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ORIGINAL PAPER

Economic Growth in the USA and Germany 1960–2013: The Underestimated Role of Energy

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Abstract Economic growth in Germany and the USA between the years 1960 and 2013 is analyzed, using two LinEx production functions $Y_{L1}(K, L, E;t)$ and $Y_{L11}(K, L, E;t)$, which depend linearly on energy and exponentially on quotients of the production factors capital $K(t)$, labor $L(t)$, and energy $E(t)$. The two production functions result from output elasticities that are the simplest, and next simplest, factor-dependent solutions of a set of partial differential equations and their asymptotic boundary conditions, which reflect technical trends in highly industrialized economies. The production functions depend on time *t explicitly*, when innovations and structural changes induce temporal variations of two technology parameters. The latter model capital effectiveness and the energy demand of the capital stock. In this article, we present some econometric evidence that, in conjunction with capital, the production factor energy provides the major contribution, and the temporal variations of the LinEx technology parameters provide the minor contribution to that part of economic growth that is assigned to "technical progress" by neoclassical economics. We also discuss assets and shortcomings of the two LinEx functions, and different aspects of using either primary energy data or exergy data for the energy variable E.

Keywords Energy · Innovation · Structural change · Output elasticities · Economic growth

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Introduction

The issue of this article is the economic weight of production factors. To be more specific, the question is how, and how much, instrumental capital $K(t)$, human labor $L(t)$, and energy $E(t)$ of a given economic system at time t contribute to the production and growth of the system's output Y, which is the gross domestic product (GDP), or parts thereof. This question matters for growth theory and policies concerning energy and the environment. Looking into it, we use macroeconomic production functions $Y(K(t), L(t), E(t); t)$, in which capital-activating energy complements capital-handling labor. We follow the usual assumption of mainstream economics that production functions are state functions.

State functions describe system properties and dynamics in the natural sciences and economics. They depend only on the actual magnitudes of the independent variables at time t, and not on the history of the system. (Examples from physics are hamiltonians in mechanics and entropy in thermodynamics.) The infinitesimal change of a state function is a total differential, whose line integral in the space of the independent variables does not depend on the chosen path.

The *implicit* time dependence of production functions is the time dependence of the factors $K(t)$, $L(t)$, $E(t)$. We will omit the time argument of the production factors and simply write $Y(K, L, E; t)$, further on. The *explicit* time dependence of the production function takes care of structural changes and innovations, or, in other words, the impact of human ideas, inventions, and value decisions.

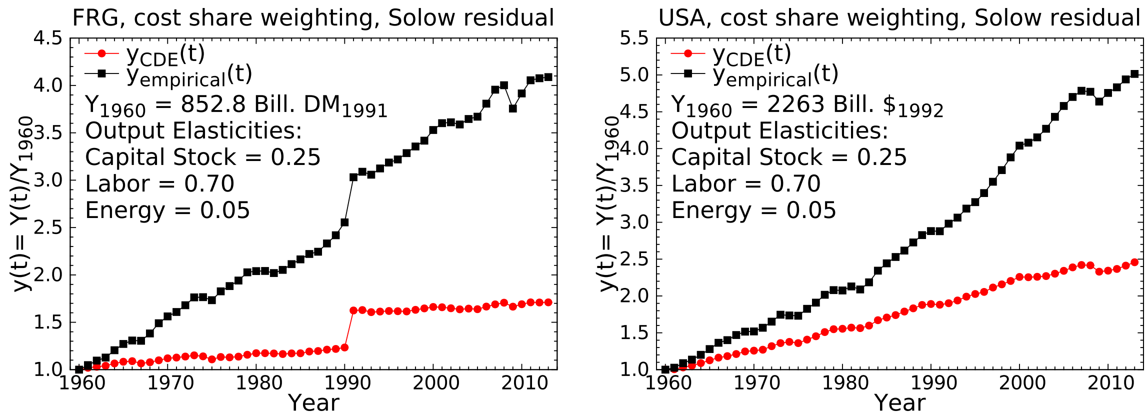


Fig. 1 Empirical outputs (*squares*) and theoretical outputs (*circles*) for the total economies of Germany (FRG) and the USA. The theoretical growth is computed with the energy-dependent Cobb–Douglas function (9) and cost-share weighting of the production factors. The differences between the empirical and the theoretical growth curves are called "Solow residuals." The empirical growth of the inputs is shown in Figs. 2c and 4c

Standard economics considers capital and labor as the factors of production[1] that are the independent variables of the neoclassical production function $A(t)f(K, L)$. Here, $A(t)$ is a measure of innovation, called "technical progress." In the wake of the first and the second oil-price shock between 1973 and 1981, economists like Hudson and Jorgenson (1974), Berndt and Jorgenson (1978), Berndt and Wood (1979), and Jorgenson (1984) introduced energy as a third factor of production. And so did Nordhaus (2008), when he considered economic problems of climate change. Then, the production function is $A(t)f_E(K, L, E)$.

In these neoclassical production functions the economic weights of the production factors, which are the factors' output elasticities defined in Eq. (2) below, are determined by the cost-share theorem. The cost-share theorem, derived from the maximization of profit or time-integrated utility without observing technological constraints on factor combinations, says that the output elasticity of a production factor must be equal to its share in total factor cost. Since in highly industrialized countries this share has been roughly 25% for capital, 70% for labor, and 5% for energy, the direct impact of the production factor energy on economic growth has been considered as marginal, at best, by mainstream economists; see e.g., Denison (1979).

With output elasticities obtained from the cost-share theorem, one has the problem of the Solow residual, which is the big difference between empirical and theoretical growth. Figure 1 shows an example. After its discovery by Solow (1957) the technical-progress functions $A(t)$ were introduced in order to take care of this residual. Later, Solow (1994) commented that the dominating role of technical progress in standard growth theory "has led to a criticism of the neoclassical model: it is a theory of growth that leaves the main factor in economic growth unexplained."

The laws of physics, especially the ones concerning energy conversion and entropy production, have led a growing number of scholars to look into energy and economic growth from the perspective of thermodynamic laws. And long ago Tryon (1927) already stated: "Anything as important in industrial life as power deserves more attention than it has yet received from economists… A theory of production that will really explain how wealth is produced must analyze the contribution of the element energy."

Saunders (2008) carefully and critically discussed energy-dependent production functions that may be appropriate for energy consumption analyses, especially with respect to the rebound effect (Sorrell and Dimitropoulos 2008; Brockway et al. 2017). Stern (2011) proposed "to modify Solow's growth model by adding an energy input that has low substitutability with capital and labor, while allowing the elasticity of substitution between capital and labor to remain at unity." He expressed *gross* output by a function that "embeds a Cobb–Douglas function of capital (K) and labor (L) in a CES function of value added and energy (E)." Time-dependent augmentation indices A_L and A_E of labor and energy reflect changes in technology and factor quality. The embedded Cobb–Douglas function weighs capital and labor with their cost shares. Since *gross* output is the sum of GDP *plus* intermediate consumption, Stern's nested CES function is a tool for research on issues different from the question which output elasticities of capital, labor, and energy are appropriate for describing the growth of GDP. Santos et al. (2016) looked

[1] In highly industrialized countries, the role of the factor "land," so important for biomass production by photosynthesis in agrarian societies, is reduced to providing sites for the capital stock, essentially.

into cointegration between output and (qualified) capital, labor, and energy inputs in Portugal. They concluded that "energy essentiality in production… is not *a priory* incompatible with the neoclassical assumptions of the cost share theorem."

But the cost-share theorem is wrong, because technological constraints on factor combinations do exist. They are the limits to capacity utilization[2] and automation and result in the destruction of the equality of output elasticities and factor cost shares. This has been shown explicitly by optimizing profit and time-integrated utility subject to these constraints (Kümmel et al. 2010, 2014, 2015).

An early study that determined output elasticities econometrically, instead of fixing them by the cost shares, is that of Tintner (1974), who, by applying an energy-dependent Cobb–Douglas function to the Austrian economy between 1955 and 1972, found energy output elasticities of more than 30%.

A set of differential equations with asymptotic boundary conditions, derived by Kümmel (1982) for output elasticities and production functions, yielded the (first) LinEx function $Y_{L1}(K, L, E;t)$, given by Eq. (14) below, as the production function with the simplest factor-dependent output elasticities. The explicit time dependence of $Y_{L1}(K, L, E;t)$ is that of two technology parameters, which formally are integration constants of the differential equations, and whose variations reflect efficiency alterations and structural changes. The requirement that output elasticities must be non-negative constrains these parameters, and possible factor combinations as well. With constant technology parameters, recalibrated once, in 1978, a nearly residual-free description of growth between 1960 and 1990 in the Federal Republic of Germany (FRG), the USA, and Japan was possible; the recessions due to the two oil-price shocks from 1973 to 1975 and 1979 to 1981 were well reproduced (Hall et al. 2001). Modeling continuous temporal changes of the technology parameters by, e.g., logistic functions, improved the agreement between theoretical and empirical growth (Kümmel et al. 2002) and even allowed to describe economic growth in Germany between 1960 and 2000, including the reunification of Eastern and Western Germany in 1990 (Kümmel 2011). From these studies resulted what we now call "the fundamental heresy" with respect to neoclassical economics: Energy's output elasticities are much larger than its cost shares, and for labor just the opposite is true.

Ayres and Warr (2005) and Ayres and Warr (2009) used the LinEx function with exergy-based useful work data U instead of primary energy E. Exergy is the useful part of energy, which can be converted into any form of useful work. Calculation of "useful work" is difficult. Brockway et al. (2014) contributed toward a common useful work accounting framework, and Serrenho et al. (2014) calculated U, including residential energy needs, for 15 EU countries. Although in principle, all fossil and non-fossil primary energy carriers—except heat-transporting media—are 100% exergy, it was useful work data U that allowed Ayres and Warr (2005) to reproduce economic growth in the USA from 1900 to the mid 1970s with constant technology parameters and small residuals. Their useful work data incorporate the efficiency changes that occurred in the US economy during the first two-thirds of the twentieth century. Up to 1998, however, the residuals increased substantially. Then Ayres and Warr (2009) applied the LinEx function with *time-dependent* technology parameters to the USA and Japan from 1900 to 2005 (except for the years of World War II), and the former residuals became very small. In both studies, the output elasticities of useful work significantly exceed those of labor after 1910.

By now, new empirical data for output, capital, labor, and primary energy have become available to us for Germany and the USA. They are presented and discussed in Appendix 3 and cover the time from the base year $t_0 = 1960$ to the end year 2013. Between these years occurred the dramatic fluctuations of the oil prices with their severe economic repercussions, the reunification of Germany in 1990, with the resulting structural break in the political entity called "Federal Republic of Germany" (FRG), the first global recession of the twenty-first century after the bursting of the US housing bubble in 2007, and considerable outsourcing of energy-intensive industries from the highly industrialized countries to industrially less developed regions of the world. The data have been obtained from the national accounts, labor statistics, and energy statistics; some care had to be dedicated to data consistency and the chaining of inflation-corrected data on output and capital.

In the following, the fundamental heresy is put to test with the new data, employing the first LinEx function Y_{L1} and a new one, Y_{L11}, whose output elasticity of capital has been modified, and which is invariant against transformations of the base year.

In "Output Elasticities and Production Functions" we briefly review the set of differential equations for production functions and output elasticities. The most general form of the production function, in terms of a line integral in factor space over *any* output elasticities that satisfy the twice-differentiability conditions for state functions, is derived. The general solution, and special solutions of the partial differential equations for the output elasticities, is given, and criticism of the resulting production functions is considered. "Technology Parameters and their Dynamics" sketches the method of determining the time-dependent

[2] The machines of the capital stock cannot operate with energy inputs that exceed the ones they are designed for.

technology parameters “capital effectiveness” and “energy demand of the capital stock” of LinEx functions by SSE minimization subject to inequality restrictions. The resulting theoretical growth curves and the time-varying and time-averaged output elasticities for Germany and the USA are presented in the "Results" section. Finally, we provide "Summary and Conclusions."

Output Elasticities and Production Functions

Macroeconomic production functions have been criticized by scholars who argued that aggregation of output and inputs should be in physical terms, exclusively (Robinson 1953, 1971; Fisher 1993; Kurz and Salvadory 1995; Felipe et al. 2003; Silverberg 2007). A response to this criticism is based on the understanding that the capital stock of industrialized economies consists of all energy-converting and information-processing machines in conjunction with all buildings and installations necessary for their protection and operation, and that output Y results from work performance and information processing by the combination of such capital with (routine) labor[3] $L(t)$ and energy $E(t)$. Strictly speaking, this understanding excludes the residential sector with its considerable energy consumption for room heating and cooling. Chapter 4, Appendix 3, of Kümmel (2011) outlines the basic *physical* aggregation principles for output and capital in terms of work performance and information processing, and their relations to constant currency.

