



Hertz's Viewpoint on Quantum Theory

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Abstract

In the nineteenth century (in the process of transition from mechanics to the field theory), the German school of theoretical physics confronted problems similar to the basic problems in the foundations of quantum mechanics (QM). Hertz tried to resolve such problem through analysis of the notion of a scientific theory and interrelation of theory and experiment. This analysis led him to the *Bild* (image) conception of theory (which was later essentially developed, but also modified by Boltzmann). In this paper, we claim that to resolve the basic foundational problems of QM, one has to use the *Bild* conception and reject the observational viewpoint on physical theory. As an example of a *Bild* theory underlying QM (treated as an observational theory), we consider prequantum classical statistical field theory (PCSFT): theory of random subquantum fields.

Keywords Hertz *Bild* theory · Descriptive and observational theories · Hidden variables in electromagnetism vs quantum mechanics · Quantum theory as observational theory · Prequantum classical statistical field theory

Introduction

During one hundred years, quantum theory has been suffering of endless debates about its meaning and interpretation. I claim that this unacceptable situation is the result of neglect by the fathers of quantum mechanics (QM) the extensive study of similar problems by the traditional German school in physics: ignoring the works of Helmholtz, Hertz, and Boltzmann (see, e.g., Hertz 1899 and Boltzmann 1905, 1974). Consciously or unconsciously, Bohr, Heisenberg, Einstein, Pauli, and other main contributors to foundations of quantum theory (but excluding Schrödinger, see, for example, D'Agostino 1992) ignored the historical lessons of the debate on the interrelation between theory and experiment which was initiated by transition from Newtonian mechanics to Maxwellian electromagnetism (Hertz 1899). In particular, in this debate, the problem of hidden variables was enlighten by Hertz—may the first time in history of science (Hertz 1899). In the light of this debate, the following debate between Bohr and Einstein can be characterized by lack of deep philosophic analysis (Einstein

et al. 1935; Bohr 1935). I am not afraid to call the latter debaters naive—by taking into account the lessons of the aforementioned debate about electromagnetism.

In this paper, I shortly present the views of Hertz; see, e.g., Hertz (1899), see also Boltzmann (1905, 1974), on scientific theory - the *Bild (image) conception*, section “*Bild Conception*.” Here, I follow the works D'Agostino (1992) and Miller (1984). At the end of this section, there are discussed various approaches to the notion of theory, *the descriptive, Bild, and observational approaches*. By speaking about a scientific theory, one has to specify its type. Then I proceed to quantum physics. I consider the present situation in quantum foundations by appealing to the Herzian *Bild* conception, section “*Hertzian Viewpoint on Foundations of Quantum Mechanics*.” In section “*Correspondence Between Mathematical Formalisms of Theories of Different Types*,” there are formulated rules of correspondence between two theories of different types (especially their mathematical structures).¹ Finally, in section “*Results: Correspondence Between Prequantum Classical Statistical Field Theory and Quantum Mechanics*,” a theory of micro-phenomena based on the *Bild* conception is presented, *prequantum classical statistical field theory* (PCSFT); see Khrennikov (2007a, b, 2014, 2017c).

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¹This paper is a continuation of my previous works in which the *Bild*-conception was explored in quantum physics (Khrennikov 2017a, b, c). It is also important to remark that similar approach to quantum theory was supported by Schrödinger (see D'Agostino (1992)).

Since this issue is devoted to ontology of quantum theory, it is useful to stress the impact of the Bild conception to the quantum foundational debate about realism, including Einstein-Bohr debate about completeness of QM. From the Bild-viewpoint, realism in physics as well as any other area of scientific research is reduced to experimental facts. This is exactly Bohr's position (see, e.g., Plotnitsky 2006, 2009).² Thus, the only realistic component of quantum physics are outcomes of measurements (Bohr's "phenomena"). Any physical theory is only about human images of natural phenomena. At the same time, these images are created on the basis of human's interaction with nature.

In Khrennikov (2017c), I tried to establish relation between the Hertz-Boltzmann Bild viewpoint and the ontic-epistemic viewpoint (Atmanspacher and Primas 2005) on the notion of scientific theory. However, this is a complex problem. Observational theories of the present paper can be definitely treated as epistemic theories. However, the Bild conception is not about reality as it is (as in an ontic theory), it is about human images of reality.

