e^{-\chi x_n}|^2 dx_n\right)|f(\tau,\xi')|^2\,\mathrm{d}\xi'\mathrm{d}\tau$$
$$= \frac{1}{2}\int_{\mathbb{R}}\int_{\mathbb{R}^{n-1}}\frac{1}{\mathrm{Re}\chi}|f(\tau,\xi')|^2\,\mathrm{d}\xi'\mathrm{d}\tau,$$

where we used Tonelli's theorem in the first identity.

ii) We calculate

$$A\left|\frac{e^{-Bx_n}-e^{-Ax_n}}{B-A}\right| = \frac{Ae^{-Ax_n}}{|B-A|}|e^{-(B-A)x_n}-1| \le \frac{Ae^{-Ax_n}}{|B-A|}|B-A|x_n$$
$$= Ax_n e^{-Ax_n} \le Ce^{-\frac{1}{2}Ax_n}.$$

In the first inequality we used that $\mathrm{Re}((B-A)x_n) > 0$. □

Proposition 2.3 *Let* $g_1 \in L^2(0,\infty;\dot H^{3/2}(\Gamma)) \cap \dot H^1(0,\infty;\dot H^{-1/2}(\Gamma))$ *with* $g_1(0)=0$ *and* $g_2 \in L^2(0,\infty;\dot H^{1/2}(\Gamma))$ *be given. Then the system*

$$\begin{aligned} \partial_t u - \mu\Delta u + \nabla p &= 0 && \text{in } \mathbb{R}^n_+ \times (0,\infty) \\ \mathrm{div} u &= 0 && \text{in } \mathbb{R}^n_+ \times (0,\infty) \\ u|_{x_n=0} &= (0', \partial_t h + g_1) && \text{on } \Gamma \times (0,\infty) \\ \partial_t^2 h + \Delta^2 h &= -p + g_2 && \text{on } \Gamma \times (0,\infty) \end{aligned}$$

together with initial conditions $u(0)=0$, $h(0)=\partial_t h(0)=0$ *has a solution*

$$\begin{aligned} &u \in \dot H^1(0,\infty;L^2(\mathbb{R}^n_+)) \cap L^2(0,\infty;\dot H^2(\mathbb{R}^n_+)), \\ &p \in L^2(0,\infty;\dot H^1(\mathbb{R}^n_+)), \\ &h \in \dot H^1(0,\infty;\dot H^{3/2}(\Gamma)) \cap \dot H^2(0,\infty;\dot H^{-1/2}(\Gamma)). \end{aligned}$$

Furthermore, we have the estimate

$$\begin{aligned} &\|u\|_{\dot H^1(0,\infty;L^2(\mathbb{R}^n_+))\cap L^2(0,\infty;\dot H^2(\mathbb{R}^n_+))} + \|p\|_{L^2(0,\infty;\dot H^1(\mathbb{R}^n_+))} \\ &\quad + \|h\|_{\dot H^1(0,\infty;\dot H^{3/2}(\Gamma))\cap \dot H^2(0,\infty;\dot H^{-1/2}(\Gamma))} \\ &\le C\big(\|g_1\|_{L^2(0,\infty;\dot H^{3/2}(\Gamma))\cap \dot H^1(0,\infty;\dot H^{-1/2}(\Gamma))} + \|g_2\|_{L^2(0,\infty;\dot H^{1/2}(\Gamma))}\big). \end{aligned}$$

The analogous statement holds if the time interval is replaced with $(0,T)$.

Proof First consider the case $g_1 = 0$ in which the solution (2.10) simplifies to

$$\begin{aligned} \hat p &= (1-\mathcal{M}(A,\tau))\hat g_2\, e^{-Ax_n}, \\ v' &= \frac{i\xi'}{A}\frac{i\tau B}{A}\mathcal{N}(A,\tau)\hat g_2\, A\frac{e^{-Bx_n}-e^{-Ax_n}}{B-A}, \\ v_n &= \frac{i\tau B}{A}\mathcal{N}(A,\tau)\hat g_2\, A\frac{e^{-Bx_n}-e^{-Ax_n}}{B-A} + i\tau\mathcal{N}(A,\tau)\hat g_2\, e^{-Bx_n}, \\ \hat h &= \mathcal{N}(A,\tau)\hat g_2. \end{aligned} \tag{2.11}$$

where by Lemma 2.1

$$\mathcal{M}(A,\tau) := \mathcal{N}(A,\tau)(A^4-\tau^2)$$

is a bounded multiplier function. By the previous two lemmata, we now show the corresponding estimates.

(i) $u' \in L^2(0,\infty;\dot H^2(\mathbb{R}^n_+))$: Concerning tangential derivatives $\partial_k\partial_i u_j$, $i,j,k \in \{1,...,n-1\}$, we estimate as follows:

$$\begin{aligned}\|\xi_k\xi_i v_j(i\tau)\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} &= \left\|\left(\frac{\xi_k}{A}\frac{\xi_i}{A}\frac{i\xi_j}{A}\right)\tau\mathcal{N}AB\,\hat g_2\,A\frac{e^{-Bx_n}-e^{-Ax_n}}{B-A}\right\|_{L^2(\mathbb{R}\times\Gamma)}\\ &\le c\left\|\tau B\mathcal{N}A\hat g_2 e^{-\frac12 Ax_n}\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} \le c\left\|\hat g_2\sqrt{A}\right\|_{L^2(\mathbb{R}\times\Gamma)}\\ &\le c\left\|g_2\right\|_{L^2(\mathbb{R},\dot H^{1/2}(\Gamma))}.\end{aligned} \tag{2.12}$$

