Improved fuzzy clustering for image segmentation based on a low-rank prior

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Abstract Image segmentation is a basic problem in medical image analysis and useful for disease diagnosis. However, the complexity of medical images makes image segmentation difficult. In recent decades, fuzzy clustering algorithms have been preferred due to their simplicity and efficiency. However, they are sensitive to noise. To solve this problem, many algorithms using non-local information have been proposed, which perform well but are inefficient. This paper proposes an improved fuzzy clustering algorithm utilizing non-local self-similarity and a low-rank prior for image segmentation. Firstly, cluster centers are initialized based on peak detection. Then, a pixel correlation model between corresponding pixels is constructed, and similar pixel sets are retrieved. To improve efficiency and robustness, the proposed algorithm uses a novel objective function combining non-local information and a low-rank prior. Experiments on synthetic images and medical images illustrate that the algorithm can improve efficiency greatly while achieving satisfactory results.

Keywords image segmentation; fuzzy clustering; non-local information; low-rank prior; medical images

1 Introduction

With the development of medical diagnostic technology, various forms of information, such as medical images and electrocardiograms, have been

adopted for use in clinical decision support systems. The combination of medical knowledge and data processing technology is an active research area which has received extensive attention. Currently, data processing technologies such as image segmentation, image registration, and 3D reconstruction play an important role in smart healthcare [1].

Generally speaking, medical image segmentation can be used to partition an image into different tissues or organs, which is helpful for clinical decision support systems. However, the complexity of medical images makes this problem difficult. In medical images, the intensity value of a pixel is influenced by adjacent pixels due to the imaging principle [2]. Therefore, the intensity value of a pixel may represent interactions with corresponding tissues or organs. Many algorithms have been proposed for image segmentation, such as threshold-based algorithms [3–5], fuzzy clustering algorithms [6], and so on. Among these algorithms, fuzzy C-means (FCM) is preferable since it is suitable for modelling the principles of formation of medical images. In FCM, each pixel is assigned membership in $[0, 1]$ to denote the degree to which it concurrently belongs to each of several clusters. Much information is thereby retained, enhancing the segmentation results.

However, the traditional FCM algorithm is sensitive to image noise as it only considers intensity information; many algorithms have been proposed to improve its robustness. For example, Bezdek [7] proposed a bias-corrected version of FCM (BCFCM), and Stelios [8] proposed a fuzzy local information C-means clustering algorithm (FLICM). In these algorithms, neighborhood information is introduced in different forms to improve performance. However, when the image is contaminated heavily, these algorithms are either ineffective or inefficient.

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To achieve satisfactory results, improved FCM algorithms based on non-local information (NLFCM) have been proposed [9]. In NLFCM, the information covering the whole image can be utilized, and is not limited to the vicinity of a given pixel. In algorithms such as BCFCM, FLICM, and NLFCM, neighboring pixels or similar pixels are made to belong to the same cluster, thus improving the insensitivity to image noise. In these algorithms, the most important problem is to measure the relatedness of pixels, which can be measured in different ways. In Ref. [7], the pixel correlation between neighboring pixels and the central one is defined as the constant α . In Ref. [10], pixel relatedness is defined as the product of spatial relatedness and intensity relatedness. Due to the limitations of spatial relatedness, pixel relatedness decreases greatly with the increase of Euclidean distance between pixels. Thus, only nearby pixels can play positive roles, resulting in poor performance. In Ref. [9], pixel relatedness is defined as the similarity between image patches, which can enhance the results to some extent, but with low efficiency.

This paper proposes an improved fuzzy clustering algorithm for segmentation, exploiting more information. Firstly, the cluster centers are initialized by peak detection. Then, a novel distance model to measure pixel relatedness is constructed, a *patch-weighted distance*. With accurate relatedness, more information can be utilized, just as in NLFCM. Finally, a low-rank prior is merged into the fuzzy clustering algorithm framework to perform image segmentation.

The rest of the paper is organized as follows: Section 2 presents the motivation and contribution. Section 3 presents the proposed algorithm in detail, including cluster center initialization, a novel pixel relatedness model, and the improved fuzzy clustering algorithm. Section 4 shows and analyses experimental results. Section 5 summarizes this paper and suggests future work.