Method

Bild Conception

Hertz' discovery of radio waves was connected with his deep analysis of the Maxwellian electromagnetism from the viewpoint of interrelation between theory and experiment. Electromagnetism, in Hertz's opinion, based on *the action at a distance principle* was only a "first approximation to the truth." And he worked hardly to approach the final true theory. From the formal viewpoint, he tried to create a mechanical model of electromagnetic phenomena. However, these studies led him to understanding that it seems to be impossible to construct such a model without invention of *hidden variables* of the mass type, so-called concealed masses. In turn, this led him to deep philosophical and methodological studies devoted to meaning of "theory" in science.³

²However, Bohr would say that a Bild-type theory has nothing to do with physics and he would refer to it as a metaphysical theory. At the same time, he was not so much interested in no-go theorems for descriptive or Bild-type theories. In principle, he could not exclude that such "beyond quantum theories" might be constructed. But, they would not have any value for physics (Plotnitsky 2006, 2009).

³He was not able to complete his project on the mechanical theory of electromagnetism. (Ironically the same fate befell Einstein who in turn spent the last 20 years of his life by attempting to create the classical field theory of quantum phenomena, see, e.g., Einstein and Infeld (1961).) However, Hertz's contribution was very valuable to the methodology of science. And it influenced strongly Boltzmann and Schrödinger and through Boltzmann's works Planck (and may be even Einstein), see Miller (1984).

The main impact of these studies was relative liberation of theory from experiment. One of Hertz' fundamental statements is that "We become convinced that the manifold of the actual universe must be greater than the manifold of the universe which is directly revealed to our senses." See Hertz (1899).

Hertz explored heavily Helmholtz's principle about a parallelism between concepts and perceptions. However, Hertz rejected Helmholtz's claim that this parallelism uniquely determines the theory consistent with experimental facts. Hertz questioned the later (so to say the strong version of the parallelism principle) and claimed that there exists a multiplicity of representations satisfying the requirement of Helmholtz's parallelism: "The images [Bilder] which we may form of things are not determined without ambiguity by the requirement that the consequents of images must be images of consequents. Various images of the same objects are possible, and these images may differ in various aspects." See Hertz (1899).

It is even more important for our present considerations that Hertz stated that Helmholtz's parallelism of laws does not even work if *a theory is limited to visible quantities*. Only the introduction of *hidden quantities* allows creation of a consistent theory: "If we try to understand the motions of bodies around us, and refer to simple and clear rule, paying attention only to what can be directly observed, our attempts will in general fail. We soon become aware that the totality of things visible and tangible do not form a universe conformable to law, in which the same result always follow from the same conditions." See Hertz (1899).

From Hertzian perspective, a theory is not a true description of nature (Boltzmann's "complete congruence with nature") or at least a best approximation of it, but a theory is "mere a representation (Bild) of a nature... which at the present allows one to give the most uniform and comprehensive account of totality of phenomena" See Boltzmann (1905).

Of course, this viewpoint on the conception of theory represents a failure from the perspective of the traditional descriptive conception of theories. However, this liberation of the theory from experiment has its big advantage, since it liberates scientists' mind from rigid constraints of the present experimental situation.

Treatment of theory as a consistent system of mental images leads to its *causality*. The latter is a consequence of causality of human reasoning. In his reasoning, a human cannot do anything else than to proceed from cause to its consequence. At the same time, causality should not be treated as a purely mental (logical) feature of a theory. We recall that Helmholtz's parallelism between sensation and perception played the fundamental role in establishing of the Bild conception. Therefore, the causal structure of human reasoning is the result of evolutionary experiencing of humans observing causality in natural processes.

Since the Hertz(-Boltzmann) viewpoint on the notion of theory has not been commonly accepted, it is useful to specify it by calling “Bild-theory.” It should be distinguished from “descriptive theory” attempting to provide “complete congruence with nature.” Besides Bild and descriptive theories, we consider “observational theory” operating only with outputs of observations. This sort of theory can also be called “sensational theory,” in contrast to Bild theory which can be called “perceptual theory.” The same experimental situation can be represented by various types of theories: descriptive, Bild, and observational.