For the normal derivatives $\partial_n^2 u'$ we have:

$$\begin{aligned}\partial_n^2 v' &= \frac{i\xi'}{A}\frac{i\tau B}{A}\mathcal{N}\hat g_2\,A\frac{B^2e^{-Bx_n}-A^2e^{-Ax_n}}{B-A}\\ &= \frac{i\xi'}{A}\frac{i\tau B}{A}\mathcal{N}\hat g_2 A\frac{(B^2-A^2)e^{-Bx_n}}{B-A} + \frac{i\xi'}{A}i\tau B\mathcal{N}A\,\hat g_2\,A\frac{e^{-Bx_n}-e^{-Ax_n}}{B-A}.\end{aligned}$$

The second summand can be treated as above. Hence we only have to consider the first one:

$$\begin{aligned}&\left\|\frac{i\xi'}{A}i\tau B\mathcal{N}\hat g_2(B+A)e^{-Bx_n}\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} \le c\left\|\tau B\mathcal{N}\hat g_2 B\frac{1}{\sqrt{\mathrm{Re}B}}\right\|_{L^2(\mathbb{R}\times\Gamma)}\\ &\le c\left\|\frac{\tau B^2\mathcal{N}}{A}\hat g_2\sqrt{A}\right\|_{L^2(\mathbb{R}\times\Gamma)} = c\left\|\mathcal{N}\left(\frac{-\tau^2\mu^{-1}}{A}+i\tau A\right)(\hat g_2\sqrt{A})\right\|_{L^2(\mathbb{R}\times\Gamma)}\\ &\le c\left\|g_2\right\|_{L^2(\mathbb{R},\dot H^{1/2}(\Gamma))}.\end{aligned} \tag{2.13}$$

For mixed derivatives $\partial_k\partial_n u_j$, $j,k \in \{1,...,n-1\}$ we have that

$$\begin{aligned}i\xi_k\partial_n v_j &= \frac{i\xi_k}{A}\frac{i\xi_j}{A}i\tau B\mathcal{N}\hat g_2 A\frac{-Be^{-Bx_n}+Ae^{-Ax_n}}{B-A}\\ &= \frac{i\xi_k}{A}\frac{i\xi_j}{A}i\tau B\mathcal{N}\hat g_2 A\frac{(A-B)e^{-Bx_n}}{B-A} - \frac{i\xi_k}{A}\frac{i\xi_j}{A}i\tau BA\mathcal{N}\hat g_2 A\frac{e^{-Bx_n}-e^{-Ax_n}}{B-A}.\end{aligned}$$

The first summand can be treated as in (2.13), the second one as in (2.12).

(ii) $u_n \in L^2(0,\infty;\dot H^2(\mathbb{R}^n_+))$: Since $\operatorname{div} u = 0$ and thus $\partial_k\partial_n u_n = -\sum_{j=1}^{n-1}\partial_k\partial_j u_j \in L^2((0,\infty)\times\mathbb{R}^n_+)$ as shown in (i), it is left to consider $\partial_k\partial_i u_n$ for $i,k \neq n$. Moreover, the first summand of v_n can be treated in the same way as v', see (2.11). Therefore, we only have to discuss the second one:

$$\begin{aligned}\|\xi_k\xi_i\tau\mathcal{N}\hat g_2 e^{-Bx_n}\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} &\le c\left\|\tau A^2\mathcal{N}\hat g_2\frac{1}{\sqrt{\mathrm{Re}B}}\right\|_{L^2(\mathbb{R}\times\Gamma)}\\ &\le c\left\|\tau A\mathcal{N}\hat g_2\sqrt{A}\right\|_{L^2(\mathbb{R}\times\Gamma)} \le c\left\|g_2\right\|_{L^2(\mathbb{R},\dot H^{1/2}(\Gamma))}.\end{aligned}$$

(iii) $p \in L^2(0,\infty;\dot H^1(\mathbb{R}^n_+))$: Since $\partial_n\hat p = -A\hat p$, it suffices to consider the tangential derivatives $\nabla' p$ where, since $\mathcal{M}$ is bounded, we get that

$$
\begin{aligned}
\|\xi'\hat{p}\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} &= \left\|\xi'(1-\mathcal{M})\hat{g}_2 e^{-Ax_n}\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} \\
&\le c\|(\hat{g}_2\sqrt{A})\|_{L^2(\mathbb{R}\times\Gamma)} \\
&\le c\|g_2\|_{L^2(\mathbb{R},\dot{H}^{1/2}(\Gamma))}.
\end{aligned}
$$

(iv) $h \in \dot{H}^1(0,\infty;\dot{H}^{3/2}(\Gamma)) \cap \dot{H}^2(0,\infty;\dot{H}^{-1/2}(\Gamma))$: This follows immediately from $\hat{h} = \frac{\mathcal{N}}{\sqrt{A}}(\hat{g}_2\sqrt{A})$ and the boundedness of $\tau A\mathcal{N}$ and $\frac{\tau^2}{A}\mathcal{N}$.

(v) $u \in \dot{H}^1(0,\infty;L^2(\mathbb{R}^n_+))$: The estimate follows from $\partial_t u = \Delta u - \nabla p \in L^2((0,\infty)\times\mathbb{R}^n_+)$.

