Multi-objective Optimization Approach Based on Deterministic and Metaheuristic Techniques to Resource Management in Health Crisis Scenarios Under Uncertainty

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Abstract

This paper presents an optimization strategy based on mathematical programming to solve resource management problems regarding hospitalization of sick patients, considering emergency scenarios, such as those that can occur in a pandemic. This optimization strategy is based on the use of efficient optimization tools in solving complex problems in which other strategies are not efficient. The mathematical modeling of epidemiological phenomena is a useful tool to predict the direction of a disease, as well as to adequately and timely manage the available resources and thus save as many lives as possible. This work uses a mathematical model formulation based on deterministic optimization developed in general algebraic modeling system (GAMS) environment. The main user interface has been developed in a Microsoft™ (MS) Excel worksheet, which is familiar to many users. The linking code to send values from MS Excel to GAMS has been programmed in visual basic for applications (VBA) and it uses GAMS data exchange (GDX) files. The proposed optimization methodology is applied to case studies based on data obtained from affected people by severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), which is the virus that causes coronavirus disease 2019 (COVID-19). The distances were taken as example from severely affected cities in the USA. The obtained results offer attractive alternatives for the specified objective function in an acceptable computation time.

Keywords Deterministic optimization · Mathematical modeling · GAMS · MS Excel · SARS-CoV-2 · COVID-19

Introduction

At present, proper management of available natural resources (Pimentel et al. 1994) is essential to guarantee the sustainability of the lifestyle despite population growth (Ehrlich and Holdren 1971) and the impact it has on the environment (Cropper and Griffiths 1994). For this purpose, different optimization strategies have been developed that allow proposing different alternative solutions to the environmental problem, for example, the distribution of supply chains (Bowling et al. 2011), processes for obtaining biofuels (Hernández-Pérez et al. 2019), adequate management of resources in habitational complexes (Fuentes-Cortés et al. 2015), production of conventional electric energy through alternative technologies

(Serrano-Arevalo et al. 2020), and the management of urban solid waste (Santibañez-Aguilar et al. 2013) and the analysis of the water-energy-food nexus (González-Bravo et al. 2018).

It is worth noticing that there are many mathematical optimization approaches (Andiappan V, 2017) from which it is possible to develop methodologies and propose models (Sun and Fan 2020) that offer us the best possible solution for a specific problem. Figure 1 shows a general classification of the different optimization procedures and global search methods (Coello-Coello et al. 2002; Devillers 1996). In general, the use of the optimization techniques mentioned above can be simplified as follows: in the first instance, deterministic search strategies are used to solve linear programming (LP) or mixed-integer non-linear programming (MINLP) formulations (Costa and Oliveira 2001) and metaheuristic or stochastic programming (Ponce-Ortega and Hernández-Pérez 2019) is used when they have very complicated (highly non-convex) problems. Another advantage of using a metaheuristic optimization algorithm is that multiple configurations can be analyzed with different values of the search variables. This is

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Fig. 1 Classification of global search and optimization

particularly useful for evaluating the performance of certain objectives in a multi-scenario analysis.

Most of the reported works that are mentioned above to propose alternative solutions to the environmental problem are part of optimization strategies through the use of mathematical programming (Horst and Tuy 1990) and deterministic solution strategies (Ponce-Ortega and Santibañez-Aguilar 2019). Computational software that allows implementing mathematical models for optimization is based on the use of mathematical tools to find the optimum of a specific problem. Two of the most used commercial optimization programs are General Algebraic Modeling System (GAMS) software (Brooke et al. 1992) and Matrix Laboratory (MATLAB).

However, there are also solution proposals through procedures in which evolutionary and differential evolution (DE) algorithms have been used to solve those problems, in which due to their complexity, it is not possible to apply conventional search strategies.

Metaheuristic optimization strategies are often referred as stochastic optimization due to the uncertain nature inherent in evolutionary algorithms (Diwekar 2008). The above is because in all these evolutionary search strategies, a randomness factor is used. With a randomness factor, new individuals (possible sets of solution values) are generated. With the use of randomness factors and mutation operators, it is avoided to get caught in local solutions. To avoid any confusion, in this text the metaheuristic term will refer to the use of different evolutionary optimization algorithms and the stochastic term to the uncertainty associated with the use of randomness in the method.

When a procedure that intends to use both optimization approaches is proposed, deterministic and metaheuristic, it is

necessary to establish a link between the algorithms of each approach. There are different alternatives for linking optimization algorithms, even if they are on different platforms or programs. In the proposal presented here, a code to link Microsoft™ (MS) Excel and GAMS software was developed in visual basic for applications (VBA) using GAMS data exchange (GDX) files. The used linking subroutine allows the transfer of parameter and variable values from one algorithm to another.

Methodological strategy refers to the set of steps that is proposed in this work, because it involves the use of multiple optimization algorithms and not just a search procedure. The optimization framework of the methodological strategy proposed here was developed to solve a complex problem in the distribution of resources. The case study where it is applied is to solve the problem of resource management in an emergency scenario such as that which occurs during a pandemic. The case study presented to test the effectiveness of the method is a resource distribution problem elaborated with real data from the current pandemic.

Severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) is the virus that causes coronavirus disease 2019 (COVID-19). According to the World Health Organization (WHO, 2020), the disease has now infected almost 50 million people and killed about 1.24 million victims worldwide. In the USA, one of the countries most affected by the pandemic, there have been registered almost 10 million confirmed positive cases for SARS-CoV-2 and the COVID-19 has claimed the life of more than 237,000 people only in the USA according to data of November 07, 2020. Analyzing the previous data, it can be seen that about a fifth of the people affected by the disease reside in the USA. Likewise, the concentration

of the greatest number of positive cases is found in states of that country, such as New York State, with more than 526,000 confirmed cases and with more than 33,000 deaths from the disease. Another state hardly affected by this new disease is New Jersey, where it has more than 254,000 positive cases and more than 16,000 deaths. Under the current trend following the COVID-19 pandemic, health care systems have already been saturated as projected by simulations (Ferguson et al. 2020).