Hertzian Viewpoint on Foundations of Quantum Mechanics

For our considerations, the most important is that Hertzian analysis of methodology of science implies:

1. Any attempt to create a consistent (causal) theory on the purely experimental basis would lead to a failure;
2. Any consistent theory of natural phenomena would contain hidden variables, quantities which are unapproachable for our perception (at least at the present time);
3. Generally, in theory, it is impossible to approach the one-to-one correspondence between theoretical concepts and experimental facts.

We state that these principles were totally ignored not only by fathers of QM (with a few exceptions such as Schrödinger), but even by practically all experts working in quantum foundations. The majority of them followed “the spirit of Copenhagen” (Plotnitsky 2016) and put tremendous efforts to proceed without taking into account Hertz 1, 2, i.e., to develop the formalism of observational (sensational) theory of micro-phenomena which is nowadays known as QM (cf. with Stapp’s analysis of the Copenhagen interpretation in Stapp 1972). This approach led to the dead-end in the form of slogan: “Shut up and calculate!” (It is commonly assigned to Feynman). Following Hertz ideas, I claim that the basic problems of quantum foundations can be resolved only by rejecting the spirit of Copenhagen and creation of a real quantum theory liberated from sensational paradigm.

At the same time, it is important to understand that Einstein and his followers also suffered from ignoring the Bild conception about the meaning of scientific theory. They followed the old-fashioned descriptive understanding of theory and missed to explore the possibilities opened by Hertz 3 statement. Attempts to establish one-to-one correspondence between theory and experiment (as, e.g., in Bohmian mechanics) led either to invention of new concepts (such as, e.g., nonlocality in Bohmian mechanics) which do not match to “natural concepts” generated by human

experience or makes the project too complicated (as in the case of Einstein’s attempts to create the classical field model matching with micro-phenomena).

Of course, the main problem is the spirit of Copenhagen. The majority of the quantum community (especially the young generation) is oriented to the observational theory—QM. This theory is powerful and convenient, but it does not provide the consistent “Bild” of micro-phenomena. The latter is disturbing. Surprisingly, it is disturbing not only for those who reject the Copenhagen interpretation (or at least understand its restrictive character), but even for its strongest and world’s famous supporters.⁴

Measurement Problem It cannot be solved in the framework of the observational theory. One has to introduce hidden variables. (Of course, this viewpoint may be surprising: one should use unobservable variables to describe generation of outputs of measurement devices.) Bell understood the role of hidden variables in description of the process of quantum measurement very well. And he started the right project, but then he was disappointed by “nonlocality catastrophe.”⁵ As was pointed out, the latter is resulted from ignoring the possibility provided by Hertz 3.

Acausality of QM Von Neumann emphasized acausality of QM (Von Neuman 1955). He also pointed to specialty of quantum randomness, as irreducible randomness. (The latter claim is heavily explored in justification of specialty of randomness generated by quantum random generators.) Acausality of quantum theory is not surprising, generally acausality is a feature of observational theories. One cannot approach causality without transition to the Bild conception. Thus, quantum acausality and specialty of quantum randomness are not the (mystical) physical features of micro-world, but the features of the use of Mach’s treatment of a physical theory.

Perfect Correlations The EPR correlations (Einstein et al. 1935) neither can be explained by the observational theory—without introducing hidden variables. Bell understood this well and his *original Bell inequality* (Bell 1964)

⁴During the 20 years of Växjö conferences on quantum foundations, I was lucky to meet in the private and relaxing atmosphere many leading experts in quantum theory and experiment, “big names.” Surprisingly, practically all of them dream for a new quantum theory which (soon or later) will replace the present quantum theory. Unfortunately, people do not like to speak openly about their dreams. (The later is understandable: typically dreams are too private and personal). Therefore, young researchers live being sure that the present quantum theory is the final theory of micro-phenomena.