If on the other hand $g_2 = 0$, but

$$
0 \ne g_1 \in L^2(0,\infty;\dot{H}^{3/2}(\Gamma)) \cap \dot{H}^1(0,\infty;\dot{H}^{-1/2}(\Gamma)), \quad g_1(0) = 0,
$$

then we make use of the boundedness of $\mathcal{M}(A,\tau) := \mathcal{N}(A,\tau)(A^4 - \tau^2)$, see Lemma 2.1. Consider the set of solutions with $g_2 = 0$, see (2.10):

$$
\begin{aligned}
\hat{p} &= \left(\frac{i\tau}{A} + \mu A + \mu B\right)\mathcal{M}(A,\tau)\hat{g}_1 e^{-Ax_n}, \\
v' &= \frac{i\xi'}{A}\frac{B}{A}\mathcal{M}(A,\tau)\hat{g}_1 A\frac{e^{-Bx_n} - e^{-Ax_n}}{B-A}, \\
v_n &= \frac{B}{A}\mathcal{M}(A,\tau)\hat{g}_1 A\frac{e^{-Bx_n} - e^{-Ax_n}}{B-A} + \mathcal{M}(A,\tau)\hat{g}_1 e^{-Bx_n}, \\
\hat{h} &= (\mathcal{M}(A,\tau) - 1)\frac{1}{i\tau}\hat{g}_1.
\end{aligned}
$$

Although the following estimates are similar to the previous ones, we present some details for the convenience of the reader:

(i) $u' \in L^2(0,\infty,\dot{H}^2(\mathbb{R}^n_+))$: For tangential derivatives $\partial_k\partial_i u_j$, $1 \le i,j,k \le n-1$, we use the interpolation inequality

$$
\|g_1\|_{\dot{H}^{1/2}(\mathbb{R};\dot{H}^{1/2}(\Gamma))} \le \|g_1\|^{1/2}_{L^2(\mathbb{R};\dot{H}^{3/2}(\Gamma))}\|g_1\|^{1/2}_{\dot{H}^1(\mathbb{R};\dot{H}^{-1/2}(\Gamma))} \tag{2.14}
$$

and that $|B| \le \left|\frac{\tau}{\mu}\right|^{1/2} + |A|$, and calculate as follows:

$$
\begin{aligned}
\|\xi_k\xi_i v_j\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} &= \left\|\left(\frac{\xi_k}{A}\frac{\xi_i}{A}\frac{i\xi_j}{A}\right)\mathcal{M}AB\hat{g}_1\left(A\frac{e^{-Bx_n} - e^{-Ax_n}}{B-A}\right)\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} \\
&\le c\left\|AB\hat{g}_1\frac{1}{\sqrt{A}}\right\|_{L^2(\mathbb{R}\times\Gamma)} \\
&\le c\left(\|g_1\|_{L^2(\mathbb{R},\dot{H}^{3/2}(\Gamma))} + \|g_1\|_{\dot{H}^{1/2}(\mathbb{R};\dot{H}^{1/2}(\Gamma))}\right) \\
&\le c\left(\|g_1\|_{L^2(\mathbb{R};\dot{H}^{3/2}(\Gamma))} + \|g_1\|_{\dot{H}^1(\mathbb{R};\dot{H}^{-1/2}(\Gamma))}\right).
\end{aligned} \tag{2.15}
$$

For the normal derivatives $\partial_n^2 u'$ we compute:

$$\partial_n^2 v' = \frac{i\xi'}{A}\frac{B}{A}\mathcal{M}\hat{g}_1 A\frac{B^2 e^{-Bx_n} - A^2 e^{-Ax_n}}{B-A}$$
$$= \frac{i\xi'}{A}\frac{B}{A}\mathcal{M}\hat{g}_1 A\frac{(B^2 - A^2)e^{-Bx_n}}{B-A} + \frac{i\xi'}{A}BA\mathcal{M}\hat{g}_1 A\frac{e^{-Bx_n} - e^{-Ax_n}}{B-A}.$$

The second summand can be treated as in (2.15). Hence we only have to consider the first one:

$$\begin{aligned}\left\|\frac{i\xi_j}{A}B\mathcal{M}\hat{g}_1(B+A)e^{-Bx_n}\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} &\le c\left\|B^2\hat{g}_1\frac{1}{\sqrt{\mathrm{Re}B}}\right\|_{L^2(\mathbb{R}\times\Gamma)}\\ = c\left\|\left(\frac{i\tau}{\mu}+A^2\right)\hat{g}_1\frac{1}{\sqrt{A}}\right\|_{L^2(\mathbb{R}\times\Gamma)}&\\ \le c\|g_1\|_{L^2(\mathbb{R},\dot{H}^{3/2}(\Gamma))} + \|g_1\|_{\dot{H}^1(\mathbb{R},\dot{H}^{-1/2}(\Gamma))}.&\end{aligned} \tag{2.16}$$

For mixed derivatives $\partial_k\partial_n u_j$, $k = 1, ..., n-1$, there holds

$$i\xi_k\partial_n v_j = \frac{i\xi_k}{A}\frac{i\xi_j}{A}B\mathcal{M}\hat{g}_1 A\frac{-Be^{-Bx_n} + Ae^{-Ax_n}}{B-A}$$
$$= -\frac{i\xi_k}{A}\frac{i\xi_j}{A}B\mathcal{M}\hat{g}_1 Ae^{-Bx_n} - \frac{i\xi_k}{A}\frac{i\xi_j}{A}BA\mathcal{M}\hat{g}_1 A\frac{e^{-Bx_n} - e^{-Ax_n}}{B-A}.$$

The first summand can be treated as in (2.16), the second one as in (2.15).

