Dynamic Compression Model Incorporating Elastic Nonlinearity of Paperboard

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Abstract

Spring of constant elasticity is a concept from theories of extension while elastic nonlinearity in compressive deformation is a general phenomenon for polymeric materials involved in offset or flexographic printing, paper board, polymer plate, and cushioning tape. This phenomenon needs therefore to be coped with by the model of printing dynamics. We hereby present an extended approach based on the Maxwell material model. In the extended approach, a compression process is subdivided into (or approximated by) sequential subprocesses. The elastic modulus may vary from one subsection to another but remains constant in each of the subprocesses. With the extended approach dynamic behaviours (compression/recovering) of paperboard can be reproduced and predicted. As a concrete example, dynamic behaviours of paper board in the print nip were simulated with satisfactory outcome. The simulation also revealed that viscoelasticity of the board is the origin of mechanical hysteresis of the stress–strain curve. Due to viscoelasticity and nonlinearity of the materials careful design is essential to simulate full-scale printing with a lab press.

Keywords Print dynamics · Fibre-based packaging · Compressibility · Nip pressure

Introduction

Mechanical contact between the print form (nip) and the paper surface is essential for ink transferring in flexography, offset or roto gravure. To overcome topographic variations of the paper surface and to promote contact a nip pressure is applied. The nip pressure in the respective printing techniques varies from fractions of a mega Pascal (MPa) to tens of MPa. Details of the nip-pressure profile, i.e. shape, peak and length or time duration of the nip pressure, govern the ink transferring and thereby print quality. However, in full-scale printing, nip pressure is never directly set. Rather it is the gap between the print cylinder and the impression cylinder that is set. It is common wisdom that nip pressure increases with a reduced nip gap. However, details of the nip profile depend on the mechanical properties of the materials involved e.g. paper board, polymer plate, and cushioning tape [1].

Paper is a viscoelastic material with nonlinear elastic components [2, 3], so are rubber and polymers [4]. The viscoelastic nature of the materials can cause variability in test data and interpretation of the test data [2]. Unlike an elastic material whose response depends solely on the deformation (compression strain), a viscoelastic material responds also to strain rate in addition to the strain itself. A typical example of viscoelasticity is speed hardening. A viscoelastic material becomes stiffer when it undergoes compression in a higher strain rate. For example, the dynamic compressive strength of the soft materials, e.g. sculpturing clay and sorbothane, increased by 4 and 3 orders of magnitude compared to their respective static values [5]. For papermaking industry, compressive measurements obtained according to ISO13820 standard [6] correspond to a strain rate around unity, which is in sharp contrast to cases like full scale printing wherein strain rate can be hundreds or even a thousand times higher. Hence, to be able to understand what happens in a printing nip of full-scale printing, techniques that enable one to measure both elastic and viscous characteristics of the materials involved in printing are needed [7].

Viscoelasticity and nonlinearity are general characteristics of polymeric and rubber materials [8, 9]. Characterization of the properties and mechanical behavior of polymeric

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materials have been considered as great challenges in the design and analysis of novel composites and polymer-based structures [10–12]. There is plenty of evidence that the elastic modulus of paper increases with increasing compressive deformation. Figure 1a shows the compressive deformation cycle of paper board, measured with a material testing system (MTS), a device commonly used in the papermaking industry. As denoted by the dashed lines, the slopes of the stress–strain curve in different compression sections vary with the corresponding compressive deformation. It is obvious that with increasing compression from section *A* to section *B* and to *C*, the slope of the curve increases. After reaching the max compression position, the board structure relaxes and partially recovers. Due to the hysteretic nature of the paper material, there is a V-shape formation around the turning point. A similar observation of elastic nonlinearity was also observed from the lab offset printing study [13]. As shown in Fig. 1b, compression was easier when paper entered the printing nip (regime *A'*) but became gradually harder on the way to approach the nip center through the sections *B'* and *C'*. Due to viscoelasticity of the paper material, max nip pressure was reached prior to the nip center as explained in the earlier work [1].