Occasionally, materials were considered as a fourth factor of production, besides capital, labor, and energy (Hudson and Jorgenson 1974; Berndt and Wood 1979). Although materials are indispensable for production, they are not active factors but rather the passive partners of the production process, during which their atoms and molecules are brought into their proper positions in the products by K, L, and E. Scarcity of key materials, e.g., phosphorus, or rare earths, will hamper production and growth. This has been pointed out in rich details by Valero and Valero (2015). These authors expect that we may enter an “Age of the Periodic Table.” A rough model of limits to growth due to materials scarcity, where the output elasticities in Eq. (1) are multiplied by recycling functions, is disregarded here; and the impact of emissions of particles like SO_2, NO_X, and CO_2 on output elasticities and economic growth has been considered elsewhere (Kümmel 2016).

The three parts of this section concern: (1) The growth equation and its integral. (2) The three partial differential equations, which result from the Cauchy integrability conditions for the growth equation, the K, L, E- dependencies of the output elasticities that solve them, and the corresponding production functions. (3) Discussion of the assets and shortcomings of the LinEx functions Y_{L1} and Y_{L11}. In order that this article is self-contained, Subsection 2.1 and parts of Subsection 2.2 contain material from prior publications of the authors; since the wording is not identical, quotation marks are omitted.

Integrating the Growth Equation

We divide the total differential of the production function $Y(K, L, E; t)$ by the production function itself and obtain the growth equation:

$$\frac{\mathrm{d}Y}{Y} = \alpha\frac{\mathrm{d}K}{K} + \beta\frac{\mathrm{d}L}{L} + \gamma\frac{\mathrm{d}E}{E} + \frac{\partial \ln Y}{\partial t}\mathrm{d}t\,. \tag{1}$$

The weights of the inputs capital, α, labor, β, and energy, γ, are the *output elasticities*, defined by

$$\alpha \equiv \frac{K}{Y}\frac{\partial Y}{\partial K},\ \beta \equiv \frac{L}{Y}\frac{\partial Y}{\partial L},\ \gamma \equiv \frac{E}{Y}\frac{\partial Y}{\partial E}\,. \tag{2}$$

$\frac{\partial \ln Y}{\partial t}\mathrm{d}t$ in Eq. (1) results from the influences of human ideas, inventions, and value decisions on economic evolution. We aggregate these influences in the concept of *creativity*, write $\frac{\partial \ln Y}{\partial t}\mathrm{d}t = \delta\frac{\mathrm{d}t}{\Delta t}$, and define the “output elasticity of creativity” as

$$\delta \equiv \frac{\Delta t}{Y}\frac{\partial Y}{\partial t}\,. \tag{3}$$

We choose $\Delta t = |t - t_0|$, where t_0 is an arbitrary base year with the factor inputs K_0, L_0, E_0. This choice may emphasize long-term effects of creativity actions more than an alternative choice like $\Delta t = 1$ year. One has to keep that in mind when comparing the growth contributions of energy and time in the end.

Being a state function, $Y(K, L, E; t)$ must be twice differentiable, i.e., its second-order mixed derivatives with respect to K, L, E must be equal. Calculating these derivatives from the growth equation (1) one obtains the integrability conditions

$$L\frac{\partial \alpha}{\partial L} = K\frac{\partial \beta}{\partial K}, \quad E\frac{\partial \beta}{\partial E} = L\frac{\partial \gamma}{\partial L}, \quad K\frac{\partial \gamma}{\partial K} = E\frac{\partial \alpha}{\partial E}. \tag{4}$$

We integrate Eq. (1) at a fixed time t, when the production factors are $K = K(t), L = L(t), E = E(t)$. The integral of the left-hand side from $Y_0(t)$ to $Y(K, L, E; t)$ is $\ln\frac{Y(K,L,E;t)}{Y_0(t)}$. It is equal to the integral of the right-hand side:

$$F(K,L,E)_t \equiv \int_{P_0}^{P}\left[\alpha\frac{\mathrm{d}K}{K} + \beta\frac{\mathrm{d}L}{L} + \gamma\frac{\mathrm{d}E}{E}\right]\mathrm{d}s. \tag{5}$$

[3] “Routine” labor in the sense that it is measured by hours worked per year.

For output elasticities that satisfy the integrability conditions (4) this integral can be evaluated along any path s in factor space from an initial point P_0 at (K_0, L_0, E_0) to the final point P at $(K(t), L(t), E(t))$. A very convenient path consists of three orthogonal straight lines parallel to the Cartesian axes of K, L, E space: $P_0 = (K_0, L_0, E_0) \to P_1 = (K, L_0, E_0) \to P_2 = (K, L, E_0) \to P = (K, L, E)$. Consequently,

$$F(K,L,E)_t = \int_{K_0,L_0,E_0}^{K,L_0,E_0} \alpha(K,L_0,E_0)\frac{dK}{K} + \int_{K,L_0,E_0}^{K,L,E_0} \beta(K,L,E_0)\frac{dL}{L} + \int_{K,L,E_0}^{K,L,E} \gamma(K,L,E)\frac{dE}{E}. \quad (6)$$

With $\ln \frac{Y(K,L,E;t)}{Y_0(t)} = F(K,L,E)_t$ the production function becomes

$$Y(K,L,E;t) = Y_0(t) \exp\left\{F(K,L,E)_t\right\}. \quad (7)$$

The integration constant $Y_0(t)$ is the monetary value of the basket of goods services at time t, if it were produced by the factors K_0, L_0, and E_0. If creativity were dormant during the time interval $|t - t_0|$, $Y_0(t)$ would be also equal to the production function at time t_0. If creativity is active, on the other hand, Y_0 may change, and so will two technology parameters (a and c) in the output elasticities and the production functions to be introduced in the following section.

Computing Output Elasticities and Production Functions

At any fixed time t the production function $Y(K, L, E; t)$ must be linearly homogeneous, because adding to an existing production system an identical one with the same inputs doubles output. Thus, $\alpha + \beta + \gamma = 1$. With that the integrability conditions (4) turn into $K\frac{\partial \alpha}{\partial K} + L\frac{\partial \alpha}{\partial L} + E\frac{\partial \alpha}{\partial E} = 0$, $K\frac{\partial \beta}{\partial K} + L\frac{\partial \beta}{\partial L} + E\frac{\partial \beta}{\partial E} = 0$, $L\frac{\partial \alpha}{\partial L} = K\frac{\partial \beta}{\partial K}$. The general solutions of these partial differential equations are the output elasticities

$$\alpha = A(L/K, E/K),\ \beta = B(L/K, E/K) = \int^{K} \frac{L}{K'}\frac{\partial A}{\partial L} \mathrm{d}K' + J(L/E),\ \gamma = 1 - \alpha - \beta; \quad (8)$$

$A(L / K, E / K)$ and $J(L / E)$ are arbitrary differentiable functions of their arguments.

The trivial solutions of the differential equations are the constants α_0, β_0 and $\gamma_0 = 1 - \alpha_0 - \beta_0$. With them, and Eqs. (6) and (7), one obtains the energy-dependent Cobb–Douglas function Y_{CDE}:

$$Y_{CDE}(K,L,E;t) = Y_0(t)\left(\frac{K}{K_0}\right)^{\alpha_0}\left(\frac{L}{L_0}\right)^{\beta_0}\left(\frac{E}{E_0}\right)^{1-\alpha_0-\beta_0}. \quad (9)$$

This is the simplest—and frequently used—production function. Figure 1 shows, how it reproduces economic growth, if output elasticities are set equal to cost shares.

The simplest *non-trivial* solutions have been constructed to satisfy two asymptotic boundary conditions, which result from the trend of factor evolution in highly industrialized countries. (1) α should vanish for $\frac{L/L_0}{K/K_0} \to 0$ and $\frac{E/E_0}{K/K_0} \to 0$. This incorporates the law of diminishing returns for the combination of labor and energy with instrumental capital. It takes care of the facts that "there are many reasons why a unit of production may not want to work at the maximum capacity" (Georgescu-Roegen 1986) and that strong increases of the capital stock, while labor and energy increase much less, contribute less and less to the increase of output. (2) β should vanish, when $(K(t), L(t), E(t))$ approach the state of *maximum* automation, characterized by the point $(K_m(t), L_m(t), E_m(t) = E_0 \cdot c(t) \cdot K_m(t)/K_0)$ in factor space, where, by definition, an additional unit of labor does not contribute to the growth of output any more. The technology parameter $c(t)$ measures the energy demand of the fully utilized capital stock $K(t)$. $K_m(t)$ is the fully automated capital stock that could be installed to produce a given output $Y_{given}(t)$ in combination with the inputs $L_m(t) << L(t)$ and $E_m(t)$, if—within the finite space of the production system—the achievable degree of automation at time t were not restricted by the volume, mass, and energy demand (VME) of the available information processors. Progress in transistor technology reduces VME. Welfonder and Frederking (2002) and Welfonder (2007) constructed a global socio-economic dynamic model and looked into technical–economic development. One of their findings is that, during the last four decades of the twentieth century, the degree of automation nearly doubled in Germany and more than doubled in the USA. They conclude that, when considering sustainable global socio-economic evolution, one must take into account further progress in automation and information technologies.

The two asymptotic boundary conditions are satisfied by two simple sets of output elasticities:

$$\alpha = a\frac{(L/L_0 + E/E_0)}{K/K_0},\quad \beta = a\left(c\frac{L/L_0}{E/E_0} - \frac{L/L_0}{K/K_0}\right),\quad \gamma = 1 - a\frac{E/E_0}{K/K_0} - ac\frac{L/L_0}{E/E_0}, \quad (10)$$

and

$$\alpha = a\frac{L/L_0}{K/K_0} + \frac{1}{c}\frac{E/E_0}{K/K_0},\quad \beta = a\left(c\frac{L/L_0}{E/E_0} - \frac{L/L_0}{K/K_0}\right),\quad \gamma = 1 - \frac{1}{c}\frac{E/E_0}{K/K_0} - ac\frac{L/L_0}{E/E_0}. \quad (11)$$

The technology parameter a, which complements the energy-demand parameter c, is a measure of capital effectiveness, with somewhat different meanings in (10) and (11). In (10) it indicates the weight, with which the ratio of labor *plus* energy to capital contributes to the output elasticity of capital, while in (11) the parameter a only affects the ratio of labor to capital (i.e., the inverse capital deepening). Since α determines the negative part of β according to Eq. (8), the output elasticity of labor in Eq. (11) has the same mathematical form as β in Eq. (10). The meaning of the technology parameters is different, though. We will come back to that in the section "Variations of Technology Parameters and Output Elasticities."

$F(K,L,E)_t$ of Eq. (6) becomes with the output elasticities (10):

$$F(K,L,E)_{t,L1} = a\left(2 - \frac{L/L_0}{K/K_0} - \frac{E/E_0}{K/K_0}\right) + ac\left(\frac{L/L_0}{E/E_0} - 1\right) + \ln\frac{E}{E_0}, \tag{12}$$

and with the output elasticities (11):

$$F(K,L,E)_{t,L11} = a\left(1 - \frac{L/L_0}{K/K_0}\right) + \frac{1}{c}\left(1 - \frac{E/E_0}{K/K_0}\right) + ac\left(\frac{L/L_0}{E/E_0} - 1\right) + \ln\frac{E}{E_0}. \tag{13}$$

Inserting these expressions for $F(K,L,E)_t$ into (7) yields the LinEx production functions

$$Y_{L1}(K,L,E;t) = Y_0(t)\frac{E}{E_0}\exp\left[a\left(2 - \frac{L/L_0 + E/E_0}{K/K_0}\right) + ac\left(\frac{L/L_0}{E/E_0} - 1\right)\right], \tag{14}$$

and

$$Y_{L11}(K,L,E;t) = Y_0(t)\frac{E}{E_0}\exp\left[a\left(1 - \frac{L/L_0}{K/K_0}\right) + \frac{1}{c}\left(1 - \frac{E/E_0}{K/K_0}\right) + ac\left(\frac{L/L_0}{E/E_0} - 1\right)\right]. \tag{15}$$

If creativity is active, the integration constant Y_0 and the technology parameters a and c may change over time.

Assets and Shortcomings of Y_{L1} and Y_{L11}

We must do with approximate solutions of the partial differential equations for the output elasticities—and thus with approximate production functions—because the *exact* output elasticities for a given economic system would have to satisfy *exact* boundary conditions. These, however, would require the knowledge of β on a boundary surface in factor space and that of α on a boundary curve in that space.[4] This is, and will be, impossible. The LinEx functions result from output elasticities that satisfy the *asymptotic* boundary conditions described above. But being approximations, they have assets and shortcomings, of course.

The principal asset is that LinEx functions reproduce economic growth with rather small residuals and that the growth contribution of their explicit time dependence, which is due to the time dependence of their technology parameters $a(t)$, $c(t)$, is much smaller than that of neoclassical technical-progress functions $A(t)$. The growth contribution from the temporal variations of the technology parameters, *plus* the larger contribution from energy in combination with capital, supersedes the growth contribution from $A(t)$. This is also indicated by the output elasticities of energy, γ, and creativity, δ, as reported by Kümmel (2011) and calculated in the "Results" section, see below. Furthermore, the temporal changes of the technology parameters point to specific efficiency alterations and structural changes, which can be checked with empirical findings. This is why we think that the LinEx functions are a useful tool for analyzing the role of energy and time in economic growth.