2 Motivation and contribution

In improved FCM algorithms based on non-local information, to ensure efficiency, a search window with a large radius is adopted instead of the whole image. In essence, the purpose of these methods is to enforce similar pixels to be classified as belonging to the same cluster. However, the improved robustness

comes at the cost of efficiency [9]. Specifically, if the radius of the search window is r , the number of pixels considered in image segmentation is $(2r + 1)^2 - 1$. When the patch-weighted distance model is introduced to measure pixel relatedness, $(2r + 1)^2 - 1$ weights must be computed first, which further reduces efficiency. To overcome this problem, this paper proposes a segmentation method based on a low-rank prior and non-local self-similarity.

As we all know, almost all images have high information redundancy either in the form of low rank or sparse representation [11, 12]: many pixels share similar features. Based on a low-rank prior or sparse representation, images can be denoised [13–17]. For medical images with limited intensity levels, the phenomenon of low rank is particularly obvious. Figure 1 illustrates the low-rank property using two medical images. The patch matrices are approximately low-rank, so most image patches share similar features. Therefore, in the image segmentation process, we can improve the efficiency by putting those similar pixels into the same cluster without considering dissimilar pixels.

In fact, the idea of a low-rank prior is widely applied in the fields of image denoising [13] and resolution enhancement [18]. In Ref. [19], an improved superpixel segmentation algorithm was proposed which updates the seeds by averaging pixels with the most homogeneous appearance; not all pixels belong to a superpixel. This can also avoid inhomogeneous intensity within a superpixel. In Ref. [18], a low-rank prior is exploited to estimate the missing pixels and reconstruct the high resolution (HR) image. In segmentation algorithms based on soft sets [20], pixels are divided into three regions: positive, boundary, and negative. In the process of image segmentation, only the pixels in the positive and boundary regions are utilized.

Furthermore, fuzzy clustering algorithms tend to fall into local minima, which also reduces efficiency. It is well known that the histogram of an image can reflect its grayscale frequency distribution well [21], and many segmentation algorithms based on histograms have been proposed [22, 23]. Peaks in the histogram are grayscales correlated with more pixels while troughs are gray levels associated with fewer pixels. Generally speaking, the peaks are close to the cluster centers while the valleys lie far away.

Fig. 1 Low-rank prior in medical images. (a) Original images (above: MR brain image, below: CT lung image). (b) Distributions of singular values of corresponding patch matrices. (c, d) Low-rank approximations with rank = 20, 30, respectively.

Therefore, the histogram peaks can be adopted for cluster initialization.

Recently, background knowledge or prior knowledge has been adopted in supervised algorithms, such as CNN-based methods, to improve accuracy [24]. However, these algorithms may provide highly inaccurate results for medical images for two reasons. First, there is physiological variability between different subjects [25]. Secondly, large numbers of samples are required to train a CNN, which is difficult due for individual privacy and other reasons. In clinical applications, accuracy and speed requirements of medical image segmentation are very high [26]. In order to achieve satisfactory results with acceptable efficiency, we combine non-local information and a low-rank prior in the framework of fuzzy clustering algorithms. Image segmentation proceeds in four steps: (i) initialize the cluster centers by peak detection, (ii) relatedness of pixels is modeled, (iii) a low-rank prior is exploited to retrieve the most related pixels, and (iv) image segmentation is performed in the framework of fuzzy clustering.

Our main contributions are as follows: (i) an initialization method which avoids the local minimum problem of traditional fuzzy algorithms, (ii) a model which can accurately measure pixel relatedness, (iii)

an efficient yet efficacious method for medical image segmentation, utilizing a low-rank prior and non-local information simultaneously, and (iv) utilization of the FLICM framework, which is free of parameter adjustment and provides easy extension to other fuzzy clustering algorithms.

3 Method

The framework of the proposed algorithm is presented in Fig. 2. It has three steps: cluster center initialization, related pixel retrieval, and image segmentation.

Fig. 2 Pipeline of the proposed approach.

3.1 Cluster center initialization

In traditional fuzzy algorithms, memberships are initialized at random, and cluster centers are computed based on intensity values and initial memberships. In fuzzy clustering algorithms, random initialization of the memberships may lead to unstable performance, and often the process becomes trapped in local minima [27]. Intuitively, cluster centers should be located in regions with greater diversity: grayscales with higher frequency are suitable for use as the initial cluster centers. In the proposed schema, the cluster centers are initialized using peak detection [2].