⁵In spite of the common opinion that Bell “enjoyed” nonlocality, in reality, nonlocality came to him as unexpected consequence of his analysis of the EPR-Bohm correlations; see Bell (1964, 1987).

was derived to analyze this problem. However, at that time, it was impossible to prepare singlet states with sufficiently high probability and to perform experiments to test the original Bell inequality. Therefore (to establish some relation to experiment), Bell was convinced to proceed with the CHSH inequality. Later, he had never mentioned the original Bell inequality and its the crucial difference from the CHSH inequality. The latter has nothing to do with the perfect correlations and the EPR-argument (Khrennikov and Basieva 2018). This paper also contains the analysis of the modern experimental situation and the novel possibilities to test the original Bell inequality as well as motivation to test it and not the CHSH inequality.

Quantum Nonlocality It is considered as the most intriguing feature of quantum theory. The nonlocality prejudice is so strong, because it is supported by both camps in quantum foundations, those who use observational theory (QM) and those who use descriptive theories (such as Bohmian mechanics). In fact, typically two (totally different) nonlocalities generated by observational and descriptive theories are identified into aforementioned “quantum nonlocality.” Genuine quantum (observational) nonlocality is encoded in the tensor product structure and the projection postulate. The descriptive nonlocality is encoded in nonlocal equations of motions, such as in Bohmian mechanics, or in violation of Bell-type inequalities (the latter issue is very delicate and we shall consider it in more detail below).

Violation of Bell Inequality By taking into account, the big impact of the debates about the Bell-type inequalities, (see, for example, Adenier (2007, 2008)) and its impact to establishing the notion of quantum nonlocality, we specially discuss Bell’s studies, from the viewpoint of the Bild conception. Bell suffered from the same problem as Einstein and Bohm. He took into account Hertz 1 and 2 statements, but ignored Hertz 3. He also tried to proceed in the old-fashioned descriptive framework and to identify the experimental correlations with correlations based on hidden variables; see Khrennikov (2017a, b, c) and Khrennikov and Basieva (2018). De Broglie (1964) understood well that such identification has no physical justification and that the Bell-type inequalities cannot be derived for experimental correlations; see Khrennikov (2017a, b).

Merging QM and General Relativity In this project, the main efforts we set to “quantize gravity.” It seems that this activity is totally meaningless. One tries to transform the descriptive theory into the observational theory. The situation is really paradoxical: one try to collect in one bottle all problems from resulting from ignoring Hertz 1, 2, and Herz 3; see Khrennikov (2017d). It is not

surprising that it does not work. Merging cannot be approached neither through quantization of gravity nor via naive descriptive “completion” of quantum theory (in the spirit of Einstein or Bohm). Both QM and general relative have to be reconsidered from the viewpoint of the Bild conception.

Correspondence Between Mathematical Formalisms of Theories of Different Types

Since each theory is based on its mathematical formalism, it is useful to establish correspondences between mathematical formalisms of different types of theories representing the same experimental data. The basic elements of the mathematical formalism of a theory τ are its state space S_τ and the space of variables V_τ , some space (may be very special) of real functions on S_τ . For two theories τ_1 and τ_2 , one can try to establish correspondence between their basic elements. This task is not straightforward. In particular, the notion of a state is different for different theories, e.g., for Bild and observational theories τ_B and τ_O (and we shall be interested in establishing correspondence between such two types of theories). A Bild-theory is causal and here the same initial condition implies the same consequence. Observational theories are often acausal. And let us consider such a case, i.e., τ_O is acausal. It would be naive to expect that it would be possible to establish straightforward correspondence between the state spaces of these theories. Causality is transformed into acausality through consideration of probability distributions. Therefore, by establishing correspondence between τ_B and τ_O , we have to consider some space (may be very special) of probability distributions P_B on S_B and map it onto the state space of τ_O . (We assume that states of τ_O are interpreted statistically.) Then we have to construct two “physically natural maps,”

$$J : P_B \rightarrow S_O, \quad J^* : V_B \rightarrow V_O. \quad (1)$$

Here, “physically natural” means consistent matching with the experimental facts. Both theories τ_B and τ_O have experimental justification through coupling to facts and the correspondence maps have to couple these experimental justifications. (We shall illustrate this statement by considering two theories of micro-phenomena, QM as τ_O and PCSFT as τ_B). Of course, theories τ_B and τ_O can differ by details of experimental justification. Therefore, in correspondence provided by the maps J, J^* , some of these details can be ignored.