Two shortcomings—from a strictly neoclassical point of view—have been discussed in the literature. Kümmel et al. (1985) point out that in the allowed factor space, defined by the restrictions that output elasticities must be non-negative, see Eq. (19) below, the $L-K$ and $E-K$ isoquants are convex; the $L-E$ isoquant, however, is concave. They explain this concavity by progress in automation. "If initially a fixed quantity of capital K of low degree of automation is combined with much labor and little energy, the labor-substituting energy at first takes over essentially the performance of hard physical work like lifting and transportation of cargos, deformation of matter, digging of holes etc. With increasing degree of automation at constant numerical value of K, the activity of workers is more and more restricted to information processing, which mainly consists of the handling of machines. In the production of a unit of output the necessary amount of information processing requires less energy than the complementary amount of physical work on matter. Therefore, less and less additional energy is needed in order to substitute the more and more exclusively information-processing workers—thus the $L-E$ isoquant is concave."

Saunders (2008) notes that Y_{L1} does not have decreasing marginal *returns* of labor, i.e., $\partial^2 Y_{L1}/\partial L^2$ is not negative. This can be easily seen from the definition (2) of labor's output elasticity, according to which the marginal product of labor is $\partial Y/\partial L = Y\beta/L$, so that $\partial^2 Y/\partial L^2 = (\partial Y/\partial L)\beta/L + Y\partial(\beta/L)/\partial L = Y\beta^2/L^2 + Y\partial(\beta/L)/\partial L$. Inserting β from Eqs. (10) or (11) into this yields $\partial^2 Y_{L1}/\partial L^2 = Y\beta^2/L^2 + 0$; this never becomes negative. This would be a serious violation of the law of

[4] How this results from the theory of partial differential equations is pointed out more explicitly in Appendix 6 of Chapter 4 of Kümmel (2011).

diminishing returns for production functions in which *L*/*K* could become arbitrarily large. This is not the case for LinEx functions, which are subject to the restrictions (19) of non-negative output elasticities. Thus, in the accessible factor space, *L* cannot exceed the upper limit imposed by the restriction that γ of Eq. (10) must be non-negative. (And in highly industrialized economies *L*/*K* has been and is decreasing strongly, anyway.)

Stern (2011) also objected to the LinEx function Y_{L1}, whose marginal *product* of labor, $Y_{L1}\beta/L$, decreases, when *E* increases. He wrote: "the marginal product of labor [...] can become negative when energy is very abundant." But, again, a restriction, this time that the output elasticity of labor, β, must be non-negative, prevents that. It imposes an upper limit on energy *E*. The sector of factor space, where β would be negative, is inaccessible. Furthermore, in highly industrialized economies, where automation of the capital stock increases, demanding increasing amounts of (mostly electrical) energy, society is faced with the problem that more and more (routine) jobs get lost. This is what one expects from a decreasing marginal product of labor.

The energy-dependent Cobb–Douglas production function, Y_{CDE}, Eq. (9), has positive marginal products and negative marginal returns everywhere in factor space. Then, why not use it, just disregarding the cost-share theorem as Tintner (1974) did? Growth analyses with CDE after 1974 confirmed that output elasticities are for energy much larger and for labor much smaller than the cost shares. But the reproductions of the economic downturns and upswings since the 1970s are insufficient. Figure 7 shows the only noteworthy result we could obtain with the new data for the German industrial sector. After 1990, the variations of output are hardly reproduced, and the statistical quality measures are poor. For the total economies of Germany and the USA, and with the new data, CDE just produces a smooth growth of GDP, which follows the growth of capital. Furthermore, the arbitrarily large factor magnitudes that are possible within the domain of the CDE may mislead studies on future economic developments into scenarios that are impossible from an engineering point of view, because the CDE allows for (asymptotically) complete factor substitutability; in other words, at given output it would be possible that capital substitutes for energy until only arbitrarily small energy inputs remain. This contradicts thermodynamics. Real-world production processes have finite minimum energy requirements.

Compared to Y_{L1}, the modified LinEx function $Y_{L11}(K, L, E; t)$ has the advantage of being invariant under the transformation from a base year t_0, with the inputs K_0, L_0, E_0 , and the technology parameters a and c, to another base year t_1, with the inputs K_1, L_1, E_1 , and the technology parameters a_1 and c_1. The proof is given in Appendix 1.

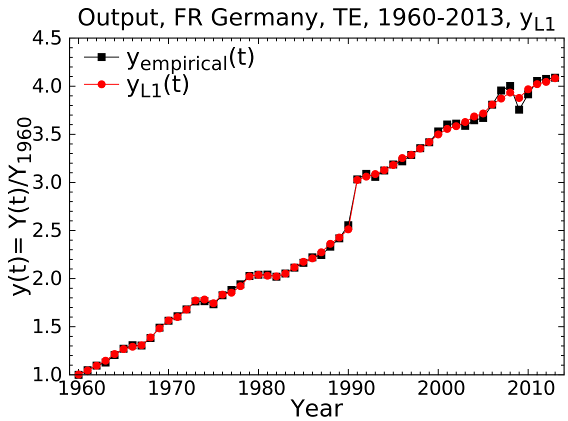
The invariance of Y_{L11} against transformations of the base year is related to the $E\ /\ c$ dependence of its output elasticities. $E\ /\ c$ is primary energy *E* divided by the energy demand *c* of the fully utilized capital stock. If at decreasing *c*(*t*) the *composition* of the output in terms of performed physical work and processed information, and the corresponding monetary valuation, would not change, the variable $E\ /\ c$ would be proportional to the *exergy* that acts on matter via capital activation with an efficiency that increases as *c*(*t*) decreases.

Finally, when wondering, whether to use Y_{L1} or Y_{L11}, one has to consider that Y_{L1} is the simplest production function with factor-dependent output elasticities. (Therefore, Occam's razor had cut it out first.) Because of its relative simplicity, the algorithms for the determination of its technology parameters are simpler and consume less computing time than the ones for Y_{L11}. Furthermore, one has to consider another trade-off, namely that between ambiguity and sub-optimal approach to the state of maximum automation. Maximum automation is defined by the asymptotic boundary condition $\beta \to 0$, when the inputs capital, labor, and energy approach K_m/K_0, L_m/L_0, and $E_m/E_0 = cK_m/K_0$. The first LinEx function Y_{L1} satisfies this boundary condition. But the factor space accessible to Y_{L1} *also* includes the range where $E/E_0 = cK/K_0$, and where β vanishes for *any* *K* and *L*. This ambiguity is removed by the modified ansatz for α in the output elasticities (11) of Y_{L11}: there, γ becomes negative, namely $-ac\frac{L/L_0}{K/K_0}$, when $E/E_0 \to cK/K_0$, and the restrictions (19) exclude this region of factor space for Y_{L11}. Of course, the state of maximum automation is excluded, too–strictly speaking. But this state is *almost* accessible to Y_{L11}, because $L_m/L_0 << K_m/K_0$ so that $\gamma \to -0$. Since the state of maximum automation is still not too close for present-day economies, although it is on the horizon of "Industry 4.0," the disadvantage that it belongs only asymptotically to the domain of Y_{L11} should not be too serious.

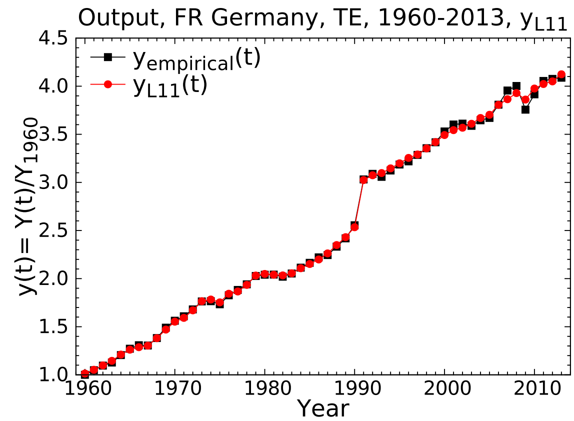
Subsequently, we will see that both LinEx functions reproduce economic growth in Germany and the USA satisfactorily for more than 50 years. Although the overall output elasticities are similar, their temporal variations reveal interesting differences in the details of how energy and creativity—the latter through time-changing technology parameters in the production function—contribute to output growth.

Technology Parameters and their Dynamics

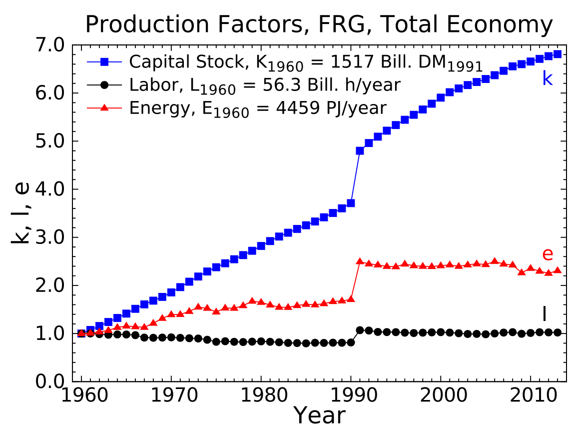
Further on, it is convenient to work with dimensionless variables, which are normalized to their numerical values in the base year t_0. For the inputs they are



(a) Output: empirical (squares), theoretical from Y_{L1} (circles)



(b) Output: empirical (squares), theoretical from Y_{L11} (circles)



(c) Empirical inputs

Fig. 2 FRG Total Economy, 1960–2013. Output, empirical, and theoretical according to LinEx functions Y_{L1} and Y_{L11}; and empirical inputs $k \equiv K(t)/K_{1960}$, $l \equiv L(t)/L_{1960}$, $e \equiv E(t)/E_{1960}$

$$k(t) \equiv K(t)/K_0, \quad l(t) \equiv L(t)/L_0, \quad e(t) \equiv E(t)/E_0, \tag{16}$$

and the normalized production function is

$$y(k, l, e; t) \equiv Y(K, L, E; t)/Y(K_0, L_0, E_0; t_0). \tag{17}$$

The differential equations, which yield the output elasticities and production functions, are invariant under these transformations.

The technology parameters a and c, and $y_0(t) \equiv Y_0(t)/Y_0(0)$, are determined by minimizing the sum of squared errors

$$\text{SSE} = \sum_i \left[y_{empirical}(t_i) - y(t_i)\right]^2 \tag{18}$$

subject to the restrictions

$$\alpha \geq 0, \quad \beta \geq 0, \quad \gamma = 1 - \alpha - \beta \geq 0\,. \tag{19}$$

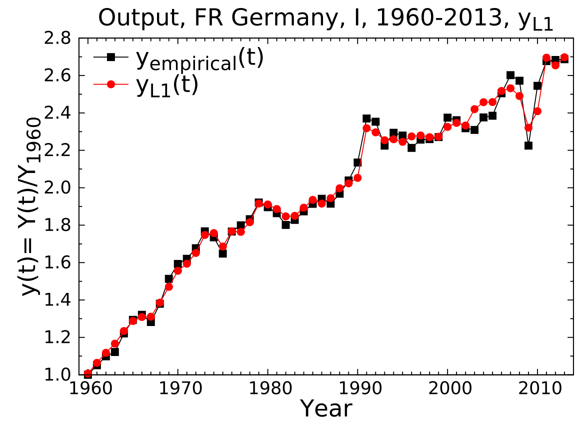
For notational simplicity, $y(t_i)$ abbreviates the normalized production function with the normalized inputs $k(t_i), l(t_i), e(t_i)$. The sum goes over all years t_i between the initial and the final observation time.

The empirical time series of output $y_{empirical}(t_i)$ and capital $k(t_i)$ are provided by the national accounts; labor and energy statistics give the empirical $l(t_i)$ and $e(t_i)$. These time series are shown in Figs. 2, 3, and 4.