Likewise, there are some states in this country that do not have such a high rate of infections, and therefore, the total number of people affected by the disease is less. Some of these states are neighbors (such as Connecticut, Pennsylvania, and Delaware) to those most affected, so the analysis of efficient patient transfer strategies can offer alternative solutions to save as many people as possible.

The proposed methodology consists in an optimization strategy based on mathematical programming to obtain the best management of resources in scenarios of a health crisis considering hospital restrictions. The mathematical modeling of a phenomenon such as the distribution of resources under crisis scenarios can be extremely complex. Therefore, the use of a methodology that allows the use of both deterministic optimization and metaheuristic tools is necessary.

Recently, the solution of a complex optimization problem has been reported, which consists in the optimal management of an essential resource, water (Hernández-Pérez et al. 2020). In that work, a general methodology that allows to link deterministic optimization with metaheuristics was used. However, one characteristic of crisis scenarios is that they are under inherent uncertainty. Therefore, the development of a new multi-objective optimization methodology that allows the use of deterministic optimization and metaheuristics considering uncertainty is essential for optimal resource management in crisis scenarios such as what occurs during a pandemic.

In the present methodological proposal that is shown in this work, it is proposed to link the computer programs that allow the use of deterministic optimization in GAMS and metaheuristics through a DE algorithm programmed in MS Excel. The link between these programs is carried out through the action of a code or subroutine developed in the VBA environment. Likewise, it offers the user a simple and easily manageable interface developed in a program with which many users are familiar.

The review of the different optimization approaches that is carried out is to show that there are multiple types of problems and various alternatives to address them according to their own nature. Likewise, to establish that despite the fact that efficient search procedures have been proposed in solving optimization problems with deterministic or metaheuristic approaches, but not both simultaneously, considers also the uncertainty associated with emergency scenarios. These types of problems are addressed because they are the ones that, due to

their complexity, require a methodological strategy that involves the simultaneous use of different optimization algorithms, but it may well be used in other types of problems with associated uncertainty.

Methodology

As mentioned previously, mathematical programming is a tool that consists in the representation of phenomena in models for their subsequent analysis and optimization. Optimization is a strategy to search for the best values that a decision variable can take with which the best possible performance of an objective function is achieved. However, the set of possibilities of configurations of a process (including those of transport) can become very exhaustive due to the large number of variables, equations, parameters, and restrictions that are involved in the mathematical model. For this, the implementation and solution of these mathematical models in a computational tool is essential.

Optimization strategy programming is possible from a root language such as C++ and Java. However, there are currently a variety of commercial programs with deterministic optimization tools, such as Lindo and Gurobi. MS Excel can be used to contain a metaheuristic optimization program (Hernandez-Perez et al. 2020). Thus, it can be used by itself for linking with the simulation program and optimizing the problem using parameters specified by the user.

In this paper, a linking program code (LPC) was implemented to link the GAMS program with the MS Excel through VBA code. This strategy is done in order to send and receive data (like DV and objective functions values) from the deterministic optimization algorithm (DOA) that is carried out in GAMS, with the metaheuristic optimization algorithm (MOA) that is previously programmed in MS Excel. Likewise, a stochastic optimization algorithm (SOA) is proposed from which uncertain parameter values are created by the action of a random values generator (RVG) code; in this way, the uncertainty is incorporated into the proposed optimization approach.

The methodological strategy proposed in this work consists of three fundamental parts, the MOA, the DOA, and the SOA, each is described below:

- **Deterministic optimization algorithm:** The DOA is where the mathematical model is contained, which is successively solved in GAMS software from the values that the MOA generates. The DOA reads the values of the decision variables that the MOA generates through the LPC.
- **Stochastic optimization algorithm:** The SOA is the part of the methodological proposal, developed in VBA environment, which is responsible for incorporating the uncertainty from the uncertain parameter values that creates the

RVG code. The SOA creates a determined number of uncertain parameter values calling RVG code for each set of decision variable values. Therefore, several new functions must be analyzed in a statistical treatment and this way select the values of the objective functions that will be sent through the LPC to the MOA for its performance evaluation.

- Metaheuristic optimization algorithm: The MOA is the algorithm based on DE previously programmed in MS Excel. The MOA generates the sets of values (individuals) of the decision variable that may be the optimal solution. Likewise, it is in the MOA where the evaluation of the objective functions is carried out to select the sets of values with the best performance and propose new values for the next iteration (generation).

These optimization algorithms are independent and are commonly used separately; however, due to the complexity of the case study that is addressed in this work, a hybrid methodology is developed that allows the use of deterministic optimization simultaneously with the metaheuristic optimization tools, while incorporating uncertainty by a stochastic algorithm. Each of the algorithms that are part of the methodology is described in detail below.

General Optimization Approach

In this work, a stochastic hybrid (deterministic and metaheuristic) multi-objective optimization algorithm is developed. Consideration of uncertainty is essential to analyze problems in which there is a high probability of change in the value of the parameters. There are several problems in which considering uncertainty offers a better picture of the general behavior of the analyzed phenomenon. Comparing the performance of different solution proposals in which the uncertainty associated with the change of parameter values within the model is considered allows a more realistic analysis of the problem to be addressed. This is because an alternative solution may well represent the best solution with fixed values of certain parameters, but the same proposal may have a lower performance if the value of these parameters changes randomly. In this way, the methodological proposal that considers the random manipulation of constants to consider the performance of a proposal calculated with the values of decision variables offers better resilient solutions to the random change of uncertain parameters. The general optimization approach is shown in Fig. 2. The proposed methodology consists of the steps described below.