3.2 Pixel relatedness model

As mentioned earlier, the measurement of pixel relatedness is a key problem in fuzzy clustering algorithms. In our opinion, only considering the most closely related pixels in image segmentation will improve efficiency. In previous work [6, 9], pixel relatedness was measured by patch distance. However,

a smaller distance between corresponding patches does not always correspond to similarity of pixels, as shown by the example in Fig. 3. It is reasonable to classify the center pixel and the pixel above in Fig. 3(a) in the same cluster, while the center pixel and the pixel below should belong to different clusters. However, the distances suggest the opposite: see Fig. 3(c). Hence, measuring pixel relatedness by the distance between image patches is an unsuitable approach.

The problem is that distance between corresponding patches does not consider edge information. Specifically, different neighbors of a pixel may have different influences on the central pixel. To tackle this problem, we present a novel relatedness model, formalized in Algorithm 1, which introduces weighting for different directions to better measure pixel relatedness. Using the novel model, the pixel relatedness between the center pixel and neighboring pixels in Fig. 3(a) is presented in Fig. 4. The relatedness computed by the novel model is more reasonable.

Fig. 3 Smaller patch distance does not mean similarity of pixels: (a) pixels in part of an image, (b) corresponding intensity values, (c) distances between two image patches.

Algorithm 1 Pixel relatedness retrieval

Input: Image I , and parameters α, γ to control relatedness.

Output: Relatedness between the central pixel p and the pixels in the search window.

1. For each image pixel p , construct image patches X_p .
2. Retrieve the difference between corresponding patches in different directions: $d_p(q) = \sum |X_p - X_q| / |N_p|$, where N_p is the set of neighboring pixels, with cardinality $|N_p|$.
3. Compute the weights in different directions: $w_p(q) = \exp(-\alpha d_p(q)) / \sum_{q \in N_p} \exp(-\alpha d_p(q))$.
4. Compute the weighted distance in different directions, $d_p^w(q) = \sum_q (w_p \cdot |X_p - X_q|) / |N_p|$, where $\cdot$ is the vector dot product.
5. Compute the relatedness between corresponding pixels: $s(p, q) = \exp(-\gamma d_p^w(q))$.

0.1161	0.2166	0.3428
0.8718	1	0.8295
0.0001	0.0001	0.0001

Fig. 4 Relatedness of pixels in Fig. 3(a).

3.3 Finding related pixels by low-rank prior

As mentioned earlier, information from a neighborhood or the whole image is used to resist the effects of image noise. More information provided by similar pixels plays a positive role in accurate performance. However, using more information reduces efficiency. To ensure efficiency, various limitations have been considered. For example, the size of the search window may be limited, and only neighboring pixels are considered in FGFCM and FLICM. In NLFCM, a large search window is used, including similar and dissimilar pixels. Since only similar pixels play a positive role, why not neglect the dissimilar pixels?

When image patches are analyzed by singular value decomposition (SVD), most of the energy is concentrated into a few, largest, singular values. Following denoising algorithms [14, 15], we utilize the most related pixels to play a positive role in image segmentation, while neglecting other pixels

in the non-local search window. As we all know, the reason for the success of low rank and sparse representations is that many pixels in the image share similar features [28]. Therefore, the number of pixels in a cluster is closely related to the rank of image patches. Specifically, a large rank means a small number of pixels in the same cluster, while a low rank means a large number of pixels in the same cluster. However, measuring the rank accurately is very difficult, and considering fewer pixels will degrade accuracy. Hence, we must consider the number of similar pixels in the search window based on the rank prior, which will be discussed in Section 4.

3.4 Image segmentation

We now present the improved FLICM algorithm in detail. FLICM introduces a fuzzy factor to replace the effect of neighboring pixels, and avoids the burden of parameter adjustment. However, when applied to complex images, FLICM has the following disadvantages: (i) when the image is severely noisy, FLICM performs poorly, (ii) the relatedness between pixels is measured by Euclidean distance, so effectively ignores far away pixels, and (iii) to improve robustness, a large search window is used, degrading efficiency. We aim to overcome these problems, using non-local information and a low-rank prior to achieve high accuracy with acceptable efficiency. In this study, the fuzzy factor is defined as