(ii) $u_n \in L^2(0,\infty;\dot{H}^2(\mathbb{R}^n_+))$: Since $\partial_k\partial_n u_n = -\sum_{j=1}^{n-1}\partial_k\partial_j u_j \in L^2((0,\infty)\times\mathbb{R}^n_+)$, it is only left to estimate the term $\partial_k\partial_i u_n$ for $i, k \neq n$. The first summand of v_n can be treated in the same way as v_j; thus we only have to consider the second one:

$$\begin{aligned}\|\xi_k\xi_i\mathcal{M}\hat{g}_1 e^{-Bx_n}\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} &\le c\left\|A^2\hat{g}_1\frac{1}{\sqrt{\mathrm{Re}B}}\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)}\\ &\le c\left\|A^{3/2}\hat{g}_1\right\|_{L^2(\mathbb{R}\times\Gamma)} \le c\|g_1\|_{L^2(\mathbb{R},\dot{H}^{3/2}(\Gamma))}.\end{aligned}$$

(iii) $p \in L^2(0,\infty;\dot{H}^1(\mathbb{R}^n_+))$. For the tangential derivative $\nabla' p$ we have

$$\begin{aligned}\|\xi'\hat{p}\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} &\le \left\|\xi'\left(\frac{i\tau}{A}+\mu A+\mu B\right)\hat{g}_1 e^{-Ax_n}\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)}\\ &\le c\left\|\left(\frac{i\tau}{A}+\mu A+\mu B\right)A\hat{g}_1\frac{1}{\sqrt{A}}\right\|_{L^2(\mathbb{R}\times\Gamma)}\\ &\le c\|g_1\|_{\dot{H}^1(\mathbb{R},\dot{H}^{-1/2}(\Gamma))} + c\|g_1\|_{L^2(\mathbb{R},\dot{H}^{3/2}(\Gamma)))};\end{aligned}$$

for the last step we also use (2.15). Since $\partial_n\hat{p} = -A\hat{p}$, we can show $\partial_n p \in L^2(0,\infty;L^2(\mathbb{R}^n_+))$ as above.

(iv) $h \in \dot{H}^1(0,\infty;\dot{H}^{3/2}(\Gamma)) \cap \dot{H}^2(0,\infty;\dot{H}^{-1/2}(\Gamma))$. This follows immediately from the boundedness of $\mathcal{M}(A,\tau) - 1$.

(v) $u \in \dot{H}^1(0,\infty;L^2(\mathbb{R}^n_+))$: Here we refer to $\partial_t u = \Delta u - \nabla p \in L^2((0,\infty)\times\mathbb{R}^n_+)$.

If $g_1 \in L^2(0,T;\dot{H}^{3/2}(\Gamma)) \cap \dot{H}^1(0,T;\dot{H}^{-1/2}(\Gamma))$ and $g_2 \in L^2(0,T;\dot{H}^{1/2}(\Gamma))$ we extend g_2 by zero to an element in $L^2(0,\infty;\dot{H}^{1/2}(\Gamma))$ and, since $g_1(0)=0$, we extend g_1 to an element in $L^2(0,\infty;\dot{H}^{3/2}(\Gamma)) \cap \dot{H}^1(0,\infty;\dot{H}^{-1/2}(\Gamma))$ by $g_1\circ\varphi$ where $\varphi \in C^0([0,\infty))$ has compact support in $[0, 2T]$ and

$$\varphi : (0,\infty) \to \mathbb{R}, \quad t \mapsto \begin{cases} t, & \text{if } t \in (0,T), \\ 2T-t, & \text{if } t \in (T,2T). \end{cases}$$

Then we apply the above case and restrict the result to the finite time interval. □

2.2 The case of non-vanishing initial data

Next we want to recover non-vanishing initial data starting with $h_0 \in \dot{H}^{7/2}(\Gamma)$. Consider the classical plate equation

$$\begin{aligned} \partial_t^2\eta_0 + \Delta^2\eta_0 &= 0 && \text{in } \Gamma\times(0,T) \\ (\eta_0, \partial_t\eta_0)|_{t=0} &= (h_0, 0) && \text{on } \Gamma. \end{aligned} \tag{2.17}$$

Applying the partial Fourier transform $\mathcal{F}$ with respect to the spatial variable $x' \in \Gamma = \mathbb{R}^{n-1}$ we get a solution

$$\begin{aligned} \eta_0(t,\xi) &= \mathcal{F}^{-1}\big(\cos(A^2t)\mathcal{F}h_0\big) \in L^\infty(0,T;\dot{H}^{7/2}(\Gamma)), \\ \partial_t\eta_0(t,\xi) &= \mathcal{F}^{-1}(-\sin(A^2t)A^2\mathcal{F}h_0) \in L^\infty(0,T;\dot{H}^{3/2}(\Gamma)). \end{aligned} \tag{2.18}$$

Now the general idea is to replace h from Proposition 2.3 with $h+\eta_0$ to satisfy the initial condition and to leave the plate equation unchanged. However this replacement alters the equation $u_n|_{x_n=0} = \partial_t h$ which is why we need the following proposition on the Stokes system with inhomogeneous Dirichlet boundary data in the nth component.

Proposition 2.4 *Let* $0<T<\infty$, $h_0 \in \dot{H}^{7/2}(\Gamma)$, *and let* $\eta_0 = \mathcal{F}^{-1}(\cos(A^2t)\mathcal{F}h_0)$ *be the solution of the plate equation* (2.17). *Then the Stokes system*

$$\begin{aligned} \partial_t w_0 - \mu\Delta w_0 + \nabla p_0 &= 0 && \text{in } \mathbb{R}^n_+\times(0,T) \\ \operatorname{div} w_0 &= 0 && \text{in } \mathbb{R}^n_+\times(0,T) \\ (w_0', (w_0)_n)|_{x_n=0} &= (0', \partial_t\eta_0) && \text{on } \Gamma\times(0,T) \\ w_0|_{t=0} &= 0 && \text{in } \mathbb{R}^n_+ \end{aligned} \tag{2.19}$$

has a solution

$$w_0 \in L^2(0,T;\dot{H}^2(\mathbb{R}^n_+)) \cap \dot{H}^1(0,T;L^2(\mathbb{R}^n_+)), \quad p_0 \in L^2(0,T;\dot{H}^1(\mathbb{R}^n_+)).$$