Dynamic behaviour of viscoelastic materials is complex and a physics model is needed for the measurement and interpretation of measured data [2, 14]. The basic components of the models are spring(s) and dashpot(s), representing elastic and viscous deformations, which are in series or parallel connections or in combinations [1, 3, 15]. We have recently developed a theoretical framework to study printing dynamics with a special focus on nip pressure that a paper board experiences inside the printing nip [1]. The expression for the nip pressure as a function of viscoelastic characteristics of the materials involved, e.g. paperboard, rubber blanket, polymer plate, cushioning tape etc., in addition to the geometrical settings, printing speed etc., has been worked

out. The viscoelastic nature of the materials was found to be responsible for the so-called speed-hardening, asymmetric nip profile, variations in the nip amplitude and effective nip length etc. It was also found that the effects of viscoelastic properties of the materials on the nip dynamics depend on how the elastic components and the viscous counter parts are connected with each other.

In the present work, we demonstrate how the earlier presented approach [1] can even be applied to cases where the elastic modulus is nonlinear or increases with increasing compressive deformations.

Compression of Materials with Nonlinear Spring

In the print nip, the paper is subjected to the compressive stress of the print nip and undergoes conformational changes by the viscoelastic flow. For easy reading of the present work, the basics of the nip-geometry model presented earlier are highlighted in this section at first. Then we demonstrate how the material models can be extended to cope with situations where the materials possess nonlinear elastic modulus.

Model of Nip Geometry

Figure 2 shows the nip geometry of a printing press. The radii of the cylinders are R_1 and R_2 , respectively. The speed of the material stack going through the nip is v . $t=0$ denotes the time when the material enters the nip, $t=t_c$ the time when it reaches the nip centre, and $t=t_{ex}$ when the material exits the nip. As explained in the previous work [1], the nip rollers are two rigid cylinders, while the material stack maybe a materials sandwich of the paper substrate and print form etc. Hence the framework is applicable to not only printing processes (offset

Fig. 1 Illustrations of non-constant Elastic modulus of paper in compressive deformation. The compressive stress–strain diagram (a) and nip pressure profile of a printing stress (b) [13]

Fig. 2 Illustration of the nip geometry and the time when the paper sheet goes through the nip

and flexography) but also gravure and calendering (hard nip) processes where the print form is omitted in the material stack.

Assume that the initial thickness of the material is D_0 and the minimum opening at the nip center is g_0 , the thickness of the board in the nip is of a parabolic form, i.e.,

$$D(t) = g_0 + \emptyset(t - t_c)^2, \quad (0 \leq t \leq t_{ex}) \quad (1)$$

and the strain of the material is expressed as,

$$\varepsilon = \frac{D(t) - D_0}{D_0} = \varepsilon_0 + \frac{\emptyset}{D_0} (t - t_c)^2, \quad (0 \leq t \leq t_{ex}) \quad (2)$$

where the quantities

$$\varepsilon_0 = -1 + \frac{g_0}{D_0}, \quad (3)$$

$$\emptyset = \frac{V^2}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right), \quad (4)$$

$$t_c = \sqrt{\frac{2(D_0 - g_0) R_1 R_2}{V^2(R_1 + R_2)}}. \quad (5)$$

The nominal deformation profiles of the paperboard were calculated with Eqs. (1)–(5). Assumed that the intrinsic thickness of the board is $D_0 = 300 \mu\text{m}$, the radius of the printing plate cylinder is $R_1 = 91 \text{ mm}$, and the speed $v = 10 \text{ m/s}$ a speed to a full-scale flexographic printing process. In the Fig. 3, the plots correspond to the radii of the impression cylinders, $R_2 = [R_1, 2R_1, \infty]$ are denoted by dot-, dashed- and solid-line, respectively. The plots that correspond to three nip openings, $g_0 = [270, 210, 150] \mu\text{m}$ form three clusters. The lengths of the parabolic plots stand for the nominal nip dwell time that is proportional to the nip length. Hence, one can say that nip length increases with the radius of the impression cylinder. The case $R_2 = \infty$ means that the

Fig. 3 The nominal profiles of the board that correspond to different impression cylinders (R_2) and different nip openings (g_0). The other parameters are denoted in the figure

impression cylinder is flat as is the case of the IGT F1 lab printer. It is also intuitive that the nip length increases with the reduction of the nip gap, namely the narrower the nip opening, the more the board compression, and the wider the nip width.