$$G'_{ij} = \sum_{r \in W_j} s(j, r) (1 - \mu_{ir})^m \|x_r - v_i\|^2 \quad (1)$$

where W_j is the set of the selected similar pixels in the search window, and $s(j, r)$ is the pixel relatedness between corresponding pixels. Compared to FLICM, this algorithm has two improvements: (i) the neighborhood window N_j is replaced with W_j , which is the set of selected similar pixels in the search window, and (ii) the link between pixels is measured by pixel relatedness, not Euclidean distance. In addition, due to the use of a low-rank prior, only the most related pixels are utilized, instead of all pixels in the search window, which improves efficiency without degrading performance. In the rest of the paper, the proposed algorithm will be denoted LRFCM, meaning FCM with low-rank prior.

Just as in other FCM-related algorithms, all pixels satisfy the constraint $\sum_{i=1}^C u_{ij} = 1$. Therefore, the following equation may be constructed by the

Lagrange multipliers method (LMM):

$$J = \sum_{i=1}^C \sum_{j=1}^n \left[\mu_{ij}^m (x_j - v_i)^2 + G'_{ij} \right] + \sum_{j=1}^n \lambda_j \left(\sum_{i=1}^C u_{ij} - 1 \right) \quad (2)$$

As $\partial J / \partial u_{ij} = 0$ and $\partial J / \partial v_i = 0$, memberships and cluster centers can be updated using:

$$u_{ij} = 1 / \sum_{k=1}^C \left(\frac{|x_j - v_i|^2 + G'_{ij}}{|x_j - v_k|^2 + G'_{kj}} \right)^{1/(m-1)} \quad (3)$$

$$v_i = \sum_{j=1}^n u_{ij}^m x_j / \sum_{j=1}^n u_{ij}^m \quad (4)$$

Note that the membership and the cluster center in the revised fuzzy factor G'_{ij} are not considered in minimizing Eq. (2), as in FLICM [29, 30]. Through this processing, the performance is not reduced, and the burden of complex computation can be avoided.

To summarize, the proposed algorithm can be formalized in Algorithm 2.

Algorithm 2 LRFCM for image segmentation

Input: Image I , pixel relatedness from Algorithm 1, number of clusters C , pre-defined threshold ε , maximum number of iterations $maxIter$.

Output: Segmented image

Initialize: Set $it = 0$; randomly initialize memberships u_{ij}^{it} to satisfy $\sum_{i=1}^C u_{ij} = 1$.

while $\max\{|u^{it+1} - u^{it}|\} > \varepsilon$ and $it < maxIter$ **do**

    Compute the cluster centers from Eq. (4);

    Compute the revised factor from Eq. (1);

    Update the membership u_{ij}^{it+1} according to Eq. (3);

    Increment it ;

end while

Assign the j -th pixel to the k -th cluster, where $k = \arg_k \max\{u_{kj}\}$.

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4 Experiments

4.1 Setting

In this section, LRFCM is applied to synthetic and medical images, and compared to other typical FCM-related algorithms, such as BCFCM, EnFCM, FGFCM, FLICM, and NLFCM. In the experiments, the values of various parameters have an important effect on the segmentation results. For example, the assignment of C will present different details. For all algorithms, the value of m is set to 2, and the threshold ε is set to 10^{-5} . The value of α in BCFCM, EnFCM, and FGFCM is 2. N_R is set to 8 in BCFCM,

EnFCM, FGFCM, and FLICM, meaning that a 3×3 neighborhood window is used.

4.2 Clustering indices

To compare the segmentation results, as well as visual inspection, there are several recognised metrics, such as segmentation accuracy SA , the partition coefficient V_{PC} and the partition entropy V_{PE} . SA is the fraction of correctly classified pixels out of all pixels in the image:

$$SA = \sum_{k=1}^C \frac{|A_k \cap D_k|}{n} \quad (5)$$

where C is the pre-defined number of clusters, A_k is the set of pixels belonging to the k -th cluster, and D_k is the set of pixels belonging to the k -th cluster in the ground truth. $|\cdot|$ denotes the cardinality of a set.

V_{PC} and V_{PE} measure the fuzziness of the segmentation results, defined as

$$V_{PC} = \sum_{i=1}^C \sum_{j=1}^n u_{ij}^2 / n \quad (6)$$

$$V_{PE} = - \sum_{i=1}^C \sum_{j=1}^n (u_{ij} \log u_{ij}) / n \quad (7)$$

In preference, segmentation results should have lower fuzziness. Therefore, an algorithm with larger V_{PC} and smaller V_{PE} is better.