1. Decision variable values generation by MOA: The MOA proposes sets of decision variable values that may be the optimal solution. The values proposed by the MOA are based on the user's specifications regarding the nature of the variable (continuous or integer) and their manipulation intervals. It is in the MOA where the number of sets of values of the decision variables (population size, PS) and the number of iterations (maximum number of generations, MNG) are declared. Likewise, in the MOA, it is possible to declare inequality constraints and parameters for the operation of the metaheuristic algorithm.
2. Uncertain parameter values creation by RVG: For each set of decision variable values proposed by the MOA, the RVG assigns a specific number of uncertain parameter values. The number of uncertain parameter values is specified by the user in the SOA programming. After the execution of the RVG code, the SOA assigns the uncertain parameter values to variables of the MOA that together with the decision variable values are sent to the DOA.
3. Sending decision variable values by LPC: The values of the decision variables proposed by the MOA and the uncertain parameter values created by RVG are sent to the SOA by the action of the LPC. For this purpose, the LPC is composed of a set of subroutines or macros developed in VBA that allow the exchange of data between the MOA and the DOA, that is, between the GAMS software and the MS Excel platform. The set of macros that allow the linking of programs can be complicated; therefore, if a more detailed explanation of the particular methodology for linking the MOA with the DOA is required, more detailed information about it can be found in Hernández-Pérez et al. (2020).
4. Objective function values calculation by DOA: In DOA, the single-objective optimization process is carried out. From the values of the decision variable proposed by the MOA and the uncertain parameter values created by the RVG of the SOA, the deterministic optimization process of a single-objective is carried out considering these variables as parameters in each iteration of DOA in GAMS. The calculation of the objective function in the DOA is made for each individual in the population and for each UPV.
5. Importation of the objective function value by LPC: After the calculation of the objective function value in the DOA, this value is sent back to the SOA by the LPC.
6. Objective function statistical treatment in SOA: Once the objective function values for each set of decision variable and uncertain parameter values are calculated and imported into the SOA. After that, a statistical treatment is carried out with the values of the objective functions. This statistical treatment consists of calculating the mean of the set of values of the objective functions. This new objective function is called statistical objective function.
7. Statistical objective function performance evaluation in MOA: Finally, for each individual in the population in each iteration, performance evaluation of the statistical

Fig. 2 General optimization approach

objective functions calculated by the SOA is made. For this purpose, the statistical objective function is considered as one of the objective functions in the MOA. In this part of the proposed methodology, the performance evaluation of the objective functions is made to propose the values of the new individuals according to the selected metaheuristic optimization procedure.

By following the steps mentioned above, it is possible to use deterministic optimization simultaneously with metaheuristic tools and consider uncertainty. This methodology can be applied in solving the problem of optimal resource management for health crisis scenarios such as those that occur during a pandemic.

Case Study: Optimal Resource Management in Health Crisis

In order to test the effectiveness of the methodological proposal that is explained in this work, a case study has been selected: the solution to the problem of resource distribution in scenarios of uncertainty due to health crises such as those that occur in a pandemic.

For the specific case study addressed in this work, real data from the current pandemic caused by SARS-CoV-2 have been used. According to WHO, most people (about 80%) recover from the disease without needing special treatment, and for the majority (especially for children and young adults), illness due to COVID-19 is generally minor. However, for some people, it can cause serious illness. Around 1 in every 5 people who are infected with COVID-19 develops difficulty in breathing and requires hospital care and about 4% end up in the intensive care units (ICUs). Most of those in intensive care require ventilators. Auld et al. (2020) found that 29.7% of COVID-19 patients who required ventilators died.

According to COVID-19 information from Centers for Disease Control and Prevention (CDC), current hospital

capacity estimates are presented in Table 1. Estimates of hospital capacity were available at the national and state levels and include data from April 1 to July 14, 2020. Data displayed on this table were submitted directly to CDC's National Healthcare Safety Network (NHSN) and do not include data submitted to other entities contracted by or within the federal government. These estimates used data that hospitals submitted to the NHSN COVID-19 Module and statistical methods that included weighting (to account for non-response) and multiple imputation (to account for missing data). The estimates (number and percentage) are shown along with 95% confidence intervals that reflect random error ([cdc.gov/nhsn/covid19](https://www.cdc.gov/nhsn/covid19)).

To know the current deficit or capacity of each state, it is necessary to calculate the number of COVID-19 patients who require specific medical care. The number of patients requiring a hospital bed or ICU is calculated from a balance to obtain active cases using data from confirmed cases, recovered cases, and deaths. The active cases are calculated using Eq. 1.

$$\text{Active Cases} = \text{Confirmed} - \text{Recovered} - \text{Deaths} \quad (1)$$

As mentioned above, the specific medical care for COVID-19 patients can be mainly of two types, hospital beds and ICUs. For the case study that is addressed in this work, these two necessary medical resources are analyzed, each of which is described below:

- Medical resource 1: The first medical equipment necessary for those COVID-19 patients who require hospitalization is a hospital bed. To calculate the number of people who require a hospital bed, the number of active cases is multiplied by an estimation factor (E1). The value of E1 is 0.2. The value of the estimation factor E1 is determined from the information provided by the WHO. The number of hospital beds needed is calculated using Eq. 2.

$$\text{Beds Needed} = E1 * (\text{Active Cases}) \quad (2)$$

Table 1 Hospital capacity estimates

Hospital capacity Location	Beds			ICUs		
	Available	Occupied	% occupied	Available	Occupied	% occupied
New York	47,995	31,533	65.70	8263	4396	53.20
New Jersey	18,702	13,933	74.50	3718	1610	43.30
Connecticut	9093	5983	65.80	1618	772	47.70
Pennsylvania	40,692	26,926	66.17	5034	2819	56.00
Delaware	2582	1771	68.60	308	195	63.40

Table 2 is constructed using the information of Table 1 (corresponding to the availability of hospital beds in each location) and using Eq. 2.

- Medical resource 2: From COVID-19 patients requiring a hospital bed, a percentage requires an ICU. To calculate the number of people who require an ICU, the number of active cases is multiplied by another estimation factor (E2). The value of E2 is 0.04. The value of the estimation factor E2 is determined from the recently reported statistical analyses (Auld et al. 2020). The number of ICUs needed is calculated using Eq. 3.

$$\text{ICUs Needed} = \text{E2} * (\text{Active Cases}) \quad (3)$$

Using the information of Table 1 corresponding to the availability of ICUs in each location and Eq. 3, it is constructed in Table 3.

The balance of patients who require specific medical care (medical resource 1 or medical resource 2) is made from the data of confirmed cases, recovered, and deaths in each location. The data and active patient balance are the same for medical resource 1 (Table 2) and medical resource 2 (Table 3). From the balance of patients, the number of active cases is obtained. With the number of active cases and using the estimation factor for each medical resource, the need for each one is obtained. Subtracting this value from the

availability of each medical resource, the capacity or deficit in each location is obtained.