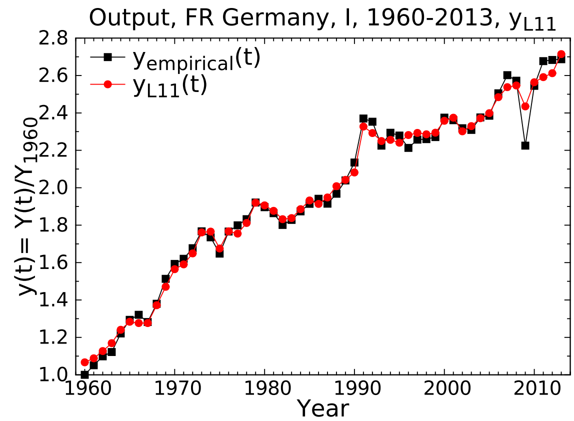
Let $p(t)$ represent either the capital-effectiveness parameter $a(t)$ or the energy-demand parameter $c(t)$ of the LinEx functions. Modeling it by logistic functions, as Kümmel et al. (2002) did, we have

$$p(t) = \frac{p_0 - p_1}{1 + \exp\left[-p_2\left(t - t_0 - p_3\right)\right]} + p_1, \tag{20}$$

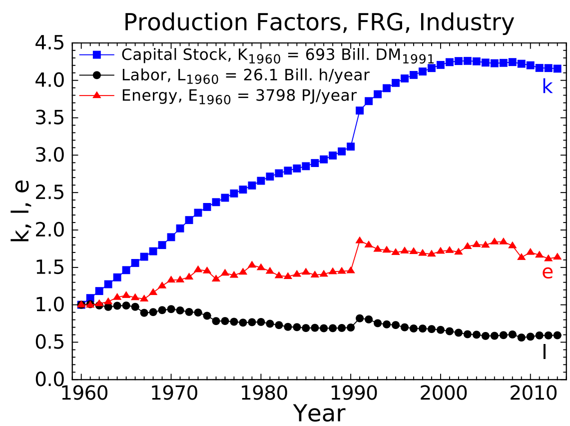
with the free (characteristic) coefficients p_0, p_1, p_2, p_3. For $p_2 \to \infty$ the logistic function turns into the step function. Examples for logistic functions are given by Winkler (2016), Fig. 2c. Alternatively, to save computing time, we also use linear functions



(a) Output: empirical (squares), theoretical from Y_{L1} (circles)



(b) Output: empirical (squares), theoretical from Y_{L11} (circles)



(c) Empirical inputs

Fig. 3 FRG Industry, 1960–2013. Output, empirical, and theoretical according to LinEx functions Y_{L1} and Y_{L11}; and empirical inputs $k \equiv K(t)/K_{1960}$, $l \equiv L(t)/L_{1960}$, $e \equiv E(t)/E_{1960}$. The contribution of the sector FRG Industry to the GDP of the FRG was 51.7% in 1970, 39.6% in 1992, and 27.1 % in 2009 (Kümmel 2011; p. 193)

$$p(t) = p_0 + p_1(t - t_0) \tag{21}$$

and combinations of them.

Minimizing the sum of squared errors (18) subject to the inequality restrictions (19) is a problem of non-linear optimization for which the Levenberg–Marquardt algorithm (Press et al. 1992) has proven to be useful. Section 3 of Winkler (2016) presents numerical details such as: (1) The mathematical structure of the algorithm, which is implemented by the Ceres Solver statistics program; (2) The equations for the statistical quality measures Durbin–Watson coefficient d_W and the adjusted coefficient of determination $\bar{R}^2$; (3) How to avoid convergence of SSE minimization in a side minimum via a new procedure of estimating the appropriate start values for the free coefficients $\{a_i\}, \{c_i\}, i = 0, 1, 2, 3$, and y_0; (4) Computation of standard deviations.

The contribution of time t to economic growth via innovations and structural change is given by creativity's output elasticity δ, defined in Eq. (3), with $\Delta t = |t - t_0|$. Using the normalized form $y_L(k, l, e; t)$, Eq. (17), for the LinEx functions one has

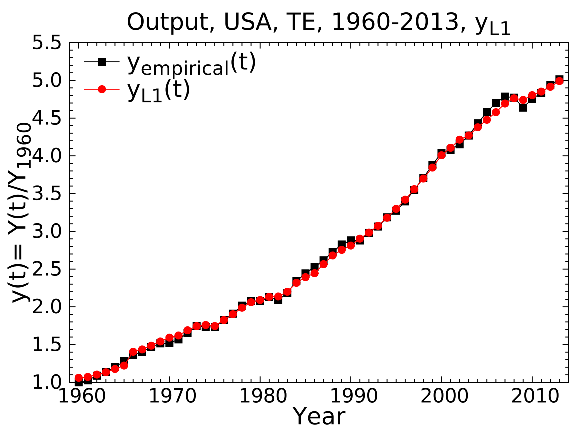
$$\delta = \frac{|t - t_0|}{y_L}\left[\frac{\partial y_L}{\partial a}\frac{\mathrm{d}a}{\mathrm{d}t} + \frac{\partial y_L}{\partial c}\frac{\mathrm{d}c}{\mathrm{d}t} + \frac{\partial y_L}{\partial y_0}\frac{\mathrm{d}y_0}{\mathrm{d}t}\right]. \tag{22}$$

For negligible $\mathrm{d}y_0/\mathrm{d}t$ one gets from Y_{L1}

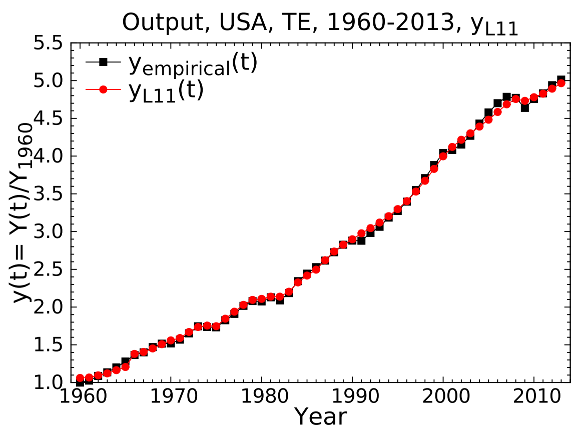
$$\delta_{L1} = |t - t_0|\left\{\left[\left(2 - \frac{l+e}{k}\right) + c(t)\left(\frac{l}{e} - 1\right)\right]\frac{\mathrm{d}a}{\mathrm{d}t} + a(t)\left(\frac{l}{e} - 1\right)\frac{\mathrm{d}c}{\mathrm{d}t}\right\}, \tag{23}$$

and Y_{L11} yields

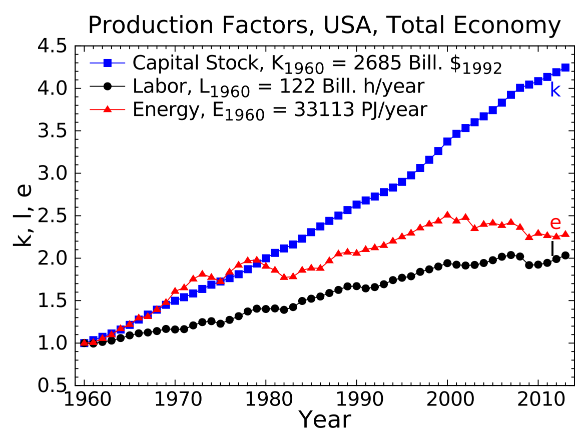
$$\delta_{L11} = |t - t_0|\left\{\left[c(t)\left(\frac{l}{e} - 1\right) - \frac{l}{k} + 1\right]\frac{\mathrm{d}a}{\mathrm{d}t} + \left[a(t)\left(\frac{l}{e} - 1\right) - \frac{1}{c(t)^2}\left(1 - \frac{e}{k}\right)\right]\frac{\mathrm{d}c}{\mathrm{d}t}\right\}. \tag{24}$$



(a) Output: empirical (squares), theoretical from Y_{L1} (circles)



(b) Output: empirical (squares), theoretical from Y_{L11} (circles)



(c) Empirical inputs

Fig. 4 USA Total economy, 1960–2013. Output, empirical, and theoretical according to LinEx functions Y_{L1} and Y_{L11}; and empirical inputs $k \equiv K(t)/K_{1960}$, $l \equiv L(t)/L_{1960}$, $e \equiv E(t)/E_{1960}$

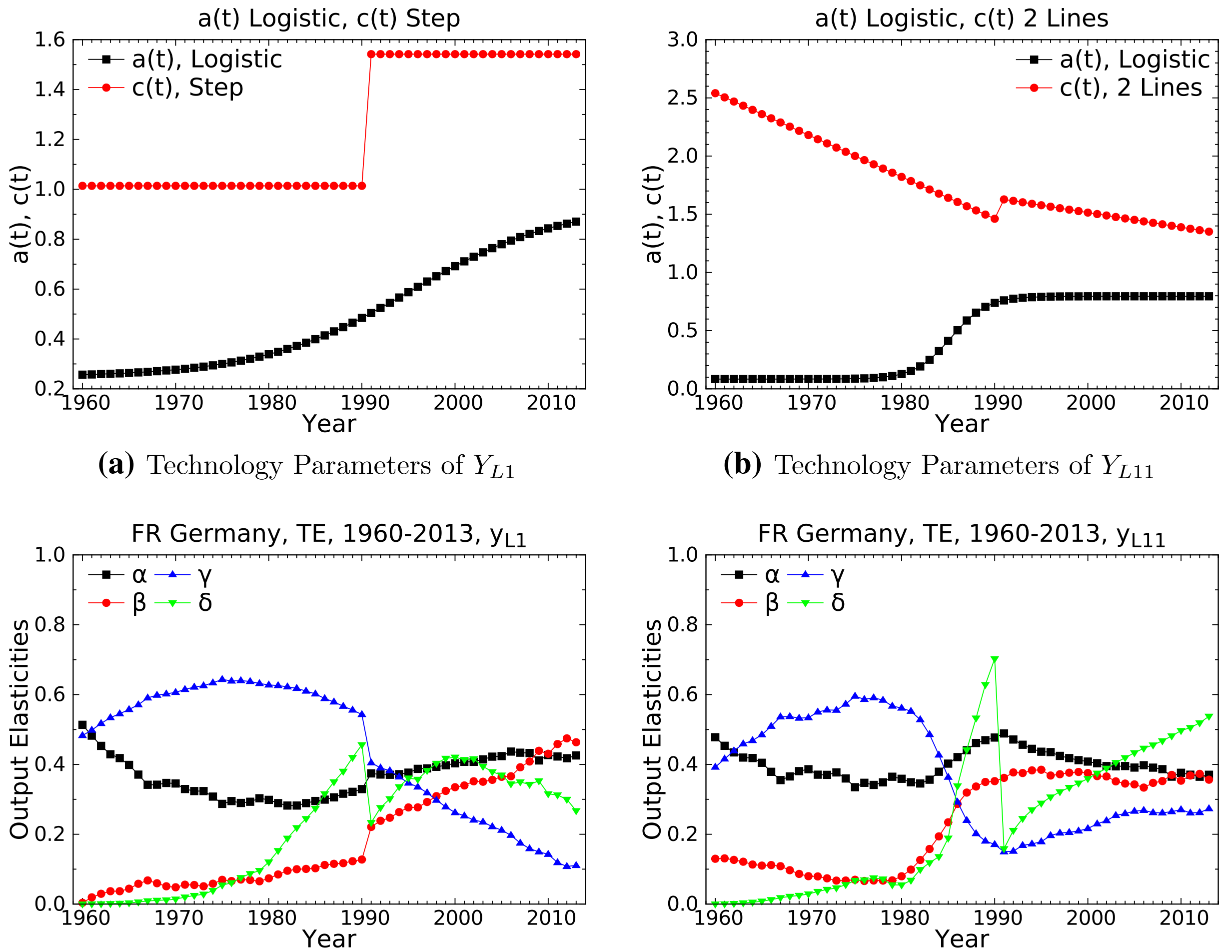
Results

The empirical growth of inputs and output in the FR of Germany (FRG) and the USA from 1960 to 2013, and the output computed with the production functions Y_{L1}, Eq. (14), and Y_{L11}, Eq. (15), are presented in Figs. 2, 3, and 4. As base year, we have chosen $t_0 = 1960$. The reproduction of economic growth in Germany and the USA by both LinEx functions is good. There is no Solow Residual. The time-dependent technology parameters $a(t)$ and $c(t)$, which, in conjunction with $y_0(t)$, are determined by SSE minimization, are given by Figs. 5, 6, and 8. These figures also show the resulting output elasticities. In Fig. 7 it is seen how the energy-dependent Cobb–Douglas function reproduces the growth of the German economic sector Industry. Table 1 lists the time-averaged LinEx output elasticities and the statistical quality measures.

Variations of Output and Inputs

Overall growth follows the growth of the capital stock, whereas the simultaneous downturns and upswings of outputs and energy inputs occur during the times of severe economic recessions between 1973–1975, 1979–1981, and 2008–2010, triggered by the first and the second oil-price shock and the bursting of the US housing bubble. The relation between output and energy input is bidirectional: When energy input decreases, fewer machines can be activated and output goes down; when demand for goods and services decreases and machines are shut down, less energy has to be inputted into the still active components of the capital stock.[5]

[5] The actually observed factor inputs result from entrepreneurial decisions influenced by the prices of inputs and output, and price expectations as well.