In addition, when binary images are segmented, another three metrics may be adopted: accuracy ($Acc.$), sensitivity ($Sen.$), and specificity ($Spe.$):

$$Acc. = (TP + TN) / (TP + TN + FP + FN) \quad (8)$$

$$Sen. = TP / (TP + FN) \quad (9)$$

$$Spe. = TN / (TN + FP) \quad (10)$$

where P , N , T , and F mean positive, negative, true, and false, respectively. Thus, TP is the number of positive samples that are classified correctly, FN is the number of positive samples that are misclassified, TN is the number of negative samples that are classified correctly, and FP is the number of negative samples that are misclassified. In essence, segmentation accuracy is the ratio of pixels that are classified correctly, including positive and negative ones. Sensitivity and specificity reveal the likelihood of classifying positive and negative pixels correctly. These three measures have values between 0 and 1, and an algorithm with higher accuracy, higher sensitivity, and higher specificity is preferable.

4.3 Parameter analysis

In this section, we discuss the effect of parameters on the performance of LRFCM, including the radius of the search window and the number of similar pixels retrieved in image segmentation. We perform LRFCM with different parameter settings on a synthetic image with different levels of noise to test the effects of these two parameters. The experiments used added Gaussian noise with variance (NV) and salt & pepper noise with different noise density (ND), levels being 5%, 10%, 15%, 20%, and 25% in each case. Figure 5 presents the *SA* of LRFCM on the synthetic image with different radii. The segmentation accuracy reaches the maximum value when the radius is 6 for Gaussian noise (see Fig. 5(a)). For salt & pepper

noise, the accuracy will not increase after the radius is greater than 6 (see Fig. 5(b)). For best efficiency and accuracy, we thus set the radius of the LRFCM search window to 6.

Figure 6 presents the *SA* of LRFCM on synthetic images with different numbers of similar pixels. As shown in Fig. 6(a), the *SA* reaches the maximum value when the number of similar pixels is set to 6×6 ; in Fig. 6(b), the *SA* does not increase too much when the number of similar pixels is larger than 6×6 . Based on these experimental results, the number of similar pixels used in LRFCM in this paper is set to 36.

4.4 Experiments on synthetic images

We first consider how LRFCM performed on two synthetic images, one binary image with intensity

Fig. 5 Segmentation accuracy (*SA*) versus radius of the search window. (a) *SA* with Gaussian noise of differing noise variance (NV); (b) *SA* with salt & pepper noise of differing noise density (ND).

Fig. 6 Segmentation accuracy (*SA*) against the num of similar pixels in image segmentation. (a) *SA* on images contaminated by Gaussian noise of different NV; (b) *SA* on images contaminated by salt & pepper noise of different ND.

values of 20 and 120, and the other having 4 clusters with intensity values 0, 85, 170, and 255. Different kinds of noise were added: for the first image we used salt & pepper noise of 15% ND and for the second, Gaussian noise of 40% NV: see Figs. 7 and 8.

Fig. 7 Segmentation results on the binary test image with added salt & pepper noise of 15% density.

Fig. 8 Segmentation results on the synthetic image with added Gaussian noise of 40% variance.

With salt & pepper noise, the results of FLICM, NLFCM, and LRFCM are less noisy; the LRFCM result is better than those of FLICM and NLFCM as it misclassifies fewer boundary pixels, due to the fact that only the most similar pixels are utilized. To compare the algorithms quantitatively, the partition coefficients, the partition entropies, and running time of the algorithms are compared, in Tables 1, 2, and 3 respectively.

Table 1 shows that the partition coefficients of LRFCM decrease with increasing noise variance or density. Table 2 shows that the partition entropies of LRFCM increase with increasing noise variance or density. These results indicate that increasing noise increases fuzziness. Compared to FLICM, NLFCM, and typical FCM-related algorithms, LRFCM has almost the largest partition coefficient and the smallest partition entropy: in other words, it results in the least fuzziness. Table 3 shows that since only the most similar pixels are considered in LRFCM, LRFCM is much quicker than NLFCM, indicating the success of utilizing a low-rank prior in image segmentation.