E1 and E2 are estimation factors that allow calculating the number of patients requiring hospital beds or ICUs, respectively. In other words, they are percentages of active cases with which the requirements of the analyzed medical resources can be estimated. Because E1 and E2 are estimation factors, they are uncertain. However, due to the random behavior inherent in a phenomenon such as a pandemic, there are many uncertain values. To consider all of them, the values that are considered uncertain in this case study are those corresponding to the number of patients who require hospitalization either for medical resource 1 or for medical resource 2 and who are on demand in hospitals originally. In this way, the deficit or capacity values obtained are used as a basis to generate uncertain values around them. As mentioned previously, E1 and E2 are estimation factors for patients requiring medical resources 1 and medical resources 2, respectively. According to the data reported by the WHO, 1 in every 5 people who are infected with COVID-19 requires a hospital bed. Therefore, $E1 = 0.2$, that is, 20% of active cases. Likewise, about 4% end up in the ICUs; therefore, $E2 = 0.04$.

On the other hand, an important aspect to analyze in the case study presented here is the amount of new equipment to be acquired, both for hospital beds and ICUs in the destination hospitals that will receive patients from localities in over-demand. The number of new equipment purchased for medical resource 1 and for medical resource 2 is variable and the

Table 2 Patients balance for beds needed

Patients balance Location	Data				Projection		
	Confirmed	Recovered	Deaths	Active cases	Beds needed	Available	Deficit/ capacity
New York	439,480	75,100	32,531	331,849	66,370	47,995	−18,374
New Jersey	193,740	33,912	15,945	143,883	28,777	18,702	−10,075
Connecticut	52,879	8967	4465	39,447	7889	9093	1203
Pennsylvania	139,376	109,900	7764	21,712	4342	40,692	36,350
Delaware	17,429	9381	605	7443	1489	2582	1093

Table 3 Patients balance for ICUs needed

Patients balance Location	Data				Projection		
	Confirmed	Recovered	Deaths	Active cases	ICUs needed	Available	Deficit/capacity
New York	439,480	75,100	32,531	331,849	13,274	8263	−5011
New Jersey	193,740	33,912	15,945	143,883	5755	3718	−2037
Connecticut	52,879	8967	4465	39,447	1578	1618	41
Pennsylvania	139,376	109,900	7764	21,712	868	5034	4165
Delaware	17,429	9381	605	7443	298	308	10

decision of the amount determines the best solution. The analysis of this aspect together with the information obtained from the transport of patients composes by itself the model of a problem to be optimized.

Although an alternative is to just acquire new resources where the cases are more frequent, it is important to note that this method not only considers that medical resources have been exhausted in a location with excess demand but also the space for improvised facilities it has been exploited and can no longer offer a greater chance of saving lives. Just make use of existing facilities in the destination hospitals is also an option, which is considered within the model in the lower limit of the search variables in the metaheuristic algorithm, that is, the optimization strategy can analyze and select the possibility of not acquiring new medical equipment.

The details of the creation of the uncertain parameters are described in detail in SOA, and how they are used to calculate the optimal distribution of transported patients is described in detail in DOA, while the variable capacity of destination hospitals is explained in MOA description. All of them are in the following parts of this text.

Due to the complexity involved in both, the modeling and the optimization process, a new hybrid strategy is necessary. In the described method, the intervention of deterministic optimization and metaheuristic tools is required to propose an alternative solution. Likewise, it is necessary to introduce randomness in the method, due to the uncertain nature with which diseases behave.

The methodological strategy proposed in this work is composed of a set of steps and algorithms described in the methodology section, each of which has its own equations, variables, and parameters (some of which are variables in an algorithm and parameters in another). The equations that it uses are included in each part of the text that describes an algorithm. The equations that are mentioned first are those of the balance of patients with the data of the case study, which allow to build the tables with which parameter values are specified in the algorithms. These patient balance equations are included in the case study because it is in this context where the analysis of two different medical resources takes relevance; likewise, the difference is made in their calculation through estimation parameters.

In the optimization model that is proposed in this case study, two objective functions are considered, one social and the other economic. The first objective function corresponds to the primary interest in a health crisis scenario, save lives. The second objective function corresponds to the economic available resource to attend the pandemic.

The social objective function is directly related to the number of patients that requires special medical care and is adequately treated according to their situation; in other words, the possibility of increasing the number of saved lives is by guaranteeing adequate medical attention to the greatest number of people that requires it. To achieve this, there are two options: transfer patients that require intensive care to a hospital that has the capacity to attend them or acquire new equipment at the hospital that is on demand. However, the number of patients in an over-demand hospital (origin hospital) is uncertain and the capacity to care for them is variable depending on the equipment acquired by each host hospital (destination hospital). The probability of saving lives increases by minimizing the number of neglected patients as is defined on objective function 1 specification in MOA.

In the formulation of the model that corresponds to the MOA, it is considered that the hospitals in the over-demand states do not have enough space to install the medical equipment that they could obtain, so the purchase of new equipment is restricted to the host hospitals. The above converts the number of equipment obtained in the host hospitals in the decision variables of the MOA model and the excess demand of saturated hospitals in the uncertain parameter values of the SOA model.

DOA Model Formulation

The DOA is the step of the methodology that allows both to raise a part of the mathematical model and to carry out the deterministic optimization process. One of the parts of the DOA is the representation of a phenomenon that occurs in reality through a mathematical model. This mathematical model consists of a set of parameters, variables, and equations

that are solved simultaneously in the deterministic optimization process. The other part of DOA consists of the optimization process that is carried out by the GAMS software. This optimization process is carried out considering the model as a single-objective problem. Therefore, if the original approach consists of a multi-objective optimization problem, it is necessary to add the calculation and the value of the other objective functions to the MOA.

For the case study presented in this work, it is necessary to address the transport model. There are some examples that can be used as a base, such as the McCarl Transport Model. This and other examples can be found in the GAMS user guide section (gams.com).