Material Model of Nonlinear Elasticity

In the recently presented printing dynamics models [1], all the materials were considered as linear with constant elastic component independent of compressive deformation. These materials models need to be extended to cope with nonlinear situations.

The basic idea of model extension is to divide the entire compression section into a few subsections and in each of the subsections, the elastic modulus is treated as constant. Figure 4 shows examples of which the compression section is approximated with 3 and 6 subsections. It is obvious that the more subsections the better the approximation. The border between the adjacent subsections, is defined by

$$\varepsilon_i = 1 - d_i \quad (i = 1, 2, \dots). \quad (6)$$

The corresponding time t_i can be calculated from Eq. (2). The Elastic modulus in the i -th subsection is denoted as E_i , ($\varepsilon_{i-1} \leq \varepsilon \leq \varepsilon_i$, $i = 1, 2, \dots$).

The concept to approximate a nonlinear system by a (sufficient) number of linear subsystems is general. Hence, in the following, we use the Maxwell model as an example to demonstrate its applicability.

Figure 5 depicts the nonlinear Maxwell material model, in which the elastic component (E) is not a constant, while the viscous component (η) is constant. When approximated by a number of linear Maxwell subsections, the mathematic expression for the model in the i -th subsection is [1],

Fig. 5 Schematic representation of nonlinear Maxwell model in which the elastic modulus varies with compression

$$\dot{\sigma}_i + \frac{E_i}{\eta_i} \sigma_i = 2 \frac{E_i \cdot \emptyset}{D_0} (t - t_c), \quad (t_{i-1} \leq t \leq t_i) \quad (7)$$

The general solution of this differential equation is

$$\sigma_i(t) = 2 \frac{\eta_i \cdot \emptyset}{D_0} \left[t - t_c - \frac{\eta_i}{E_i} \right] t + C_i \cdot \exp \left(-\frac{E_i}{\eta_i} t \right), \quad (t_{i-1} \leq t \leq t_i) \quad (8)$$

In this expression, C_i is a constant that can be determined from initial stress at t_{i-1} . Because $\sigma_i(t_{i-1}) = \sigma_{i-1}(t_{i-1})$, one receives the following relationship,

$$C_i = \left\{ \sigma_{i-1}(t_{i-1}) - 2 \eta_i \frac{\emptyset}{D_0} \left[t_{i-1} - t_c - \frac{\eta_i}{E_i} \right] \right\} \exp \left(\frac{E_i}{\eta_i} t_{i-1} \right), \quad (i = 1, 2, \dots). \quad (9)$$

Because the initial stress at $t_0=0$ when the material enters the nip (the first subsection) is $\sigma_0=0$. Thus, the expressions for the consecutive subsections can be obtained.

Fig. 4 Illustration of piece-wise approach: the nonlinearity is approximated by 3 (left) or 6 (right) linear regimes

Results

The model presented in the preceding section can be used to study the dynamic compressive behaviours of paperboard in printing and calendering processes. In the processes, interactions between the nip and the paperboard is nominally defined by the nip-gap that is the distance between the print and the impression cylinders [1]. The actual nip pressure that exerts onto the paperboard depends on the viscoelasticity and non-linearity of the paperboard because the nip

pressure equals to the resistance of the paperboard against the compressive deformation forced by the nip according to Newton's third law. In the simulations, the radii of the print and impression cylinders are $R_1 = 91$ mm and $R_2 = 2 \times R_1$, respectively, and the velocity of the press, v , is given in the respective figures.