4.5 Experiments on medical images

We now consider the application of LRFCM to medical images, including pulmonary computed tomography (CT) images and brain magnetic resonance (MR) images. Medical images provide key information for treating corresponding diseases, including lung cancer and Alzheimer's disease. For example, accurate detection of pulmonary nodules in pulmonary CT images can assist doctors in the early diagnosis of lung cancer, which is crucial to improving survival chances.

First, we consider use of LRFCM to find pulmonary nodules. Pulmonary nodules often appear in different forms, such as pleural adhesion, solitary pulmonary nodules (SPN), ground glass opacity (CGO), and vascular adhesion. Also, different medical specialists may give different determinations. For example, five medical specialists present different segmentation proposals for the same pulmonary CT image shown in Fig. 9. To balance the proposals of different imaging specialists, a 50% rule [18] is adopted for the reference nodule: if a pixel is located in the results of more than one half of all specialists, it is considered to belong to a reference nodule.

Table 1 V_{PC} for the synthetic images with various noise levels, for different algorithms

Image	Noise variance/density	FCM	BCFCM	EnFCM	FGFCM	FLICM	NLFCM	LRFCM
Fig. 7(a)	Gaussian 15%	0.899347	0.890279	0.854787	0.977020	0.978255	0.978729	0.978872
	Gaussian 20%	0.897047	0.888389	0.87464	0.975003	0.978053	0.978624	0.978763
	Gaussian 30%	0.895687	0.885539	0.853282	0.973682	0.977420	0.978190	0.978442
	Gaussian 40%	0.898585	0.890340	0.798682	0.973306	0.978162	0.9787759	0.978281
	Salt&pepper 15%	0.955578	0.757233	0.733819	0.978166	0.906730	0.934470	0.935043
	Salt&pepper 20%	0.938112	0.693446	0.740691	0.965776	0.878473	0.917678	0.917775
	Salt&pepper 30%	0.874738	0.583499	0.753729	0.936455	0.787255	0.889891	0.895331
	Salt&pepper 40%	0.836736	0.522764	0.765399	0.758558	0.865334	0.879699	0.888197
Fig. 8(a)	Gaussian 15%	0.873874	0.848987	0.745730	0.963526	0.954918	0.949699	0.963700
	Gaussian 20%	0.865818	0.721211	0.760163	0.957356	0.951179	0.946778	0.959745
	Gaussian 30%	0.869063	0.807500	0.809175	0.917179	0.934112	0.934183	0.937777
	Gaussian 40%	0.896264	0.822705	0.768758	0.933176	0.939371	0.921836	0.945576
	Salt&pepper 15%	0.914623	0.623908	0.81487	0.946408	0.857431	0.887091	0.944991
	Salt&pepper 20%	0.894291	0.546232	0.806339	0.918317	0.811726	0.857621	0.923852
	Salt&pepper 30%	0.862423	0.408195	0.795892	0.856927	0.704051	0.783498	0.898437
	Salt&pepper 40%	0.841986	0.335888	0.793045	0.799909	0.590116	0.694143	0.781164

Table 2 V_{PE} for the synthetic images with various noise levels, for different algorithms

Image	Noise variance/density	FCM	BCFCM	EnFCM	FGFCM	FLICM	NLFCM	LRFCM
Fig. 7(a)	Gaussian 15%	0.259389	0.311032	0.347091	0.077777	0.069256	0.063234	0.061916
	Gaussian 20%	0.264119	0.315188	0.307122	0.083304	0.070019	0.063733	0.063899
	Gaussian 30%	0.266685	0.321476	0.349268	0.085874	0.071634	0.064952	0.064291
	Gaussian 40%	0.260673	0.310419	0.463982	0.086131	0.069591	0.063313	0.064524
	Salt&pepper 15%	0.126445	0.539029	0.595000	0.056896	0.238843	0.198705	0.191500
	Salt&pepper 20%	0.173113	0.667667	0.584019	0.087555	0.302847	0.220559	0.222974
	Salt&pepper 30%	0.309961	0.868407	0.563511	0.162059	0.482266	0.286158	0.295273
	Salt&pepper 40%	0.357001	0.966003	0.545029	0.233355	0.550156	0.340687	0.388275
Fig. 8(a)	Gaussian 15%	0.364426	0.463391	0.705751	0.122410	0.144173	0.152608	0.123734
	Gaussian 20%	0.381606	0.758325	0.671778	0.139061	0.154500	0.160915	0.134129
	Gaussian 30%	0.379798	0.530326	0.564839	0.244734	0.197553	0.194184	0.162361
	Gaussian 40%	0.295549	0.470185	0.725430	0.180793	0.184629	0.214473	0.168085
	Salt&pepper 15%	0.244096	1.039818	0.561595	0.3690330	0.438400	0.356155	0.400681
	Salt&pepper 20%	0.303587	1.231628	0.578750	0.450168	0.564780	0.444922	0.427946
	Salt&pepper 30%	0.399474	1.56464	0.698233	0.658130	0.844623	0.653764	0.652714
	Salt&pepper 40%	0.463025	1.752415	0.990631	0.964364	0.985934	0.958883	0.936927