In general, the transport model consists of finding the optimal distribution of resources according to some known values of the problem. The information that is entered into the model that works as a known parameter consists of the distance between the locations, a demand (hospital capacity in this case), and an offer (patients requiring hospitalization). Likewise, in the transportation model, a fixed transportation cost is used. For the case study addressed in this work, the study of two states with over-demand of patients (New York and New Jersey) and three states with the capacity to serve said patients (Connecticut, Pennsylvania, and Delaware) is carried out. The superstructure of the possible distribution of patients among the states selected in the study is shown in Fig. 3. Table 4 shows the used distance between states, expressed in miles. For the particular case study that is presented in this work, the DOA specifications are as shown below:

- DOA sets: origin hospitals (i) until maximum number of origin hospitals (n) and destination hospitals (j) until maximum number of destination hospitals (m).
- DOA parameters: patients who exceeded the maximum capacity of the origin hospital i (a_i), patients that can be treated at the destination hospital j due to the new equipment acquired (b_j), distances between origin hospital i and destination hospital j (d_{ij} in mile/case), and transport cost (c_1 in \$/case/1000 miles). The value of c_1 is 30 US\$ per mile for medical resource 1 and 50 US\$ per mile for medical resource 2.
- DOA variables: number of patients who are transferred from hospital origin i to hospital destination j (x_{ij}).

- DOA objective functions: the objective function of the DOA is expressed in Eq. 4:

$$\text{Min } f(x_{ij}) = c_1 \sum_{i=1}^n \sum_{j=1}^m d_{ij} x_{ij} \quad (4)$$

- DOA restrictions: the constraints for objective function in DOA depend on the medical equipment. For medical resource 1, the restrictions are defined as C_1 and C_2 ; for medical resource 2, the restrictions are defined as C_3 and C_4 . Medical resource 1 and medical resource 2 are similar in nature; however, they are different in terms of the number of people who require one or the other with respect to the amount that is possible to offer. In other words, the demand for medical resource 1 is less than that which is possible to offer by other destination hospitals if patients are transferred; on the other hand, the demand for medical resource 2 is greater than what is possible to offer by destination hospitals, still increasing their capacity by acquiring new equipment. Therefore, it is necessary to specify the restrictions expressed in Eq. 5 and in Eq. 6 for medical resource 1, and expressed in Eq. 7 and in Eq. 8 for medical resource 2 into the mathematical model so that it works properly.

$$C_1 \equiv \sum_{j=1}^m x_{ij} \geq a_i \forall i \quad (5)$$

$$C_2 \equiv \sum_{i=1}^n x_{ij} \leq b_j \forall j \quad (6)$$

$$C_3 \equiv \sum_{j=1}^m x_{ij} \leq a_i \forall i \quad (7)$$

$$C_4 \equiv \sum_{i=1}^n x_{ij} \geq b_j \forall j \quad (8)$$

$$C_5 \equiv x_{ij} \geq 0 \forall i, j \quad (9)$$

Following the mathematical model presented above, it is possible to solve the patient transport problem using deterministic optimization through GAMS software. For the DOA, in this case study, a LP model using the CPLEX solver was specified.

SOA Parts and Sequence

The SOA is the part of the methodology that allows considering the uncertainty through the creation of UPV by the RVG code. Likewise, in the SOA, the DOA call and the import of objective function value are carried out in each deterministic optimization process in GAMS. With the objective function values of multiple sets of decision variable values and

Table 4 Distance between states in miles

Distance	Connecticut	Pennsylvania	Delaware
New York	97.7	200	188
New Jersey	165	235	156

Fig. 3 Patient distribution super structure

uncertain parameter values, a statistical treatment is made to calculate the arithmetic mean that composes the SOF. Subsequently, the SOF is declared as objective function in the MOA for the evaluation of its performance, and with these, new values are proposed for the decision variables in the next iteration. Therefore, MOA calls the SOA for each individual, which in turn calls the DOA for each uncertain parameter value created by the RVG code. The SOA consists of the following key parts:

1. **Declaration of DV and objective function:** First, it is necessary to declare the decision variables that are part of the SOA; these search variables correspond to the decision variable values that are generated in the MOA. The objective function that is declared in the SOA is the one that will be obtained in each DOA optimization process.
2. **RVG code:** The RVG code creates a set number of uncertain parameter values to introduce the uncertainty in the method. The parameter used by the RVG code to generate values is a random manipulation of 5% of the current deficit values in the destination hospitals. The RVG code does not use historical data on the behavior of uncertainty to introduce unpredictable changes to the model, evaluating the performance of possible solutions with highly uncertain values. The result of the complete optimization procedure is the selection of the best solutions resistant to drastic changes in some of the parameters in the model. These uncertain parameter values take random values in a previously established interval. For each of these uncertain parameter values, the DOA is called through the LPC.
3. **SOF calculation:** Once the value of multiple objective functions in the DOA has been calculated, they are imported into the SOA through the LPC. After that, a statistical treatment is carried out that consists of calculating the arithmetic mean of the said objective functions.

This arithmetic mean that is calculated conforms the SOF and is declared as objective function in the MOA.

Each of the parts of the SOA followed in the sequence mentioned above allows the incorporation of uncertainty into the methodology. This is through the creation of uncertain parameter values by the RVG and the subsequent statistical analysis of the objective function values obtained in each DOA optimization process to calculate the SOF, which is declared as objective function in the MOA.

For the case study that is presented in this work, the SOA specifications are as shown below.

The number of patients requiring hospitalization is considered as the uncertain parameters to properly consider the uncertainty within the method. It is considered that some parameters are uncertain in the deterministic mathematical model. For this particular case, the number of active COVID-19 patients is assumed to be uncertain and may vary by $\pm 5\%$ of the calculated balance. The manipulation range of the RVG code for the uncertain parameter values is 5% around the original value of the uncertain parameter.

For the case study analyzed in this work, uncertain parameter values are created by the RVG code around the base value of the number of patients who require special medical care (either hospital beds or ICUs). Depending on the medical resource to be analyzed, the data in the column “Deficit/capacity” are taken.