Generally, these maps are neither one-to-one nor onto. Let us consider this situation in more detail.

- A cluster of probability distributions on S_B can be mapped into a state from S_O (generally states of

τ_B and probability distributions of such states are unapproachable by τ_O .

- A cluster of variables of τ_B can be mapped into a variable of τ_O (the observational description is often operational; it does not distinguish variables of a causal theory).
- Not all elements of S_O and V_O belong to the images $J(P_B)$ and $J^*(V_B)$. (Even observational theory τ_O can contain its own ideal elements which need not be reflected in τ_B).

In a Bild theory, V_B is some space of functions on the state space S_B , maps $f : S_B \rightarrow \mathbf{R}$. Such theory is causal, the state ϕ uniquely determines the values of all physical variables belonging $V_B : \phi \rightarrow f(\phi)$.

Results: Correspondence Between Prequantum Classical Statistical Field Theory and Quantum Mechanics

In QM, states are given by density operators acting in complex Hilbert space H (endowed with scalar product $\langle \cdot | \cdot \rangle$) and physical variables (observables) are represented by Hermitian operators in H . Denote the space of density operators by S_{QM} and the space of Hermitian operators by V_{QM} .

In PCSFT (Khrennikov 2007a, b, 2014, 2017c), states are given by vectors of H (in general non-normalized), i.e., $S_{PCSFT} = H$. Physical variables are represented by quadratic forms on H , i.e., maps of the form $f(\phi) = \langle \phi | A | \phi \rangle$, where $A \equiv A_f$ is a Hermitian operator. Denote the space of quadratic forms by the symbol V_{PCSFT} . Consider the space of probability distributions on H with zero first momentum, i.e.,

$$\int_H \langle \phi | a \rangle dp(\phi) = 0 \tag{2}$$

for any $a \in H$, and finite second momentum, i.e.,

$$\mathcal{E}_p \equiv \int_H \|\phi\|^2 dp(\phi) < \infty. \tag{3}$$

Denote this space of probability distributions by the symbol P_{PCSFT} . We remark that, instead of probability distributions, we can consider H -valued random vectors with zero mean value and finite second moment: $\xi = \xi(\omega)$, where ω is the chance parameter, such that $E[\xi] = 0$ and $E[\|\xi\|^2] < \infty$. Denote this space by the symbol R_{PCSFT} . We remark that if H is finite-dimensional, these are usual complex vector-valued random variables; if H is infinite-dimensional, then the elements of R_{PCSFT} are random fields. For the latter, the basic example is given by the choice $H = L_2(\mathbf{R}^n)$. Here, each Bild-state ϕ is an L_2 -function, $\phi : \mathbf{R}^n \mapsto \mathbf{C}$.

Hence, each element of R_{PCSFT} can be represented as a function of two variables, $\xi = \xi(x; \omega) : \text{chance parameter } \omega \text{ and space coordinates } x$. This is a random field (RF). We shall use the same terminology, “random fields,” even in the finite-dimensional case.

We remark that, for the state space $H = L_2(\mathbf{R}^n)$, the quantity \mathcal{E}_p can be represented as $\mathcal{E}_p = \int_H \mathcal{E}(\phi) dp(\phi)$, where $\mathcal{E}(\phi) = \|\phi\|^2 = \int_{\mathbf{R}^n} |\phi(x)|^2 dx$ is field’s energy. Hence, \mathcal{E}_p is the average of the field energy with respect to the probability distribution p on the space of fields. We can also use the random field representation. Let $\xi = \xi(x; \omega)$ be a random field. Then its energy is the random variable $\mathcal{E}_\xi(\omega) = \int_{\mathbf{R}^n} |\xi(x; \omega)|^2 dx$ and \mathcal{E}_p is the average of the latter (here p is the probability distribution of the random field).

For any $p \in P_{PCSFT}$, its (complex) covariance operator B_p is defined by its bilinear (Hermitian) form:

$$\langle a | B_p | b \rangle = \int_H \langle a | \phi \rangle \langle \phi | b \rangle dp(\phi), \quad a, b \in H, \tag{4}$$

or, for a random vector ξ , we have: $\langle a | B_\xi | b \rangle = E[\langle a | \xi \rangle \langle \xi | b \rangle]$. Generally, a probability distribution (a random field) is not determined by its covariance operator (even under condition of zero average, see Eq. 2). We remark that such complex covariance operator has the same mathematical properties as a density operator, besides normalization by the trace one; it is Hermitian, positively semidefinite, and trace class. (The latter property is important in the infinite-dimensional case, e.g., for the state space $H = L_2(\mathbf{R}^n)$).