Furthermore, there holds the estimate

$$\|w_0\|_{L^2(0,T;\dot H^2(\mathbb{R}^n_+))\cap \dot H^1(0,T;L^2(\mathbb{R}^n_+))} + \|p_0\|_{L^2(0,T;\dot H^1(\mathbb{R}^n_+))} \le CT^{1/2}\|h_0\|_{\dot H^{7/2}(\Gamma)}. \quad (2.20)$$

Proof Choose $\varphi \in C_c^1([0,\infty))$ such that $\varphi = 1$ on $[0, T]$, $\varphi(t) = 0$ for $t > 2T$ and $t|\varphi'(t)| \le c$ on $\mathbb{R}$ with $c > 0$ independent of T. Obviously, we can replace $\partial_t\eta_0(t,\xi)$ with $\varphi(t)\,\partial_t\eta_0(t,\xi)$ in $(2.19)_3$. We want to ensure that

$$\begin{aligned} &r := \varphi\,\partial_t\eta_0 \in L^2(0,\infty;\dot H^{3/2}(\Gamma)) \cap \dot H^1(0,\infty;\dot H^{-1/2}(\Gamma)),\\ &\|r\|_{L^2(0,\infty;\dot H^{3/2}(\Gamma))\cap \dot H^1(0,\infty;\dot H^{-1/2}(\Gamma))} \le cT^{1/2}\|h_0\|_{\dot H^{7/2}(\Gamma)}. \end{aligned} \quad (2.21)$$

For the first property we have

$$|\varphi\,\partial_t\hat\eta_0\,A^{3/2}| = |A^{7/2}\hat h_0\ \sin(A^2t)\varphi| \le |A^{7/2}\hat h_0\varphi| \in L^2((0,\infty)\times\Gamma).$$

Concerning the space $\dot H^1(0,\infty;\dot H^{-1/2}(\Gamma))$ we use the estimate $|\sin s| \le |s|$, $s \in \mathbb{R}$, to get that

$$\begin{aligned} |A^{-1/2}\partial_t(\varphi\,\hat\eta_0)| &= |A^{-1/2}\varphi\,\partial_t^2\hat\eta_0 + A^{-1/2}\varphi'\,\partial_t\hat\eta_0| \\ &= |-A^{-1/2}A^4\hat h_0\cos(A^2t)\varphi - A^{-1/2}A^2\hat h_0\sin(A^2t)\varphi'| \\ &\le |A^{7/2}\hat h_0\varphi| + |A^{7/2}\hat h_0 t\varphi'| \in L^2((0,\infty)\times\Gamma). \end{aligned}$$

Now it is easily seen that r satisfies (2.21).

Since $\partial_t\eta_0(0,x')\,\varphi(0) = 0$, it is possible to extend $r = \partial_t\eta_0(t,x')\,\varphi$ by zero to a function in $L^2(\mathbb{R};\dot H^{3/2}(\Gamma)) \cap \dot H^1(\mathbb{R};\dot H^{-1/2}(\Gamma))$ still satisfying an estimate similar to $(2.21)_2$. Then we apply in (2.19) the Fourier transform with respect to time and the first $n-1$ spatial variables to obtain that

$$\begin{aligned} (i\tau + \mu|\xi'|^2)\hat w_0' - \mu\partial_n^2\hat w_0' + i\xi'\hat p_0 &= 0 && \text{in } \mathbb{R}\times\mathbb{R}^n_+\\ (i\tau + \mu|\xi'|^2)(\hat w_0)_n - \mu\partial_n^2(\hat w_0)_n + \partial_n\hat p_0 &= 0 && \text{in } \mathbb{R}\times\mathbb{R}^n_+\\ i\xi'\cdot\hat w_0' + \partial_n(\hat w_0)_n &= 0 && \text{in } \mathbb{R}\times\mathbb{R}^n_+\\ (\hat w_0', (\hat w_0)_n)|_{x_n=0} &= (0', \hat r) && \text{on } \mathbb{R}\times\Gamma. \end{aligned} \quad (2.22)$$

The following calculations resemble those at the beginning of Sect. 2.1, see (2.3)-(2.5). Since $\mathrm{div} w_0 = 0$ we get from (2.22) that $\Delta p_0 = 0$ is solved by

$$\hat p_0 = \gamma e^{-Ax_n}$$

for a function $\gamma = \gamma(\xi',\tau)$. Using this identity and $(2.22)_1$ we deduce

$$\hat w_0' = \frac{\xi'\gamma}{\tau}\left(e^{-Bx_n} - e^{-Ax_n}\right).$$

Now using $(2.22)_3$ we see that

$$\partial_n(\hat{w}_0)_n = -i\xi' \cdot \hat{w}'_0 = \frac{A^2\gamma}{i\tau}\left(e^{-Bx_n} - e^{-Ax_n}\right),$$
$$\partial_n^2(\hat{w}_0)_n = \frac{A^2\gamma}{i\tau}\left(-Be^{-Bx_n} + Ae^{-Ax_n}\right).$$