In case of the analysis of medical resource 1 (hospital beds), the data from Table 2 are taken. Following the indications above for the uncertain parameter values, the RVG code proposes integer random values for the numerical constant (18,374) between 17,455 and 19,293 ($18,374 \pm 5\%$) for the case of New York and integer random values for the numerical constant (10,075) between 9571 and 10,579 ($10,075 \pm 5\%$) for the case of New Jersey.

In case of the analysis of medical resource 2 (ICUs), the data from Table 3 are taken. Following the indications above for the uncertain parameter values, the RVG code will propose integer random values for the numerical constant (8329) between 7913 and 8745 ($8329 \pm 5\%$) for the case of New York and integer random values for the numerical constant (3476) between 3302 and 3650 ($3476 \pm 5\%$) for the case of New Jersey.

From the uncertain values and the capacity of each destination hospital, it is possible to determine the number of patients that will be rejected. Minimizing this function is expected to increase the chance of saving lives. The mathematical relationship to determine the number of rejected patients is expressed in Eq. 10 for medical resource 1.

$$\text{Min } g(a_i, b_j) = \sum_{j=1}^m b_j - \sum_{i=1}^n \mu a_i \quad (10)$$

Subject to:

$$C_6 \equiv \sum_{j=1}^m b_j \geq \sum_{i=1}^n \mu a_i \quad (11)$$

The mathematical relationship to determine the number of rejected patients is expressed in Eq. 12 for medical resource 2.

$$\text{Min } g(a_i, b_j) = \sum_{i=1}^n \mu a_i - \sum_{j=1}^m b_j \quad (12)$$

For medical resource 2, subject to:

$$C_7 \equiv \sum_{i=1}^n \mu a_i \geq \sum_{j=1}^m b_j \quad (13)$$

where

g = neglected patients
 i = origin hospitals
 j = destination hospitals
 a_i = patients who exceeded the maximum capacity of the origin hospital i
 b_j = patients that can be treated at the destination hospital j due to the new equipment acquired
 μ = uncertain parameter value (excess demand of saturated hospitals)

The statistical objective function consists in the arithmetic mean, that is, the average of a set of numerical values, calculated by adding them together and dividing by the number of terms in the set. In this methodology, these numerical values correspond to the performance of the objective function calculated in the DOA for each uncertain parameter values created by the RVG code. Therefore, the number of objective functions values that will have this statistical treatment to calculate the SOF is equal to the number of uncertain parameter values (K).

SOF₁ mathematical definition is expressed in Eq. 14.

$$\text{SOF}_1(g) = \frac{1}{K} \sum_{k=1}^K g_k \quad (14)$$

where

SOF₁ = statistical objective function 1, the arithmetic mean of a set of g

g = neglected patients

k = uncertain parameter value number k

K = set of numerical values (number of uncertain parameter values created by RVG in SOA)

SOF₂ mathematical definition is expressed in Eq. 15.

$$\text{SOF}_2(f) = \frac{1}{K} \sum_{k=1}^K f_k \quad (15)$$

where

SOF₂ = statistical objective function 2, the arithmetic mean of a set of f

f_k = uncertain objective function calculated in the DOA using the uncertain parameter value number k , uncertain transport cost

K = set of numerical values (number of uncertain parameter values created by RVG in SOA)

Using the statistical objective function, it is possible to analyze the average of a set of multiple objective functions calculated by DOA for each uncertain parameter values created in SOA. The statistical objective function is only a part of the economic objective function that must be declared in MOA.

MOA Description

The MOA is the tool that allows the solution of complex mathematical models with non-linear equations and highly non-convex problems. One of the characteristics of metaheuristic optimization algorithms is that they allow the “what if” analysis to be carried out. The “what if” analysis is one of the most useful functions that the algorithms based on DE have, since they allow comparing the performance of the objective functions with different values of the decision variables and in different scenarios. In this work, the MOA is used to perform a “what if” analysis of different scenarios by changing decision variable values.

Part of the methodological strategy presented in this work is developed in MS Excel through VBA. MS Excel is a computer software that allows to perform simple numerical (like accounting and financial) tasks due to its included functions, developed specifically to help create and work with spreadsheets. One of the strengths of MS Excel is that it gives its users the ability to customize their spreadsheets by programming their own functions, performing specific tasks, adjusted

to the needs of each one, and that have not been included in the original package.

For developers who are looking for specific objectives, or for a complexity greater than the calculation of common functions, the solution lies in the use of visual basic. It is a language with a relatively high degree of abstraction (which is considerably different from the machine language, the way in which the processor works) and which, like the one used in MS Excel, works by events (this means that the user must perform some action to trigger the execution of the program). VBA is the language of the MS operating system (Windows) that is used for program applications. Many of the programs and add-ons that are used in Windows are developed in this language so there are common elements that can be manipulated through this platform.

The combination of MS Excel and visual basic also has its limitations, and in no way can exceed the level of customization and precision possible through the creation of a program from scratch, but it is very effective and comfortable to the user. Due to the characteristics mentioned above, MS Excel represents a viable alternative for the development of stochastic optimization algorithms (Martin-Martin 2019).

For the case study that is presented in this work, the MOA specifications are as shown below:

- I-MODE algorithm: There are a wide variety of MOAs that can be used in the proposed optimization methodology. The selected MOA is the I-MODE algorithm (Sharma and Rangaiah 2013) developed in MS Excel through VBA. The multi-objective optimization metaheuristic method called I-MODE works with different termination criteria: chi-squared termination criterion (ChiTC), steady state termination criterion (SSTC), and after MNG. The main program user interface of I-MODE algorithm in MS Excel is presented in Fig. 4.
- MOA variables: The I-MODE variables correspond to the decision variables that are mentioned in the case study explanation. For MOA purposes, only the number of patients that can receive the assigned hospitals or destination hospitals are considered as search variables. (b_1 , b_2 , and b_3). The value of these variables will determine the objective function performance. Regarding the search interval, it depends on the medical resource analyzed. In the case of medical resource 1 (hospital beds), a maximum increase of 10% of the current capacity of each locality is considered. Regarding medical resource 2 (ICUs), a maximum increase of 5% of the current capacity of each locality is considered. The limits to the number of new material acquisitions were arbitrarily selected considering a moderate increase for the two medical resources according to the cost associated with each one, that is, for medical resource 1, an increase of 10% is considered, and for medical resource 2 (which is more expensive), a smaller increase of

5% is considered. These limit values in the acquisition of new medical resources are specified for illustrative purposes of the method and other percentages of increase may be well specified.