PCSFT (the Bild-type theory) is connected with QM through the following formula. For $p \in P_{PCSFT}$ and $f \in V_{PCSFT}$, we have

$$\langle f \rangle_p = \int_H f(\phi) dp(\phi) = \text{Tr} B_p A_f, \tag{5}$$

where $A_f = \frac{1}{2} f^{(2)}(0)$, i.e., $f(\phi) = \langle \phi | A_f | \phi \rangle$. We remark that the covariance operator and the energy average are coupled through the simple formula:

$$\mathcal{E}_p = \int_H \|\phi\|^2 dp(\phi) = \text{Tr} B_p; \tag{6}$$

in particular, by normalizing the covariance operator of a random field by average of field’s energy, we obtain a density operator $\rho_p = B_p / \mathcal{E}_p$.

Let us consider the following maps J and J^* ; see Eq. 1, from PCSFT to QM,

$$J(p) = \rho_p, \quad J^*(f) = A_f. \tag{7}$$

This correspondence connects the averages given by the Bild and observational theories:

$$\frac{1}{\mathcal{E}_p} \langle f \rangle_p = \text{Tr} \rho_p A_f, \tag{8}$$

i.e., the QM and PCSFT averages are coupled with the scaling factor which is equal to the inverse of the average energy of the random field. Thus, density operators are normalized (by average field energy) covariance operators of random fields and the Hermitian operators representing quantum observables correspond to quadratic forms of fields. We can also write the relation (8) in the form: $\langle \frac{f}{\mathcal{E}_p} \rangle_p = \text{Tr} \rho_p A f$. If $\mathcal{E}_p \ll 1$, we can consider the quantity $g_p(\phi) \equiv \frac{f(\phi)}{\mathcal{E}_p}$ as amplification of the PCSFT physical variable f . Thus, through coupling with PCSFT, we can be treat QM as an observational theory describing averages of amplified “subquantum” physical variables.

In contrast to QM, PCSFT is causal: selection of a vector (“field”) $\phi \in H$ determines the values of all PCSFT-variables, quadratic forms of classical fields: $\phi \rightarrow \langle \phi | A | \phi \rangle$.

For physical variables, the correspondence map J^* is one-to-one, but the map J is not one-to-one. But it is a surjection, i.e., it is on-to map.

Discussion

The aim of this paper is to remind to the quantum foundational community studies of Hertz (and Boltzmann) on the Bild conception of physical theory, especially Hertz analysis of connection between theory and experiment. We emphasize the similarity of the problems discussed by Hertz in the process of transition from Newtonian mechanics to Maxwellian electromagnetism and the problems of interrelation between classical and quantum physical theories (including the problem of hidden variables). The Bild conception can be explored to resolve the basic problems of quantum foundations: measurement problem, acausality and irreducible quantum randomness, quantum nonlocality, merging QM, and general relativity.

As an example of a Bild-type theory preceding QM (the latter is treated as an observational theory), we consider prequantum classical statistical field theory—PCSFT. In contrast to QM, PCSFT is not based solely on the observational data. It contains images which cannot be coupled straightforwardly to data. In particular, the EPR-Bohm correlations cannot be identified with the corresponding PCSFT correlations, although numerically they coincide. There exists a natural correspondence between the mathematical entities of PCSFT and QM, the correspondence is not one-to-one. The same Bild theory can be coupled to a variety of observational theories and a variety of observational theories can represent the same experimental data. PCSFT can be coupled not only to QM, but to another observational theory based on threshold detection of random signals; see Khrennikov (2012).

Finally, we stress that the Bild conception can be used to develop a consistent theory of quantum(-like) cognition and interrelation between matter and mind; see Khrennikov (2010), cf. Stapp (2004).

Compliance with Ethical Standards

Conflict of Interest The author declares that he has no conflict of interest.

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