Next, from $(2.22)_2$ we conclude that

$$(\hat{w}_0)_n = \gamma\left(\frac{A}{i\tau}e^{-Ax_n} - \frac{A^2}{Bi\tau}e^{-Bx_n}\right),$$

cf. (2.5). Finally, $(2.22)_4$ implies $\hat{r} = \gamma\left(\frac{A}{i\tau} - \frac{A^2}{i\tau B}\right) = \gamma\frac{A}{i\tau}\frac{B-A}{B}$. Hence

$$\gamma = \hat{r}\frac{i\tau}{A}\frac{B}{B-A} = \hat{r}\mu\frac{B(B+A)}{A}.$$

In summary, we obtain a solution

$$\begin{aligned}
\hat{p}_0 &= \frac{\mu B(B+A)}{A}\hat{r}e^{-Ax_n},\\
\hat{w}'_0 &= \frac{i\xi'}{A}\frac{B}{A}\hat{r}\left(A\frac{e^{-Bx_n} - e^{-Ax_n}}{B-A}\right),\\
(\hat{w}_0)_n &= \hat{r}\frac{i\tau}{A}\frac{B}{B-A}\left(\frac{A}{i\tau}e^{-Ax_n} - \frac{A^2}{Bi\tau}e^{-Bx_n}\right)\\
&= -\frac{B}{A}\hat{r}\left(A\frac{e^{-Bx_n} - e^{-Ax_n}}{B-A}\right) + \hat{r}e^{-Bx_n}.
\end{aligned} \tag{2.23}$$

Now we can prove *a priori* estimates of this solution in L^2:

(i) $w'_0 \in L^2(0,\infty;\dot{H}^2(\mathbb{R}^n_+))$: Concerning tangential derivatives $\partial_k\partial_i(w_0)_j$, $i,j,k \in \{1,\dots,n-1\}$ there holds, since $|B| \le c|\tau|^{1/2} + A$,

$$\begin{aligned}
\|\xi_k\xi_i(\hat{w}_0)_j\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} &= \left\|\frac{\xi_k}{A}\frac{\xi_i}{A}\frac{\xi_j}{A}BA\hat{r}\left(A\frac{e^{-Bx_n} - e^{-Ax_n}}{B-A}\right)\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)}\\
&\le c\left\|BA\hat{r}\frac{1}{\sqrt{A}}\right\|_{L^2(\mathbb{R}\times\Gamma)}\\
&\le c\|r\|_{L^2(\mathbb{R},\dot{H}^{3/2}(\Gamma))} + c\|r\|_{\dot{H}^{1/2}(\mathbb{R};\dot{H}^{1/2}(\Gamma))}.
\end{aligned}$$

For mixed derivatives $\partial_i\partial_n(w_0)_j$ we see that

$$
\begin{aligned}
&\|\xi_i\partial_n(\hat{w}_0)_j\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)}\\
=&\left\|\frac{\xi_i}{A}\frac{\xi_j}{A}B\hat{r}\left(A\frac{-Be^{-Bx_n}+Ae^{-Ax_n}}{B-A}\right)\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)}\\
\le&\left\|AB\hat{r}\left(-e^{-Bx_n}-A\frac{e^{-Bx_n}-e^{-Ax_n}}{B-A}\right)\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)}\\
\le&2c\left\|AB\hat{r}\frac{1}{\sqrt{A}}\right\|_{L^2(\mathbb{R}\times\Gamma)}\\
\le&c\|r\|_{L^2(\mathbb{R},\dot{H}^{3/2}(\Gamma))}+c\|r\|_{\dot{H}^{1/2}(\mathbb{R};\dot{H}^{1/2}(\Gamma))}.
\end{aligned}
$$

Finally, the normal derivative $\partial_n^2(w_0)_j$ satisfies the estimate

$$
\begin{aligned}
\|\partial_n^2(\hat{w}_0)_j\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)}\le&\left\|\frac{B}{A}\hat{r}\left(A\frac{B^2e^{-Bx_n}-A^2e^{-Ax_n}}{B-A}\right)\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)}\\
\le&\left\|\frac{B}{A}\hat{r}A\frac{B^2-A^2}{B-A}e^{-Bx_n}\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)}\\
&+\left\|\frac{B}{A}\hat{r}A^2\left(A\frac{e^{-Bx_n}-e^{-Ax_n}}{B-A}\right)\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)}\\
\le&c\left\|B\hat{r}\frac{B+A}{\sqrt{A}}\right\|_{L^2(\mathbb{R}\times\Gamma)}+c\left\|\frac{B}{A}\hat{r}\frac{A^2}{\sqrt{A}}\right\|_{L^2(\mathbb{R}\times\Gamma)}\\
\le&3c\left\|B^2\hat{r}\frac{1}{\sqrt{A}}\right\|_{L^2(\mathbb{R}\times\Gamma)}\\
\le&c\|r\|_{L^2(\mathbb{R};\dot{H}^{3/2}(\Gamma))}+c\|r\|_{\dot{H}^1(\mathbb{R};\dot{H}^{-1/2}(\Gamma))},
\end{aligned}
$$

since $A\le|B|$ and $|B|^2\le c|\tau|+A^2$.