- MOA algorithm parameters: For the optimization process, in this study, the values for the parameters associated to the used I-MODE algorithm are the following: PS: 10 individuals, MNG: 100 generations, taboo list size (TLS): 50 individuals, taboo radius (TR): 0.01, crossover fraction (CF): 0.5, and mutation fraction (F): 0.5.
- Objective function specification in MOA: In the MOA, the model is considered as a multi-objective optimization problem. The objective functions that are considered correspond, one to a social aspect and the other to economic interests. The social objective function consists in minimizing the number of patients who are left without medical attention, thereby seeking to increase the chances that a greater number of patients will have the necessary medical care. The objective function 1 (OF_1) is expressed in Eq. 16.

$$\text{Min } OF_1(g) = \text{SOF}_1(g) \quad (16)$$

where

OF_1 = objective function 1 declared in MOA, rejected patients

g_k = uncertain objective function calculated in the MOA using the UPV number k , uncertain rejected patients

b_j = patients that can be treated at the destination hospital j due to the new equipment acquired

k = uncertain parameter (excess demand of saturated hospitals)

On the other hand, the economic objective function consists in minimizing the total costs, which are composed of the costs of transporting patients and the costs allocated to the acquisition of specialized medical equipment. The objective function 2 (OF_2) is expressed in Eq. 17.

$$\text{Min } OF_2(b, f) = c_2 \sum_{j=1}^m b_j + \text{SOF}_2(f) \quad (17)$$

where

OF_2 = objective function 2 declared in MOA, total cost

SOF_2 = statistical objective function 2, the arithmetic mean of a set of f

j = destination hospitals

b_j = patients that can be treated at the destination hospital j due to the new equipment acquired

c_2 = cost of the new equipment acquired (US\$/unit). The value of c_2 is 1200 US\$ for medical resource 1 and 27,000 US\$ for medical resource 2

Main Program Interface

Algorithm: I-MODE (Improved Multi-Objective Differential Evolution) for problems with continuous/integer variables and inequality constraints

Objective Functions:

Value Cell	Add Objective Functions	F1	F2	Linking Subroutine	SOA
Goal		12168.60	174465.23		
NumF	2	Minimize	Minimize		

Design Variables:

Value Cell	Add Decision Variables	X1	X2	X3
Minimum		1249	38166	1159
Maximum		1203	38150	1147
Type of Variables		Integer	Integer	Integer
NumX	3			

Inequality Constraints:

Set Cell	Add Constraints	C1	C2	C3	C4	C5
Compare Type						
Constraint Limit						
NumC	0					

Algorithm Parameters:

Population Size	10	Max. No. of Generations	100	Tabu List	5	VAR-limit (GD)	0.0003	SSTC: A1	0.1	Termination Criterion	MNG
Cross Over Probability	0.8	Mutation Probability	0.5	Tabu Radius	0.01	VAR-limit (SP)	0.1	SSTC: A2	0.1		
								SSTC: A3	0.1		
								SSTC: Rcrit	0.9		

Interval Number of Generations for Saving Intermediate Results (This should be less than the maximum number of generations): 1

Run I-MODE

Fig. 4 Main program user interface of I-MODE algorithm in MS Excel

The two objective functions that are specified in the MOA must correspond to those that are expressed in Eq. 9 and in Eq. 10. Likewise, the goal for each objective function in the MOA for this study case must be minimizing.

In general, the fundamental parts of the MOA have been shown; these specifications can be modified according to the characteristics of the problem in the case study to be solved as well as the specifications necessary in the evolutionary algorithm selected as MOA.

LPC Modules

To successfully link GAMS to MS Excel, the use of a GDX file is necessary using code written in VBA. A linking code usually includes different instructions that are grouped into modules. These modules are sub routines that perform a specific task. Subsequently, a main module calls each sub routine in the proper order for them to work correctly. These lines of code are grouped into sub routines that are called by a main module. More detailed information about it can be found in Hernández-Pérez et al. (2020).

Results and Discussion

This section presents the results of the stochastic hybrid (deterministic and metaheuristic) multi-objective optimization approach to solve the resource management problem in health crisis scenarios under uncertainty such as those that occur during a pandemic. The methodological strategy presented in this work is made up of three optimization algorithms, the DOA, the SOA, and the MOA. The aforementioned algorithms communicate and send and receive different values of

variables and parameters. Likewise, they are run in a simultaneous and orderly manner to solve a complex problem such as the one presented in the specific case study.

The entire optimization framework was performed on an Intel™ Core™ i7-6700HM CPU @ 2.6 GHz, 32 GB computer. The computation time to solve the specific case study considering the analysis of medical resource 1 (hospital beds) was 37.07 min. On the other hand, the computation time to solve the specific case study considering the analysis of medical resource 2 (ICUs) was 40.84 min.

The termination criteria specified in the selected MOA (I-MODE) was MNG. Once the MOA reaches the MNG, it offers results with which it is possible to build the graphs presented in Fig. 5 for the analysis of medical resource 1 (hospital beds) and in Fig. 6 for analysis of medical resource 2 (ICUs). The series of points shown in Figs. 5 and 6 all correspond to optimal solutions. Only feasible solutions from the best/first Pareto-optimal front in the last generation are presented. From this set of points, the decision-maker can select the one that best reconciles both objective functions, the social and the economic. The points that best reconcile the performance of the objective functions are usually on the curve that is formed in the graph. These points have been marked as A in Fig. 5 and as B in Fig. 6.

The optimal values of the decision variables and the performance of the objective functions obtained with these values are presented in Table 5. The values presented in the said table are of points A and B and correspond to the analysis of medical resource 1 and medical resource 2, respectively.