Nip Pressure Profile of Non-Linear Materials

Figure 6 depicts the nip pressure profiles of three model paperboards whose viscosities are shown in the legends in the figure. The nonlinear compressive deformations of these model materials were divided into three and six subsections, as denoted in Fig. 6a and b. The details of the subsections, e.g. the strain range and the corresponding Elastic modulus are listed in Table 1, in which ϵ_M is the strain of maximal deformation. In each of the subsections the Elastic modulus was treated as constant that is also denoted in the respective figures.

It is clear that the simulated nip pressure profiles shown in Fig. 6 share indeed similarity to the nip pressure obtain from the experiment, shown in Fig. 1b. As expected, the profiles also become smoother when approximated with six subsections compared with that of three subsections. As seen from the figure both the peaks and the widths of the nip pressure profiles depend on the viscosity component. Both increase with increasing viscosity as the dashpot is in series connection with the spring. The stronger the viscous component, the harder the dashpot to be compressed. In an extreme case, an infinite viscosity ($\eta \rightarrow \infty$) stands for a non-compressive dashpot and the material system becomes purely elastic while the opposite extreme, $\eta \rightarrow 0$, corresponds to a purely viscous system.

Figure 7 depicts the nip pressure profiles of the paperboards that are identical to those in Fig. 6b but in a lower printing speed, $v = 1$ m/s, a speed similar to a Lab press. As shown, both the peak nip pressure and the nip length or nip duration decrease with reduced printing speed because of the lower strain rate of the paperboards, a phenomenon often known as speed hardening.

Despite general similarities between the pressure profiles shown in Fig. 6a and b, there are also remarkable differences between them. The magnitudes of the peaks in Fig. 6a

Fig. 6 Simulated nip pressure profiles of paperboards whose nonlinearity is approximated by three (a) or six (b) subsections in Maxwell material model. The elastic modulus (E) in these subsections, the viscosity (η), and printing speed (v) are denoted in the figure

Table 1 Descriptions of subsections: the strain range and Elastic modulus in each of the subsections

Subsections	1	2	3
Strain range	$0 \leq \epsilon < 0.4\epsilon_M$	$0.4\epsilon_M \leq \epsilon < 0.6\epsilon_M$	$\epsilon \geq 0.6\epsilon_M$
Elastic mod. (MPa)	1	4	10

Subsections	1	2	3	4	5	6
Strain range	$0 \leq \epsilon < 0.1\epsilon_M$	$0.1\epsilon_M \leq \epsilon < 0.25\epsilon_M$	$0.25\epsilon_M \leq \epsilon < 0.35\epsilon_M$	$0.35\epsilon_M \leq \epsilon < 0.45\epsilon_M$	$0.45\epsilon_M \leq \epsilon < 0.6\epsilon_M$	$\epsilon \geq 0.6\epsilon_M$
Elastic mod. (MPa)	0.5	1.5	2.5	3.5	5	10

Fig. 7 Nip pressure profiles of paperboards of the same board properties as in Fig. 6b but in different printing velocity, $v = 1$ m/s

Fig. 8 Nip pressure profiles of paperboards having different E modulus in the compressive subsections

differ from their counter parts in Fig. 6b. In both cases, the peaks lie in the final subsections (3 and 6) whose definitions (compression range and mechanical properties) are identical as listed in Table 1. This implies that the magnitudes of the peaks depend not only on the subsection where the peak is located but also on the preceding sections. This is further demonstrated in Fig. 8. As shown in the legend in the figure, the simulations correspond to cases of identical geometrical subsections and the same elastic modulus in the third subsection while their elastic modules of the first two subsections differ. The case of higher elastic modulus in the preceding sections led to higher nip pressure in the following section. This observation can be understood with the help of the mathematical expressions for the nip pressure, e.g. Eqs (8) and (9). The integration constant of the final section, C_3 , became greater when the elastic modulus of the preceding subsections were greater.