(a) Technology Parameters of Y_{L1} **(b)** Technology Parameters of Y_{L11}

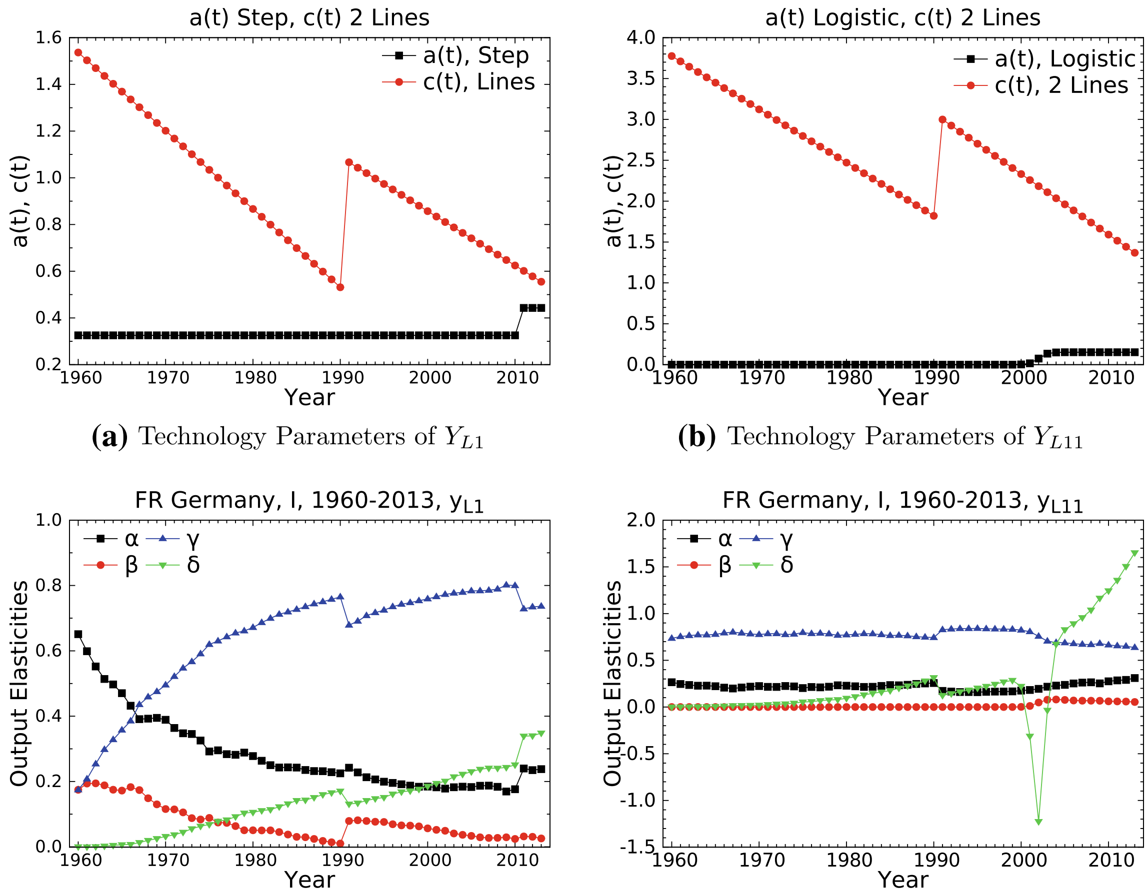
(c) Output Elasticities **(d)** Output Elasticities

Fig. 5 FR Germany Total Economy, 1960–2013. Technology parameters and output elasticities of the production functions Y_{L1}, Eq. (14), and Y_{L11}, Eq. (15)

When East Germans decided to abandon the planned economy of the former German Democratic Republic (GDR) and merge their production system with the market economy of the Federal Republic of Germany, reunification in 1990—an unprecedented structural change and challenge to socio-economic creativity—resulted in the jumps of inputs and outputs between 1990 and 1991 that are shown in Figs. 2 and 3. Trends in the "old" FRG before reunification continue in the "new" FRG: labor decreases, while energy tends to flatten out. In the USA, as shown in Fig. 4, quite to the contrary, labor grows, and so does energy, at least until 2000, and except for the times of the oil-price shocks. Nevertheless, despite the opposite trends in labor evolution, the LinEx functions work well for both systems. The diminishing output growth of the German industrial sector, reproduced in Fig. 3, is due to the diminishing and even negative growth of this sector's capital stock—a consequence of shifting energy-intensive production from Germany to other countries.

Variations of Technology Parameters and Output Elasticities

The technology parameters $a(t)$ and $c(t)$ have different meanings in Y_{L1} and Y_{L11}. This leads to the different temporal variations of the technology parameters and output elasticities of these two production functions in Figs. 5, 6, and 8. The difference originates from the different technological modeling of how capital-handling labor and capital-activating energy enter the law of diminishing returns in the output elasticities of capital, given by Eqs. (10) and (11). (1) In the output elasticities (10) of Y_{L1} the parameter $a(t)$ is the weight with which the ratio of "labor *plus* energy to capital" contributes to the output elasticity of capital, α, and the parameter $c(t)$ measures the energy demand of the capital stock $K(t)$ (at full capacity utilization) in the technology model represented by that α. (2) In the output elasticities (11) of Y_{L11} the parameter $a(t)$ gives (only) the weight with which the ratio of "labor to capital" contributes to the



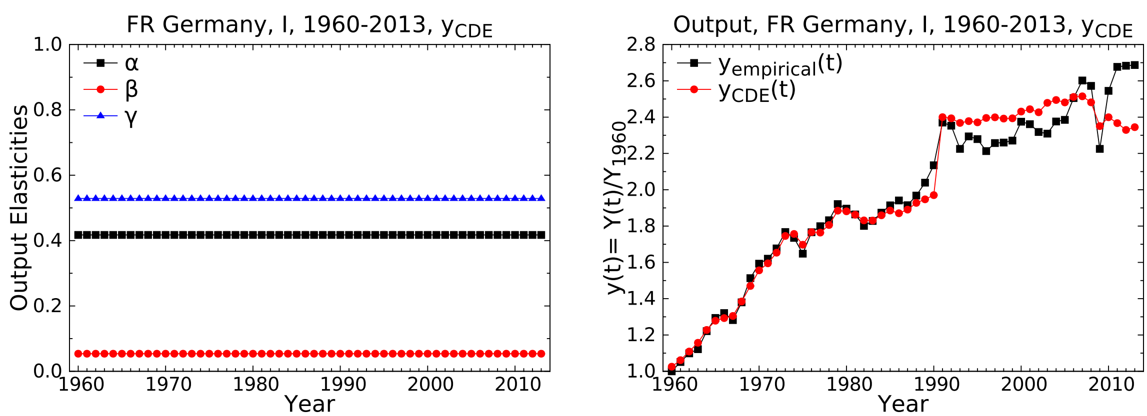
(a) Technology Parameters of Y_{L1}

(b) Technology Parameters of Y_{L11}

(c) Output Elasticities

(d) Output Elasticities

Fig. 6 FR Germany Industry, 1960–2013. Technology parameters and output elasticities for the production functions Y_{L1}, Eq. (14), and Y_{L11}, Eq. (15)

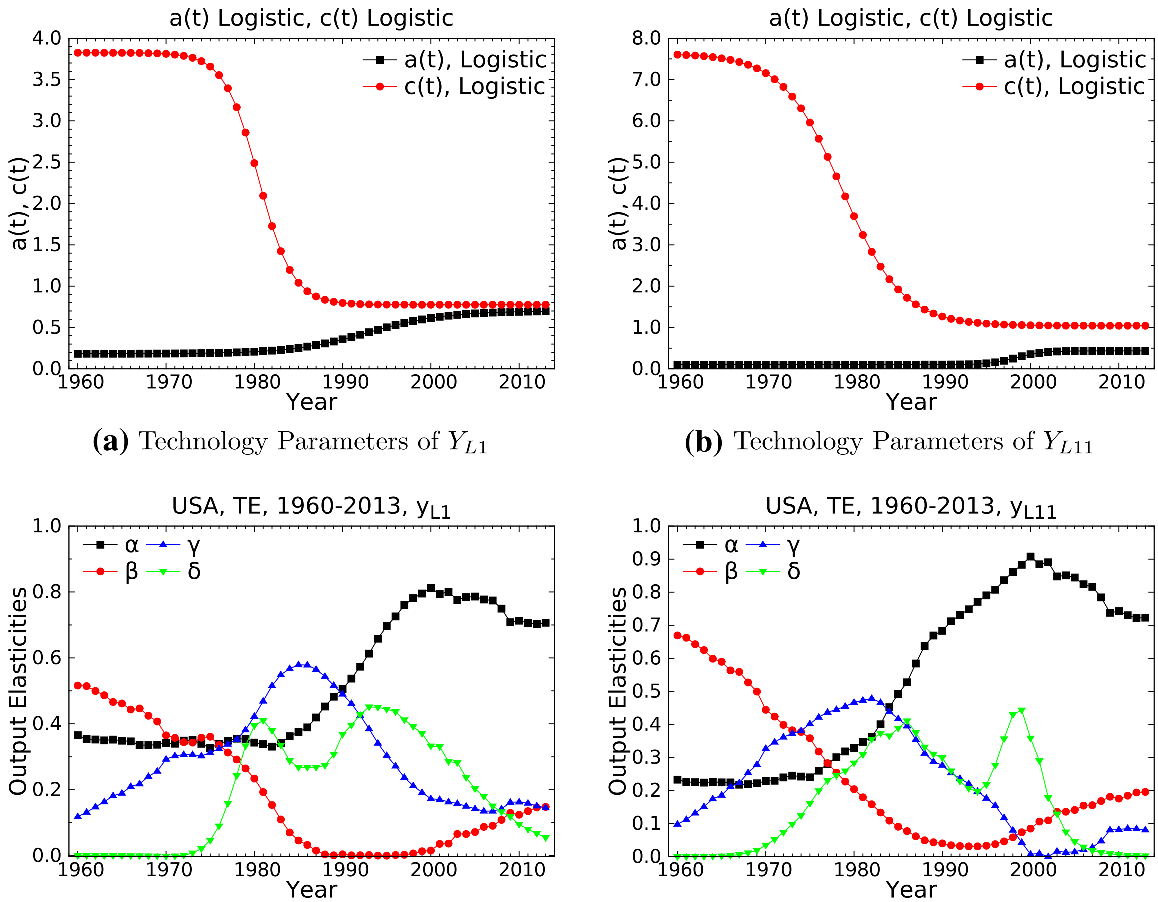


(a) Output Elasticities

(b) Fit

Fig. 7 FR Germany Industry, 1960–2013. Output elasticities and growth according to the energy-dependent Cobb–Douglas function, Eq. (9); $\bar{R}^2$ =0.939, $d_W = 0.416$

output elasticity of capital; the weight of the contribution of the ratio "energy to capital" is given by 1 / $c(t)$. Here $c(t)$ measures the energy demand of the (fully utilized) capital stock $K(t)$ in the technology model represented by the modified α. The degree of automation of $K(t)$ and the professional qualification of labor $L(t)$, which handles $K(t)$,



(a) Technology Parameters of Y_{L1}
(b) Technology Parameters of Y_{L11}
(c) Output Elasticities
(d) Output Elasticities

Fig. 8 USA Total Economy, 1960–2013. Technology parameters and output elasticities for the production functions Y_{L1}, Eq. (14), and Y_{L11}, Eq. (15). Constant technology parameters until the mid-seventies were also found by Ayres and Warr (2005), Ayres and Warr (2009) in their exergy-based analyses with Y_{L1}. ($y_0(t)$ is 1.06 until 1965 and 1.17 afterwards for both Y_{L1} and Y_{L11})

are different in the two models. (To model these differences in more detail, one would need output elasticities that are more complex than the ones in Eqs. (10) and (11).)

The difference in the meanings of the technology parameters of Y_{L1} and Y_{L11} shows most clearly in Germany's total economy before and after reunification in 1990. In Fig. 5a, the energy-demand parameter $c(t)$ jumps from the constant level at 1.01 before 1990 to the constant magnitude 1.52 since 1991. This jump is due to the incorporation of the capital stock of the former GDR, with its relatively low energy efficiency of factories and energy networks, into the capital stock of the "new" FRG. The logistic increase of $a(t)$ from 0.23 in 1960 to nearly 0.9 in 2013, with the turning point in the mid 1990s, on the other hand, indicates a gradual increase of capital effectiveness with respect to labor *and* energy and its diffusion from the "old" FRG into the eastern parts of the reunited country. In Fig. 5b, however, $a(t)$ indicates only capital's effectiveness with respect to labor. It rises from 0.08 to 0.8, and the turning point is before 1990. The linearly declining $c(t)$ before and after the jump between 1990 and 1991 indicates efficiency improvements in the activation of the capital stock by energy. Here, the decline before 1990, which starts from 2.45 in 1960, is steeper than that after 1990, when the average $c(t)$ is close to the constant $c(t) = 1.52$ of Y_{L1} in Fig. 5a.