Table 3 Run time (in second) for the synthetic images with various noise levels, for different algorithms

Image	Noise variance/density	FCM	BCFCM	EnFCM	FGFCM	FLICM	NLFCM	LRFCM
Fig. 7(a)	Gaussian 15%	0.296402	0.733205	0.015600	0.140401	3.822025	213.003765	30.482596
	Gaussian 20%	0.234001	0.717605	0.031200	0.124801	3.478822	247.089984	32.214207
	Gaussian 30%	0.265202	0.686404	0.031200	0.093601	3.712824	235.187108	32.526208
	Gaussian 40%	0.234001	0.702004	0.015600	0.078000	3.541223	239.711137	32.510608
	Salt&pepper 15%	0.234001	0.936006	0.015600	0.093601	10.530067	438.924414	32.682209
	Salt&pepper 20%	0.218401	1.357209	0.015600	0.873606	7.597249	506.223245	37.845843
	Salt&pepper 30%	0.374402	2.246414	0.015600	0.156001	14.008890	837.77217	38.98465
	Salt&pepper 40%	1.279208	6.583242	0.015600	0.093601	13.790488	846.570627	55.879558
Fig. 8(a)	Gaussian 15%	2.667617	7.004445	0.078000	0.5304030	32.931811	3424.565152	173.363912
	Gaussian 20%	3.151220	27.066174	0.046800	0.499203	25.350162	2564.562840	168.309479
	Gaussian 30%	8.065252	16.099303	0.046800	0.499203	76.456090	2221.454240	237.807924
	Gaussian 40%	5.163633	19.484525	0.031200	0.561604	41.324665	5603.789921	263.516889
	Salt&pepper 15%	1.794012	8.611255	0.046800	0.546003	51.105928	2205.620139	190.945224
	Salt&pepper 20%	2.839218	11.887276	0.124801	0.452403	51.792332	2500.134426	214.719776
	Salt&pepper 30%	2.636417	20.514132	0.093601	0.483603	76.50289	3225.788678	225.483846
	Salt&pepper 40%	2.511616	32.463808	0.062400	0.670804	118.62316	6755.716906	319.521248

Fig. 9 Segmentation scheme provided by different imaging specialists. (a) Original pulmonary CT image; (b)–(f) segmentation proposals by 5 different imaging specialists.

As noted, the predefined number of clusters is important in fuzzy clustering algorithms, since different numbers of clusters can lead to different details. To emphasize the pulmonary nodules, the pre-defined number for pulmonary nodule segmentation was uniformly set to 2. The pulmonary CT images adopted in the experiments are shown in Figs. 10(a)–10(d), which includes pulmonary nodules

of different types. Nodules in Figs. 10(a), 10(b), and 10(d) are solid, while the nodule in Fig. 10(c) has ground-glass appearance. Also, lobulations or spiculations appear in Fig. 10(a), while Fig. 10(b) is accompanied by ural retraction, and signs of vessel convergence emerge in Fig. 10(d). Using the 50% rule, the reference images determined are presented in Figs. 10(e)–10(h).

Fig. 10 Pulmonary CT images used in experiments. (a)–(d) CT images, (e)–(h) reference images of contained nodules, using the 50% rule.