Fig. 9 The stress–strain curves from simulations with 6 subsections, corresponding to different printing speed, **a** 1 m/s; **b** 10 m/s

Viscoelastic and Mechanical Hysteresis

The validity of the proposed approach was further examined by evaluations of the stress–strain curves. Figure 9 depicts the curves obtained from the simulations with 6 subsections and two printing speeds, 1 m/s and 10 m/s, respectively. As seen from the figures the characteristics resulted from the elastic nonlinearity of the paperboards were also reasonably well reproduced by the simulations compared to the experiment shown in Fig. 1a, indicating the feasibility of the proposed approach.

Another important observation from the simulated curves is that the hysteresis effect of the paperboard in the compression & relaxing cycle responded clearly to the viscosity of the paperboards and printing speed. Because all the paperboards simulated possess identical elastic characteristics, the differences observed solely originated from their differences in viscosity. As seen from Fig. 9a, a paperboard of higher viscous coefficient (η) had a higher peak nip pressure and a narrower hysteresis feature. As shown in Fig. 9b, these effects became more pronounced when the strain rate was higher as the strain

rate is proportional to the printing speed. One can anticipate that in an extreme case of infinite viscosity ($\eta \rightarrow \infty$), the model paperboard is purely elastic, and the hysteresis effect disappears.

Due to the series connection of the dashpot with the elastic spring in the Maxwell model, one receives the following relationship,

$$\sigma = \eta \dot{\varepsilon}_v = E \varepsilon_e \quad (10)$$

and

$$\varepsilon = \varepsilon_v + \varepsilon_e \quad (11)$$

where ε_v and ε_e are the strains corresponding to the deformations of the dashpot and the spring, respectively. This means that the compressive behavior of the Maxwell material is largely governed by the material's viscosity and the strain rate. The lower the viscous coefficient or the lower the strain rate the stronger the viscous behavior of the material. The elastic deformation becomes important only when the resistance to the dashpot-deformation or the product, $\eta \varepsilon_v$, is sufficiently strong.

Based on the preceding observations and reasoning one can say that material's viscoelasticity is responsible for the hysteresis effect of the paperboard.

Discussion

Mechanical nonlinearity is a general phenomenon of fibre-based material in compressive deformation. Figure 10 shows a SEM image of the cross-section of a coated paperboard sample prepared with ion beam slope cutting [16]. From top to bottom, the paper sample consists of multiple plies

Fig. 10 SEM image of cross-section of a coated paper sample prepared with ion beam slope cutting [16]

or subsections: a dense coating layer, a porous top layer of fibre, a more porous middle layer, and a porous bottom layer. When compressed the porous structure is expected to be densified. As the property of paperboard relates close to its geometrical structure and material & mass distributions, it is natural that its properties change along with the compressive deformation. A typical example is calendering. Being compressed in Z-direction (Z-D) both pore size and porosity decrease while density increases. Consequently, the mechanical and optical properties of the paperboard change upon calendering, even though the material composition of the paperboard remains unchanged.

The approach presented in this work has its natural connection to the paperboard's structure. Figure 11a visualizes the approach with three subsections which may be regarded as three plies of the paperboard. These subsections have different elastic modulus and finite range of compressive deformation, i.e. not infinitely compressible. When compressed the subsection of the lowest elastic modulus, for instance, the middle ply which is of bulky structure, is first deformed. While being compressed in Z-D, the paperboard is densified and fibre–fibre contact area grows. Consequently, resistance or the elastic modulus to compressive deformation increases. After the middle ply becomes eventually incompressible as illustrated in Fig. 11b, deformations in the other plies become dominant and this process continuous in case of sufficiently strong compressive pressure.

Even though it is less intuitive, the illustration shown in Fig. 11 can also be used to describe what happens in a single-ply paper material. When compressed, the paper structure is uniformly deformed in Z-D. One subsection in Fig. 11 corresponds to one compressive states of the total paper thickness. In other words, a section of low elastic modulus corresponds to a bulky or porous paper structure, while a section of higher elastic modulus to a more densified paper structure.