The variations of $a(t)$, $c(t)$ for Y_{L1} and Y_{L11} in the German industrial sector, whose contribution to GDP has nearly been halved during the four decades since 1970, are shown in Figs. 6a, b. Here, $c(t)$ declining linearly from different initial levels, with upward jumps between 1990 and 1991, in combination with (almost) constant $a(t)$ before 2000, suffices for good fits. Again, $a(t)$ is larger for Y_{L1} than for Y_{L11}, and $c(t)$ is larger for Y_{L11} than for Y_{L1}. (One may wonder, whether the jump of $a(t)$ in Fig. 6a between 2010 and 2011 can be interpreted as a reaction to the burst of the US housing bubble, and whether the increase of $a(t)$ in Fig. 6b after 2000 may be understood as a response to the burst of the dot.com internet bubble. To answer such questions, one

Table 1 Time-averaged output elasticities of capital, $\bar{\alpha}$, labor, $\bar{\beta}$, energy, $\bar{\gamma}$, creativity, $\bar{\delta}$, adjusted coefficient of determination $\bar{R}^2$, and Durbin–Watson coefficient d_W obtained with the LinEx production functions Y_{L1} and Y_{L11} for the systems FR Germany Total Economy (FRG TE), FR Germany Industry (FRG I), and USA Total Economy (USA TE).

Y_{L1}, System:	FRG TE	FRG I	USA TE
$\bar{\alpha}$	0.367 ± 0.006	0.248 ± 0.008	0.518 ± 0.023
$\bar{\beta}$	0.188 ± 0.004	0.076 ± 0.008	0.188 ± 0.041
$\bar{\gamma}$	0.445 ± 0.007	0.640 ± 0.011	0.294 ± 0.047
$\bar{\delta}$	0.217 ± 0.006	0.132 ± 0.007	0.200 ± 0.023
R^2	0.999	0.989	0.998
d_W	1.650	1.747	0.715
Y_{L11}, System:	FRG TE	FRG I	USA TE
$\bar{\alpha}$	0.399 ± 0.008	0.221 ± 0.020	0.533 ± 0.016
$\bar{\beta}$	0.236 ± 0.012	0.015 ± 0.009	0.242 ± 0.035
$\bar{\gamma}$	0.365 ± 0.014	0.765 ± 0.022	0.226 ± 0.039
$\bar{\delta}$	0.236 ± 0.032	0.27 ± 0.16	0.168 ± 0.019
R^2	0.999	0.988	0.999
d_W	1.508	1.581	0.762

Observation time is 1960–2013. Appendix 3 comments on the d_W of the USA

would need a model of consumer and investor responses to bubble bursts.)

For the USA, the logistic variations of $a(t)$, $c(t)$ in Figs. 8a, b show that the two technology models yield quantitative differences between the $a(t)$ and $c(t)$ of Y_{L1} and the $a(t)$ and $c(t)$ of Y_{L11} that are similar to the ones in Germany: For Y_{L1}, $a(t)$ increases from 0.18 to 0.70 while $c(t)$ decreases from 3.8 to 0.78; for Y_{L11} the decrease of $c(t)$ is much stronger, from 7.6 to 1.04, and the increase of $a(t)$ is much smaller, from 0.10 to 0.43. A significant part of the parameter variations should be related to the first and the second oil-price shock. The growth of the internet- and information-based sectors of the US economy may have contributed as well. Both influences are related to the economic effects of human creativity through ideas, inventions, and value decisions: Investors decided to introduce more energy-efficient technology and innovative information technology into the capital stock, which contributed to an increase of capital effectiveness and a decrease of the capital stock's energy demand, as reflected by the evolution of the technology parameters a and c.

The time-varying output elasticities of capital, α, labor, β, and energy, γ, in Figs. 5, 6, 7, and 8 result from Eqs. (10) and (11) with the empirically given time-varying factor inputs and the computed technology parameters $a(t)$, $c(t)$. The output elasticities of creativity, δ, have been computed from Eqs. (23) and (23). (Narrow, sharp peaks from derivatives of step functions are omitted.)

In FR Germany, Total Economy, α varies between 0.5 and 0.3; β is below 0.2 between 1960 and 1980 and then rises up to roughly 0.4 in 2013; γ rises from roughly 0.4 in 1960 to roughly 0.6 in 1980, and then decreases—more strongly, down to 0.1, for Y_{L1} than for Y_{L11}, whose γ finishes somewhat below 0.3 in 2013. The contribution of the service sector to the GDP of the FR of Germany grew from 44.9% in 1970 to 72.0% in 2009 (Kümmel 2011, p. 193). We think that this structural change is represented by the variations of β and γ. Furthermore, the rise of renewable energy sources leads to a systematic error in the energy data that can lead to decreasing γ.[6]

Figures 6c, d show the dominating role of energy in industrial production. In the industrial sector of the FRG, energy's output elasticity γ is much larger than the output elasticities of capital and labor, except for Y_{L1} in the 1960s.

In the total economy of the USA, on the other hand, capital's output elasticity α dominates after 1990, β declines significantly between 1960 and 1990, and then starts to rise again, and γ has its maximum shortly after the two oil-price shocks. These dynamics of the output elasticities, exhibited in Figs. 8c, d, show no significant differences between Y_{L1} and Y_{L11}.

The result that in the US economy the output elasticity of labor is first declining and then rising indicates that different mechanisms are at work. While the mentioned structural change towards a rising share of services increases labor's output elasticity, increasing automation, mainly in the industrial part of the economy, decreases it. In the USA the latter effect seems to dominate until the 1990s, while the further gained importance since then. In order to separate the effects of rising shares of services on the one hand and increasing automation on the other, sectorally more disaggregated application of the theory would be desirable. This is left to future research.

Table 1 lists the time-averaged output elasticities. They confirm the fundamental heresy: The time-averaged output elasticities of labor, $\bar{\beta}$, are much smaller and those of energy, $\bar{\gamma}$, are much larger than the respective factor cost shares. The discrepancies are more pronounced in Germany than in the USA. Furthermore, capital's output elasticity $\bar{\alpha}$ exceeds the cost share of capital in Germany's total economy by roughly 50%, and it is about two times larger

[6] The energy statistics include a systematic error when it comes to accounting renewable energy generation. For conventional energy sources like coal, gas, oil, and nuclear power, the primary energy used to generate the electricity is reported. For renewables, however, only the generated electricity is reported as primary energy input implicitly assuming 100% efficiency. In reality, renewables have an efficiency below 100%. If the share of renewables increases, energy input decreases, affecting the contribution of the factor energy to growth.

than the cost share in the USA. The temporal variations of the technology parameters result in time-averaged output elasticities of creativity, $\bar{\delta}$, that are smaller, or significantly smaller, than energy's $\bar{\gamma}$. Thus, in conjunction with capital, the production factor energy provides the major contribution, and the temporal variations of the LinEx technology parameters provide the minor contribution to that part of economic growth that is assigned to "technical progress" by neoclassical economics. We feel that the cost-share theorem should no longer be used in the economics of highly industrialized countries.

Summary and Conclusions

Capital's growth determines the overall growth of economic output in highly industrialized countries like Germany and the USA. Economic recessions and recuperations are more strongly correlated with the downturns and upswings of capital-activating energy than with the variations of capital-handling labor.[7] Between 1960 and 2013, theoretical, inflation-corrected GDP grew without Solow residuals by similar factors in both countries: 4.1 in the FR Germany and 5 in the USA. During the same time, empirical labor inputs decreased somewhat in the FRG before and after reunification, and doubled in the USA.

How much output a given capital stock, at certain degrees of automation and capacity utilization, can produce is only a matter of technology. The same holds for the limits to capacity utilization and automation. (Factor prices, however, influence decisions on future investments.) Therefore, the economic weights of capital, labor, and energy should be independent from the factor cost shares. And precisely this engineering conclusion is confirmed by the output elasticities computed with the two LinEx functions used in the analysis of this work: Those of energy are much larger and those of labor are much smaller than the cost shares of these factors.

Technological and structural changes such as increasing automation, outsourcing of energy-intensive industries, changing shares of the industrial and service sectors in GDP, and the absorption of a centrally planned economy by a market economy, as it occurred in German reunification, show empirically in factor inputs and theoretically in the time-changing technology parameters and output elasticities of LinEx production functions. The growth contribution from the temporal variations of the technology parameters, *plus* the larger contribution from energy in combination with capital, supersedes the growth contribution from neoclassical technological progress functions. We think that LinEx functions can serve as a tool for clarifying the contributions of energy and time to technical progress, and structural changes as well. Of course, every tool can be improved. It would be interesting to see what can be achieved by shaking off the chains of the cost-share theorem from energy-dependent CES and translog production functions.[8]

We feel that a better understanding of the economic growth observed in highly industrialized economies will foster creative responses to the environmental challenges from emissions, which are intimately coupled to energy conversion by the inevitability of entropy production, whenever something happens. Based on a proper understanding of the past, one should develop scenarios for the future. They concern factor inputs according to entrepreneurial optimizing expectations, models for technology parameters in the output elasticities of capital, labor, and energy, and the resulting economic growth. Then society will gain improved visions of technologically and economically feasible paths into the future.

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Appendix 1: LinEx Function Transformations under Base-Year Changes from t_0 to t_1

With $t_0 \rightarrow t_1, \quad \alpha \rightarrow \alpha_1, \quad \beta \rightarrow \beta_1$ one has

$$\alpha_1 = a_1 \frac{L/L_1}{K/K_1} + \frac{1}{c_1} \frac{E/E_1}{K/K_1}, \quad \beta_1 = a_1 \left(c_1 \frac{L/L_1}{E/E_1} - \frac{L/L_1}{K/K_1} \right). \tag{25}$$

The transformation of the technology parameters a and c to

$$a_1 = a(L_1/L_0)/(K_1/K_0), \quad c_1 = c(E_0/E_1)/(K_0/K_1), \tag{26}$$

[7] In business cycles, legal and social constraints make the firing and hiring of people more difficult than the reduction and increase of energy inputs into the capital stock.

[8] In a recent paper by Heun et al. (2017) focusing on the estimation of CES production functions for the UK and Portugal 1960–2009, various modeling choices are analyzed and evaluated, including (a) rejecting (or not) the cost share principle, (b) including (or not) energy, (c) quality-adjusting (or not) factors of production, and (d) CES nesting structure. The effects of those choices on the interpretation of the economy and possible policy recommendations are assessed. The paper concludes that this "raises the possibility that energy-economy modeling with aggregate production functions… may tell us more about theory and modeling than about the economy." Maybe LINEX functions can contribute to this debate.

results in $\alpha_1 = \alpha, \quad \beta_1 = \beta$.

The invariance of the output elasticities α and β implies the invariance of the production function Y_{L11}. To see this also explicitly, Eq. (15) is rewritten as

$$Y_{L11}(K,L,E;t) = Y_0(t)\exp[A]\frac{E}{E_0} \times \exp\left[-a\frac{L/L_0}{K/K_0} - \frac{1}{c}\frac{E/E_0}{K/K_0} + ac\frac{L/L_0}{E/E_0}\right], \tag{27}$$

with $A \equiv a + 1/c - ac$. The transformation (26) changes Eq. (27) to

$$Y_{L11}(K,L,E;t) = Y_1(t)\exp[A]\frac{E}{E_1} \times \exp\left[-a_1\frac{L/L_1}{K/K_1} - \frac{1}{c_1}\frac{E/E_1}{K/K_1} + a_1c_1\frac{L/L_1}{E/E_1}\right], \tag{28}$$

where $Y_1(t) \equiv Y_0(t)\frac{E_1}{E_0}$. Thus, observing the transformations of the three integration constants one sees that the LinEx function Y_{L11} with the base year t_1 is the same as Y_{L11} with the base year t_0.

The output elasticity α of the first LinEx function $Y_{L1}(K,L,E;t)$, however, is not invariant to base-year changes: For $t_0 \to t_1$ and $a \to a_1 \equiv a(L_1/L_0)/(K_1/K_0)$ one gets $\alpha = a_1(L/L_1)/(K/K_1) + a_1(E/E_1)/(K/K_1) \cdot [(E_1/E_0)/(L_1/L_0)]$. The term $[(E_1/E_0)/(L_1/L_0)]$ destroys the invariance of α. (The base-year change does not affect β.) As a consequence of the α-transformation, the transformed $Y_{L1}(K,L,E;t)$ has not only transformed parameters but also acquires an additional term in the exponent that depends on E/E_1 and K/K_1. Nevertheless, the numerical values of the output elasticities, which previously have been obtained with Y_{L1} for different base years, do agree within the error margins.