(ii) $(w_0)_n\in L^2(0,\infty;\dot{H}^2(\mathbb{R}^n_+))$: Since $\partial_j\partial_n(w_0)_n=-\sum_{k=1}^{n-1}\partial_k\partial_j(w_0)_k$, $j\in\{1,...,n-1\}$, we have to consider only $\partial_k\partial_i(w_0)_n$ for $i,k\in\{1,...,n-1\}$. The first summand in (2.23)$_4$ can be treated as $(\hat{w}_0)_j$ above; so we inspect the second one:

$$
\|\xi_i\xi_k\hat{r}e^{-Bx_n}\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)}\le\left\|A^2\hat{r}\frac{1}{\sqrt{A}}\right\|_{L^2(\mathbb{R}\times\Gamma)}\le\|r\|_{L^2(\mathbb{R},\dot{H}^{3/2}(\Gamma))}.
$$

(iii) $p_0\in L^2(0,\infty;\dot{H}^1(\mathbb{R}^n_+))$: Since $\partial_n\hat{p}_0=-A\hat{p}_0$, it suffices to consider the tangential gradient $\nabla' p_0$. Here we get that

$$\begin{aligned}\|\xi'\hat{p}_0\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)} &= \left\|\mu\frac{\xi'}{A}B(B+A)\hat{r}e^{-Ax_n}\right\|_{L^2(\mathbb{R}\times\mathbb{R}^n_+)}\\ &\le c\left\|B(B+A)\hat{r}\frac{1}{\sqrt{A}}\right\|_{L^2(\mathbb{R}\times\Gamma)}\\ &\le c\|r\|_{L^2(\mathbb{R},\dot{H}^{3/2}(\Gamma))}+c\|r\|_{\dot{H}^1(\mathbb{R},\dot{H}^{-1/2}(\Gamma))}.\end{aligned}$$

(iv) $w_0 \in \dot{H}^1(0,\infty;L^2(\mathbb{R}^n_+))$: This is immediately implied by the identity $\partial_t w_0 = \mu\Delta w_0 - \nabla p_0 \in L^2((0,\infty)\times\mathbb{R}^n_+)$.

Restricting ourselves to the time interval $(0, T)$, recalling the interpolation inequality (2.14) and also (2.21) we deduce the proposition. □

Now we reconstruct the initial data $u_0 \in \dot{H}^2(\mathbb{R}^n_+)$ with $\operatorname{div} u_0 = 0$ and $u_0'|_{x_n=0} = 0$, as well as the right-hand side $f \in L^2((0,T)\times\mathbb{R}^n_+)$.

Proposition 2.5 *Let* $T \in (0,\infty)$, $f \in L^2((0,T)\times\mathbb{R}^n_+)$, *and let* $u_0 \in \dot{H}^2\left(\mathbb{R}^n_+\right)$ *satisfy* $\operatorname{div} u_0 = 0$, $u_0'|_{\partial\mathbb{R}^n_+} = 0$. *Then the Stokes system*

$$\begin{aligned}\partial_t z - \mu\Delta z + \nabla p_1 &= f && \text{in } \mathbb{R}^n_+\times(0,T)\\ \operatorname{div} z &= 0 && \text{in } \mathbb{R}^n_+\times(0,T)\\ (z', z_n)|_{x_n=0} &= (0', \mathcal{F}^{-1}(\cos(A^2t)(\mathcal{F}u_0)_n)) && \text{on } \Gamma\times(0,T)\\ z(0) &= u_0 && \text{in } \mathbb{R}^n_+\end{aligned}\tag{2.24}$$

admits a solution

$$z \in L^2(0,T;\dot{H}^2(\mathbb{R}^n_+)) \cap \dot{H}^1(0,T;L^2(\mathbb{R}^n_+)), \quad p_1 \in L^2(0,T;\dot{H}^1(\mathbb{R}^n_+)).$$

Furthermore, we have the estimate

$$\begin{aligned}&\|z\|_{L^2(0,T;\dot{H}^2(\mathbb{R}^n_+))\cap\dot{H}^1(0,T;L^2(\mathbb{R}^n_+))} + \|p_1\|_{L^2(0,T;\dot{H}^1(\mathbb{R}^n_+))}\\ &\quad\le C\|f\|_{L^2((0,T)\times\mathbb{R}^n_+)} + CT^{1/2}\|u_0\|_{\dot{H}^2(\mathbb{R}^n_+)}.\end{aligned}\tag{2.25}$$

Proof To get rid of the terms involving u_0 define

$$w_2(t,x',x_n) := \mathcal{F}^{-1}\left(\cos(|\xi'|^2t)\,\hat{u}_0(\xi',x_n)\right) \in L^2(0,T;\dot{H}^2(\mathbb{R}^n_+))$$

which satisfies $(2.24)_{2,3,4}$. Moreover, since, in $L^2((0,T)\times\mathbb{R}^n_+)$,

$$\begin{aligned}\partial_t w_2(t,x',x_n) &= \mathcal{F}^{-1}\left(-\sin(|\xi'|^2t)|\xi'|^2\,\hat{u}_0(\xi',x_n)\right),\\ \Delta w_2(t,x',x_n) &= \mathcal{F}^{-1}\left(\cos(|\xi'|^2t)(-|\xi'|^2\hat{u}_0 + \partial_n^2\hat{u}_0)\right),\end{aligned}$$

w_2 can be estimated as z in (2.25) (with $f=0$, $p_1=0$).

Next we consider the Stokes system

$$\begin{aligned} \partial_t w_1 - \mu\Delta w_1 + \nabla p_1 &= f - (\partial_t - \mu\Delta) w_2 && \text{in } \mathbb{R}^n_+ \times (0,T) \\ \operatorname{div} w_1 &= 0 && \text{in } \mathbb{R}^n_+ \times (0,T) \\ w_1|_{x_n=0} &= 0 && \text{on } \Gamma \times (0,T) \\ w_1|_{t=0} &= 0 && \text{in } \mathbb{R}^n_+. \end{aligned}$$

Since the right-hand side lies in $L^2((0,T)\times\mathbb{R}^n_+)$, there exists a strong solution $w_1 \in L^2(0,T;H^2(\mathbb{R}^n_+)) \cap H^1(0,T;L^2(\mathbb{R}^n_+))$ and $p_1 \in L^2(0,T;\dot{H}^1(\mathbb{R}^n_+))$ with the corresponding maximal regularity estimate (see e.g. [9, Corollary 3.6]) with a constant $C > 0$ independent of T.