Maxwell material model incorporating material's nonlinearity and viscoelasticity can explain the experimental observations of paperboard materials. Both peak pressure and nip length (or nip duration) in the print nip increase

Fig. 11 Illustration of densification and nonlinearity of paperboard. The line width of the spring indicates the strength of the elastic modulus

with increasing viscosity, a common phenomenon known as speed hardening. Even the hysteretic behaviour originates from the viscoelastic nature of the paperboard. Due to a change in sign of strain rate around the turning point in the stress–strain curve, where maximal compressive strain occurs, the curve that corresponds to relaxing section separates from that of the compression section. Due to series connection of the elastic and viscos components, the viscoelastic phenomenon has more pronounced effect at low strain rate while an infinitely strong viscos component means a pure elastic material system. It is worth noticing that the simulations shown in this work made no attempt to quantitatively reproduce the experimental data presented in Fig. 4, as this figure was only used as an example to illustrate non-linear elasticity of a true paperboard. The aim of this work is to demonstrate that it is possible to account for non-linearity of a viscoelastic material like paperboard by a simple physical model, here Maxwell. Quantitative reproduction of the experimental data will be reported elsewhere.

Hysteresis originates from material's viscoelasticity and is the general characteristic of paper and polymeric materials. For an elastic material, its stress only relies on its strain, thus there is a one to one mapping between the stress and strain, e.g. the stress–strain curve corresponding to the compression section is identical to that of the recovering section (zero hysteresis). But for a viscoelastic material, its stress depends on not only the elastic strain but also on the strain rate of the viscous deformation. This leads to two different stress–strain paths around the turning point where the strain rate changes its sign. Even though the detailed shape of the hysteretic curve may be influenced by the paper's composition and structure, Hysteresis effect exists in single-ply and multiple-ply. Nevertheless, a paperboard having multiple-ply may exhibit richer non-linear elastic structure, e.g. the stress–strain curve needs to be divided into more subsections.

To simulate full-scale printing with a Lab print press is a challenging task. Materials involved in conventional printing, e.g. paperboard, polymer/rubber plate, and cushioning tape, are viscoelastic. Material's resistances or responses to compressive deformation in the printing nip depend not only on the strain but also on the strain rate that is proportional to the printing speed. In the Lab press like IGT F1 the nip pressure is directly set while in the full-scale printing it is the nip gap rather than nip pressure that is set. Hence, it is unlikely that by chance the nip pressure of the Lab press is the same as the full-scale printing processes. In addition, due to a significant difference in printing speed, the strain rates in the respective printing nip differ significantly. This means that the responses from different materials sections of the materials sandwich, board-plate-tape, may differ significantly in the Lab press compared to the full-scale printing.

Moreover, due also to the viscoelastic nature of the materials involved, the nip pressure profile (peak, duration, etc.) which is the key for ink-transferring and thereby print quality may also differ significantly between the Lab and full-scale printing processes. Hence, one cannot by chance simulate/predict a full-scale printing by just running Lab printing trials. Careful designs with help of print dynamics simulation are needed so that the Lab trial is tuned in such conditions that simulate the full-scale process in the best possible way.

Conclusion

Materials involved in flexographic/offset printing, paperboard, polymer plate or rubber blanket, and cushioning tape, are viscoelastic and nonlinear. These characteristics govern various aspects of the materials' dynamic performance, from testing and interpretation of the materials' properties to predicting the material behaviours in a dynamic process, e.g. printing nip pressure profile. For fibre-based packaging materials, e.g. paperboards, we found that the extended approach based on the Maxwell model works well to account for both viscoelastic and nonlinear characteristics. With this approach, the dynamic behaviours of the paperboard can be interpreted and predicted. The underlying origin of the elastic nonlinearity is material densification upon compressive deformation. As concrete examples, the nonlinear elasticity of the paperboard has successfully been coped with by three and six subsections and in each of the subsections, the elasticity is treated as constant. The simulation also reveals that viscoelasticity of the board is the origin of mechanical hysteresis of the stress–strain curve. The viscous component has been assumed as constant in the simulations, which is a possible limitation of the simulation results. However, this limitation can be removed by also allowing the viscous component to take multiple values as does the elastic component.

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