Appendix 2: Measures for Inputs and Output

In order to select appropriate measures for capital, labor, energy, and output, it is important to recall the conceptual foundations of the employed production functions. Production functions are supposed to describe the physical transformation of inputs to output by "technology," i.e., they are supposed to be a physical or engineering concept.[9] What can we say about the physical nature of production on the rather abstract level of aggregate production functions? On the physical level, value is generated through work performance and information processing. During production the industrial capital stock generates a flow of work performed and information processed per unit time. More specifically, capital is productive and generates value through work performance and information processing to the extent that it is activated by energy and handled by labor. This physical interpretation of production has implications with respect to the choice of measures for inputs and output of the production function. Regarding capital, it follows from the above that we are looking for a measure of capital's productive capacity. This is because it is the productive capacity that has to be utilized through energy and labor. Note that within this interpretation there is no room for a service-adjustment of the capital variable, because the service flowing from the productive capacity, i.e., the work performed and information processed per unit time, is determined by the very quantities of energy and labor that activate and handle this capital. In other words, utilization-adjusted capital would not be an independent variable besides energy and labor. Given this interpretation, the appropriate capital variable is "pure" capital stock as a measure of productive capacity. For further details and data issues on capital stock data, see Appendix 3. Regarding energy, we are looking for a measure consistent with the system boundaries of the modeled production system. In our case, analyzing German Total Economy, German Industries, and US Industries, all of which include the energy conversion sector (whose capital stock contains power plants, refineries, etc.), it is primary energy that crosses the systems' boundaries to drive the respective capital stocks. Hence, we use primary energy as input variable.[10] For further details on the use of primary energy data, see Appendix 3. Regarding labor, our model explicitly separates two components. We assume that the productive contribution of human labor can be divided into (i) the contribution of routine labor and (ii) the contribution of human creativity. While we assume that the contribution of routine labor can be measured by actual hours worked per year, the contribution of human creativity through ideas, inventions, and value decisions can be measured only ex post: It manifests itself through the explicit time dependence of the production functions' technology parameters. It may be noted that this approach is consistent with doing without a quality-adjustment for (routine) labor and seems to be sufficient to fully resolve the Solow residual, although models that differentiate labor with respect to

[9] As Dorfman et al. (1957), and Solow (1957) remark "…there seems to have been a misunderstanding somewhere because the technologists do not take responsibility for production functions either. They regard the production function as an economist's concept, and, as a matter of history, nearly all the production functions that have actually been derived are the work of economists rather than of engineers."

[10] It may be noted that in case of the most important primary fuels, namely coal, oil, gas, and nuclear fuels, primary energy is nearly 100 % exergy (essentially useful work).

(educational) quality are of course also possible.[11] Regarding output, it is important to once again recall the physical interpretation of value creation through work performance and information processing. This has an important implication with respect to both the adequate measure of output and the role of materials in the production process. Materials, despite being indispensable for production, neither perform work nor process information. Hence, they do not actively contribute to the generation of value, but rather are the passive partners in the production process, onto which value is imprinted upon. This is why it is consistent with our interpretation of value creation through work performance and information processing to disregard materials as factors of production and to use (not gross output including the value of preprocessed materials but) net output or value added as output measure.[12] And this output measure is the GDP, whose growth is in the center of economic and political interest.[13]

Traditional economics treats only capital and labor as primary production factors and interprets energy as an intermediate. Treating energy as an intermediate may be consistent with national accounting conventions, however it raises conceptual issues: Is the treatment of energy as an intermediate (produced by some combination of capital and labor only) consistent with the laws of thermodynamics according to which any process is driven by energy conversion (economic production being no exception)? How can the process of value creation, based on work performance and information processing, be interpreted physically, if energy does not contribute to this process by assumption? Finally, what about the argument that energy ought to be considered as an intermediate since it is an output of the extraction or energy conversion sector: Indeed, if a production system excluding the extraction or energy conversion sector is considered, then energy plays a double role. It is then both an intermediate (as output of the extraction or conversion sector) and a primary production factor that contributes to value creation through work performance and information processing within the economic sector under consideration, and therefore should be included in the production function to explain the generation of value added by this economic sector.

[11] Needless to say, if one constructs (educational) indices in order to quality-adjust labor data, additional assumptions are necessary for this construction. Educational index data are provided, e.g., by Barro and Lee (2001).

[12] Kander and Stern (2014) consider energy as an intermediate in the production function, and use value added plus the cost of energy as output measure.

[13] There is a small difference between GDP and Gross Value Added, where the former includes subsidies and taxes and the latter does not.

Appendix 3: Data

In this Appendix, the construction of the new time series of capital, labor, energy, and output is described. The empirical data for the FR Germany, Total Economy (FRG TE), FR Germany, Industry (FRG I), and the USA, Total Economy (USA TE), whose numbers are listed in tables by Winkler (2016), are shown in graphical form. Data uncertainties for the USA are discussed.

Conventions for Monetary Aggregates

Historically, monetary aggregates like the Gross Domestic Product (GDP), gross value added, or fixed assets were expressed in constant prices of a reference year to adjust the aggregates for inflation. Instead of using a fixed reference year, monetary aggregates are calculated nowadays by using the previous-year price basis in Germany and the United States. This method has the advantage that a periodical change of the reference year is no longer required. Furthermore, the problem of the so-called substitution bias is solved by the new method (Quaas 2009).

Output In Germany, chained price indexes $CI_G(y)$ for aggregates like the GDP or gross value added are reported for each year y.[14] The aggregates consist of n goods and services $q_1, q_2, \ldots q_n$ with prices $p_1^y, p_2^y, \ldots p_n^y$ of a specific year y. The quotient of the chained price indexes can be interpreted as follows (Quaas 2009):

$$\frac{CI_G(y)}{CI_G(y-1)} = \frac{\sum_{i=1}^{n} q_i^y \cdot p_i^{y-1}}{\sum_{i=1}^{n} q_i^{y-1} \cdot p_i^{y-1}} = \frac{A_{Real}(y)}{A_{Nom}(y-1)}.$$

In this notation, $A_{Real}(y)$ is the real aggregate of the year y in prices of the previous year $y-1$. $A_{Nom}(y-1)$ is the nominal aggregate of the year $y-1$.

We introduce a reference year y_{ref} in which we set $CI_G(y_{ref}) = 1$. This choice of reference year should not be confused with the base year of constant prices in the former methodology. Absolute values of chained indexes can be changed by redefining the reference year of the index. However, relative changes of the indexes are independent of the choice of the reference year.

In the United States, chained indexes $CI_U(y)$ are reported. The quotient of the chained index of a year $CI_U(y)$ and of the previous year $CI_U(y-1)$ in the US convention is the so-called Fisher Index $QI_F(y)$, which is defined as follows (Whelan 2000):

[14] Index subscripts refer to countries (G = Germany, U = USA) or type of index (Q = Quaas, F = Fischer) applied by those countries.

$$QI_F(y) = \frac{CI_U(y)}{CI_U(y-1)} = \sqrt{\frac{\sum q_i^y \cdot p_i^{y-1}}{\sum q_i^{y-1} \cdot p_i^{y-1}} \cdot \frac{\sum q_i^y \cdot p_i^y}{\sum q_i^{y-1} \cdot p_i^y}}.$$

In the context of modeling production and growth, we require a measure for output adjusted for inflation. If the prices of the quantities constituting the output do not change relative to each other, the growth in constant prices of a certain year and in prices of the previous year would be the same. However, if a change in the price structure occurs (for instance, computers become cheaper relative to the prices of other products), the growth rate differs depending on the method applied.

OECD (2001) states: “In a time-series context, i.e. for the measurement of the rates of change of outputs, inputs and productivity, there is a strong preference in the literature in favor of chained indexes.” Therefore, we use chained indexes and update the already existing data in Kümmel (2011).

The relation of the GDP and gross value added in producer prices (excluding taxes on products and including subsidies on products) is:

Gross Domestic Product = Gross value added + Taxes on Products – Subsidies on Products.

Since we are not interested in the effect of subsidies and taxes, we use gross value added (GVA) in producer prices in our analysis when it is available. It can be interpreted as a measure for the value of the produced goods and services in a sector of the economy.

Capital Our model needs a measure of the productive capacity of the capital stock. The best available such measure is gross capital stock (Bruttoanlagevermögen). Gross capital stock does not account for wear and tear, but it does account for the physical retirement of assets (see e.g., OECD 2001, 2009). We use chained indexes of gross capital stock for the German time series to be consistent with our measure of output. In the case of the United States, there is no publication of a measure for the gross capital stock. Therefore, we use chained indexes of net capital stock. This corresponds to the assumption that the age efficiency profile (time dependence of wear and tear) corresponds to the time profile of depreciation.

Data Sources by Systems

FRG TE

Capital The German national accounts (VGR des Bundes—Anlagevermögen nach Sektoren) available at Statistisches Bundesamt (2015) give nominal and real (chained index) values for the total invested capital (Alle Anlagegüter) and for the components (Nutztiere und Nutzpflanzungen, Ausrüstungen, Wohnbauten, Nichtwohnbauten, Geistiges Eigentum) for 1991–2014. We need to subtract the component “residential fixed capital” (Wohnbauten) from the value for total invested capital, since residential buildings are not part of the productive capital. Since we cannot just add or subtract chained quantities $CI_Q(y)$, we derive real capital in prices of the previous year $Q_{Real}(y)$ with the nominal capital in prices of the previous year $Q_{Nom}(y-1)$:

$$Q_{Real}(y) = \frac{CI_Q(y)}{CI_Q(y-1)} \cdot Q_{Nom}(y-1).$$

We know the values for the chained indexes $CI_Q(y)$, $CI_Q(y-1)$ and for the nominal capital $Q_{Nom}(y-1)$ from the above-mentioned national accounts.

After deriving real values for total capital $T_{real}(y)$ and for the residential fixed capital $R_{real}(y)$ in this way, we calculate the change of a chained index for the productive capital $CI_{PC}(y)$ with the nominal total capital $T_{nom}(y-1)$ and the nominal residential fixed capital $T_{nom}(y-1)$ in prices of the previous year:

$$\frac{CI_{PC}(y)}{CI_{PC}(y-1)} = \frac{T_{real}(y) - R_{real}(y)}{T_{nom}(y-1) - R_{nom}(y-1)}.$$

$CI_{PC}(1991)$ is set to the value of the capital in 1991, as given by Kümmel (2011).

Labor We are interested in total hours worked per year (Arbeitszeitvolumen). Those data are collected by the Institut für Arbeitsmarkt und Berufsforschung (Arbeitszeitrechnung) and are part of the German national statistics (VGR des Bundes: Arbeitsstunden, WZ2008). Data for 1991–2014 are available. The values are normalized to the value of L_{1960} in Kümmel (2011).

Energy Arbeitsgemeinschaft Energiebilanzen (2015) publishes yearly energy balances for Germany. Data are available for 1991–2013. In 1995, the data aggregation was changed. In the years after 1994, the energy input for production is calculated as follows:

Energy Input for Production = Primary Energy Consumption – Residential Final Energy Consumption – Non-Energetic Usage.

(In the original German notation, this reads as follows: Primärenergieverbrauch—Haushalte endenergieverbrauch—Nichtenergetischer Verbrauch.)

This means that the total energy-converting sector (e.g., refineries, power plants) is fully considered in the energy input for the production function. This is consistent with the definition of productive capital.

Before 1995, the residential final energy consumption was not declared separately, but as part of “Residential Consumers and All Small Consumers” (Haushalte und Kleinverbraucher insgesamt). We use the 1995 share of residential final energy consumption in the energy consumption of households, trades, and services (Haushalte, Gewerbe, Handel, und Dienstleistungen) and assume this share to be equal the share of residential final energy consumption in the energy consumption of residential consumers and all small consumers before 1995. With this assumption, we can reproduce the values from Kümmel (2011). The values are normalized to the value of E(1960) in Kümmel (2011).

Output The gross value added (Bruttowertschöpfung) is used as a measure for the output which is part of the German national statistics (VGR des Bundes—Bruttowertschöpfung (preisbereinigt): Deutschland, Jahre). The changes of the chain index available for 1991–2014 are used as a measure for output growth. *CI*(1991) is set to the value for the output in 1991 in Kümmel (2011).

FRG I

The considered sectors are the producing industries (in German: Produzierendes Gewerbe). This term is clearly defined in the German national accounts by the classification system WZ 2008.

Capital The total invested capital of the industrial sector is given by the German national accounts (VGR des Bundes: Bruttoanlagevermögen, Deutschland, Jahre, Wirtschaftsbereiche, Anlagearten) for 1991–2014 as a chained index. Since residential fixed capital is not part of the producing industries, no further adjustment needs to be done. *CI*(1991) is set to the value for the capital in 1991 in Kümmel (2011).

Labor We are interested in total hours worked per year (Arbeitszeitvolumen) in the industrial sector. Those data are collected by the "Institut für Arbeitsmarkt und Berufsforschung, Arbeitszeitrechnung" and are part of the German national statistics (VGR des Bundes—Arbeitsstunden, WZ2008). Data are available for 1991–2014. The values are normalized to the value of L(1960) in Kümmel (2011).

Energy Arbeitsgemeinschaft Energiebilanzen (2015) publishes yearly energy balances for Germany. The energy input for production in the industry sector is calculated as follows for years after 1994:

Energy Input for Production in the sector FRG I = Primary Energy Consumption – Non-Energetic Usage – Final Energy Consumption of Traffic minus Final Energy Consumption of Residential Consumers and All Small Consumers. (In the original German notation, this reads as: Primärenergieverbrauch–Nichtenergetischer Verbrauch–Verkehr insgesamt–Haushalte, Gewerbe, Handel und Dienstleistungen.)

Before 1995, we use:

Energy Input for Production in Sector Industry = Primary Energy Consumption – Non-Energetic Usage – Final Energy Consumption of Traffic – Final Energy Consumption of Residential Consumers and All Small Consumers – Final Energy Consumption of Military Services. (In the original German notation, this reads as: Primärenergieverbrauch–Haushalte und Kleinverbraucher insgesamt–Nichtenergetischer Verbrauch–Verkehr insgesamt– Militärische Dienststellen.)