So we deduce that $z := w_1 + w_2$ and p_1 satisfy (2.24), (2.25). □

Corollary 2.6 *Let $T \in (0,\infty)$, $f \in L^2((0,T)\times\mathbb{R}^n_+)$, $h_0 \in \dot{H}^{7/2}(\Gamma)$, $u_0 \in \dot{H}^2(\mathbb{R}^n_+)$ with* $\operatorname{div} u_0 = 0$ *and* $u_0'|_{\partial\mathbb{R}^n_+} = 0$. *Then the Stokes system*

$$\begin{aligned} \partial_t z_1 - \mu\Delta z_1 + \nabla q &= f && \text{in } \mathbb{R}^n_+ \times (0,T) \\ \operatorname{div} z_1 &= 0 && \text{in } \mathbb{R}^n_+ \times (0,T) \\ z_1|_{t=0} &= u_0 && \text{in } \mathbb{R}^n_+, \end{aligned} \tag{2.26}$$

together with the boundary condition

$$(z_1', (z_1)_n)|_{x_n=0} = \left(0', \mathcal{F}^{-1}(-\sin(A^2 t)A^2 \mathcal{F}h_0 + \cos(A^2 t)(\mathcal{F}u_0)_n)\right) \tag{2.27}$$

on $\Gamma \times (0,T)$, has a solution

$$z_1 \in L^2(0,T;\dot{H}^2(\mathbb{R}^n_+)) \cap \dot{H}^1((0,T);L^2(\mathbb{R}^n_+)), \quad q \in L^2(0,T;\dot{H}^1(\mathbb{R}^n_+)).$$

Furthermore, we have the corresponding estimate as in (2.25), including the term $CT^{1/2}\|h_0\|_{\dot{H}^{7/2}(\Gamma)}$.

Proof Let (w_0, p_0) be a solution of (2.19) and (z, p_1) be a solution of (2.24). Then define

$$z_1 := z + w_0 \in L^2(0,T;\dot{H}^2(\mathbb{R}^n_+)) \cap \dot{H}^1(0,T;L^2(\mathbb{R}^n_+))$$

and $q := p_0 + p_1 \in L^2(0,T;\dot{H}^1(\mathbb{R}^n_+))$ to get the desired solution. □

2.3 Proof of Theorem 1.3

Let

$$(z_1, q) \in L^2(0,T;\dot{H}^2(\mathbb{R}^n_+)) \cap \dot{H}^1(0,T;L^2(\mathbb{R}^n_+)) \times L^2(0,T;\dot{H}^1(\mathbb{R}^n_+))$$

be a solution of (2.26)-(2.27). Due to $q|_{\partial\mathbb{R}^n_+} \in L^2(0,T;\dot{H}^{1/2}(\Gamma))$ Proposition 2.3 implies the existence of

$$
\begin{aligned}
(u^\star, p^\star) &\in L^2(0,T;\dot H^2(\mathbb{R}^n_+)) \cap \dot H^1(0,T;L^2(\mathbb{R}^n_+)) \times L^2(0,T;\dot H^1(\mathbb{R}^n_+)),\\
h^\star &\in \dot H^1(0,T;\dot H^{3/2}(\Gamma)) \cap \dot H^2(0,T;\dot H^{-1/2}(\Gamma))
\end{aligned}
$$

such that

$$
\begin{aligned}
\partial_t u^\star - \mu\Delta u^\star + \nabla p^\star &= 0 && \text{in } \mathbb{R}^n_+ \times (0,T)\\
\operatorname{div} u^\star &= 0 && \text{in } \mathbb{R}^n_+ \times (0,T)\\
u^\star|_{x_n=0} &= (0', \partial_t h^\star + g_1) && \text{on } \Gamma \times (0,T)\\
\partial_t^2 h^\star + \Delta^2 h^\star &= -p^\star + (g_2 - q) && \text{on } \Gamma \times (0,T)
\end{aligned}
$$

satisfying the initial conditions

$$
u^\star|_{t=0} = 0 \ \text{ on } \ \mathbb{R}^n_+, \quad (h^\star, \partial_t h^\star)|_{t=0} = (0,0) \ \text{ on } \ \Gamma.
$$

To finish the proof define

$$
\begin{aligned}
&u = u^\star + z_1 \in L^2(0,T;\dot H^2(\mathbb{R}^n_+)) \cap \dot H^1(0,T;L^2(\mathbb{R}^n_+)),\\
&p = p^\star + q \in L^2(0,T;\dot H^1(\mathbb{R}^n_+)),\\
&h = h^* + \mathcal{F}^{-1}(\cos(A^2 t)\mathcal{F}h_0) + \mathcal{F}^{-1}\left(A^{-2}\sin(A^2 t)(\mathcal{F}u_0)_n\right),
\end{aligned}
$$

where $h \in \dot H^1(0,T;\dot H^{3/2}(\Gamma)) \cap \dot H^2(0,T;\dot H^{-1/2}(\Gamma))$, to get a solution to (1.2). □

Remark 2.7 Despite $h \in \dot H^2(0,T;\dot H^{-1/2}(\Gamma))$ we did not show that h is very regular with respect to space, let alone $h(t) \in \dot H^{7/2}(\Gamma)$ for almost all $t \in (0,T)$. This is due to the fact that while $\tau^{3/2}\mathcal{N}(A,\tau)$ is bounded, see Lemma 2.1, the multiplier function $A^3\mathcal{N}(A,\tau)$ is not uniformly bounded in A, τ.

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Declarations

Conflict of interest: The authors declare that they have no conflict of interest

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