The segmentation results of various algorithms are presented in Fig. 11, and the *SAs* of the algorithms are presented in Table 4. LRFCM performs best in lung CT images with lobulations or spiculations, while FCM and BCFCM perform best in CT images with ural retraction, EnFCM performs best for ground-glass CT images, and NLFCM performs best for CT images with signs of vessel convergence. As can be seen from Table 4, LRFCM performs in the top two of all algorithms for lung CT images of any kind, indicating that the principle behind the proposed algorithm is reasonable.

To further compare performance on medical images, brain images from Brainweb [31] were used to evaluate these algorithms. There are 3 main pixel clusters in brain images, belonging to gray matter (GRY), white matter (WHT), and cerebral spinal fluid (CSF). The images used are 30 brain region slices in the axial plane generated with T1 modality and 1 mm slice

thickness. To illustrate the robustness of LRFCM, 5% Rice noise was added, and the intensity non-uniformity parameter was set to 40%. Segmentation results of several algorithms are presented in Fig. 12 for the 77th slice, and the corresponding *SAs* for GRY, WHT, and CSF are tabulated in Table 5.

Figure 12 shows that image noise still exists in the results of FCM, BCFCM, EnFCM, and FGFCM. The results of FLICM and NLFCM lose many details. Comparatively, LRFCM is not only insensitive to image noise, but can retain image details. This is also indicated by the comparison of segmentation accuracy in Table 5. The data there are the average values over the 30 slices used in the experiments. As shown in Table 5, LRFCM gets more accurate GRY and CSF scores, and a WHT score a little lower than BCFCM. The running time of the algorithms is presented in Table 6. LRFCM is much quicker than NLFCM, meeting our goals. In addition, the

Fig. 11 Pulmonary CT image results produced by various algorithms.

Table 4 Segmentation accuracy (%) of various algorithms

Image	FCM	BCFCM	EnFCM	FGFCM	FLICM	NLFCM	LRFCM
Fig. 10(a)	89.3194	89.6175	87.6304	89.7665	90.9091	90.9588	91.5549
Fig. 10(b)	94.7342	94.7342	94.4362	94.9081	94.0636	93.4178	94.6846
Fig. 10(c)	98.3376	98.2523	98.5934	98.2950	97.4851	96.0358	98.3376
Fig. 10(d)	95.6981	95.8333	95.2381	95.9686	96.5368	97.4838	96.7532

Fig. 12 Segmentation results on the 77th slice for various algorithms.

Table 5 *SAs* of different algorithms on brain tissues (%)

Image	FCM	BCFCM	EnFCM	FGFCM	FLICM	NLFCM	LRFCM
WHT	0.921431	0.928560	0.921996	0.927398	0.925495	0.918785	0.926177
GRY	0.857252	0.856792	0.814118	0.860465	0.837990	0.813755	0.862061
CSF	0.879993	0.881704	0.525906	0.875929	0.825389	0.792881	0.883101
Average	0.886225	0.889019	0.754006	0.887930	0.862958	0.841808	0.890446

Table 6 Average running time (in second) for different algorithms on brain images

FCM	BCFCM	EnFCM	FGFCM	FLICM	NLFCM	LRFCM
2.279694	9.875383	0.060840	0.435763	53.017980	528.497068	174.318637

brain tissues reconstructed based on the segmentation results of all algorithms are shown in Fig. 13. The 3D reconstruction results of LRFCM retain more details while improving robustness, justifying combining a low-rank prior and non-local information in LRFCM.

5 Conclusions

In this study, an improved algorithm for image segmentation is proposed, which combines non-local information and a low-rank prior into the framework of fuzzy clustering. In the proposed algorithm, a

novel pixel relatedness model is presented, by which non-local information can be utilized to improve the robustness. With the help of a low-rank prior, only the information provided by the most similar pixels is utilized, which improves the efficiency of our algorithm based on non-local information. Experiments on synthetic and medical images illustrate the advantages of the proposed algorithm over other FCM-related algorithms.

In our future work, the ideas of this study will be extended to medical image series segmentation. Relatedness will be measured by similarity within

Fig. 13 3D reconstruction of WHT and GRY by different algorithms. (a) FCM; (b) BCFCM; (c) EnFCM; (d) FGFCM; (e) FLICM; (f) NLFCM; (g) LRFCM. Above: reconstructed WHT. Below: reconstructed GRY.

pixel cubes, to utilize information covering the whole image series. We hope that 3D reconstruction of tissues or organs be achieved directly, to guide disease diagnosis.

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