Output Gross value added of the sector FRG I (Bruttowertschöpfung des produzierenden Gewerbes) is used as a measure for the output which is part of the German national statistics (VGR des Bundes, Bruttowertschöpfung (preisbereinigt): Deutschland, Jahre, Wirtschaftsbereiche). The changes of the chain index are used as a measure for output growth. Data are available for 1991–2014. *CI*(1991) is set to the value for the output in 1991 in Kümmel (2011).

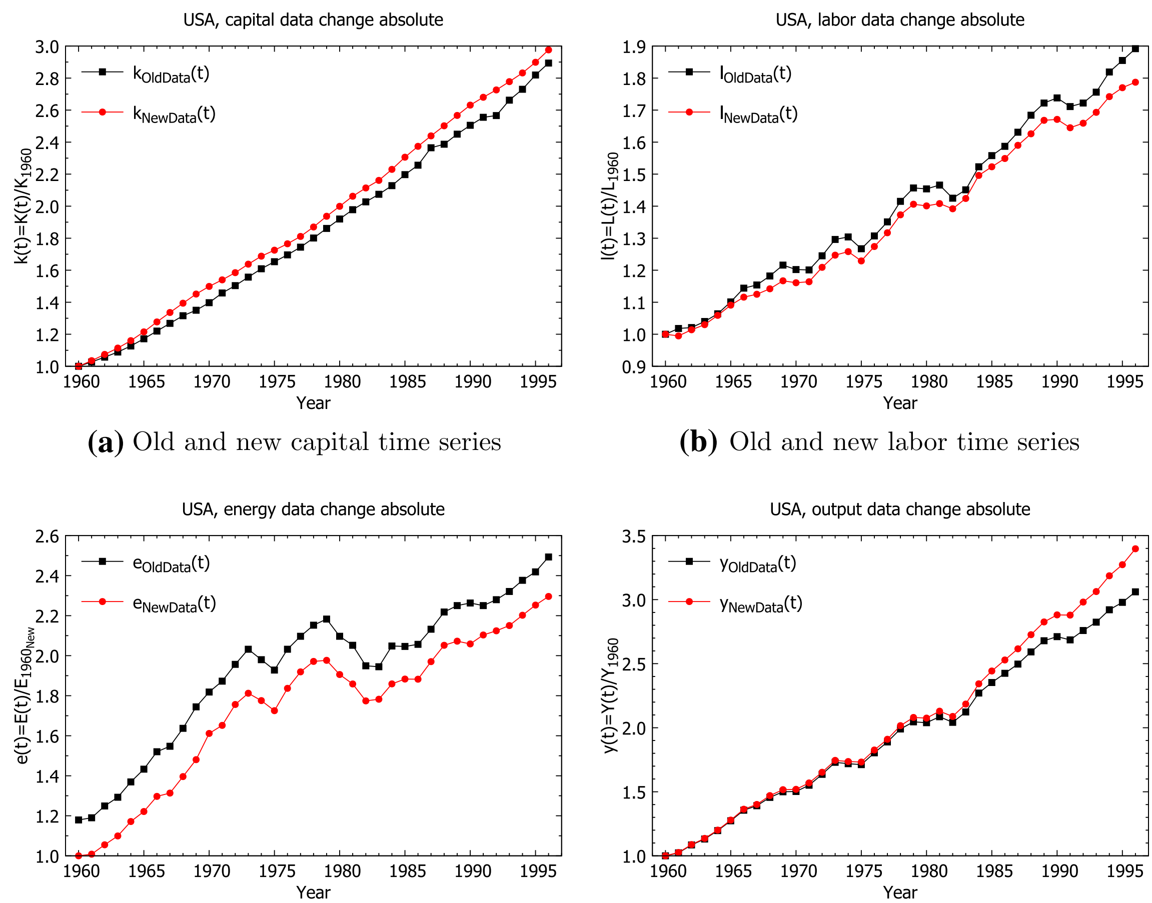
USA TE

Capital According to OECD (2001), two sources for capital measures are available: (1)The Bureau of Economic Analysis publishes data about net capital stocks. (2) The Bureau of Labor Statistics publishes data about capital services.

Since capital services implicitly contain a measure of the degree of capacity utilization, whereas we need a measure of productive capacity, we use the net capital stock ((BEA 2015), Chain-Type Quantity Indexes for Net Stock of Fixed Assets and Consumer Durable Goods). Since we need to exclude residential fixed capital, we use the component "Nonresidential" of "Private and government fixed assets." *CI*(1960) is set to the value of 1.

Labor The Federal Reserve Bank of St. Louis (2015) publishes data series about "Average Annual Hours Worked by Persons Engaged for United States" starting in 1950. The US Bureau of Labor Statistics (2015) publishes data about the employment level/total number of employed persons as part of the "Labor Force Statistics from the Current Population Survey." Multiplying both time series (average hours worked per person per year multiplied by the total number of employed persons) yields the total number of hours worked in a year. The values derived in this way match the data of Kümmel (2011) well. The data by the Federal Reserve Bank of St. Louis end in 2011. The Bureau of Labor Statistics publishes an index of aggregate weekly hours of all employees starting in 2009. The change rate of the total hours worked derived by us matches the change rate of this index in 2009, 2010, and 2011. Therefore, we continue our data with the same change rate as this index in the year 2012–2015.

Energy OECD publishes yearly "Energy Balances of OECD Countries" (OECD 1960–1985 and OECD 1980–2014). We calculate the energy input as: Energy Input for Production = Total Primary Energy Supply – Residential Sector Final Energy Consumption – Non-Energy Usage.



(a) Old and new capital time series **(b)** Old and new labor time series

(c) Old and new energy time series **(d)** Old and new output time serie

Fig. 9 USA Total Economy, 1960–1996. Old data (*squares*) and new data (*circles*). The old energy data are normalized to the 1960 value (33113 PJ) of the new energy data. (The 1960 value of the old energy data was 39051 PJ.)

Kümmel (2011) used the "Monthly Energy Review" of the US Energy Information Administration, complemented by other data sources recommended by US energy experts. These publications convert renewable energy generation like hydroelectricity or solar thermal/photovoltaic electricity generation to a primary energy equivalent "using the fossil-fueled plants heat rate." However, OECD (1980–2014) use the electricity generated by hydro, wind, tide/wave/ocean, and solar photovoltaic as the relevant contribution to the primary energy, and do not calculate the "hypothetical amount of energy necessary to generate an identical amount of electricity in conventional thermal power plants." We follow the international standards of OECD. Therefore, our values are systematically lower than the values in Kümmel (2011), see Fig. 9c.

In OECD (1960–1985), the old convention of calculating the primary energy equivalent of renewable energy is used. Therefore, we apply a correction to consider only the actual electricity produced. Furthermore, the energy balances in 1960–1969 do not report the "Residential Sector Final Energy Consumption" explicitly but only "Total Other Sectors." We make the assumption that the share of the residential sector of "Total Other Sectors" is before 1970 the same as the average of this share in the years 1970–1976.

Output The Bureau of Economic Analysis (BEA) publishes chained quantity indexes for the real gross domestic product and value added by sector. However, BEA uses purchasers' prices (including taxes and excluding subsidies) instead of producer's prices to calculate the monetary aggregates. Therefore, the above given relation between GDP and value added (Gross Domestic Product = Gross value added + Taxes on Products – Subsidies on Products) does not hold for BEA's data. Instead, here we have: GDP = total value added. In this sense, the definition (or rather price basis) of "value added" is different in the national accounts of Germany and the United States. Therefore, we use the GDP as a measure for output in the US (Table 1.1.3.

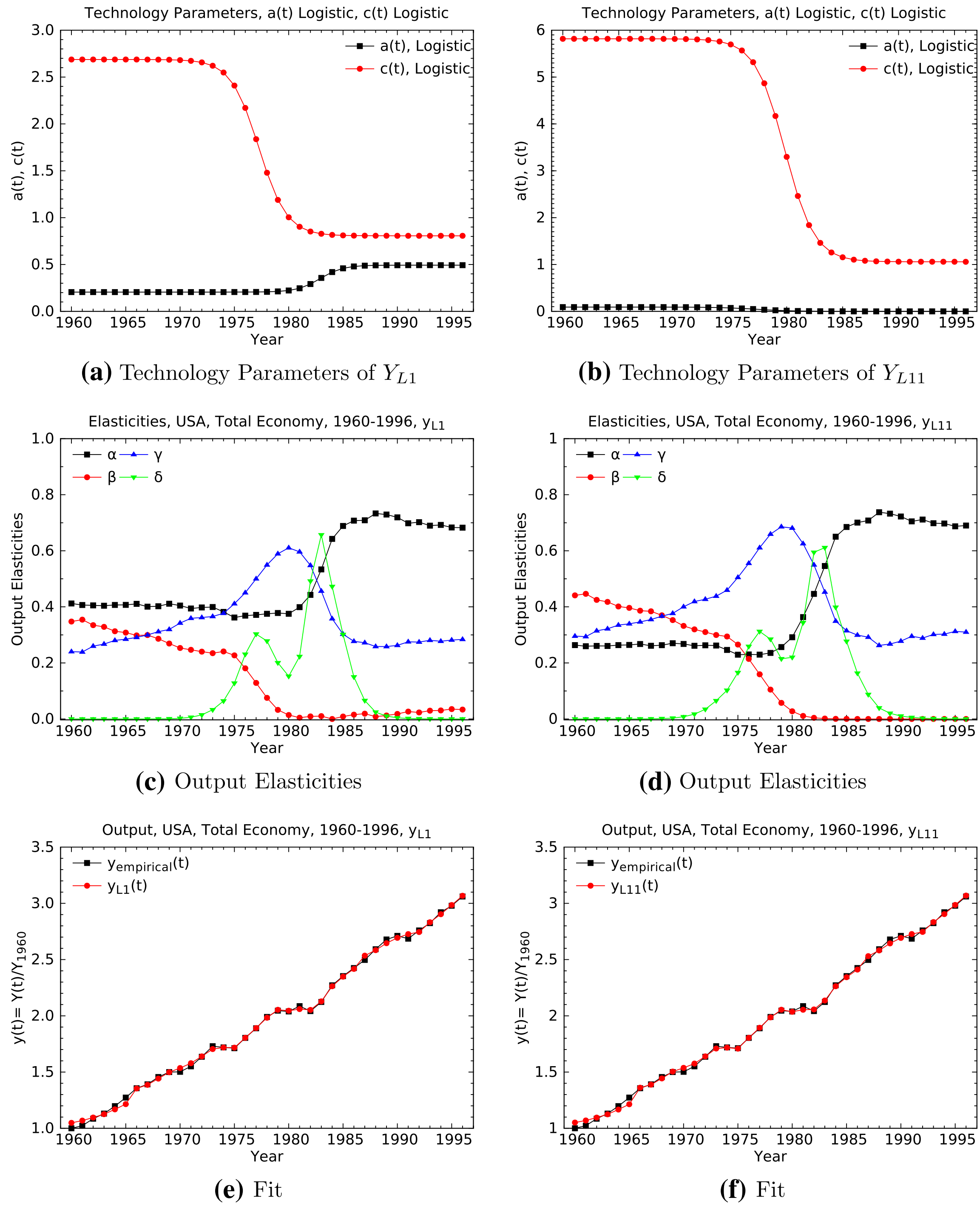


Fig. 10 USA Total Economy, 1960–1996. Technology parameters, output elasticities, and output growth are obtained by using the production functions Y_{L1} from Eq. (14) and Y_{L11} from Eq. (15)

Real Gross Domestic Product, Quantity Indexes). $CI(1960)$ is set to the value of 1.

Data Uncertainties for the USA

Figure 9 shows the time series of the old data on US inputs and output between 1960 and 1996 in comparison with the new data for the same time interval.[15] The new capital data available for the USA refer to the net capital stock, whereas

[15] The old data were assembled and updated by one of us (R.K.) during several personal visits to US research facilities between 1981 and 1999, with kind help from colleagues in economics.

the old capital data, used in Kümmel (2011), refer to the gross capital stock. Thanks to normalizing all inputs to the base year 1960, the difference between the old and new capital time series is relatively small, see Fig. 9a. The biggest difference is in the energy data, Fig. 9c; the (or: one) reason for that is indicated in the previous subsection. The labor and output data differ noticeably, too.

Despite the data differences, the overall output elasticities obtained with the old and the new data are essentially the same. However, some details, like the creativity responses to the first and the second oil-price shock 1973–1975 and 1979–1981 are closer to the shock ti mes in Fig. 10 than in Fig. 8. Outputs, output elasticities, and technology parameters in Fig. 10 are computed for Y_{L1} and Y_{L11} with the old data 1960–1996, with *all* input data normalized to their 1960 values. The uncertainties in the new US data may be responsible for the relatively low US Durbin–Watson coefficient given in Table 1. The US $d_W = 1.46$ in Table 4.5 of Kümmel (2011) is closer to the optimum at 2.0.

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