Likewise, with the values of the decision variables that are presented in Table 5 and with the current demand without considering the uncertainty in the demand for medical equipment, Table 6 is obtained for the analysis of medical resource 1 and Table 7 for the analysis of the medical resource 2. These

Fig. 5 Results after MNG for medical resource 1 (hospital beds)

tables show the optimal distribution of patients according to the excess demand in the origin hospitals and the capacity to attend the same patients in the destination hospitals.

As can be seen in Table 6 and in Table 7, the optimal distribution of patients corresponds to the one that allocates the largest number of patients from the places with the highest demand to those who have the capacity to receive them and who are within a short distance. However, the balance used can be updated and analyzed different scenarios as the pandemic progresses.

Conclusions

This paper has presented a new multi-objective stochastic optimization framework based on deterministic optimization and metaheuristic tools to solve complex problems considering uncertainty. Also, the user is offered a familiar and easy-to-use interface to quickly manipulate the values and make appropriate decisions in each possible scenario that the advance of the pandemic may form.

The methodology presented in this work consists of solving the problem using different algorithms: DOA, SOA, and MOA. The MOA proposes values of the decision variables,

the total capacity to attend patients with the current capacity plus the capacity due to the medical equipment acquired. The SOA creates random values of the uncertain parameters to bring the uncertainty to the model. Based on the values of the decision variables of the MOA and the uncertain parameter values of the SOA, the DOA solves the patient distribution from the origin hospitals on demand to the destination hospitals that can serve them, either with the medical equipment available or acquiring new.

To test the effectiveness of this method, the case study of the optimal distribution of resources in health crisis scenarios such as those that occur during a pandemic is addressed. The data used to make the analysis of the specific case study correspond to those of the current COVID-19 pandemic caused by SARS-CoV-2. It also considers a balance of patients who require hospitalization to update the distribution requirements. If this methodology is used, it is possible to establish the best patient distribution.

For the solution of the scenarios that are addressed in the case study, real data reported by the WHO of the COVID-19 pandemic were used. These data were collected on September 1, 2020; however, it is possible to update the numbers and run the optimization methodology presented here again. Likewise, it is possible to change the data of distances between locations,

Fig. 6 Results after MNG for medical resource 2 (ICUs)

Table 5 Optimal values for DV and for OFs

Optimal values	X1	X2	X3	OF1 (patients)	OF2 (MUS\$)
Beds	1248	38,158	1157	12,358	173.20
ICUs	41	4165	11	7381	42.52

capacities, and demands. As for the uncertainty, a different interval can be manipulated to create uncertain parameters.

Optimization functions obey the fulfillment of two simultaneous objectives, one social and the other economic. The social objective corresponds to minimizing the number of rejected patients, thereby increasing the chances of saving lives, while the economic objective corresponds to minimizing the total costs of both transportation and acquisition of medical equipment (hospital beds and ICUs). The results show the optimal configuration of the medical equipment acquired by analyzing medical resource 1 (hospital beds) and medical resource 2 (ICUs) as well as the adequate distribution of patients from hospitals on demand to destination hospitals that can attend them considering the uncertainty associated with changes in demand.

The methodological proposal presented in this work is general and can be applied to solve other problems in which it is necessary to analyze different solution alternatives where there is uncertainty associated with parameters within the model. This methodology can be used in different deterministic models with other data and by specifying search variables different from those specified in the metaheuristic tool. This makes possible to analyze the sustainability of multiple processes looking for the most efficient solutions with social, economic, and even environmental objectives.

Table 6 Optimal solution for patient transport problem analyzing medical resource 1 (hospital beds)

Beds	Connecticut	Pennsylvania	Delaware
New York	1248	17,044	82
New Jersey	0	0	1075

Table 7 Optimal solution for patient transport problem analyzing medical resource 2 (ICUs)

ICUs	Connecticut	Pennsylvania	Delaware
New York	414	4165	0
New Jersey	0	0	11

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Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no competing interests.

Nomenclature i , origin hospitals; n , maximum number of origin hospitals; j , destination hospitals; m , maximum number of destination hospitals; k , uncertain parameter number; K , set of numerical values (number of uncertain parameter values created by RVG in SOA); a_i , patients who exceeded the maximum capacity of the origin hospital i (using μ , uncertain parameter values in SOA, parameter in DOA); b_j , patients that can be treated at the destination hospital j due to the new equipment acquired (variable in MOA, parameter in DOA); c_1 , transport cost (\$/case/1000 miles) (parameter in DOA); c_2 , cost of the new equipment acquired (\$/unit) (parameter in MOA); d_{ij} , distances between origin hospital i and destination hospital j (mile/case) (parameter in DOA); x_{ij} , number of patients who are transferred from hospital origin i to hospital destination j (variable in DOA); μ , uncertain parameter value (in SOA), (excess demand of saturated hospitals); f_k , uncertain objective function calculated in the DOA using the uncertain parameter value number k ; g_k , uncertain objective function calculated in the SOA using the uncertain parameter value number k ; OF₁, objective function 1 declared in MOA, rejected patients; OF₂, objective function 2 declared in MOA, total cost; SOF₁, statistical objective function 1, the arithmetic mean of a set of g ; SOF₂, statistical objective function 2, the arithmetic mean of a set of f ; E1, estimation factor to calculate beds needed (0.2) (parameter in patients balance); E2, estimation factor to calculate ICUs needed (0.04) (parameter in patients balance); CDC, Center for Disease Control and Prevention; CF, crossover fraction; ChiTC, Chi-squared Termination Criteria; COVID-19, coronavirus disease 2019; DOA, deterministic optimization algorithm; GAMS, general algebraic modeling system; F , mutation fraction; GDX, GAMS Data eXchange; ICU, intensive care unit; LP, linear programming; LPC, linking program code; MINLP, mixed-integer nonlinear programming; MNG, maximum number of generations; MOA, metaheuristic optimization algorithm; MS, Microsoft™; NHSN, National Healthcare Safety Network; PS, population size; RVG, random values generator; SARS-CoV-2, severe acute respiratory syndrome coronavirus 2; SOA, stochastic optimization algorithm; SOF, statistical objective function; SSTC, steady state termination criteria; TLS, taboo list size; TR, taboo radius; VBA, visual basic for applications; WHO, World Health Organization

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